CERN-TH.5003/88

OFF-SHELL EFFECTIVE ACTIONS IN STRING THEORY

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A B S T R A C T

A new, "off-shell", approach to conformal invariance, β -functions and field equations in first quantized string theory is presented. The method, valid to all orders in the weak-field expansion and non-perturbatively in α' , is illustrated for a tachyon background at genus zero and one level. For a particular choice, we derive a Zamolodchikov-type equation for the effective action.

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CERN-TH.5003/88

March 1988

1. - INTRODUCTION

A crucial element in the first quantized approach to string theory consists of exploiting the conditions of conformal invariance. For Polyakov's path integral [1] describing a string moving in a background [2], these conditions guarantee the decoupling of the conformal factor σ of the two-metric $g_{\alpha\beta}$ and are crucial for the eventual consistency (causality and unitarity) of the theory.

What is remarkable is that the above equations, which can be expressed as the conditions for vanishing β -functions, not only determine, for a string in flat space-time, the dimensionality of the latter but even enforce, in the general case, the equations of motion (and/or mass-shell conditions) for the background fields in which the string propagates. Furthermore, order by order in α' , these equations happen to coincide with the Euler-Lagrange equations derived from an effective action, Γ_{conf} . Finally, this action also generates the string S-matrix, precisely like an off-shell effective action does in usual field theory. Much work has been devoted to understand these welcome, yet somewhat mysterious, coincidences. (For a partial list of relevant papers, see Refs. [3-8].)

In this note we shall present a new viewpoint on this relationship. It is based on the assumption, which is largely supported by some old [9] as well as by recent results [10], that one can construct an off-shell extension of the theory in a way which, while spoiling conformal invariance, still preserves certain crucial properties. It is not clear, at the moment, whether such an extension can be derived by giving a meaning to Polyakov's path integral away from its conformal invariance points (hence off-shell).

In any event, this assumption allows us to define two types of generating functionals, related to one another by a non-local field redefinition, a modified Legendre transformation. In general, both kinds of functionals depend on σ . The conditions of σ -independence can be almost trivially studied, however, for one functional and then translated in terms of the other by standard functional derivative techniques. In this way we are able to identify what we can define as β -functions and to relate these to appropriate derivatives of the effective action functional. Our results fit nicely with (and extend) those discussed already or partly conjectured in the literature. In particular, for one particularly appealing choice of the Legendre transformation, we derive a Zamolodchikov-type equation [11,12] for the effective action.

We shall treat, for simplicity, the case of a closed bosonic string moving in a tachyon background at the sphere and torus level, but to all orders in α' . We are thus able to discuss the (finite) mass, coupling and S-matrix renormalizations induced by string loops, confirming and extending previous results [13]. We see no

reason why our procedure should not be generalizable to any background field in any string theory and at any (finite) genus.

This note is supposed to give the essential ideas and main results of our approach. More details and explicit proofs will be given elsewhere [14].

2. - THE BASIC ASSUMPTIONS

Our starting point will be the formal expression of Polyakov's path integral for the (closed, bosonic) string partition function:

$$Z[\phi_a^{(0)}] = \int \mathcal{D}X^\mu(z) \mathcal{D}g_{\alpha\beta}(z) \exp \left\{ -S_0 + \sum_a \int d^D p \phi_a^{(0)}(p) V_a^{(0)}(p) \right\} \quad (1)$$

where S_0 is the free Euclidean action and $\phi_a^{(0)}(V_a^{(0)})$ are the unrenormalized background fields (vertices), with a labelling the various backgrounds (tachyon, graviton, dilaton...).

We would like to give a precise meaning to Z for all ϕ_a (fields to be related to $\phi_a^{(0)}$ by some appropriate renormalization) at least as a Taylor series:

$$Z = T[\phi_a] = \sum_n \int \prod_{i=1}^n d^D p_i \prod_{i=1}^n \phi_{a_i}(p_i) T_{a_1, \dots, a_n}^{(c)}(p_1, \dots, p_n), \quad (2)$$

where c stands for connected. Since the one- and two-point functions require some special attention, let us consider first T' corresponding to Eq. (2) with the sum starting from $n = 3$.

