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TOTAL CROSS-SECTIONS FOR HEAVY FLAVOUR PRODUCTION
IN HADRONIC COLLISIONS AND QCD

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ABSTRACT

The total cross-sections for top, bottom and charm production in hadronic collisions at energies of practical interest are studied using a recent complete calculation of the next-to-leading QCD corrections. A set of proton structure functions with QCD evolution at next-to-leading accuracy with variable Λ and including updated experimental information is used. A careful discussion of theoretical errors due to scale dependence and ignorance of Λ , input gluon density and so on is given. It is found that particularly reliable predictions can be given for top or b' production at Sp $\bar{p}$ S and Tevatron energies. By comparison with the UA1 experimental bounds, we find the limits $m_t > 41$ GeV and $m_{b'} > 34$ GeV. The cross-sections for heavy quark production at supercolliders (LHC, SSC) are estimated. Bottom and charm production in fixed target, ISR and collider experiments are considered. It is found that the corrective terms tend to improve the agreement with the data. In particular, the computed amount of b production agrees with the measurements by UA1 in $p\bar{p}$ collisions at $\sqrt{s} = 0.63$ TeV and by NA10 and WA78 in π^-N at $\sqrt{s} = 23\div 24.5$ GeV. The charm production data in pN collisions at fixed target energies are in general agreement with the corrected QCD prediction with $m_c = 1.5$ GeV, i.e., without need of assuming a small effective charm mass.

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1. - INTRODUCTION

The problem of obtaining a precise determination of the predicted cross-sections for heavy flavour quark production in hadron-hadron collisions is very important, both for practical and for theoretical reasons. From the practical point of view, this problem is relevant for the search of new heavy quarks, in particular of the top quark and for establishing (if not found) the corresponding bounds on the masses. It is also important for the interpretation of single and double-charged lepton signals, for background estimates in the search for new physics and so on. On the theoretical side, it is essential to aim at a precise quantitative test of the parton approach and of the QCD specific predictions in a highly non-trivial dynamical situation ($2 \rightarrow 2$ and $2 \rightarrow 3$ strongly interacting parton subprocesses). Quite remarkable progress has recently been accomplished in this problem by the complete calculation in Ref. [1] of the next-to-leading corrections to the total cross-sections of heavy flavour production. Corresponding results for the rapidity and transverse momentum distributions are also to be published shortly by the same authors. As a consequence, a much more solid quantitative analysis of heavy flavour production is now possible.

The present work is devoted to a systematic application of the results of Ref. [1] to the phenomenology of heavy flavour production. The predictions for the total cross-sections of top, bottom and charm production are studied in detail at all energies of practical interest at present and in the near future. In particular the implications on the lower bounds on m_t and $m_{b'}$ (where b' is a fourth generation down quark) that can be derived from the UA1 data [2] are analyzed, as well as the implications for the search of the t quark at the SppS (with improved luminosity provided by ACOL) and the Tevatron colliders. The comparison with the existing data for c and b production is also considered. The theoretical problems which appear to exist in predicting the c and b cross-sections (or in general when $m/\sqrt{s} \ll 1$, m being the heavy quark mass) are discussed.

The sophisticated level of accuracy of the calculations of Ref. [1] demands an adequate treatment of all input elements in the derivation of the heavy flavour cross-sections and a critical discussion of the related theoretical and experimental sources of uncertainty. The total error on the resulting heavy flavour cross-sections is in part due to our ignorance of the different input quantities [uncertainty on Λ , on the parton structure functions, on the heavy quark mass, e.g., $m_b = (4.5 \div 5)$ GeV or $m_c = (1.2 \div 1.5)$ GeV] and in part to intrinsic theoretical ambiguities (for example, the choice of the renormalization and/or factorization scales). An estimate of the effect of higher order terms should also in principle be included, but in practice for this part only a guess can be made.

The error on Λ is important because the lowest order cross-section is of order

α_s^2 . Within the $\overline{\text{MS}}$ renormalization prescription [3], the precise definition of Λ at two-loop accuracy depends mainly on the number of excited flavours but also on the procedure adopted for making α_s continuous across the thresholds. In Section 3, we specify our definition of Λ and discuss the available experimental information on α_s . As a result for $f = 5$, where f is the number of excited flavours, we adopt the range

$$\Lambda_{\overline{\text{MS}}}^{f=5} \equiv \Lambda_5 = (170 \pm 80) \text{ MeV} \quad (1)$$

In most of the domain of μ^2 of interest here ($\mu \gtrsim 2 \text{ GeV}$) [with μ^2 being the scale in $\alpha_s(\mu^2)$] this range approximately corresponds to:

$$\Lambda_{\overline{\text{MS}}}^{f=4} \equiv \Lambda_4 = (260 \pm 100) \text{ MeV} \quad (2)$$

in terms of Λ for $f = 4$. Although both the central value and the error on Λ are to some extent debatable, certainly the stated range is indicative of the present uncertainty on Λ .

For our purposes, we need a set of structure functions with the following characteristics. First, the set of densities should also take into account the most recent data. This especially applies to the gluon structure function because gluon fusion is a particularly important parton subprocess in heavy flavour production. Sets of structure functions only based on the old CDHSW [4] data for the gluon density should be to some extent revised. In fact, the shape of the gluon density at $Q^2 \sim 5 \text{ GeV}^2$ derived from deep inelastic scattering is now considerably softer than it used to be, according to the results obtained mainly by the CHARM [5] and BCDMS [6] collaborations (while the old CDHSW structure functions at low x have been revised [7,8]). Second, the QCD evolution should be performed at next-to-leading accuracy, with due care to the problem of bringing the definitions of parton densities beyond the leading order in agreement with the conventions of Ref. [1]. Finally, for error evaluations, there should be the possibility of varying Λ in the QCD evolution. In most of the available parametrizations of parton densities [9,10], Λ is fixed from a best fit of the input data at different Q^2 (or, in some cases, a double choice is given). The value of Λ so obtained is not necessarily what one wants to use in a given case. Moreover in the QCD analysis of scaling violations in deep inelastic scattering, there is a correlation between the value of Λ and the form of the gluon density. Thus, if we want to vary Λ in a given range, for example that of Eq. (1), the input gluon density must be accordingly modified and then the QCD evolution must be performed for each Λ using the corresponding set of values of α_s and of the gluon density. A set of structure functions that satisfies all the above requirements has been obtained following the work of Ref. [11]. Thus we have been able to study the variation of the heavy

flavour cross-sections in a given range of Λ by changing at the same time α_s , the input gluon and the QCD evolution.

