QUARK-GLUON PLASMA AND MIXED PHASE
IN AN AGGREGATION-PERCOLATION MODEL

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ABSTRACT

The mixed phase occurring in the formation, and hadronization, of the quark-gluon plasma is studied within the framework of percolation approaches. Starting from a gas of nucleon bags, and, at finite temperature, of pion bags, the transition from a hadronic gas to a quark-gluon plasma is calculated using a restructuring aggregation model. The key parameter of this approach is the quark confinement radius of the nucleon. In the hadronization process we start from the MIT bag model description of the quark-gluon plasma and study the mixed phase using a three-dimensional site-bond percolation model on a cubic lattice. The key parameter of this approach is the bag constant B. These two complementary descriptions of the mixed phase are discussed and compared together.

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1. Introduction

1.1 Physical conditions

The formation of a deconfined phase of quarks and gluons in ultra-relativistic heavy ion collisions has been a subject of intense research during these last few years, both theoretically and experimentally. From a theoretical point of view, a direct manifestation of the otherwise confined constituents of the nucleons building up nuclei is certainly a very exciting subject. The large density of matter in heavy nuclei, together with the high bombarding energies which are available now for heavy projectiles, may be sufficient to create the conditions for a total or partial deconfinement of quarks and gluons in heavy ion collisions. This has been already predicted a long time ago and several experiments are now hunting for this new phenomenon. From an experimental point of view, the observation of this phase is a very difficult task, and one needs specific signatures of the formation of the plasma.

Monte Carlo simulations based on lattice gauge theory have clearly predicted that a quark-gluon plasma should be produced at high temperature and/or high baryon density through a phase transition from hadronic matter to quark-gluonic matter. More phenomenological approaches are useful to study particular points of this phase transition. In particular, it is interesting to know to which extent geometrical aspects play some role in the problem of quark deconfinement. This will be the aim of this paper where we concentrate mainly on the description of the transition within a percolation-aggregation approach.

In order to understand more precisely what is really going on in ultrarelativistic heavy ion collisions, several aspects of the problem should be cleared up. There is now little doubt that a phase transition should occur at some critical density and temperature [1]. We shall use the predictions based on the simplest bag model (MIT bag model) [2], in order to illustrate this point. However, in order to be sure that this transition can be reached experimentally, one has first to understand how the flow of

matter behaves under these extreme conditions (200GeV per nucleon for ^{16}O and ^{32}S on heavy nuclei for the first experiments at CERN) [3]). This leads to a first class of studies, connected with the (relativistic) hydrodynamical evolution of the flow of matter to model the change of the density and temperature of the medium (plasma). One of the key issues of these studies is to know the experimental conditions under which one can deconfine hadronic matter.

A second class of studies is related to the signatures of the quark-gluon plasma formation. Because of the total deconfinement of quarks and gluons in the plasma, it is expected that the physical properties of the system, after hadronization, will be changed compared to a usual gas of hot dense nucleons. This is how, for instance, the abundance of charmed particles is changed [4].

In this paper we shall concentrate on a third class of studies which deals with what is called the intermediate or "mixed" phase which appears before the formation of the quark-gluon plasma, or during its hadronization. In this work, we propose to describe the properties of the mixed phase in the framework of percolation theory. Depending on the physical conditions of the system, its temperature and baryon density, a total deconfinement of quarks and gluons may be impossible. However, a partial deconfinement may occur over a limited region of space only, giving rise to clusters of quarks, antiquarks and gluons surrounded by a gas of dense nucleons and pions. We shall investigate this particular situation in a percolation-aggregation model at zero and finite temperature. In this latter case, the excitation of real pions contributes to the formation of a percolation cluster at a density much lower than at $T=0$. At the critical temperature T_c and zero baryon density, the percolation cluster is composed of pions only (if one neglects the excitation of heavier resonances). Within the percolation model one can study several quantities like the cluster distribution, or the increase of the mean-squared radius of the bags in the medium. Such model may also be used to study the inner core of neutron stars.