T' should be, first of all, a finite functional of its arguments. As is well known in ordinary field theory, even forgetting ultra-violet divergences, gauge fixing is needed to obtain a finite result from a gauge theory path integral. Reparametrization invariance can be used to fix the gauge to be of the orthonormal type

$$g_{\alpha\beta} = \delta_{\alpha\beta} e^{\sigma(z)},$$

$$g_{\alpha\beta} dx^\alpha dx^\beta = e^{\sigma(z)} dz d\bar{z}, \quad (3)$$

where, for the time being, we limited ourselves to the sphere level.

Weyl invariance would allow us to fix σ as well, and would guarantee that the result is independent of the particular choice adopted. Vice versa, we can study the conditions of Weyl invariance by studying the dependence of Z from the choice of σ .

When the external states are on shell, even after fixing the conformal factor, Möbius (projective) transformations remain as residual symmetry and induce, at the sphere level, the well-known infinite $SL(2, \mathbb{C})$ volume factor Ω_M in T_n for $n \geq 3$.

It appears to us that Möbius invariance is a crucial property if we want off-shell finiteness and factorization. Indeed, if and only if there is Möbius invariance, are we allowed to factor out Ω_M and to fix three integration variables. The number of integration variables is then the correct one for factorizing T_n as:

$$T_n \longrightarrow T_{m+1} (s-m^2)^{-1} T_{n-m+1} \quad (4)$$

since $(n-3) = (m+1-3) + (n-m+1-3) + 1$, where the last +1 stands for the integration yielding the pole.

As already mentioned, old [9] as well as recent [10] results show that, indeed, projective invariance can be maintained off shell. This allows us to make the following educated basic assumptions.

Assumptions

There exists a gauge fixed definition of $T'[\phi_a; \sigma]$ such that:

- i) $SL(2, \mathbb{C})$ invariance holds for all ϕ_a ;
- ii) The amplitudes T_n factorize properly even off shell, possibly within an enlarged Hilbert space.

As we have already pointed out, the n -Reggeon amplitudes constructed in Refs. [9,10] satisfy explicitly property i). As to property ii), we have verified its validity at the tachyon pole for the tachyonic amplitudes given in these papers at the sphere level. Actually, the pole positions are unchanged for all levels.

Both properties use the fact that the dependence upon the conformal factor σ is only through its value at a fixed, but otherwise arbitrary, point z_0 . The best way to justify this from the point of view of Polyakov's path integral is to take σ to be constant (almost everywhere). At least for the case of tachyons considered here, this restriction is not important: as it will become obvious later, invariance under a global rescaling of σ will imply invariance under any local change.

The amplitudes T_n ($n \geq 3$) thus obtained are made finite by:

- a) removal (cancellation) of the infinite $SL(2, C)$ volume factor Q_M ;
- b) regularization of products of operators at the same point through an invariant cut-off

$$\Delta \bar{z} \Delta \bar{z} > \exp(-\sigma) a^2 . \quad (5)$$

This second step allows us to regularize the functional T' via a simple renormalization. From now on, we shall restrict ourselves to a tachyon background and omit the label a . Defining:

$$\phi = (a^2)^{\frac{\alpha'}{4} p^2} \phi^{(0)} , \quad V = (a^2)^{-\frac{\alpha'}{4} p^2} V^{(0)} , \quad (6)$$

one finds:

$$\phi^{(0)} V^{(0)} \longrightarrow (e^{-\sigma})^{\frac{\alpha'}{4} p^2} \phi V . \quad (7)$$

As indicated above, in the process of renormalization, T' inherits an extra σ dependence (besides the one already present in the classical vertex). Such a dependence, which is of crucial importance for us, appears already in Refs. [10], but was neglected there since it disappears when one goes, eventually, on shell. It is now easy to verify that the following equation holds:

$$\frac{\partial T'}{\partial \sigma} = K \phi \frac{\delta T'}{\delta \phi} , \quad (8)$$

where we have defined

$$K \phi \frac{\delta T'}{\delta \phi} \equiv \int d^D p \, K(p^2) \phi(p) \frac{\delta T'}{\delta \phi(p)} , \quad (9)$$

and $K(p^2) = (1 - \alpha' p^2/4)$ is the kinetic operator for the tachyon. This equation is basically the same as Weinberg's [15], who derived the Virasoro conditions on physical vertices by requiring σ -independence. Note at this point that our factorization assumption ii) contains the non-trivial (bootstrap) condition that

the (tachyon) poles are precisely at the point $K = 0$ where conformal invariance holds.