Another important source of errors is introduced by the scale dependence of the result. In principle, one can distinguish the renormalization scale [i.e., the scale μ in $\alpha_s(\mu)$] and the factorization scale [i.e., the scale μ in the parton densities $q(x, \mu)$ or $g(x, \mu)$]. In the following, we shall normally identify these two scales. On physical grounds the scale μ should be of the order of the heavy flavour mass m . If one works at leading order, a change of μ around m introduces an error of order $\alpha_s^3(m)$. At next-to-leading accuracy, the variation of μ in the leading term is automatically compensated at order α_s^3 by the corresponding change in the correction, so that the resulting error is in this case of order $\alpha_s^4(m)$. Thus, the ambiguity in the result for a given change of μ is normally decreased when going from the leading to the next-to-leading accuracy (assuming that there are no accidental cancellations for example between the scale dependence of parton densities and the scale dependence of α_s). In the following, for each case of interest, we shall study in detail the μ -dependence of the result and the comparison of the leading with the next-to-leading formulae [with the same $\alpha_s(\mu)$, i.e., α_s will always be computed at two loop accuracy with a given Λ also when the leading order formulae are considered]. Various optimization procedures [12-14] have been suggested in the literature, i.e., criteria for selecting in each case the "best" value of μ . For example, it has been advised [12] to take the scale μ that makes $d\sigma/d\mu = 0$ where σ is the physical quantity of interest at next-to-leading accuracy (criterion of "minimal sensitivity"). This is the scale around which the residual scale dependence is smallest. The idea inspiring this suggestion is that the exact σ at all orders cannot depend on μ . Another proposal [13] ("fastest convergence") is to use the scale where the next-to-leading correction vanishes (i.e., the so-called K factor is one). More complicated optimization procedures which separately adjust the factorization and renormalization scales are also being used [14]. In a healthy case, all these scales are of order m and are equally acceptable. In a pathological case, the optimized scales very much deviate from m and the result is suspect. In general, we can affirm without possibility of being contradicted that there is no serious reason to prefer one or the other scale around m . In the following, we shall exhibit the μ -dependence of the results explicitly. The reader, if he so wishes, can obtain from our plots the values of the optimized scales and the corresponding results for the heavy flavour cross-sections. We prefer to quote an indicative error, associated with the scale dependence, which corresponds to a change of μ in the range of $m/2$ up to $2m$.

2. - BASIC FORMULAE

The total cross-section for the inclusive production of a heavy quark pair

(e.g., $AB \rightarrow Q\bar{Q}X$) is given by [1]:

$$\sigma(S) = \sum_{ij} \int dx_1 dx_2 \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i^A(x_1, \mu) F_j^B(x_2, \mu) \quad (3)$$

where F_i^A are the densities of parton i in the hadron A , m is the heavy quark mass, $\sqrt{S}$ is the total centre-of-mass energy of the $A+B$ system, μ is the factorization scale taken as coincident with the renormalization scale. The partonic cross-section $\hat{\sigma}_{ij}$ is given by:

$$\hat{\sigma}_{ij}(s, m^2, \mu^2) = \frac{\alpha_s^2(\mu^2)}{m^2} f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) \quad (4)$$

where $s = x_1 x_2 S$ is the partonic centre-of-mass energy squared and $\rho = 4m^2/s$ ($\rho \leq 1$ in the physical region). The dimensionless functions f_{ij} are written in the form:

$$f_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = f_{ij}^{(0)}(\rho) + 4\pi\alpha_s(\mu^2) \left[f_{ij}^{(1)}(\rho) + \bar{f}_{ij}^{(1)}(\rho) \ln \frac{\mu^2}{m^2} \right] + o(\alpha_s^2) \quad (5)$$

The explicit expressions of $f^{(0)}$, $f^{(1)}$ and $\bar{f}^{(1)}$ are given in Ref. [1] and will not be repeated here. The following points should be remarked. The functions $f^{(1)}$ depend on the renormalization scheme. In Ref. [1], the results for $f^{(1)}$ are given in two cases: a specified version of the $\overline{MS}$ prescription [3] and a more physical procedure [15] where quark densities are defined in terms of the structure function F_2 of deep inelastic scattering at the scale $|Q^2| = \mu^2$. We adopt here the latter definition which agrees with the one used for the QCD evolution of parton densities beyond the leading order in Ref. [11]. The corrections bringing the definitions of the gluon density in Refs. [1] and [11] in agreement have been included, but their effect is numerically irrelevant. While in Ref. [1] the expressions given for $f^{(1)}$ are parametrized fits to the exact formulae which are not available in analytic form, it has been checked in a number of typical cases that the exact expressions lead to practically identical results for the cross-sections of interest (i.e., the associated error is negligible with respect to the quoted errors). The functions $\bar{f}^{(1)}$ which multiply $\ln(\mu^2/m^2)$ in Eq. (5) are entirely determined by the requirement of compensating at order α_s^3 the μ -dependence of the lowest order cross-section:

$$\bar{f}_{ij}^{(1)}(\rho) = \frac{1}{8\pi^2} \left[4\pi b_0 f_{ij}^{(0)}(\rho) - \int_{\rho}^1 dz_1 f_{kj}^{(0)}\left(\frac{\rho}{z_1}\right) P_{ki}(z_1) - \int_{\rho}^1 dz_2 f_{ik}^{(0)}\left(\frac{\rho}{z_2}\right) P_{kj}(z_2) \right] \quad (6)$$

[where P_{ij} are the lowest order splitting functions [16] and b_f is the beta function coefficient defined in Eq. (11)] so that a variation of μ around m only induces a change of σ at order α_s^4 . The results of the integrations in Eq. (6) are given explicitly in Ref. [1]. It is easy to prove that the scale where the correction of order α_s^3 vanishes (schematically $f^{(1)} + \bar{f}^{(1)} \ln(\mu^2/m^2) = 0$) differs from the scale where $d\sigma/d\mu = 0$ (at leading order in α_s) only by terms proportional to the two-loop beta function coefficient b_f' [given in Eq. (11)] or the splitting function kernels at two-loop accuracy. In most cases, these terms are numerically not very important and the two scales mentioned above are not much different, being essentially determined by the same condition of cancellation of the large terms. This effect is clearly observed in Figs. 2-4 where the μ -dependence of the cross-section is shown: the maximum of the corrected result is for a scale μ almost coincident with the scale where the lowest order and the corrected cross-sections coincide.

