Addendum
CERN-TH.5005/88
MAD/TH/88-10
MIU-THP-88/31

AN IMPROVED FLIPPED $SU(5) \times U(1)$ MODEL FROM FOUR-DIMENSIONAL STRING

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Note the following two features of the fermion mass matrices (5), (6) and (7):

(A) Before the reduction of physical states due to the projection induced by the boundary condition vector α of Eq. (2g), the model has an $SO(10)$ symmetry. The couplings of the states which survive the α projection are still related by $SO(10)$. This implies that the top left-hand side entries in the matrices $M_{1/3}(5)$ and $M_{1/3}(7)$ are equal, and hence that $m_b = m_\tau$ [17] at the superunification scale. Renormalization effects [16] then yield $m_b \sim 5$ to 6 GeV for the physical bottom quark mass.

(B) Although the $F_1 \bar{F}_2 \bar{H}_{12}$ and $F_2 \bar{F}_1 \bar{H}_{12}$ couplings are equal at the superunification scale because of the same $SO(10)$ symmetry, this does not mean that $m_c = m_t$. One reason is that the $F_{1,2}$ fields have as yet undetermined mixings with the $H_{1,2}$ fields. Since the (F_1, H_1) and (F_2, H_2) diagonalizations are independent, the couplings of the physical t and c quarks may be different. Moreover, the renormalization group equations for the couplings of F_1 and F_2 are different because of the large $\lambda_1 F_1 F_1 h_1$ coupling, implying that $m_c \neq m_t$ after renormalization.

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CERN-TH.5005/88
MAD/TH/88-10
MIU-THP-88/31
March 1988

CERN-TH.5005/88
MAD/TH/88-10
MIU-THP-88/031

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ABSTRACT

We discuss a four-dimensional string model whose effective field theory is a supersymmetric flipped $SU(5) \times U(1)$ GUT with the following properties.

- The quark and lepton mass matrices have a hierarchical structure and all Cabibbo-Kobayashi-Maskawa mixing angles can be non-zero.
- There is a natural splitting of Higgs doublets and triplets.
- A novel seesaw mechanism gives light left-handed neutrinos.
- The gauge group is reduced to the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ at a large mass scale close to M_P .

Extensive use is made of non-renormalizable superpotential couplings which may arise from couplings to identifiable massive modes, and are restricted by an R symmetry and the requirements of flatness in some field directions.

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March 1988

The general principles for the construction of string theories directly in four dimensions are now understood [1-3]. They make available a wide variety of effective low-energy field theories with different gauge groups. Presumably, the theory reduces to the supersymmetric Standard Model at energies $\lesssim m_W$, but it is an open question whether there are other gauge symmetry-breaking scales between ~ 1 TeV and M_P . These might be associated with additional U(1) factors and/or the incorporation of the Standard Model into a Grand Unified Theory (GUT). Four-dimensional gauge groups of the form $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^{n-1}$ [4,5] and $SU(3)^3$ [6] have often been discussed in the context of manifold and orbifold compactifications of the ten-dimensional heterotic string, and have many attractions [5,6]. However, in many of these models the problems of baryon decay and neutrino masses either are not addressed or are disastrous [7]. Both these problems may be solved by introducing a large mass scale which is $\gg m_W$, but may be $< M_P$, like the grand unification scale M_G in GUTs. However, most GUTs cannot be obtained from string theory because they need adjoint Higgs representations to break the gauge symmetry, and the effective field theories obtainable from strings do not contain adjoint Higgs fields [8].

There is only one known GUT which does not need adjoint Higgs fields [9], namely flipped $SU(5) \times U(1)$ [10]. This model also has a simple and elegant mechanism for splitting the light doublet Higgses from their heavy colour triplet partners thereby guaranteeing a long proton lifetime, and a seesaw mechanism giving light left-handed neutrino masses [9]. The superpotential for the simplest form of the model is

$$\begin{aligned}
 W = & \lambda_1 F F h + \lambda_2 F \bar{F} \bar{h} + \lambda_3 \bar{F}^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\
 & + \lambda_6 F \bar{H} \phi + \lambda_7 h \bar{h} \phi + \lambda_8 \phi^3
 \end{aligned} \tag{1}$$