A mixed phase also occurs in the hadronization of the quark-gluon plasma when the energy density, or baryon density and temperature, are not large enough to sustain a stable plasma. In that case, one starts having

bubbles of non-perturbative vacuum (the vacuum of nuclear physics) popping up from the plasma. When the physical conditions are sufficiently far away from the stability curve, these bubbles may isolate islands of quark-gluon plasma, creating thus clusters of quarks and gluons. When the temperature and density are low enough, i.e. below what we shall call the percolation temperature and density, only small clusters survive, giving rise to ordinary hadrons (mainly nucleons and pions). We shall study this mechanism using a 3-dimensional site-bond percolation model on a cubic lattice. In this approach the sites are occupied by the quarks and antiquarks, with a probability which depends upon the baryon density and temperature. Similarly, the bonds between occupied sites are function of the temperature and the energy density of the system.

The paper is organized as follows : in section 2 we study a restructuring aggregation model of a gas of nucleons and pions in order to evaluate the conditions under which one can form a plasma of quarks and gluons. In section 3 a site-bond percolation model is applied to study the mixed phase occurring in the hadronization of the quark-gluon plasma. Before going into a more detailed analysis of the mixed phase, let us briefly recall here the basic equations for the stability of the quark-gluon plasma as described in the simplest MIT bag model.

1.2 Quark-gluon plasma in the MIT bag model

In the simplest MIT bag model [2] at zero temperature, without any corrections from gluon exchange nor from zero-point motion, the total energy of the bag is equal to the kinetic energy of the free quarks inside the cavity plus the energy necessary to create the cavity in the non-perturbative vacuum. The energy density to create the bag is a fundamental non-perturbative property of QCD, which one calls the bag constant B . By minimizing the total energy of the bag with respect to its radius, one can eliminate the bag radius R_b , and get for the energy density of the bag [1,2] :

$$\epsilon = 4 B \quad (1)$$

In the above value of ϵ , $3B$ are coming from the kinetic energy of the quarks and the amount B left is just, by definition, the energy density

necessary to create the bag. Condition (1) is also equivalent to say that the pressure from the quarks inside the cavity is exactly compensated by the pressure from the non-perturbative vacuum outside the bag, equal to B .

In the deconfined phase at high temperature, T , and baryon density, ρ , we shall assimilate the plasma to a free gas of quarks and gluons (gauge particles). We consider here only the light quarks u and d , and take their mass equal to zero. The energy density, $\bar{\epsilon}$, internal pressure, P and entropy density, s , of the constituents of the plasma are thus [1] :

$$\bar{\epsilon} = \gamma_g \epsilon_g + \gamma_q (\epsilon_q + \epsilon_{\bar{q}}) \quad (2a)$$

$$P = \frac{1}{3} \bar{\epsilon} \quad (2b)$$

$$s = \frac{1}{3} \frac{\partial \bar{\epsilon}}{\partial T} \quad (2c)$$

The total energy density of the system, ϵ , is the sum of the energy density of the constituents, $\bar{\epsilon}$, plus the energy density necessary to create the cavity, i.e., in the MIT bag model :

$$\epsilon = \bar{\epsilon} + B \quad (3)$$

For simplicity, we shall not consider, in the following, the correction from quark-gluon interaction[†]. Assimilating the gluons to an ideal relativistic gas of massless bosons at temperature $T=1/\beta$, one can write its energy density as :

$$\gamma_g \epsilon_g = \frac{16\pi^2}{30} T^4 \quad (4)$$

[†] Inclusion of these corrections does not change any of the qualitative aspects of this description. It only shifts the stability line of the plasma to higher temperature and density.

where γ_g is the degeneracy of the gluons ($\gamma_g = 8 \text{ colors} \times 2 \text{ helicities}$). Similarly, the density of quarks and antiquarks is given by [1]:

$$\gamma_q (\varepsilon_q + \varepsilon_q^-) = \frac{7\pi^2}{10} T^4 + 3 \mu^2 T^2 + \frac{3}{2\pi^2} \mu^4 \quad (5)$$

where γ_q is the degeneracy of the quarks ($\gamma_q = 3 \text{ colors} \times 2 \text{ spins} \times 2 \text{ flavors}$), while μ is the chemical potential of the quarks (it is $-\mu$ for the antiquarks). It is calculated by demanding that the total baryonic density ρ is fixed. Thus, one has :

$$\rho = 2 \mu \frac{T^2}{3} + 2 \frac{\mu^3}{3\pi^2} \equiv \frac{1}{3} (n_q - n_q^-) \quad (6)$$

where n_q (n_q^-) is the density of quarks (antiquarks) given by :

$$n_q = 12 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(p-\mu)} + 1} \quad (7)$$