3. - THE QCD COUPLING

Throughout this paper, we shall use the $\overline{MS}$ definition of α_s [3]. In addition, we shall always refer to the expression of α_s at two-loop accuracy, even when discussing the lowest order formulae for heavy flavour production. This is because we want to compare leading and non-leading formulae at the same values of $\alpha_s(\mu)$ for all μ . According to the renormalization scheme adopted in Ref. [1] for top, bottom and charm production we need to use α_s for five, four and three flavours respectively. Defining $\Lambda = \Lambda_5$, i.e., the one appropriate for five flavours, the relevant definitions of α_s for five, four and three flavours, accurate to next-to-leading order are:

$$\alpha'_{s,5}(Q) = \alpha_s(Q, 5) \quad (7)$$

$$\frac{1}{\alpha'_{s,4}(Q)} = \frac{1}{\alpha_s(Q, 4)} + \frac{1}{\alpha_s(a m_b, 5)} - \frac{1}{\alpha_s(a m_b, 4)} \quad (8)$$

and

$$\frac{1}{\alpha_{s,3}(Q)} = \frac{1}{\alpha_s(Q,3)} + \frac{1}{\alpha_s(am_c,4)} + \frac{1}{\alpha_s(am_b,5)} - \frac{1}{\alpha_s(am_b,4)} - \frac{1}{\alpha_s(am_c,3)} \quad (9)$$

where

$$\alpha_s(Q, f) = \frac{1}{b_f \ln \frac{Q^2}{\Lambda_f^2}} \left[1 - \frac{b'_f \ln \ln Q^2 / \Lambda_f^2}{b_f \ln Q^2 / \Lambda_f^2} \right] \quad (10)$$

and the beta function coefficients b_f and b'_f are given by [17]:

$$b_f = \frac{33-2f}{12\pi} \quad ; \quad b'_f = \frac{153-19f}{2\pi(33-2f)} \quad (11)$$

Setting $a = 1$ in Eqs. (8) and (9), one has

$$\alpha_{s,5}(m_b) = \alpha_{s,4}(m_b) \quad ; \quad \alpha_{s,4}(m_c) = \alpha_{s,3}(m_c) \quad (12)$$

The above matching conditions are the appropriate ones for the $\overline{MS}$ scheme at next-to-leading order [18] and those that must be used to be consistent with Ref. [1]. We find, however, that varying a from 1 to 2, i.e., changing the matching conditions from $Q = m$ to $Q = 2m$, only gives negligible corrections on α_s , as can be seen from Fig. 1. For a comparison with the experimental data on α_s we now define:

$$\alpha_s(Q) = \alpha_{s,5}(Q) \theta(Q - am_b) + \alpha_{s,4}(Q) \theta(am_b - Q) \theta(Q - am_c) + \alpha_{s,3}(Q) \theta(am_c - Q) \quad (13)$$

For comparison one can also introduce Λ_4 , i.e., Λ for $f = 4$ defined in a completely analogous way, starting from the solution of the equation

$$\alpha_{s,4}(Q) = \frac{1}{b_4 \ln Q^2 / \Lambda_4^2} \left[1 - \frac{b'_4 \ln \ln Q^2 / \Lambda_4^2}{b_4 \ln Q^2 / \Lambda_4^2} \right] \quad (14)$$

for large values of Q .

We now consider the experimental information on α_s expressed in terms of Λ_5

(or Λ_4) with the aim of determining a reasonable range for these quantities. A complete review of the available data is clearly beyond the scope of this paper. We therefore concentrate on three experimental sources, i.e., quarkonium decays, deep inelastic scattering and e^+e^- annihilation, which are certainly the most important ones.

Quarkonium decays [19] have been recently reanalyzed in Ref. [20]. Their conclusion is that

$$\alpha_s'(m_b) = 0.185 \pm 0.006 \quad (m_b = 4.9 \text{ GeV}) \quad (15)$$

from $\Gamma(T \rightarrow \gamma gg)/\Gamma(T \rightarrow ggg)$ combined with the results on $\Gamma(T, J/\psi \rightarrow ggg)/\Gamma(T, J/\psi \rightarrow \mu^+\mu^-)$. We observe that this value of α_s could be underestimated. In fact, from the observed spectrum of the photon in the decay $T \rightarrow \gamma gg$, one sees that important deviations from the short distance prediction are present. An important effect could be the effective mass attributed to the gluon jet by the process of hadronization. The three-body phase space is particularly sensitive to masses and is reduced by this gluon mass. The three-gluon phase space is more suppressed than the γgg phase space. As a consequence, the value of α_s obtained by the perturbative expansion for massless partons is possibly underestimated. Certainly the quoted error is to be increased to include theoretical errors (non-perturbative effects as discussed above, relativistic corrections and so on).

In deep inelastic scattering, the BCDMS [6] and EMC [21] collaborations present the muon data with largest statistics and largest Q^2 values. The QCD analysis by BCDMS shows really an impressive agreement between the observed and the predicted pattern of scaling violations. The resulting value from the combined analysis by BCDMS of the data on hydrogen and carbon targets is:

$$\alpha_s'(Q) \approx 0.16 \pm 0.01 \quad (Q \approx 10 \text{ GeV}) \quad (16)$$

(The Q values quoted here and in Fig. 1 are only very loosely related to the average Q of the corresponding experiment.) The EMC data show a well-known discrepancy at small x with respect to the BCDMS data, well outside the quoted systematic errors. The results of EMC on α_s are consistent with those of BCDMS but with a larger error:

$$\alpha_s'(Q) \approx 0.160 \pm \begin{matrix} 0.032 \\ 0.022 \end{matrix} \quad (Q \approx 5 \text{ GeV}) \quad (17)$$

Among neutrino data (at lower Q^2) we quote the result of the CHARM collaboration [5]

$$\alpha_s(Q) \approx 0.20 \pm 0.03 \quad (Q = 6 \text{ GeV}) \quad (18)$$

The CHARM analysis is also important for their determination of the gluon density which is used in the structure functions of Ref. [11]. The CDHSW collaboration has not yet published the final results on the high statistics sample of events. At the Uppsala Conference [8], they quoted a range of Λ which corresponds to:

$$\alpha_s(Q) \approx 0.12 \div 0.18 \quad (Q \approx 8 \text{ GeV}) \quad (19)$$

Results from the CCFR collaboration [22] are also consistent with the ones reported here.

The determination of α_s in e^+e^- annihilation experiments was summarized at the Hamburg Conference [23]. From the hadronic cross-section by fitting the experimental value of R as a function of energy with fixed $\sin^2\theta_W \approx 0.23$, one finds

$$\alpha_s(Q) \approx 0.145 \pm 0.020 \quad (Q \approx 34 \text{ GeV}) \quad (20)$$

Finally, a number of experiments [23] have determined α_s from a study of the asymmetry in energy-energy correlations (EECA). The data quoted in Ref. [23] are summarized in Fig. 1, together with the previously mentioned results in Eqs. (15)-(20). Note that the errors on the EECA data do not include the unknown systematic uncertainty introduced by the models of parton fragmentation and hadronization. For example, the MARK J data are based on a different model than the other ones. These errors are certainly quite substantial and their neglect must be kept in mind.

Clearly it is not simple to extract a conclusion from the available set of data. We tentatively combine the determination of α_s from deep inelastic scattering by BCDMS with the value obtained from R in e^+e^- annihilation and obtain

$$\alpha_s(Q) \approx 0.13 \pm 0.01 \quad (Q \approx 34 \text{ GeV}) \quad (21)$$

which corresponds to the values of $\Lambda_{4,5}$ given in Eqs. (1) and (2). As seen from Fig. 1, the corresponding values of α_s as function of Q provide a reasonable interpolation of the available data.