where the $F_i \equiv (10, \frac{1}{2})$, $\bar{F}_i \equiv (\bar{5}, -3/2)$ and $\bar{F}_i^c \equiv (1, 5/2)$ are three generations of matter fields, $H \equiv (10, \frac{1}{2})$ and $\bar{H} \equiv (\bar{10}, -\frac{1}{2})$ are Higgses breaking $SU(5) \times U(1) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$, $h \equiv (5, -1)$ and $\bar{h} \equiv (\bar{5}, 1)$ are Higgses breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, and the $\phi_{0,i} \equiv (1, 0)$ are invariant under $SU(5) \times U(1)$. The roles of the different Yukawa couplings are the following: $\lambda_1 \rightarrow m_{d,s,b}$, $\lambda_2 \rightarrow m_{u,c,t}$, $\lambda_3 \rightarrow m_{e,\mu,\tau}$, $\lambda_{4,5} \rightarrow$ doublet/triplet splitting, $\lambda_6 \rightarrow$ seesaw mechanism, $\lambda_7 \rightarrow$ absence of massless pseudoscalar, $\lambda_8 \rightarrow$ absence of unwanted flat direction [9].

In a previous paper [11], we used the fermionic formulation of strings in four dimensions to obtain an $SU(5) \times U(1)$ model along the lines of (1), but with the following differences and defects: a surplus U(1) gauge factor is unbroken down to energies $O(m_W)$, there are no couplings of the λ_2 type to give $m_{u,c,t}$, there are no couplings of the λ_6 type to give large masses to the conjugate neutrinos, and there

are no off-diagonal terms in the $m_{u,d,s}$ matrix which could give Cabibbo-Kobayashi-Maskawa mixing angles [11].

In this paper, we present an improved $SU(5) \times U(1)$ model in which the fermionic boundary conditions are slightly changed^{*}). This model contains masses for all the quarks and leptons, with hierarchical mass matrices that yield small Cabibbo-Kobayashi-Maskawa angles. The previous simple mechanism for natural doublet-triplet splitting is maintained and extended. The left-handed neutrinos are again light, thanks to a seesaw mechanism of a novel kind. Radiative corrections break the gauge group down to the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ at a large mass-scale close to M_P . Thanks to the nature of the doublet-triplet splitting mechanism, which also suppresses dimension-five operators, proton decay is slow enough to be compatible with experiment. Key rôles in the fermion mass matrices are played by non-renormalizable terms which make contributions suppressed by $O(M_G/M_P)^{n \geq 1}$ relative to renormalizable terms [12]. Many of these non-renormalizable terms are mediated by the exchanges of identifiable massive modes of the theory. Their forms are significantly constrained by R symmetry [13,14] and by string requirements that certain field directions in the effective field theory must be completely flat [15].

We now construct our four-dimensional string model. We use the fermionic formulation [3], which is characterized by a set $\mathbb{E}$ of boundary conditions for all the world-sheet fermions. In an (in general complex) diagonal basis, the elements of $\mathbb{E}$ are vectors $\alpha(f) \in (-1,1]$: the fermions f pick up phases $f \rightarrow -e^{i\pi\alpha(f)} f$ when parallel transported around the string. The set $\mathbb{E}$ forms a group under addition (mod 2), and is generated in our case by the following basis with eight elements $B = \{1, S, b_1, \dots, b_4, b_5', \alpha\}$ where (see also Ref. [11]):

$$S \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{1, \dots, 6}}, 0, \dots, 0; 0, \dots, 0) \quad (2a)$$

$$b_1 \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{1,2}, y^3, \bar{y}^3, \dots, y^6, \bar{y}^6}, 0, \dots, 0; \underbrace{1, \dots, 1}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^1}, 0, \dots, 0) \quad (2b)$$

$$b_2 \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{3,4}, y^{1,2}, \bar{y}^{1,2}, \omega^{5,6}, \bar{\omega}^{5,6}}, 0, \dots, 0; \underbrace{1, \dots, 1}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^2}, 0, \dots, 0) \quad (2c)$$

^{*}) This model was mentioned briefly in paragraph (g) of Ref. [11].