and similarly for n_q^- with μ changed into $-\mu$. In the expression (4-7) we assumed that the quarks are massless. In this simple description, the domain of stability of the plasma corresponds to the region, in the $(T-\rho)$ plane, for which the internal pressure from the quarks and gluons, given by (2.b), can balance the external vacuum pressure determined by the bag constant B , as introduced in the MIT bag model. On the critical line $P = B$ (which we shall call in the following the stability line of the plasma). One thus has :

$$B = \pi^2 T_c^4 \left[\frac{37}{90} + \left(\frac{\mu_c}{\pi T_c} \right)^2 + \frac{1}{2} \left(\frac{\mu_c}{\pi T_c} \right)^4 \right] \quad (8)$$

with μ obtained from (5). The subscript c (standing for critical) indicates in (8) that the values of the density, temperature and chemical potentials are evaluated on the critical curve. Fig.1 shows the corresponding diagrams calculated for three different values of the bag constant B (or $B^{1/4}$ expressed in MeV). Note that condition (8) corresponds also to an energy

density of the plasma (ϵ) equal to $4B$, as in the MIT bag description of the nucleon at zero temperature.

In the spirit of the restructuring aggregation model we will be using in sect.2, the fundamental parameter is the bag radius of the nucleon R_b rather than the bag constant. However, using (1), one can directly relate the bag constant B to the volume of the quark core of the nucleon, for a total energy of the nucleon of about 1 GeV. The three values chosen in fig.1 correspond roughly to a bag radius of 0.5, 0.65 and 0.8 fm for $B^{1/4} = 245, 190$ and 155 MeV, respectively. We shall come back in sect.3 to the precise value chosen for B . As expected, the quark-gluon plasma occurs at much lower densities and temperatures when the bag constant is low, i.e. when the bag radius is large. In the original MIT bag model, with a bag constant of about $B^{1/4} \approx 100$ MeV, the quark-gluon plasma is already stable at zero temperature and ordinary nuclear matter density. However, for such a small bag constant, gluon corrections have to be included which prevent the plasma to be stable at such low densities.

It will be useful, for our forthcoming discussion, to keep in mind the simple results recalled here since they correspond to the limiting case of total deconfinement of quarks and gluons.

2. Restructuring aggregation of a hadron gas

2.1 Physical picture

It is already known that the finite volume of the hadrons plays a major role in the transition to a quark-gluon plasma [5]. Otherwise, the transition temperature (the so-called "Hagedorn" temperature) will be only a limiting temperature which would never be reached experimentally. In this section, we would like to see to which extent geometrical ideas might be able to qualitatively describe the formation of a mixed phase. We propose a simple restructuring aggregation model based on the fact that nucleons and pions are bags of quarks [6,7]. We shall see that, with minimal assumptions, one can get a qualitative evaluation of the conditions under which there is a transition from a hadronic gas to a large cluster of quark-gluon plasma (percolation cluster). Because nuclei contain a limited

number of nucleons (and hence valence quarks), one expects large fluctuations for various quantities. This is at variance to macroscopic systems which are made of an almost infinite number of particles. As a consequence, the transition from a hadronic gas to a quark-gluon plasma can only be very smooth. Let us now briefly describe the basic features of the model we used. A nucleon is made up of three quarks enclosed into a bag of radius R_b . This latter quantity is the free parameter of the model, and the conditions under which quark deconfinement takes place depend strongly upon this parameter (as we already mentioned, it is directly related to the bag constant B in the MIT bag model). Let us first start with the simplest case where we consider a nucleus as a sphere of radius R (volume V) at zero temperature. Suppose that there exists a process by which we can compress it while keeping its temperature equal to zero. Then R (and V) decreases as the compression increases. If the compression is large enough, all the nucleon bags will overlap to form a color conductor system and quark deconfinement will be reached. This situation occurs above a critical density which we shall evaluate within the following model.

We build the nucleus in a statistical way by putting nucleons at random inside the volume V . Each time we add a new nucleon we check if its bag overlaps with a bag already present in the volume V . If it is so, we form a larger bag which contains all the quarks belonging to the overlapping bags. In this operation, we assume that the volume of the new bags is proportional to the number of quarks, i.e. we assume that the quark density remains constant. Depending upon the volume V , which is related to the compression of the system, and on the value of the free parameter R_b , one gets either a big cluster (which we shall call percolation cluster) or several small size clusters. In the first case one has achieved a partial or total deconfinement of the system and formed a quark-gluon plasma in an extended region of space. In the second case, quarks are still confined and the system remains color insulator. It is clear that R_b is the key parameter of the model. If R_b increases, the density at which the percolation cluster appears decreases and vice-versa. A similar model has already been used to study the distribution of multiquark states in nuclei [8] but without restructuration of the clusters during the aggregation process. The model we propose differs however by the assumption that once two bags overlap, there is a rearrangement of the bags to form a new spherical bag. This avoid having unrealistic long and complicated clusters.