4. - TOP PRODUCTION

In this section, we start the applications to phenomenology by studying the production of a heavy quark with mass $m \geq 25 \text{ GeV}$ at energies $\sqrt{s} \geq 630 \text{ GeV}$ in $p\bar{p}$ (or

pp) collisions. This case is not only important for the search of new quarks (t, b', ...) at present and future accelerators, but is also interesting because the heavy flavour mass and the energy $\sqrt{S}$ of the SppS and Tevatron colliders are in a ratio which is particularly favourable for a relatively precise theoretical prediction.

First we consider the μ -dependence of the computed cross-sections at a fixed value of Λ and for a given set of parton densities. In Figs. 2-4, we compare the μ -dependence of the cross-section at leading and next-to-leading accuracy. The figures refer to $\sqrt{S} = 630$ GeV for $m_t = 40$ and 80 GeV and to $\sqrt{S} = 1.8$ TeV for $m_t = 80$ GeV. The QCD running coupling is specified by $\Lambda_5 = 200$ MeV and the set of structure functions is DFLM [11]. The same function $\alpha_s(\mu)$ and the same set of parton densities are used for the computation of the cross-section as a function of μ at the leading order [$f^{(1)} = \bar{f}^{(1)} = 0$ in Eq. (5)] or at the next-to-leading level [by including $f^{(1)}$ and $\bar{f}^{(1)}$]. The range of μ explored in Figs. 2-4 is $0.2m \div 5m$. It is immediately seen that the variation of the results in this range of μ is much larger for the lowest order cross-section (which is monotonically decreasing with μ) than for the corrected cross-section. Thus, there is an increase of stability versus changes of μ in going from the lowest order to the corrected cross-section, as it should be. One sees that the cross-section at next-to-leading accuracy shows a maximum at $\mu \sim m/2$. Note that this maximum approximately coincides with the point where the leading and next-to-leading cross-sections intersect each other. In other words the optimized scale ($d\sigma/d\mu = 0$) and the scale where the K factor is one are very close. As already mentioned this is due to the fact that $|K-1|$ tends to be large, for example $K \sim 2$ at large μ and $K \sim 1/3$ at small μ (in the explored range). The fact that the scale where $K = 1$ is not too far from $\mu \sim m$ (the natural physical scale) shows that the perturbative expansion, up to order α_s^3 , is quite all right.

From Figs. 2-4 it is apparent that in this case it makes little sense to quote values for the K factor because it very much depends on μ . The situation is different from the Drell-Yan case where the notion of the K factor was introduced [15,24]. For Drell-Yan processes, the lowest order cross-section does not depend on α_s . It is mainly determined by the electroweak interactions. It is true that the exact form of the next-to-leading correction depends on the definition of parton densities. However, the relative change, for reasonable choices of the factorization scale around the natural value $Q = M_{\mu^+\mu^-}$, is small. The corresponding ambiguity is well within the experimental errors on the value of parton densities. As a result, the K factor has a well-defined physical meaning. On the contrary, in the case of heavy flavour production (and of all other processes which vanish with α_s), the K factor is strongly dependent on the definition and the scale of α_s , because the lowest order cross-section is of order α_s^2 . For example,

the K factor very much depends on whether the lowest order cross-section is evaluated with the leading or the next-to-leading expression for α_s (with a given Λ). Necessarily it also rapidly varies with the choice of scale. For example, consider a quark of mass m . If the K factor is one at $\mu \sim m/2$, it must be ~ 1.6 or ~ 2 at $\mu = 2m$ for $m = 40$ GeV or $m = 5$ GeV respectively simply because this is the order of magnitude of $\alpha_s^2(m/2)/\alpha_s^2(2m)$ in the two cases. On the other hand, it is clear from Figs. 2-4 that the real improvement obtained by the inclusion of next-to-leading terms in the cross-section consists in a smaller error being associated to the variation of μ in a given range around m . Thus, the quantity that should be quoted is not the K factor but the predicted value of the cross-section. In the following, we shall take the range $m/2 < \mu < 2m$ as a reference for error estimates. The cross-section at non-leading accuracy for heavy quark production at $\sqrt{s} = 630$ GeV as function of m is plotted in Fig. 5 (see also Table 1). The three curves are for $\mu = m/2, m, 2m$ at fixed $\Lambda_5 = 170$ MeV [the central value in Eq. (1)] and with the DFLM structure functions.

While the μ -dependence corresponds to an intrinsic theoretical error, there are additional ambiguities which are due to our ignorance of the exact input structure functions and of the precise value of Λ . We now study the dependence on Λ . As already stated in the introduction, when varying Λ , we have correspondingly changed not only the QCD evolution of the parton densities, but also the input gluon density whose form is correlated to Λ in the fits to scaling violations in deep inelastic scattering. For this purpose, the DFLM set of structure functions has been expressly rederived and evolved separately [25] for $\Lambda_5 = 90, 170, 260$ MeV. The results shown in Fig. 6 were obtained for the heavy quark cross-section at $\sqrt{s} = 630$ GeV as a function of m for the three mentioned values of Λ_5 with μ fixed, at $\mu = m$. By comparison with Fig. 5, we see that the total uncertainty from the variation of Λ in the given range is comparable to the effect of changing μ by a factor of two around m on either side. In Table 1 the μ and Λ dependences are presented for several values of $\sqrt{s}$. Also reported is the total error, obtained by combining in quadrature the uncertainty from μ and from Λ . This is the error which we attribute to our estimate of the heavy flavour cross-sections. Plots of the resulting cross-section bands for $\sqrt{s} = 630$ GeV and $\sqrt{s} = (1.8 \div 2)$ TeV are given in Figs. 7 and 8 for two different ranges of the heavy quark mass.

The previous error estimate was obtained within the set of DFLM structure functions. However, it must be stressed that by varying Λ and the input gluon density actually a whole family of sets was generated. One might still wonder what changes are to be expected if one takes a different parametrization of parton densities among those available in the literature. We find that the main source of difference is the form of the gluon density. The dependence on the gluon density for a given m, Λ and $\sqrt{s}$ is presented in Fig. 9. In particular, we point out the

difference induced on the cross-section by the CDHSW gluon density with respect to those obtained by the CHARM collaboration [5]. A form of the gluon density softer than that of CDHSW is also supported by the BCDMS deep inelastic scattering experiments [6] and by the production of photons at large p_T [26,27]. Thus all sets of parton densities based on the old CDHSW data tend to predict a larger value of the heavy flavour cross-sections than we obtained. For example, we show in Table 2 a comparison of the cross-sections at $\sqrt{s} = 630$ GeV and $\sqrt{s} = 1.8$ TeV obtained for fixed Λ and $\mu = m$ by using the set of parton densities by DFLM and ELHQ (set I) [10].