$$b_3 \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{5,6}, \omega^1, \bar{\omega}^1, \dots, \omega^4, \bar{\omega}^4}, 0, \dots, 0; \underbrace{1, \dots, 1}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^3}, 0, \dots, 0) \quad (2d)$$

$$b_4 \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{1,2}, y^{3,6}, \bar{y}^{3,6}, \omega^{4,5}, \bar{\omega}^{4,5}}, 0, \dots, 0; \underbrace{1, \dots, 1}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^1}, 0, \dots, 0) \quad (2e)$$

$$b_5' \equiv (\underbrace{1, \dots, 1}_{\psi^\mu, \chi^{3,4}, \omega^{1,5}, \bar{\omega}^{1,5}, y^{2,6}, \bar{y}^{2,6}}, 0, \dots, 0; \underbrace{1, \dots, 1}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^2}, 0, \dots, 0) \quad (2f)$$

$$\alpha \equiv (\underbrace{1, \dots, 1}_{\chi^{1, \dots, 4}, y^1, \bar{y}^1, y^3, \bar{y}^3, y^4, \bar{y}^4, \omega^2, \bar{\omega}^2}, 0, \dots, 0; \underbrace{\frac{1}{2}, \dots, \frac{1}{2}}_{\bar{\psi}^{1, \dots, 5}, \bar{\eta}^{1,2,3}}, \underline{A}) \quad (2g)$$

As previously [11], the left-movers (denoted by $\psi^\mu, \chi^i, y^i, \omega^i$: $i = 1, \dots, 6$) and 12 right-movers ($\bar{y}^i, \bar{\omega}^i$) are real fermions, whilst the remaining right-movers ($\bar{\psi}^1, \dots, \bar{\psi}^5, \bar{\eta}^{1,2,3}$) are complex fermions, separated from the real ones by the semi-colons in (2). The basis vectors $\{S, b_1, b_2\}$ guarantee $N = 1$ space-time supersymmetry [11]. Combined with just b_3 , they would yield an observable gauge group $SO(10) \times SO(6)^3$ and a hidden sector gauge group E_8 . The basis vectors $\{b_4, b_5', \alpha\}$ break $SO(10)$ to $SU(5) \times U(1)$ and $SO(6)^3$ to $U(1)^3$ [11]. An appropriate choice of $\underline{A}$ in the basis vector α above guarantees that the only massless states transforming non-trivially under the observable gauge group are the space-time fermions from the sectors $S, b_1, b_2, b_3, b_4, b_5', S+b_4+b_5'$ and their bosonic superpartners. The particular form of $\underline{A}$ also fixes the final form of the hidden gauge group. There are no massless states transforming non-trivially under both the hidden and observable gauge groups. The only difference in (2) from our previous [11] set of boundary conditions is in b_5' (2f), where $\bar{\psi}^1, \dots, \bar{\psi}^4$ are taken as antiperiodic, instead of periodic as before.

It is simple to obtain from (2) the spectrum of massless chiral fields, which is similar to that in Ref. [11] but with some additional Higgs fields. The sectors b_i ($i=1,2,3$) contain three generations $M_i = F_i + \bar{F}_i + \lambda_i^C$ of quarks and leptons with charges $(\frac{1}{2}, 0, 0)$, $(0, \frac{1}{2}, 0)$ and $(0, 0, -\frac{1}{2})$ respectively under the three additional $U(1)$ gauge factors. The sector b_4 gives rise to GUT Higgses $H_1 \equiv (\underline{10}, \frac{1}{2})_{(\frac{1}{2}, 0, 0)}$ and $\bar{H}_1 \equiv (\underline{\bar{10}}, -\frac{1}{2})_{(-\frac{1}{2}, 0, 0)}$. In a change from the previous model, the modified sector b_5' gives rise to more GUT Higgses $H_2 \equiv (\underline{10}, \frac{1}{2})_{(0, \frac{1}{2}, 0)}$ and $\bar{H}_2 \equiv (\underline{\bar{10}}, -\frac{1}{2})_{(0, -\frac{1}{2}, 0)}$. As before, there are three pairs of electroweak Higgses: $h_1 \equiv (\underline{5}, -1)_{(-1, 0, 0)}$, $h_2 \equiv (\underline{5}, -1)_{(0, -1, 0)}$ and $h_3 \equiv (\underline{5}, -1)_{(0, 0, 1)}$ as well as the corresponding conjugate

representations $\bar{h}_{1,2,3}$. In another change from the previous model, the modified sector $S+b_4+b_5$ gives an additional pair of electroweak Higgses: $h_{12} \equiv (5, -1)_{(\frac{1}{2}, \frac{1}{2}, 0)}$ and $\bar{h}_{12} \equiv (5, 1)_{(-\frac{1}{2}, -\frac{1}{2}, 0)}$. Finally, as before there are three pairs of $SU(5) \times U(1)$ singlet fields: $\phi_{23} \equiv (1, 0)_{(0, 1, 1)}$, $\phi_{13} \equiv (1, 0)_{(1, 0, 1)}$ and $\phi_{12} \equiv (1, 0)_{(1, -1, 0)}$ as well as their conjugate representations $\bar{\phi}_{23}$, $\bar{\phi}_{13}$ and $\bar{\phi}_{12}$.