The idea of using percolation to describe the transition from a hadronic gas to a quark-gluon plasma has been first pointed out by Baym [7] within the framework of lattice percolation models.

If one assumes now that pions are also present in the nuclear medium, as it is the case at non zero temperature, one has to add new quark bags to the medium. Consequently, for a given volume, one expects to have a larger overlap at finite temperature between the bags, compared to a gas of nucleons only. A direct consequence of this is to decrease the critical density and temperature at which partial or total quark deconfinement takes place. For simplicity we consider only the possibility of exciting pions and neglect all other mesons which are of much higher mass. The effect of these latter should be only to slightly decrease the critical temperature. In the simplest approximation where one assimilates the pions to a free gas of bosons, their density at temperature T is given by :

$$\rho_{\pi}(T) = 3 \int_0^{+\infty} \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta \sqrt{p^2 + m_{\pi}^2}} - 1} \quad (9)$$

where m_{π} is the pion mass.

One can thus extend the zero temperature restructuring aggregation model as follows. We start with a spherical system of uniform baryon density ρ at temperature T . From eq.(9) one can calculate the number of pions in the medium. Then, one puts at random nucleons and pions one after the other in the volume V in such a way that the center of mass of the final system is at the center of the sphere. Each time one adds a new particle, one looks if it overlaps with quark bags already present in the medium. If it is so, one forms, as in the zero temperature case, a larger bag with all the overlapping bags. In this transformation, one makes the assumption that the volume of the resulting spherical bag is proportional to the number of quarks it contains. When all the particles, nucleons and pions, have been added to the system, the two cases discussed at $T=0$ may occur depending upon the initial conditions : one gets either a large color conducting percolation cluster, or the system remains color insulator. The process described above is then repeated several times to generate an ensemble of events which allows to perform ensemble averages.

In this simple picture, one may note that we have made no

considerations about the energy. This could only been done in a dynamical calculation of the quark-gluon plasma formation within a given model for the nucleon structure. We assume here that the thermal and compressional energies of the system are large enough to allow for multi-quark cluster formation. This might not be too bad an approximation at large compressions and large temperatures. However, close to normal nuclear matter density and small T values, the energy might be the dominant constraint. Unfortunately, it is rather difficult to have a realistic evaluation of the energy necessary to form multi-quark systems. Furthermore, one does not know to which extent one can extrapolate the theoretical equations of state used around normal densities and low temperatures. Therefore, the picture that we propose is applicable only in the case where the energy deposited into the system is large enough for the transformations we described above to take place. In other words, our results indicate what are the geometrical constraints (and finite size effects) in the formation of a mixed phase containing a large cluster of plasma (percolation cluster) and smaller clusters.

2.2 Results

Let us now discuss the results that we have obtained within this schematic approach and specify a few more details about the model. The nucleons are small qqq bags of radius R_b , while the pions are $q\bar{q}$ bags. We have assumed that the quark density is the same in both bags, i.e. the radius of the pion bag is taken to be $(2/3)^{1/3} R_b$. More generally, the radius of a n -quark cluster is equal to $(n/3)^{1/3} R_b$.

We show in fig.2, in the T - ρ plane, the curve where the transition from a hadronic gas to a quark-gluon plasma takes place, for three typical values for the bag radius, $R_b = 0.5, 0.65$ and 0.8 fm and for a system composed of 200 nucleons at $T=0$. These curves have been calculated at the point where the "susceptibility", as defined in ref.[9], is maximum. In our present description, the susceptibility $\mathcal{M}$ is the mean value of N^2 calculated without the largest cluster, where N is the number of particles in a given cluster, i.e. :

$$\mathcal{M} = \sum' N^2 n(N) \quad (10)$$

In this equation, $n(N)$ is the distribution of clusters as a function of N . The prime in the superscript of the summation symbol means that we remove the largest value of N in the summation. The quantity $\mathcal{M}$ is analogous to the susceptibility associated with magnetisation in a Ising model and is directly related to the fluctuations of the system. The maximum of the susceptibility corresponds to a situation where the percolation cluster contains, in the average, about half the total number of quarks and antiquarks present in the system. Let us discuss in some detail the properties of the curve corresponding to $R_b = 0.5$ fm. At $T=0$, one has to reach a critical baryon density of $\rho_c^0 = 0.6$ nucleons/fm³ (or 4 times the normal nuclear matter density, taken to be $\rho_0 = 0.15$ N /fm³ for simplicity), in order to deconfine half of the quarks contained in the system. On the other hand, at normal density, one needs to heat the system up to $T_c = 240$ MeV to get the same transition.