We finally compare our results on the heavy quark cross-section in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV with the UA1 95% confidence level limits on the $t\bar{t}$ and $b'\bar{b}'$ cross-sections [2]. The comparison is shown in Fig. 10. The theoretical prediction and its error band, obtained as explained for Table 1 and Figs. 7 and 8, is superimposed on the UA1 limits. For the limit on the top quark we obtain

$$m_t > 41 \text{ GeV} \quad (22)$$

The value of the lower limit varies between 41 and 48 GeV if the theoretical prediction is moved from the lower to the upper edge of the error band. On the b' quark, we find

$$m_{b'} > 34 \text{ GeV} \quad (23)$$

or a limit between 34 and 40 GeV. Thus, we essentially confirm the UA1 values of the limits as originally stated [2] ($m_t > 44$ GeV and $m_{b'} > 32$ GeV). However, this limit is now much better justified. The reason why a better limit is not found (although the theoretical error band is smaller for the cross-section computed at order α_s^3 than it was for the lowest order case) is that a softer gluon density is now preferred. Note that by using other sets of parton densities based on harder gluon densities, as for example ELHQ, we would obtain larger values of the minimum t or b' masses. This is also seen from Fig. 10 where the dashed line is the one relevant for the limit with the ELHQ parton densities. Also, for a given Λ , computed at two-loop accuracy [Eq. (10)] α is smaller than what would generally be used in a lowest order calculation. A better determination of the limit from the data can only be made when the corrected p_T and y distributions (including terms of order α_s^3) will be used to directly relate the experimental numbers, obtained in the UA1 acceptance, with the t and b' masses.

Finally a plot of the central values of the production cross-sections at pp supercolliders (e.g., LHC, SSC) is presented in Fig. 11, for different values of the heavy quark mass. Note, however, that the cross-section estimate becomes less and less reliable when $\sqrt{s}$ increases at fixed m for reasons which we will discuss in

the next section.

5. - b PRODUCTION

For b production the importance of the corrective terms of $O(\alpha_s^3)$, at fixed $m/\sqrt{s}$, is a priori expected to be larger than for t production because $m_b \ll m_t$. A posteriori the corrections are found to be particularly large, especially for $m/\sqrt{s} \ll 1$, for example at collider energies. In general, the bulk of the corrections arise from two regions of the integration domain in Eq. (3): the threshold region $2m/\sqrt{s} \sim 1$, and the large $\sqrt{s}$ region, $m/\sqrt{s} \ll 1$, where $\sqrt{s}$ is the centre-of-mass energy of the parton subprocess. The contribution of the threshold region is important because of large logarithmic corrections. Note also that the functions $f_{gg}^{(1)}$ and $f_{qq}^{(1)}$ approach constant limits while the lowest order terms $f_{gg}^{(0)}$, $f_{qq}^{(0)}$ vanish for $2m/\sqrt{s} \rightarrow 1$. In addition, the correction terms of order α_s^3 become very large when $m/\sqrt{s} \ll 1$, for all values of m . The origin of this effect can be traced back to the diagrams of Fig. 12, where a quasi real gluon is exchanged in the t channel of gg or gq parton subprocesses. While at lowest order, a fermion of spin- $\frac{1}{2}$ is exchanged in the t channel, at next-to-leading accuracy a spin-1 gluon is also present in the t channel. At large values of $\sqrt{s}/m$ the asymptotic behaviour is dominated by the fixed pole in the t channel with largest spin. This fact, together with the large gluon luminosity at small x , leads to a large next-to-leading correction. This diffraction-like production of a $Q\bar{Q}$ pair is a new mechanism with characteristic rapidity and p_T distributions, so that the presence of anomalously large corrections is not to be necessarily interpreted as a failure of the perturbative expansion, in the sense that the corrections of still higher order can well be of moderate size.

This phenomenon is already important for b (and, even more for c) production at present colliders. In Figs. 13-15, analogous to Figs. 2-4 for top production, we consider the μ -dependence of the b -production cross-section both at lowest order and including the next-to-leading corrections. Consider first Fig. 13 where the (academic) case of $p\bar{p}$ collisions with $\sqrt{s} = 62$ GeV is considered. We see that the μ -dependence of the corrected cross-section very much resembles the plots already seen for the top quark. This is not surprising because the ratio $m/\sqrt{s}$ is comparable in the two cases. In Fig. 14 the same type of plot is reproduced for $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV. Here the situation is completely different. In the range $m/5 < \mu < 5m$ there is no maximum of the corrected cross-section and no scale where $K = 1$. In other words, the scale where $K = 1$ is not of order m . Not only that, but the uncertainty associated to a variation of μ in a given range, for example $m/5 < \mu < 5m$, is a bit larger for the corrected cross-section than for the lowest

order one. This last fact is due to the complete dominance of the terms of order α_s^3 and also to an accidental cancellation in the lowest order cross-section between the scale dependence of the parton densities (mainly the gluon density) and that of $\alpha_s(\mu)$. It is interesting to observe that at $\sqrt{s} = 62$ GeV the $p\bar{p}$ and pp cases behave differently. In pp collisions (Fig. 15) the $q\bar{q}$ component is smaller and the regime of gluon dominance is already visible. However, in this case the μ -dependence is more pronounced for the lowest order, as normally is the case.

Thus from a study of the total cross-sections including next-to-leading corrections as functions of $\sqrt{s}$, it appears that b production can be reliably computed (although with large errors) at ISR energies and below. On the contrary, at collider energies and above, the situation becomes more uncertain because the corrective terms are dominant over the lowest order cross-section at all values of order m of the scale μ . A study of the distribution in phase space of the correction of order α_s^3 (which is underway) will presumably better clarify the origin of the problem and the possible remedies.

For the total cross-sections we present in Table 3 the results obtained for b production at $\sqrt{s} = 41$ GeV and $\sqrt{s} = 62$ GeV in pp collisions. In Table 3, we also report the results obtained in $p\bar{p}$ collisions at $\sqrt{s} = 0.63$ TeV which are to be taken with the words of caution specified above. The central values of the b production cross-sections, as functions of $\sqrt{s}$ (obtained for $\mu = m_b = 4.75$ GeV, $\Lambda_5 = 170$ MeV) in pp and $p\bar{p}$ collisions are shown in Fig. 16.

The most extensive studies of b production in $p\bar{p}$ collisions are due to the UA1 Collaboration [28] at CERN. By extrapolating outside the UA1 acceptance the measured $b\bar{b}$ inclusive cross-section, by using the p_T and y distributions calculated by QCD (with next-to-leading corrections also taken into account), one obtains [29] at $\sqrt{s} = 0.63$ TeV:

$$\sigma(p\bar{p} \rightarrow b\bar{b} X) = (16 \pm 5) \mu b \quad (24)$$

Thus, by comparing with Table 3 it is impressive that in spite of the large theoretical uncertainties affecting b production at collider energies, a remarkable agreement with experiment is obtained.