3. - LEGENDRE TRANSFORMATION AT THE SPHERE LEVEL

We would now like to go from the functional T' , which generates connected, truncated amplitudes, to another functional Γ , which is one-tachyon-irreducible (1TI). This procedure has an intrinsic ambiguity related to the possibility of subtracting some finite part together with the pole terms. This ambiguity, which is the analogue of a renormalization ambiguity in the non-linear σ -model approach, induces also an ambiguity in the definition of the quadratic terms in T and Γ .

Indeed, let us add to T' a quadratic term and define

$$T = T' + \frac{1}{2} \phi \pi \phi . \quad (10)$$

Provided that $\pi K^{-1} \rightarrow 1$ as $K \rightarrow 0$, one finds that the modified Legendre transformation defined by

$$\bar{\phi} \equiv \pi^{-1} \frac{\delta T}{\delta \phi} , \quad (11a)$$

$$\Gamma[\bar{\phi}] = T[\phi] - \bar{\phi} \pi \phi \quad (11b)$$

defines a 1TI functional that can be identified with the off-shell effective action in the sense that it generates back T from tree diagrams with π^{-1} propagators. Our modification of the usual Legendre transformation, consisting of the extra factors $\pi^{-1}(\pi)$ in the definitions of $\bar{\phi}(\Gamma)$, is required by the fact that T generates truncated Green functions, i.e., amplitudes without external propagators. A similar observation has been also made in [7].

From these definitions, it is easy to check that:

$$\frac{\delta \Gamma}{\delta \bar{\phi}} = - \pi \phi . \quad (12)$$

ϕ and $\bar{\phi}$ differ only by interaction terms. Indeed one finds:

$$\bar{\phi} = \phi + \pi^{-1} \frac{\delta T'}{\delta \phi} = \phi + \mathcal{O}(\phi^2) . \quad (13)$$

Using Eqs. (10) and (11), we can also extract the quadratic terms in the Γ 's

$$\Gamma = -\frac{1}{2} \bar{\phi} \pi \bar{\phi} + \Gamma' , \quad \Gamma' = \mathcal{O}(\bar{\phi}^3) , \quad (14)$$

and verify that:

$$\frac{\delta \Gamma'}{\delta \bar{\phi}} = \pi (\bar{\phi} - \phi) = \frac{\delta T'}{\delta \phi} . \quad (15)$$

We finally come to the main purpose of all this exercise: σ -dependence. This does actually depend on the choice of Π . In general, one finds:

$$\begin{aligned} \frac{\partial \Gamma}{\partial \sigma} &= \kappa \phi \pi (\bar{\phi} - \phi) + \frac{1}{2} \phi \frac{\partial \pi}{\partial \sigma} \phi \\ &\quad - \bar{\phi} \frac{\partial \pi}{\partial \sigma} \phi . \end{aligned} \quad (16)$$

Let us discuss at this point three possible choices for Π . They should ultimately be equivalent in the sense of giving the same on-shell theory, but one of them (or maybe a fourth one?) could be most convenient for an off-shell continuation.

i) "Minimal" subtraction

We define this by simply taking $\Pi = K$, i.e., a σ -independent Π . Using Eqs. (14)-(16), we obtain

$$\frac{\partial \Gamma}{\partial \sigma} = \frac{\partial \Gamma'}{\partial \sigma} = - \frac{\delta \Gamma}{\delta \bar{\phi}} \frac{\delta T'}{\delta \phi} = - \frac{\delta \Gamma}{\delta \bar{\phi}} \frac{\delta \Gamma'}{\delta \bar{\phi}} . \quad (17)$$

ii) "Covariant" subtraction

If we define Π to satisfy

$$\frac{\partial \Pi}{\partial \sigma} = 2 K \Pi , \quad (18)$$

T and T' transform in a similar way under the change of σ

$$\frac{\partial T}{\partial \sigma} = K \phi \frac{\delta T}{\delta \phi} \quad (19)$$