Since a nucleus is a finite system, we do not expect to observe a sharp phase transition but rather a smooth one. In order to study this effect, we have calculated the transition curves with three different assumptions corresponding to a mean size of the percolation cluster equal to 70%, 90% or 100% of the total number of quarks and antiquarks (fig.3) using $R_b = 0.5$ fm. As expected, one observes a rather large shift of the transition curve, depending upon the definition we adopt for the transition. It is essentially due to the small number of nucleons involved and is a characteristics of the finite size of the system under consideration. This is certainly a drawback for a clear observation of a phase transition because this latter will be washed out. The limit 100% indicated in fig.3 corresponds to the ideal situation where there is only one cluster left. In an infinite system, this will happen, at $T=0$, for a density $\rho_c^0(\infty) = 3/4\pi R_b^3$, i.e. about $13 \rho_0$ for $R_b = 0.5$ fm, since we have made the assumption that there is rearrangement of the cluster to a sphere. In the finite system we are dealing with, the critical density is much reduced and is given by :

$$\rho_c^0 = \frac{A}{K^3} \rho_c^0(\infty) \quad (11)$$

with :

$$K = \frac{A-1}{A} \left[1 + (A-1)^{1/3} \right] \quad (12)$$

so that $\rho_C^0 = 8.1 \rho_0$ for $R_b = 0.5$ fm. The expressions (11) and (12) are obtained by demanding that one cluster of volume V containing A particles is formed from a cluster of $A-1$ particles and one nucleon. The critical density corresponding to other values of the bag radius R_b are reported in table 1. For an infinite system, it is interesting to note that this critical density, in a model which does not include any short-range repulsion between nucleons, is larger than the critical density in a model with infinite correlations (hard spheres). In this latter case $\rho_C^0(\infty) = 1/(4\sqrt{2} R_b^3)$ (close compact configuration of the nucleons), i.e. $\rho_C^0 = 9.4 \rho_0$ for $R_b = 0.5$ fm.

Since the only dimension in our description is the radius of the bag, the critical density at zero temperature, ρ_C^0 , is given by :

$$\rho_C^0 = \frac{\lambda_C}{R_b^3} \quad (13)$$

where λ_C is a dimensionless quantity characteristic of the restructuring aggregation model. Its value depends upon the conditions where we define the phase transition (size of the percolation cluster) and the number of particles. If we define it at the point where the "susceptibility" is maximum, and for $A=200$ nucleons, it turns out that $\lambda_C = 3/40$. At finite temperature, one can calculate the baryon density, $\rho_C(T)$ at which the transition occurs from the critical density $\rho_C^0 \equiv \rho_C(T=0)$ (which we checked to be independent of the number of particles for not too small A , but A taking values in the mass range of nuclei) at $T=0$, by the following scaling relation :

$$\rho_C^0 = \rho_C(T) + \frac{2}{3} \rho_\pi(T) \quad (14)$$

where the pion density at temperature T is given by eq.(9). In order to study the influence of the bag radius, one can just draw a series of transition lines associated to different values of ρ_C^0 (or R_b). Some of them are displayed in fig.2. As expected, quark deconfinement becomes easier if

R_b increases. We have summarized all the characteristics of the percolation transition in table 1 for three typical values of the bag radius. We give in table 2 the distribution of multi-quark clusters for cold nuclei and for three different bag radii. It is interesting to note that, for $R_b = 0.8$ fm, the critical density to form a percolation cluster is just ρ_0 , so that the results indicated in the table for this radius correspond to a different phase to those corresponding to $R_b = 0.5$ and 0.65 fm. This is why the percentage of high multi-quark states is lower in that case than for smaller radii.