On b production at ISR energies, we recall the strange result of Ref. [30], which finds:

$$\sigma_{\text{partial}} \cdot B(1_b \rightarrow p D^0 \pi^-) = (150 \div 500) \mu b \quad (25)$$

where σ_{partial} refers to $\sqrt{s} = 63$ GeV, $y(p(K^-\pi^+)\pi^-) > 1.4$ and $x_F(p) > 0.3$. Certainly, for all reasonable estimates of $B(\Lambda_b \rightarrow pD^0\pi^-)$ it is difficult to reconcile this value of the cross-section with the QCD production mechanisms discussed in the present paper (see Table 3).

It is interesting to recall that the experiment WA78 [31] at CERN has recently studied multimuon events by dumping 320 GeV π^- in uranium. From the study of muon events, assuming a linear A dependence, WA78 finds a cross-section for beauty production (at $\sqrt{s} = 24.5$ GeV) given by

$$\sigma(\pi^- N \rightarrow b\bar{b} X) = (2.0 \pm 0.3 \pm 0.9) \text{ nb} \quad (26)$$

where N is an isoscalar nucleon [i.e., $(p+n)/2$]. A larger value of the same cross-section had previously been reported by the NA10 Collaboration [32] at CERN. Their result found in π^- -N at $\sqrt{s} \approx 23$ GeV, is

$$\sigma(\pi^- N \rightarrow b\bar{b} X) = (14 \pm 7)_6 \text{ nb} \quad (27)$$

In view of the small amount of experimental information on b production in hadronic collisions, it is worthwhile to compare these data with the improved QCD predictions. Of course, we need to input the π -structure functions which are much less known than the parton densities in the proton. If we take the π -structure functions given in Ref. [33], we obtain the results quoted in Table 3. We see that the agreement with WA78 is especially good for $m_b \approx 5$ GeV while the NA10 result prefers the value $m_b \approx 4.5$ GeV.

6. - CHARM PRODUCTION

It is clear that the prediction of charm cross-sections is affected by particularly large uncertainties because m_c is so small. Actually, if we would attempt to present a plot of the μ dependence of the cross-sections in the range $m/5 \lesssim \mu \lesssim 5m$, as was done for t and b quarks, we would have to go down at scales much below $\mu = 1$ GeV which we really consider as the lowest conceivable scale for perturbative QCD calculations. As an example, the μ dependence of the charm cross-section for $\sqrt{s} = 30$ GeV and $m_c = 1.5$ GeV is shown in Fig. 17 only for $\mu > 1$ GeV. Rather by applying simple scaling arguments, one understands from the discussion of the previous section that charm cross-sections can only be estimated with large errors for moderate values of $\sqrt{s}$. Actually, most of the present data are at fixed target energies so that a comparison with theory can be directly done.

In Fig. 18 we plot the charm cross-section in pp collisions for energies in

the range $\sqrt{s} \approx (10 \div 62)$ GeV. The results are given for $m_c = 1.5$ GeV. For comparison, the results with $m_c = 1.2$ GeV are also presented. For fixed m_c the error band shown includes the variation of μ between 1 GeV and $2m_c$ and that of Λ_5 in the range given by Eq. (1). The predicted cross-sections are compared to the experimental data, as compiled in Ref. [34]. Clearly not all available data are mutually consistent within the stated errors. Note that the band of values corresponding to $m_c \approx 1.5$ GeV provides a reasonable description of the size and of the general trend shown by the data. This is interesting because lower values of m_c , somewhat marginal on physical grounds (e.g., $m_c = 1.2$ GeV), are needed [34] to reproduce the experimental data in terms of lowest order cross-sections. Thus, in spite of the large numerical uncertainties, there are strong indications that much of the observed abundant charm production in pp collisions can indeed be attributed to rather large non-leading perturbative effects. It will be particularly interesting to compare the rapidity and p_T distributions of the produced charmed particles with the perturbative formulae corrected at order α_s^3 , in order to try to disentangle perturbative vs. non-perturbative effects.

7. - SUMMARY AND CONCLUSION

We have analyzed the physical implications of the calculation in Ref. [1] of the next-to-leading QCD corrections to the total cross-sections for heavy quark production in hadronic collisions. From a numerical study of the corrections as functions of m and $\sqrt{s}$, it appears that not only large values of m are necessary, but also a certain balance between m and $\sqrt{s}$ is required for a good convergence of the perturbative expansion to the order α_s^3 . As a consequence, the most precise predictions can be made for top production at present collider energies. The resulting cross-sections at SppS and Tevatron energies were computed and a discussion of the errors was given. From a comparison with the 95% c.l. bounds on the t and b' cross-sections reported by UAL, we obtained the limits $m_t > 41$ GeV and $m_{b'} > 34$ GeV. When $2m/\sqrt{s}$ is too close to zero, large corrections arise mainly from the t -channel exchange of a gluon in gg or gq subprocesses (Fig. 12). The complete calculation of the terms of order α_s^3 in the total cross-section provides us with the theoretically correct description of processes like gluon splitting, flavour excitation and diffractive production, which previously could only vaguely be discussed. The quantitative effect of these terms is large for b production at ISR and collider energies or for c production at fixed target energies. Thus the prediction of the corresponding cross-sections is affected by large uncertainties. However, it is interesting to remark that in all cases the computed corrections, taken at face value, lead to a better agreement between the theoretical calculations and the data. This is true both for b production at the SppS collider and

for charm production at fixed target energies. Within the considerable theoretical errors, there is a remarkable agreement between theory and experiment. The rates of b production observed by UA1 at $\sqrt{s} = 0.63$ TeV and those measured by NA10 and WA78 in π^-N interactions at $\sqrt{s} = (23 \pm 24.5)$ GeV are in good agreement with the QCD predictions. We also find good general agreement of the computed cross-sections for charm production in pN collisions at fixed target and ISR energies. The observed cross-sections do not need a particularly small value of the effective charm mass. We find that $m_c \approx 1.5$ GeV is consistent with the data.

Further theoretical studies are needed in order to extend the domain of energies where reliable calculations of the heavy flavour cross-sections can be made. As a first step, it is important to study the rapidity and p_T distributions of the $O(\alpha_s^3)$ corrections and compare them with the data. This analysis will determine the regions of phase-space where the corrections are particularly large, and help a better understanding of the underlying physical mechanisms. As a second step, one should finally develop appropriate techniques for handling the problems which are clearly present for $2m/\sqrt{s} \ll 1$ or $2m/\sqrt{s} \approx 1$.