[compare Eq. (8)]. In this case, Eq. (16) gives

$$\frac{\partial \Gamma}{\partial \sigma} = -K \phi \Pi \bar{\phi} = K \bar{\phi} \frac{\delta \Gamma}{\delta \bar{\phi}} = -K \phi \frac{\delta T}{\delta \phi} . \quad (20)$$

iii) "Intermediate" subtraction

Something amusing happens if we define Π to satisfy

$$\frac{\partial \Pi}{\partial \sigma} = K \Pi , \quad (21)$$

i.e., σ -dependence in between choices i) and ii). In this case Γ satisfies

$$\frac{\partial \Gamma}{\partial \sigma} = -\frac{1}{2} \frac{\delta \Gamma}{\delta \bar{\phi}} K \Pi^{-1} \frac{\delta \Gamma}{\delta \phi} , \quad (22)$$

which has the same structure as the equation proposed by Zamolodchikov [11,12] for the central charge. Since the latter is expected to be closely related to the effective action, our result should not be surprising. One could ask why it should work only in the intermediate subtraction. The answer could be that only in such a case the following simple renormalization group-like equation can be shown to hold.

$$\frac{\partial}{\partial \sigma} (\bar{\phi} - \phi) = K \phi \frac{\delta}{\delta \phi} (\bar{\phi} - \phi) = -\beta(\phi) \frac{\delta}{\delta \phi} (\bar{\phi} - \phi) . \quad (23)$$

Equations (17), (20) and (22) are our main results at the sphere level. They show clearly that the free equations of motion for ϕ become, through the modified Legendre transformation, the fully interacting equations for $\bar{\phi}$ irrespective of the choice of subtraction. This observation makes precise earlier claims by Tseytlin [16] that one set of equations goes into the other through the renormalization process. In our approach, however, the renormalization of UV infinities was done once and for all at the level of the functional T and, from thereon, we have only considered finite field redefinitions.

Finally, we can re-express our result as follows: since stability of the ground state requires $\delta \Gamma / \delta \bar{\phi} = 0$, we see that vacuum stability guarantees, through Eqs. (17), (20) and (22) conformal invariance.

4. - EXTENSION TO THE TORUS

It is relatively simple to extend our method to the genus 1 (torus) level and in principle to higher genera as well. We shall have to continue to assume that off-shell amplitudes satisfying factorization can still be defined. Also, by consistency, on- and off-shell amplitudes will be given by integrals over the same region of moduli space (this requirement replaces that of Möbius invariance discussed at the sphere level).

Apart from these assumptions, which are presumably realized if string loops are constructed by the procedure of Refs. [9,10], we have to be careful about the fact that, unlike in field theory, in string theory we cannot resum self-energy diagrams to obtain dressed propagators. To each finite order g in the string loop, expansion poles are always at the tree-level position but their order can go up to g ($g-1$ for external masses). A related point is that we expect the conditions for conformal invariance to be unchanged by loops as far as the functional T is concerned. Yet we would like to recover the expected loop effects on masses, couplings and scattering amplitudes.

Let us denote now by the subscripts 0 and 1 objects that belong to the sphere and torus level respectively. T_1 contains double- and external-line poles which can be both taken into account by a term:

$$T_{\text{DEP}} \equiv \frac{1}{2} \frac{\delta T_0}{\delta \phi} \pi_0^{-1} \Pi_1 \pi_0^{-1} \frac{\delta T_0}{\delta \phi} , \quad (24)$$

where Π_1 is the torus two-point function. It is possible to take the σ -dependence to be the same for Π_1 as for Π_0 :

$$\pi_1^{-1} \frac{\partial \Pi_1}{\partial \sigma} = \pi_0^{-1} \frac{\partial \Pi_0}{\partial \sigma} . \quad (25)$$