The presence of multi-quark bags in the medium leads to an apparent increase of the size of the nucleon. We have indicated in table 2 the relative increase of the bag radius. Since we did not include any short range correlations in our description, this increase, together with the probabilities to have multi-quark states in the medium is a bit overestimated. At this point, one can notice that we have assumed that two bags fuse if they overlap, and in particular if they just touch. One could have used another prescription, namely that they fuse if they have at least one quark in common. In order to get the same result as those discussed so far, one would need to increase R_b by about a factor of two. Therefore, all the results corresponding to $R_b = 0.5$ fm discussed previously would correspond to $R_b \approx 1$ fm with this new assumption.

While the main characteristics of the percolation transition we studied above are rather close to what one can expect about the formation of the quark-gluon plasma, one may note that for low baryon density the transition curves show a different behaviour compared to those evaluated using the MIT bag model (see fig.3). Since the only parameter in our model is the bag radius, this behaviour may be associated with our assumption that the bag radius remains constant with the temperature. At finite temperature, we only considered, in our simple model, the possibility to create pion bags from the non-perturbative vacuum. However, it is also possible to create $q\bar{q}$ pairs (and gluons) from the perturbative vacuum itself inside the bag ('heating' the bag). In this case, since we assume the energy density to be constant inside the cluster, the radius of the cluster should increase with the temperature. It is thus obvious that the critical temperature to form a percolation cluster will decrease, while the critical density at $T=0$ will be unchanged.

To estimate the increase in radius of a cluster of baryon density ρ , we assume that the variation of the volume is proportional to the increase in the energy density, namely :

$$R_B(T, \rho) = R_B^0 [1 + \delta_B(T, \rho)]^{1/3} \quad (15)$$

with :

$$\delta_B(T, \rho) = \frac{1}{\varepsilon_0} [\gamma_g \varepsilon_g(T) + 2 \gamma_q \varepsilon_q(T, \rho)] \quad (16)$$

and ε_0 is the energy density at $T=0$, taken from eqs.(2-5). The energy density for the $q\bar{q}$ pairs ($2\gamma_q \varepsilon_q$) is calculated numerically. Note that the increase in the radius depends upon the baryon density inside the cluster through the chemical potential of the quarks. This dependence is however very small and may be neglected.

The percolation transition is thus calculated as above. The scaling relation (12) becomes in that case :

$$\rho_c^0 = \rho_c(T) [1 + \delta_B(T, \rho_c)] + \frac{2}{3} \rho_\pi(T) [1 + \delta_\pi(T)] \quad (17)$$

where $\delta_\pi(T) = \delta_B(T, 0)$. We have indicated the new transition diagram by dashed curves in fig.4, and the corresponding critical temperature and density are reported in table 2. As expected, the critical temperature is very much reduced, and is now quantitatively comparable with the critical temperature of the quark-gluon plasma in the MIT bag model description.

3. Quark-gluon plasma hadronization

3.1. Physical picture

We would like to turn now to the discussion of the mixed phase formation once the plasma has already been produced. This happens during the hadronization of the plasma, while the case we discussed in the preceding section concerned the formation of the plasma.

In the hadronization process, the starting point is no longer a (free) gas of nucleons and pions, but the quark gluon plasma itself. To describe this phenomenon, it turns out that a different type of percolation mechanism should be used. We propose here to tentatively describe the very beginning of the hadronization of the plasma within the framework of a 3-dimensional site-bond percolation model on a cubic lattice [10]. This kind of percolation approach has already been applied with some success in another context to describe the multifragmentation of nuclei in heavy-ion collisions [11]. Let us recall the main ideas of this description.

There are two key parameters in the site-bond percolation approach : the concentration of sites, p , which is the ratio between the number of occupied sites and the total number of sites of the lattice, and the concentration of bonds, q (not to be confused with the quark symbol), which is the ratio between the number of unbroken bonds over the total number of possible bonds in the lattice. These two parameters vary between 0 and 1. A cluster is defined as a group of occupied sites linked together by unbroken bonds. Different configurations can be generated by computer simulation for given values of p and q . By studying a large number of configurations, one obtains the distribution of cluster $n(N)$ (where N is the number of quarks and antiquarks in the clusters) for each value of p and q (p and q varying between 0 and 1). Within this approach, it turns out that the distribution of cluster exhibits interesting features that we shall now briefly discuss.