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$\sqrt{s} = 0.63 \text{ TeV}$					
$m(\text{GeV})$	$\sigma(\text{nb})$	$\mu = m/2$ $\Lambda = 170 \text{ MeV}$	$\mu = 2m$ $\Lambda = 170 \text{ MeV}$	$\mu = m$ $\Lambda = 90 \text{ MeV}$	$\mu = m$ $\Lambda = 250 \text{ MeV}$
20	$25.6 \pm_{7.9}^{5.6}$	30.9	20.2	19.8	27.3
30	$3.04 \pm_{0.94}^{0.62}$	3.62	2.47	2.29	3.26
40	$0.643 \pm_{0.20}^{0.11}$	0.738	0.532	0.481	0.696
50	$0.188 \pm_{0.056}^{0.025}$	0.208	0.158	0.142	0.204
60	$(0.669 \pm_{0.190}^{0.067})10^{-1}$	$0.718 \cdot 10^{-1}$	$0.569 \cdot 10^{-1}$	$0.508 \cdot 10^{-1}$	$0.716 \cdot 10^{-1}$
70	$(0.267 \pm_{0.074}^{0.022})10^{-1}$	$0.284 \cdot 10^{-1}$	$0.229 \cdot 10^{-1}$	$0.204 \cdot 10^{-1}$	$0.281 \cdot 10^{-1}$
80	$(0.114 \pm_{0.031}^{8.009})10^{-1}$	$0.122 \cdot 10^{-1}$	$0.989 \cdot 10^{-2}$	$0.880 \cdot 10^{-2}$	$0.118 \cdot 10^{-1}$
90	$(0.511 \pm_{0.136}^{0.037})10^{-2}$	$0.548 \cdot 10^{-2}$	$0.443 \cdot 10^{-2}$	$0.394 \cdot 10^{-2}$	$0.517 \cdot 10^{-2}$
100	$(0.234 \pm_{0.062}^{0.019})10^{-2}$	$0.254 \cdot 10^{-2}$	$0.203 \cdot 10^{-2}$	$0.181 \cdot 10^{-2}$	$0.232 \cdot 10^{-2}$
110	$(0.110 \pm_{0.029}^{0.009})10^{-2}$	$0.119 \cdot 10^{-2}$	$0.946 \cdot 10^{-3}$	$0.847 \cdot 10^{-3}$	$0.107 \cdot 10^{-2}$
120	$(0.517 \pm_{0.142}^{0.044})10^{-3}$	$0.562 \cdot 10^{-3}$	$0.441 \cdot 10^{-3}$	$0.398 \cdot 10^{-3}$	$0.494 \cdot 10^{-3}$
$\sqrt{s} = 1.8 \text{ TeV}$					
20	$228 \pm_{60}^{55}$	277	190	182	252
40	$9.63 \pm_{2.5}^{1.7}$	11.4	7.97	7.74	9.97
60	$1.27 \pm_{0.35}^{0.19}$	1.46	1.06	1.01	1.31
80	$0.285 \pm_{0.077}^{0.037}$	0.322	0.241	0.222	0.296
100	$(0.873 \pm_{0.230}^{0.109})10^{-1}$	$0.974 \cdot 10^{-1}$	$0.755 \cdot 10^{-1}$	$0.675 \cdot 10^{-1}$	$0.910 \cdot 10^{-1}$
120	$(0.331 \pm_{0.086}^{0.034})10^{-1}$	$0.362 \cdot 10^{-1}$	$0.289 \cdot 10^{-1}$	$0.257 \cdot 10^{-1}$	$0.346 \cdot 10^{-1}$
140	$(0.144 \pm_{0.037}^{0.013})10^{-1}$	$0.155 \cdot 10^{-1}$	$0.127 \cdot 10^{-1}$	$0.112 \cdot 10^{-1}$	$0.150 \cdot 10^{-1}$
160	$(0.691 \pm_{0.173}^{0.047})10^{-2}$	$0.732 \cdot 10^{-2}$	$0.607 \cdot 10^{-2}$	$0.540 \cdot 10^{-2}$	$0.712 \cdot 10^{-2}$
180	$(0.352 \pm_{0.086}^{0.019})10^{-2}$	$0.369 \cdot 10^{-2}$	$0.311 \cdot 10^{-2}$	$0.276 \cdot 10^{-2}$	$0.359 \cdot 10^{-2}$
200	$(0.187 \pm_{0.045}^{0.008})10^{-2}$	$0.195 \cdot 10^{-2}$	$0.166 \cdot 10^{-2}$	$0.147 \cdot 10^{-2}$	$0.189 \cdot 10^{-2}$
$\sqrt{s} = 2.0 \text{ TeV}$					
20	$275 \pm_{71}^{67}$	334	231	219	307
40	$12.2 \pm_{3.1}^{2.3}$	14.4	10.1	9.86	12.7
60	$1.66 \pm_{0.44}^{0.25}$	1.91	1.38	1.32	1.71
80	$0.378 \pm_{0.101}^{0.051}$	0.429	0.319	0.296	0.390
100	$0.117 \pm_{0.031}^{0.015}$	0.131	0.100	0.091	0.121
120	$(0.446 \pm_{0.116}^{0.048})10^{-1}$	$0.491 \cdot 10^{-1}$	$0.387 \cdot 10^{-1}$	$0.346 \cdot 10^{-1}$	$0.464 \cdot 10^{-1}$
140	$(0.196 \pm_{0.051}^{0.018})10^{-1}$	$0.212 \cdot 10^{-1}$	$0.172 \cdot 10^{-1}$	$0.153 \cdot 10^{-1}$	$0.204 \cdot 10^{-1}$
160	$(0.952 \pm_{0.240}^{0.071})10^{-2}$	$0.101 \cdot 10^{-1}$	$0.836 \cdot 10^{-2}$	$0.743 \cdot 10^{-2}$	$0.986 \cdot 10^{-2}$
180	$(0.496 \pm_{0.122}^{0.029})10^{-2}$	$0.522 \cdot 10^{-2}$	$0.438 \cdot 10^{-2}$	$0.388 \cdot 10^{-2}$	$0.509 \cdot 10^{-2}$
200	$(0.271 \pm_{0.065}^{0.013})10^{-2}$	$0.283 \cdot 10^{-2}$	$0.240 \cdot 10^{-2}$	$0.213 \cdot 10^{-2}$	$0.275 \cdot 10^{-2}$

Table 1

Heavy quark (mass m) production cross-sections in $p\bar{p}$ collisions. The cross-sections (in nb) in the second column are expressed by a "central" value, obtained for $\mu = m$ and $\Lambda_5 = 170 \text{ MeV}$, and errors derived from varying μ and Λ_5 in the ranges $m/2 \leq \mu \leq 2m$ and $90 \text{ MeV} \leq \Lambda_5 \leq 250 \text{ GeV}$ (columns 3-6) by adding the corresponding uncertainties in quadrature.