We analyze the phenomenology of this model in several steps, starting with the renormalizable trilinear terms in the superpotential which are allowed by $SU(5) \times U(1)^4$ symmetry^{*}:

$$\begin{aligned}
 W_R = & \lambda_1 (F_1 F_1 h_1, F_2 F_2 h_2, F_3 F_3 h_3) + \lambda_2 (\underline{F_1 \bar{f}_2 \bar{h}_{12}}, \underline{F_2 \bar{f}_1 \bar{h}_{12}}) \\
 & + \lambda_3 (\bar{f}_1 \ell_1 h_1, \bar{f}_2 \ell_2 h_2, \bar{f}_3 \ell_3 h_3) + \lambda_4 (H_1 H_1 h_1, \underline{H_2 H_2 h_2}) \\
 & + \lambda_5 (\bar{H}_1 \bar{H}_1 \bar{h}_1, \underline{\bar{H}_2 \bar{H}_2 \bar{h}_2}) \\
 & + \lambda_7 (h_1 \bar{h}_2 \phi_{12}, h_1 \bar{h}_3 \phi_{13}, h_2 \bar{h}_3 \phi_{23}, \bar{h}_1 h_2 \bar{\phi}_{12}, \bar{h}_1 h_3 \bar{\phi}_{13}, \bar{h}_2 h_3 \bar{\phi}_{23}) \\
 & + \lambda_8 (\phi_{12} \phi_{23} \bar{\phi}_{13}, \bar{\phi}_{12} \bar{\phi}_{23} \phi_{13})
 \end{aligned} \tag{3}$$

where we have classified the Yukawa couplings using the notation of Eq. (2), and the new couplings of the additional h_{12} , $\bar{h}_{12}$, H_2 and $\bar{H}_2$ fields not included in Ref. [11] are underlined. The other a priori possible $SU(5) \times U(1)^4$ -invariant superpotential terms of the forms $F_1 H_1 h_1$ and $H_1 \bar{f}_2 \bar{h}_{12}$, $H_2 \bar{f}_1 \bar{h}_{12}$ are forbidden by a generalized R-parity $(\bar{H}_{1,2} \leftrightarrow (\bar{H}_{1,2})^{**})$. Such an R-symmetry is a necessary feature of $N = 1$, $d = 4$ supergravity theories with a $U(1)$ factor [13], such as those in this model [11], and is an automatic symmetry of the massless modes coming from an underlying string theory [14].

We analyze the first stage of gauge symmetry breaking by looking for flat directions of the potential derived from (3). We first note that as before the λ_8 terms have three D- and F-flat directions: $\langle \phi_{23} \rangle = \langle \bar{\phi}_{23} \rangle \equiv V_{23} \neq 0$ or $\langle \phi_{12} \rangle = \langle \bar{\phi}_{12} \rangle \equiv V_{12} \neq 0$ or $\langle \phi_{13} \rangle = \langle \bar{\phi}_{13} \rangle \equiv V_{13} \neq 0$. The symmetry of the model between $i = 1, 2$ means that only two of these choices are inequivalent: V_{23} (or V_{13}) $\neq 0$ and $V_{12} \neq 0$. Turning to the λ_7 terms in (3), we see that if $V_{23} \neq 0$, then the $(\bar{h}_{2,3})$ are massive, whereas if $V_{12} \neq 0$ then the $(\bar{h}_{1,2})$ are massive. In the latter

^{*}) As we discuss later, one combination of the $U(1)$'s is anomalous, but the other $U(1)$'s alone suffice to fix the superpotential. Moreover, the anomalous $U(1)$ remains as a global symmetry of the theory.

^{**}) Even if this R-parity were absent, it would be possible to find a basis for the fields in which there is manifestly no lepton-Higgs mixing, and the light spectrum contains the three desired charge +1/3 antiquark triplets.