For a system composed of an infinite number of particles, there is a sharp phase transition (percolation transition) on a critical line $p_c(q)$ which divides the p - q plane in 2 different regions. For small p and q values ($p < p_c(q)$) one has only small size clusters while, for large p and q values ($p > p_c(q)$) one forms a big cluster (percolation cluster). This well separated situation for an infinite system is no longer observed for a system containing a small number of particles. There, the transition is more progressive and one can hardly speak about a phase transition in the usual sense. One moves smoothly from the small cluster region to the percolation region. The lines, in the p - q plane, where the transition takes place have to be defined arbitrarily. For instance, the percolation transition may be determined as in the preceding section by looking at the maximum of the "susceptibility". This is almost equivalent to taking the point at which a cluster containing 50% of the total number of particles is

formed. We shall use this latter prescription in the following developments.

One can now understand how this approach may be used to describe the hadronization of the quark-gluon plasma close to the stability line (as defined in sect.1.2). On the stability line of the plasma, described here with the MIT bag model, the energy density of quarks and gluons is equal to $4B$. All the sites of the lattice are occupied by quarks and antiquarks ($p=1$). Their number is proportional to $(n_q + n_{\bar{q}})$ and is calculated numerically as a function of the temperature and baryon density (or chemical potential) using eq.(7). Moreover, because the plasma is stable under these conditions, this means that all the neighbouring sites are linked together ($q=1$).

When the temperature and/or the density decrease, the plasma is no longer stable, and, as we already mentioned in the introduction, bubbles of ordinary (non-perturbative) vacuum appear in the medium. In our scheme, this corresponds to a decrease of the value of the p and q parameters. Two things may happen when we go away from the stability line. First of all, the number of particles (quarks and antiquarks here) decreases as the baryon density and temperature decrease. For zero baryon density, the number of particles is not zero but is just given by the number of quark-antiquark pairs created from the QCD vacuum at the critical temperature. We can thus relate the occupation probability p of the sites to the total density of particles (quarks and antiquarks, noted ρ_Q ($=n_q + n_{\bar{q}}$), at a given baryon density and temperature. To normalize p to unity on the stability line, we choose to follow the line of equal entropy of the plasma. Therefore :

$$p = \frac{\rho_Q(\rho, T)}{\rho_Q^c(\rho_c, T_c)} \Big|_{s=s(\rho, T)} \quad (18)$$

where s has been given in (2.c).

The second mechanism which prevents the plasma to be stable is the fact that the links between the sites can be broken. This is a natural consequence of the appearance of bubbles of ordinary vacuum inside the plasma which will break links between the quarks. In the percolation scheme, this leads to a decrease of q . To describe this situation we have

chosen the following ansatz :

$$q = \frac{\varepsilon(\rho, T)}{4B} \quad (19)$$

where $\varepsilon(\rho, T)$ is given by eq.(2b). In our description, the gluon degrees of freedom appear only indirectly in the definition (19) of the concentration of links between the sites occupied by the quarks and antiquarks. From eq.(2a) and (3), they contribute at most to 1/3 of the total energy at the critical temperature and zero baryon density.

The above choice for p and q is of course not unique, but it is anyhow suggested by the physical situation under consideration. We think that one can get at least a qualitative feature of the mixed phase, as it appears at the beginning of the hadronization of the quark-gluon plasma.

3.2. Results

The formation of the percolation cluster, i.e. the critical line $p_c(q)$ in the p, q plane, depends only upon the number of sites, and upon the configuration of the lattice. We use here for simplicity a cubic centered lattice of $7 \times 7 \times 7$ sites and fig.5 shows the transition line for these conditions. For small p and q , only small clusters are present with an exponential distribution. At the percolation transition, a percolation cluster is formed (containing about 50% of the total number of particles), while small clusters have a distribution which follows a power law distribution [10]. For p and q values above the transition the size of the percolation cluster increases and the distribution of the small clusters is again exponential. This behaviour is illustrated in fig.6 where we have plotted the distribution of clusters of size N for a particular case : $q = 0.6$ and three values of p : $p = 0.4$ (below the percolation transition), $p = 0.6$ (around the transition) and $p = 0.8$ (above the percolation transition).