The double-pole-subtracted T_1'

$$T_1' = T_1 - T_{\text{DEP}} \quad (26)$$

can be used to construct the quantify

$$\hat{T} \equiv T_0 + T_1' . \quad (27)$$

Since $\hat{T}$ factorizes with just simple poles in the internal lines, we can use for it the modified Legendre transform used at the sphere level defining:

$$\hat{\Gamma} = \hat{T} - \bar{\phi} \pi_0 \phi ,$$

$$\bar{\phi} \equiv \pi_0^{-1} \frac{\delta \hat{T}}{\delta \phi} = \phi + O(\phi^2) \quad (28)$$

and obtain, after a straightforward but lengthy calculation, the σ -dependence:

$$\begin{aligned} \frac{\partial \hat{\Gamma}}{\partial \sigma} = & \bar{\phi} (K \pi_0 - \frac{\partial \pi_0}{\partial \sigma}) \phi + \frac{1}{2} \phi (\frac{\partial \pi_0}{\partial \sigma} - 2K \pi_0) \phi \\ & + \frac{1}{2} \bar{\phi} (\frac{\partial \pi_1}{\partial \sigma} - 2K \pi_1) \bar{\phi} + \phi (2K \pi_1 - \frac{\partial \pi_1}{\partial \sigma}) \bar{\phi} . \end{aligned} \quad (29)$$

However, the LTI functional which generates the full T_{0+1} amplitudes is not $\hat{\Gamma}$ but:

$$\Gamma = \hat{\Gamma} + T_{\text{DEP}} \Big|_{\phi=\phi[\bar{\phi}]} = \hat{\Gamma} + \frac{1}{2} \bar{\phi} \pi_1 \bar{\phi} + (\text{higher genus}) \quad (30)$$

and satisfies:

$$\begin{aligned} \frac{\partial \Gamma}{\partial \sigma} = & \bar{\phi} (K \pi_0 - \frac{\partial \pi_0}{\partial \sigma}) \phi + \frac{1}{2} \phi (\frac{\partial \pi_0}{\partial \sigma} - 2K \pi_0) \phi \\ & + \bar{\phi} (\frac{\partial \pi_1}{\partial \sigma} - K \pi_1) \bar{\phi} + \phi (2K \pi_1 - \frac{\partial \pi_1}{\partial \sigma}) \bar{\phi} . \end{aligned} \quad (31)$$

In the three subtraction schemes defined previously, this equation becomes

$$\text{Minimal:} \quad \frac{\partial \Gamma}{\partial \sigma} = - \frac{\delta \Gamma}{\delta \bar{\phi}} \{ K(\bar{\phi} - \phi) + \pi_1 \bar{\phi} \} , \quad (32a)$$

$$\text{Covariant:} \quad \frac{\partial \Gamma}{\partial \sigma} = K \bar{\phi} \frac{\delta \Gamma}{\delta \bar{\phi}} , \quad (32b)$$

$$\text{Intermediate: } \frac{\partial \Gamma}{\partial \sigma} = -\frac{1}{2} \frac{\delta \Gamma}{\delta \bar{\phi}} K \pi_0^{-1} \frac{\delta \Gamma}{\delta \bar{\phi}} . \quad (32c)$$

This shows that, in all cases, conformal invariance holds on the string-loop corrected equations of motion $\delta \Gamma / \delta \bar{\phi} = 0$. Furthermore, for the covariant and intermediate subtractions, the equation is not changed from the sphere case. In particular, the Zamolodchikov's form keeps its validity up to the $g = 1$ level.

Let us discuss how the conditions of conformal invariance at the $g = 1$ level imply the renormalized values for masses, couplings and scattering amplitudes. Expanding in a Taylor series:

$$\Pi_1(p^2) = \Pi_1(-m_0^2) + (p^2 + m_0^2) \Pi_1'(-m_0^2) + \dots , \quad (33)$$

one finds from (30) a mass shift given by:

$$m_1^2 = m_0^2 + \frac{4}{\alpha'} \Pi_1'(-m_0^2) . \quad (34)$$

Also, the correctly normalized tachyon field becomes:

$$\hat{\phi} = \left\{ 1 + \frac{4}{\alpha'} \Pi_1'(-m_0^2) \right\}^{1/2} \phi \equiv Z^{-1/2} \phi , \quad (35)$$

so that the S-matrix is generated by Γ (plus pole terms) as

$$S_n \sim Z^{n/2} \left. \frac{\delta^n \Gamma}{\delta \bar{\phi}^n} \right|_{p_i^2 = -m_1^2} . \quad (36)$$