case, none of the $\lambda_{4,5}$ terms in (3) can realize the elegant doublet-triplet mass splitting mechanism, so we discard the possibility that $V_{12} \neq 0$ and focus on the case $V_{23} \neq 0$. The model (3) has other flat directions for the GUT Higgses: $\langle H_1 \rangle = \langle \bar{H}_1 \rangle \equiv V_1 \neq 0$ and $\langle H_2 \rangle = \langle \bar{H}_2 \rangle = V_2 \neq 0^{**}$. Values of $V_{1,2,23}$ which are close to, but significantly smaller than M_p , can be generated by a first stage of radiative corrections [16]. We now observe that, in contrast to the previous version of this string model, now $SU(5) \times U(1) \times U(1)_1 \times U(1)_2 \times U(1)_3 \rightarrow SU(3)_C \times SU(3)_L \times U(1)_Y$ at this first stage of GUT symmetry breaking. This is because one combination of the generation-dependent $U(1)$'s is anomalous: $U(1)_1 + U(1)_2 - U(1)_3^{**}$, one combination is broken by V_{23} : $U(1)_2 + U(1)_3$, one combination is broken by V_1 : $U(1) + U(1)_1$ and one combination is broken by V_2 : $U(1) + U(1)_2^{***}$.

Despite the fact that $V_{23} \neq 0$ gives large masses to the $h_{2,3}^{(-)}$ in this new model electroweak Higgses are available to give masses both to the charge $-1/3$ quarks and charged leptons as before: $\langle h_1 \rangle \equiv v_1 \neq 0$, and also to the charge $+2/3$ quarks: $\langle \bar{h}_{12} \rangle \equiv \bar{v}_{12} \neq 0$. These v.e.v.'s are expected to be generated by a second stage of radiative corrections. However, it is apparent from the form of the superpotential (3) that only one charge $-1/3$ quark, one charged lepton and two charge $2/3$ quarks can acquire masses at this stage. Any resemblance to the physical fact that $m_{t,b,c,\tau} \gg m_{s,\mu,d,u,e}$ is surely coincidental.

To analyze the model further, we study possible non-renormalizable superpotential terms in the fermion mass matrices [12] and elsewhere. These are generic features of string theory, and many can be identified as originating from the exchanges of specific massive modes. They are restricted not only by $SU(5) \times U(1)^4$ invariance, but also by R-invariance [13,14] and by string theory arguments [15] which identify flat directions at the string tree level. The criterion of Ref. [15], although not directly applicable to our model, suggests that the above mentioned flat directions of the trilinear superpotential remain flat in the presence of non-renormalizable terms. The lowest order non-trivial contributions to the fermion mass matrices come from

*) Note, however, that there is no λ_6 term in (3) which could use these large v.e.v.'s and generate large masses for the conjugate neutrinos as required for the seesaw mechanism [9].

**) We note that the form of this $U(1)$ is unique within our framework.

***) As discussed later, the alignment of $\langle H_{1,2} \rangle$ in the same v_H^c direction is guaranteed by non-renormalizable terms in the superpotential.

$$W_1 = \lambda_{11} (F_1 F_2 h_1) (H_1 \bar{H}_2) + \lambda_{12} (F_2 F_2 h_1) (H_1 \bar{H}_2)^2 + \lambda_{13} (F_3 F_3 h_1) (H_1 \bar{H}_2)^2 \phi_{23}^{(4_1)}$$

$$W_2 = \lambda_{21} (F_1 \bar{F}_1 \bar{h}_{12}) (\bar{H}_1 H_2) + \lambda_{22} (F_2 \bar{F}_2 \bar{h}_{12}) (\bar{H}_2 H_1) + \lambda_{23} (F_3 \bar{F}_3 \bar{h}_{12}) (H_1 \bar{H}_2) \phi_{23}^{(4_2)}$$

$$W_3 = \lambda_{31} (\bar{F}_1 l_2^c h_1) (H_1 \bar{H}_2) + \lambda_{32} (\bar{F}_2 l_1^c h_1) (H_1 \bar{H}_2) \\ + \lambda_{33} (\bar{F}_2 l_2^c h_1) (H_1 \bar{H}_2)^2 + \lambda_{34} (\bar{F}_3 l_3^c h_1) (H_1 \bar{H}_2)^2 \phi_{23} \quad (4_3)$$

Since we expect V_1 , V_2 and V_{23} to be large, but significantly smaller than the string unification scale (taken here as the unit of the mass scale), the terms (4) make contributions to the mass matrices which are hierarchically smaller than the renormalizable $\lambda_{1,2,3}$ terms in (3). Thus we can hope to obtain a realistic hierarchical mass matrix with small Cabibbo-Kobayashi-Maskawa mixing angles [12]. The charge -1/3 quark mass matrix is

$$M_{-1/3} = U_1 \begin{pmatrix} \lambda_1 & \lambda_{11} V_1 V_2 & 0 \\ \lambda_{11} V_1 V_2 & \lambda_{12} V_1^2 V_2^2 & 0 \\ 0 & 0 & \lambda_{13} V_1^2 V_2^2 V_{23} \end{pmatrix} \quad (5)$$

and the charge +2/3 quark mass matrix is

$$M_{+2/3} = \bar{U}_{12} \begin{pmatrix} \lambda_{21} V_1 V_2 & \lambda_2 & 0 \\ \lambda_2 & \lambda_{22} V_1 V_2 & 0 \\ 0 & 0 & \lambda_{23} V_1 V_2 V_{23} \end{pmatrix} \quad (6)$$