$\sqrt{s} = 0.63 \text{ TeV}$		
$m_t(\text{GeV})$	$\sigma^{\text{EHLQ}}(\text{nb})$	$\sigma^{\text{DFLM}}(\text{nb})$
20	33.3	25.6
40	0.838	0.643
60	$0.802 \cdot 10^{-1}$	$0.669 \cdot 10^{-1}$
80	$0.133 \cdot 10^{-1}$	$0.114 \cdot 10^{-1}$
100	$0.274 \cdot 10^{-2}$	$0.234 \cdot 10^{-2}$
120	$0.606 \cdot 10^{-3}$	$0.517 \cdot 10^{-3}$
$\sqrt{s} = 1.8 \text{ TeV}$		
20	271	228
40	12.4	9.63
60	1.68	1.27
80	0.375	0.285
100	0.113	$0.873 \cdot 10^{-1}$
120	$0.414 \cdot 10^{-1}$	$0.331 \cdot 10^{-1}$
140	$0.175 \cdot 10^{-1}$	$0.144 \cdot 10^{-1}$
160	$0.817 \cdot 10^{-2}$	$0.691 \cdot 10^{-2}$
180	$0.410 \cdot 10^{-2}$	$0.352 \cdot 10^{-1}$
200	$0.216 \cdot 10^{-2}$	$0.187 \cdot 10^{-2}$

Table 2

Comparison of heavy quark (mass m) production cross-sections in $p\bar{p}$ collisions at Sp $\bar{p}$ S and Tevatron energies, computed for $\mu = m$ and $\Lambda_5 = 170 \text{ MeV}$ from the parton densities of Ref. [10] (ELHQ) and of Ref. [11] (DFLM).

		$\sqrt{s} = 41 \text{ GeV}$		pp	
$m_b(\text{GeV})$	σ	$\mu = m/2$ $\Lambda = 170 \text{ MeV}$	$\mu = 2m$ $\Lambda = 170 \text{ MeV}$	$\Lambda = 90 \text{ MeV}$ $\mu = m$	$\Lambda = 250 \text{ MeV}$ $\mu = m$
4.5	$23 \pm_{15}^{21} \text{nb}$	40	12	13	34
5	$9.0 \pm_{5.9}^{8.4}$	16	4.7	4.9	14
		$\sqrt{s} = 62 \text{ GeV}$		pp	
4.5	$142 \pm_{80}^{98} \text{nb}$	231	81	91	182
5	$66 \pm_{38}^{47}$	109	37	41	86
		$\sqrt{s} = 630 \text{ GeV}$		$p\bar{p}$	
4.5	$19 \pm_8^{10} \mu\text{b}$	27	15	12.5	25
5	$12 \pm_4^7$	18	10	9	16
		$\sqrt{s} = 24.5 \text{ GeV}$		$\pi^- N$	
4.5	$7.6 \pm_{3.8}^{4.7} \text{nb}$	12	4.6	5.2	10
5	$3.1 \pm 1.5 \text{nb}$	4.4	1.9	2.2	3.9

Table 3

Same as Table 1, for b production in pp, $p\bar{p}$ and $\pi^- N$ collisions.

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FIGURE CAPTIONS

- Fig. 1: The two-loop QCD coupling $\alpha_s(Q)$ for different value of $\Lambda_5 \equiv \Lambda_{\overline{MS}}^{f=5}$, where f is the number of excited flavours. Below $Q = 2m_b \approx 10$ GeV each curve is split into two branches (corresponding to a different extrapolation across the threshold region) depending on the parameter a defined in Eqs. (7)-(9). The upper branch is for $a = 2$, the lower branch is for $a = 1$. A representative sample of the most recent experimental determinations of α_s is shown.
- Fig. 2: μ -dependence of the lowest order (dashed) and the complete (at order α_s^3) cross-sections (solid) for heavy quark production (with mass $m = 40$ GeV) in $p\bar{p}$ collisions at $\sqrt{S} = 630$ GeV. μ is the scale at which both $\alpha_s(\mu)$ and the parton densities are evaluated.
- Fig. 3: Same as Fig. 2 but $m = 80$ GeV.
- Fig. 4: Same as Fig. 3 but $\sqrt{S} = 1.8$ TeV.
- Fig. 5: Uncertainty about the corrected heavy flavour production cross-section at $O(\alpha_s^3)$ in $p\bar{p}$ collisions at $\sqrt{S} = 630$ GeV associated with a variation of the scale μ in the interval $m/2 < \mu < 2m$ when m is the heavy quark mass.
- Fig. 6: Uncertainty about the corrected heavy flavour production cross-section at $O(\alpha_s^3)$ in $p\bar{p}$ collisions at $\sqrt{S} = 630$ GeV associated with a variation of Λ_5 in the interval $90 \text{ MeV} < \Lambda_5 < 250 \text{ MeV}$.
- Fig. 7: Predicted heavy quark total cross-sections at $\sqrt{S} = 0.63, 1.8, 2$ TeV for $30 \text{ GeV} < m < 100 \text{ GeV}$. The bands include the uncertainties associated with independent variations of μ and Λ_5 in the given ranges, added in quadrature.
- Fig. 8: Same as Fig. 7 for $80 \text{ GeV} < m < 220 \text{ GeV}$.
- Fig. 9: Dependence of the heavy quark (of mass $m = 80$ GeV) total cross-section at the Sp $\bar{p}$ S collider on the choice of the gluon structure function. Softer gluons correspond to smaller cross-sections.
- Fig. 10: A comparison of the predicted cross-sections for heavy quark production at the Sp $\bar{p}$ S collider with the UA1 95% c.l. upper bounds on $t\bar{t}$ and $b'\bar{b}'$ pair production cross-sections. The shaded area is obtained by combining in quadrature the uncertainties from $m/2 < \mu < 2m$ and $90 \text{ MeV} < \Lambda_5 < 250 \text{ MeV}$. One obtains $m_t > 41 \text{ GeV}$, $m_{b'} > 34 \text{ GeV}$. The dashed line corresponds to the lower edge of the cross-section band, if the EHLQ parton densities are used.
- Fig. 11: Central values of total cross-sections for heavy quark production in pp collisions at supercollider energies.
- Fig. 12: Diagrams with gluon exchange in the t -channel.
- Fig. 13: μ -dependence of the lowest order (dashed) and the complete (at order α_s^3) cross-sections (solid) for b production in $p\bar{p}$ collisions at $\sqrt{S} = 62$ GeV. μ is the scale at which both $\alpha_s(\mu)$ and the parton densities are evaluated.
- Fig. 14: Same as Fig. 13 but with $\sqrt{S} = 630$ GeV.

Fig. 15: Same as Fig. 12 but for pp collisions with $\sqrt{S} = 62$ GeV (ISR).

Fig. 16: Central values of the b production cross-sections in pp and $p\bar{p}$ collisions as functions of $\sqrt{S}$.

Fig. 17: μ -dependence of the lowest order (dashed) and the complete (at order α_s^3) cross-sections (solid) for c production in pp collisions at $\sqrt{S} = 30$ GeV. μ is the scale at which both $\alpha_s(\mu)$ and the parton densities are evaluated.

Fig. 18: Total cross-section for charm production in pp collisions. The data compilation is taken from Ref. [34]. The solid (dashed) curves determine the band, obtained for $m_c = 1.5$ GeV (1.2 GeV), by combining the theoretical uncertainties deriving from independent variations of μ and Λ_5 in the given ranges (added in quadrature).

Fig. 1

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Fig. 18