The result (34) appears to be in agreement with the prescription obtained by several authors [13] in rather indirect ways. As for (35), this is less obvious since our off-shell amplitudes differ from those of [13]. Indeed, we are not sure that the latter can be derived by a Möbius invariant object and that it obeys factorization.

5. - CONCLUSIONS

String theory was born as an S-matrix theory. Even after twenty years it seems to remember (suffer from?) this inheritance reflected in its resistance to being extended off shell. Yet, we believe that an off-shell extension of string theory not only is desirable, but could even turn out to be essential to its eventual success.

Apart from the (purely technical?) difficulty in defining finite-loop amplitudes while sitting on-shell all the time, the on-shell theory is, by definition, a theory in one vacuum. When such a theory shows no disease order by order (as it seems to be the case for the superstring), one may be happy with such a limitation, but this means to give up any possibility of: i) dealing with theories with either perturbative (bosonic string) or non-perturbative (superstring?) vacuum instabilities and ii) relating different consistent theories (and at present we have too many of them!) as realizations of one and the same theory around different vacua and thus being able to select one of them dynamically.

In this paper we have made an effort to go off-shell which, so far, seems to work. It rests on the basic assumption, which is, however, supported by some explicit results, that an off-shell continuation preserving factorization and Möbius (resp. modular) invariance at the sphere (resp. higher genus) level exists. Of course, conformal (Weyl) invariance is lost off-shell and thus the off-shell extension depends on a conformal factor σ (on which we do not integrate). The conditions of σ -independence imply the full interacting equations of motion defined by the off-shell extension and vice versa, the equations of motion required by stability of the vacuum imply that no on-shell quantity (i.e., no physics) depends on σ .

The analogy between two-dimensional and space-time concepts goes even further, at least in the case of the intermediate subtraction defined by Eq. (21). It is clear from the space-time action point of view that a stable vacuum corresponds to a minimum of Γ and not just to a stationary point. From the point of view of two-dimensional field theory the same conclusion is reached by demanding that the conformal invariant point is an ultra-violet fixed point so that a small perturbation does not make the system run away in the limit of infinite (momentum) cut-off ($\Delta z \rightarrow 0$ or $\sigma \rightarrow \infty$). It is an easy exercise to solve Eq. (22) for small fluctuations around a conformal invariant point (see Ref. [14]) and to verify explicitly that for $\sigma \rightarrow \infty$ one runs away from any point except for the true minima of the effective action. There is even an amusing analogy between Eq. (22) and the equations of potential flow in hydrodynamics which helps understanding the "time" evolution of the system.

The only puzzle that seems to be left is that, while the action cannot change from a conformal point to another if the two are connected by a path of conformal backgrounds, this is obviously not the case in general as implied by the existence of saddle points. The question then is whether two conformal theories with different action have the same central charge or not. On the one hand, one would say that they do since, in string theory, the central charge is fixed by conformal invariance; at the same time, this would spoil the claimed [11,12] relation between central charge and effective action. Having not studied yet such a relation, we are not able to say more at present on this point.

In this paper, we have not dealt with the question of how the critical dimensions originate in our scheme. Since D-26 (or D-10 for superstrings) is known to appear in the dilaton β -function, we certainly expect to recover these conditions as soon as we shall add all the massless backgrounds. Similarly, the well-known tadpole divergences which are known to occur for the bosonic string at the torus level will presumably induce in Γ finite linear terms proportional to some combination of massless fields. This would be our way of recovering the results of Ref. [17].

ACKNOWLEDGEMENTS

The authors would like to thank D. Amati, P. Di Vecchia, M. Martellini, N. Sakai and P. West for useful discussions. One of us (T.K.) wishes to acknowledge the partial support of the Swiss National Science Foundation and of the Japan Society for the Promotion of Science.

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