At this stage there is still no mixing between generations 1 and 2, to be identified with (t,b,c,s), and generation 3, to be identified with (u,d). This is because of a discrete symmetry P_3 : $M_3 \leftrightarrow -M_3$ which is respected by all the interaction terms in the low-energy effective theory. We will discuss later how this defect may be remedied. The charged lepton mass matrix is

$$M_{-1} = U_1 \begin{pmatrix} \lambda_3 & \lambda_{31} V_1 V_2 & 0 \\ \lambda_{32} V_1 V_2 & \lambda_{33} V_1^2 V_2^2 & 0 \\ 0 & 0 & \lambda_{34} V_1^2 V_2^2 V_{23} \end{pmatrix} \quad (7)$$

which has a very similar structure to the charge $-1/3$ quark mass matrix (5). This gives us hope that when the magnitudes of the λ_i are calculated, the successful SU(5) GUT prediction $m_b = m_\tau$ [17] will be re-obtained. However, the terms giving rise to m_d and m_e are sufficiently different that one need not expect them to be equal at the GUT scale.

Fermion mass matrices with artfully distributed zeros are familiar from previous attacks on the flavour problem. However, there the zeros were imposed by global symmetries, whereas here they are imposed by local symmetries. This is because in general string theory abhors global symmetries in four dimensions [18], and promotes global world-sheet symmetries to local space-time symmetries.

Next we discuss the neutrino mass matrix in this model. Although there are no λ_6 terms in the renormalizable superpotential (3), the following non-renormalizable terms provide large masses for the conjugate neutrinos and a Dirac mass for the ν_3 :

$$W_6 = \lambda_{611} (F_1 \bar{H}_1)^2 + \lambda_{612} (F_1 \bar{H}_1) (F_2 \bar{H}_2) + \lambda_{621} (F_1 \bar{H}_2) (F_2 \bar{H}_1) \\ + \lambda_{622} (F_2 \bar{H}_2)^2 + \lambda_{623} (F_3 \bar{F}_3 \bar{H}_{12}) (H_1 \bar{H}_2) \phi_{23} + \lambda_{633} (F_3 \bar{H}_2)^2 \phi_{23} \quad (8)$$

where the parentheses denote SU(5) contractions. Note that there is a massive state $\chi \equiv (1,0)_{(-\frac{1}{2}, \frac{1}{2}, 0)}$ which generates a coupling of the form λ_{621} via the coupling $F_1 \bar{H}_2 \chi$, and that a λ_{633} coupling is generated by a massive state $\phi \equiv (1,0)_{(0, \frac{1}{2}, \frac{1}{2})}$ via the coupling $F_3 \bar{H}_2 \phi$. The terms (8) yield a novel form of seesaw neutrino mass matrix for generations 1 and 2^{*)}:

^{*)} There are also terms of the form $(F \bar{H} \phi)(\bar{H} H)^2$ and of higher order which mix the $\nu_{1,2}^c$ with the $\phi_{12,13}$, but since $m_{\phi_{12,13}} \sim \lambda V_{23}$ these only give small mixing angles and negligible contributions to $m_{\nu_{1,2}}$.

$$(\nu_1, \nu_1^c, \nu_2^c, \nu_2) \begin{pmatrix} 0 & 0 & \lambda_2 \bar{V}_{12} & 0 \\ 0 & \lambda_{611} V_2^2 & (\lambda_{612} + \lambda_{621}) V_1 V_2 & \lambda_2 \bar{V}_{12} \\ \lambda_2 \bar{V}_{12} & (\lambda_{612} + \lambda_{621}) V_1 V_2 & \lambda_{622} V_1^2 & 0 \\ 0 & \lambda_2 \bar{V}_{12} & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_1^c \\ \nu_2^c \\ \nu_2 \end{pmatrix} \quad (9)$$

which clearly gives two eigenstates of mass $\sim \lambda V_{1,2}^2$ and two eigenstates of mass $\sim \lambda_{612}^2 / V_{1,2}^2$ corresponding to $m_{\nu_{1,2}} \sim m_{\nu_{\mu,\tau}} \sim \text{eV}$. On the other hand, the generation three neutrino mass is of the traditional seesaw type^{*}):

$$(\nu_3, \nu_3^c) \begin{pmatrix} 0 & \lambda_{23} \bar{V}_{12} V_1 V_2 V_{23} \\ \lambda_{23} \bar{V}_{12} V_1 V_2 V_{23} & \lambda_{633} V_2^2 V_{23} \end{pmatrix} \begin{pmatrix} \nu_3 \\ \nu_3^c \end{pmatrix} \quad (10)$$

with eigenstates of mass $\sim \lambda V_2^2 V_{23}$ and $\sim \lambda_{633}^2 V_1^2 V_{23}$, corresponding to $m_{\nu_3} \sim m_{\nu_e} \sim 10^{-8} \text{ eV}$.

Next we return to the problem of doublet-triplet splitting, incorporating the following non-renormalizable terms:

$$W_4 = \lambda_{4122} (H_1 H_2 h_2) (\bar{H}_1 H_2) + \lambda_{4123} (H_1 H_2 h_3) (\bar{H}_1 H_2 \bar{\phi}_{23}) \quad (11)$$

and W_5 defined analogously in terms of the conjugate states^{**}). We also make use of non-renormalizable terms induced by the following trilinear couplings to massive states:

^{*}) Note that the discrete symmetry P_3 discussed earlier forbids any connection between the $\nu_{1,2}^{(c)}$ and $\nu_3^{(c)}$ mass matrices.

^{**}) These terms and another $\propto (H_{1\alpha\beta} \bar{H}_1^\beta \gamma_{H_2\gamma\delta} \bar{H}_2^{\delta\alpha} - 2(H_1 \bar{H}_2)(H_2 \bar{H}_1))$ guarantee the alignment of $\langle H_{1,2} \rangle$ in the same ν_H^c direction.

$$W_H = \lambda_{H_{12}} H_1 H_{12} h_2 + \lambda_{H_{13}} H_1 H_{13} h_3 + \lambda_{\bar{H}_{12}} \bar{H}_1 \bar{H}_{12} \bar{h}_2 + \lambda_{\bar{H}_{13}} \bar{H}_1 \bar{H}_{13} \bar{h}_3 + \lambda_{H_5} H_1 H_5 h_{12} + \lambda_{\bar{H}_5} \bar{H}_1 \bar{H}_5 \bar{h}_{12} \quad (12)$$

where H_{12} , H_{13} and H_5 are specific massive states with the quantum numbers $(\underline{10}, \frac{1}{2})_{(-\frac{1}{2}, 1, 0)}$, $(\underline{10}, \frac{1}{2})_{(-\frac{1}{2}, 0, -1)}$ and $(\underline{10}, \frac{1}{2})_{(-1, -\frac{1}{2}, 0)}$ respectively. The exchange of $H_5/\bar{H}_5$ gives an effective quadrilinear interaction

$$W_q = \lambda_{H_5} \lambda_{\bar{H}_5} \epsilon^{\alpha\beta\gamma\delta\epsilon} H_{1\alpha\beta} h_{12\gamma} \epsilon_{\kappa'\beta'\delta'\epsilon'} \bar{H}_{1\kappa'\beta'} \bar{h}_{12\delta'} \quad (13)$$

where we have indicated explicitly the SU(5) indices, so as to make clear that when $\langle \bar{H}_1 \rangle = V_1$, only the triplet components of h_{12} and $\bar{h}_{12}$ are coupled by a large mass term of magnitude $\lambda_H \lambda_{\bar{H}} V_1^2$. The $H_{12}/\bar{H}_{12}$ and $H_{13}/\bar{H}_{13}$ give analogous mass terms coupling h_2 to $\bar{h}_2$ and h_3 to $\bar{h}_3$ respectively. Using (3), (11) and (13) we arrive at the following triplet mass matrix^{*}:

$$(h_1, h_2, h_3, h_{12}, \bar{H}_1, \bar{H}_2) \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \lambda_4 V_1 & \cdot \\ \cdot & \lambda_{H_{12}} \lambda_{\bar{H}_{12}} V_1^2 & \lambda_7 V_{23} & \cdot & \lambda_{4122} V_2^2 V_1 & \lambda_4 V_2 \\ \cdot & \lambda_7 V_{23} & \lambda_{H_{13}} \lambda_{\bar{H}_{13}} V_1^2 & \cdot & \lambda_{4123} V_2^2 V_1 & \lambda_{4123} V_1 V_2 V_{23} \\ \cdot & \cdot & \cdot & \lambda_H \lambda_{\bar{H}} V_1^2 & \cdot & \cdot \\ \lambda_5 V_1 & \lambda_{5122} V_1^2 V_2 & \lambda_{5123} V_2^2 V_1 V_{23} & \cdot & \cdot & \cdot \\ \cdot & \lambda_5 V_2 & \lambda_{5123} V_1 V_2 V_{23} & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \bar{h}_3 \\ \bar{h}_{12} \\ H_1 \\ H_2 \end{pmatrix} \quad (14)$$

It is clear from the form of this matrix that all the unwanted Higgs triplets acquire large masses $\gg m_W$ as required.

Penultimately, we address the question of mixing among the light doublet components of the $(\bar{h}_{1,12})$ multiplets. In most supersymmetric models, similar mixing is needed to avoid an electroweak axion and/or to ensure non-zero v.e.v.'s for all Higgs doublets and hence masses for all quarks and leptons. In our model, charge $-1/3$ quarks and charged leptons acquire masses from h_1 , whilst charge $+2/3$ quarks acquire masses from $\bar{h}_{12}$. However, the gauge symmetries of this model preclude an effective direct $h_1 \bar{h}_{12}$ coupling from either renormalizable or non-renormalizable

^{*}) Some of the dots in the matrix (14) may represent other, smaller non-renormalizable terms which are not needed for the present discussion.

interactions. This is not necessarily a problem, since v.e.v.'s for both h_1 and $\bar{h}_{1,2}$ could be generated independently by radiative corrections. If there were superpotential couplings of the form $(\bar{H}H)^m \phi_{23}^n h_{1,12} \bar{h}_{1,12}$ where $m+n \geq 4$, they would generate mixing of the desired magnitude $\sim m_W$. However, such terms would destroy flatness in a field direction which is known from the arguments of Ref. [15] to be flat at the string tree level. One can only generate $h_{1,12} \bar{h}_{1,12}$ mixing in the presence of supersymmetry breaking, and scenarios have been suggested in conventional supergravity for generating a supersymmetry-breaking $h\bar{h}$ mixing term with a coefficient $\sim m_W$ from a suitable hidden sector [19]. In the absence of a clear understanding of supersymmetry breaking in the type of string model discussed here, we cannot tell whether such a mechanism may be applicable.

Finally, we discuss the need for terms in the quark mass matrices (5,6) which mix generations 1 and 2 with generation 3, so as to obtain all the required Cabibbo-Kobayashi-Maskawa mixing angles. A mechanism for generating these terms, which are forbidden a priori by the discrete symmetry $P_3: M_3 \rightarrow -M_3$ mentioned earlier, has been suggested in Ref. [20]. In our model, this mechanism would use a v.e.v. for the sneutrino component of $\bar{f}_3$ to mix λ_3^C with h_1 , leading to terms in the charge +2/3 quark mass matrix $(M_{+2/3})_{31,32} \sim \lambda^2/32\pi^2 \tilde{m}$, where $\tilde{m} \sim 100$ GeV to 1 TeV is the scale of supersymmetry breaking in the observable sector. The required magnitude of $\langle \nu_3 \rangle$ can be reconciled with upper limits on $\Delta L \neq 0$ processes [20], but it is not clear whether the conventional radiative correction mechanism can generate $\langle \nu_3 \rangle \neq 0$.

Although certain points remain unclear, we believe the model outlined here represents a significant advance on previous GUT and superstring model-building [4-7,11]. The fact that M_G is close to, but significantly below, M_P has been exploited to give a hierarchical structure for fermion masses and mixing angles [12]. Previous $SU(5) \times U(1)$ successes [9] such as natural doublet-triplet splitting and seesaw neutrino masses have been retained and extended. The gauge group is reduced to the Standard Model at a large mass-scale close to M_P . Next steps in the development of this model - possibly accompanied by further adjustment of the underlying world-sheet fermion boundary conditions (2) if needed - include calculating the superpotential couplings, understanding supersymmetry breaking and exploring the breaking of gauge symmetry by radiative corrections [16]. We will return to these issues in future publications.

ACKNOWLEDGEMENT

The work of D.V. Nanopoulos was supported in part by Department of Energy grant DE-AC02-76ER0081 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

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