The physical interpretation of the percolation transition is indicated in fig.7. This follows from the definition (18) and (19) of p and q in terms of the physical parameters T and ρ of the system. We show only the

case corresponding to $B^{1/4} = 245 \text{ MeV}$ ($R_b = 0.5 \text{ fm}$). The thin solid lines indicate the lines of equal- q (or equal energy density), for $q = 0.6, 0.7, 0.8$ and 1 , the last one being just the stability line of the plasma we have already shown in fig.1 (line for which $p = q = 1$). At $T=0$, one can easily calculate the density of particles, since in that case it is just three times the baryon density, therefore $p = \rho$ (point K)/ ρ_c^0 (point L). The point K is thus chosen so that the value of p and q associated to it lies on the percolation line of fig.5. For non-zero temperature (point I for example), one has first to fix q arbitrarily and the exact position of the point on the line $q = c^{te}$ is given by the value of p . To calculate this probability, one has first to calculate numerically the quark and antiquark densities along the lines $q = c^{te}$. Then one determines the entropy at which the ratio p of the densities for, respectively, q and $q=1$ (i.e. $p = \rho_Q$ (point I) / ρ_Q^c (point J)) is precisely the value of the parameter p_c on the percolation line of fig.5 associated to q . One thus proceeds further on with a different value of q . One obtains in this way the transition shown by a dashed line in fig.7. It corresponds to the physical conditions where one has a mixed phase containing the percolation cluster and small size clusters.

The critical density, and to a lesser extent the temperature, necessary to have a percolation cluster is, as expected, much smaller than the temperature and density of the stable plasma. We have summarized these values in table 3 for the bag constants we already used. The value of the bag constant is chosen in order to get the same critical density ρ_c^0 at $T=0$ as the 100% limit of the aggregation model presented in sect.2 (see fig.3). It is thus possible to compare quantitatively the two complementary mechanisms proposed in this work for the formation of the mixed phase.

These two approaches, starting either from a free gas of nucleons and pions (aggregation model presented in sect.2) or directly from the quark-gluon plasma (hadronization) are rather compatible. This can be seen, for instance, by comparing the critical density and temperature corresponding to the situation where a percolation cluster containing 50% of the quarks and antiquarks is formed. These quantities are given in table 1 and 3 and the transitions are shown by the dashed lines in figs.4 and 7. In the restructuring aggregation mechanism, the critical densities are somewhat smaller, and the critical temperatures a bit larger than in the

hadronization mechanism by about 20%. This is certainly not very significant given the uncertainties inherent to our description. It is in fact rather satisfactory that both mechanisms lead to approximately the same configuration. In order to get to this point, we have to emphasize here that it is essential to take into account the increase of the nucleon bag radius due to the temperature or, in other words, to take into account the pressure coming from gluons and quark-antiquark pairs inside the bag at non-zero temperatures. Otherwise, the critical temperature of the mixed phase is much too large and even larger than the critical temperature of the stable plasma evaluated in the MIT bag model.

4. Conclusion

We have proposed in this paper a simple description, based on percolation concepts, of the mixed phase as it occurs in the formation and hadronization of the quark gluon plasma. This phase appears in the formation of the quark-gluon plasma when the temperature and density of the hadronic gas is increased. Provided one has formed the quark-gluon plasma, it is also present at the beginning of the hadronization of the system when the temperature and density decrease. It is certainly very difficult to describe in detail the microscopic behaviour of the system under these extreme conditions. In particular, because the temperature of the system is rather large (of the order of 100 MeV or more), one creates a large number of particles (mainly pions) in the medium which make any theoretical treatment very difficult. Given this complexity, we have proposed in this work to study the mixed phase in the framework of percolation theory. This involves only a limited number of assumptions about the physical conditions and emphasizes the geometrical aspects of the process. The results presented in this work allow to evaluate the importance of these latter aspects and should be understood as a very first attempt in the direction of an understanding of the mixed phase.

We have first described the formation of the quark-gluon plasma with a restructuring aggregation model of nucleons and pions. At a given density and temperature, one may observe the formation of a percolation cluster, in which the quarks and gluons are deconfined. If the experimental conditions are not strong enough, a total deconfinement of quarks and gluons is not

possible, but there may still exist a percolation cluster which is large enough to say that one has reached a partial deconfinement of the system. For a nucleon bag radius of 0.5 fm for instance, and at zero temperature, 50% of the quarks are deconfined at about 4 times the normal nuclear matter density while one needs to reach about $8\rho_0$ for their total deconfinement. At higher temperatures, the critical density at which the preceding phenomenon takes place decreases due to an increase of the number of particles (formation of pions).

A microscopic description of the hadronization of the plasma is also a very difficult problem since it involves non-perturbative aspects of QCD. In this paper we have tentatively proposed to describe the main features of the beginning of the hadronization with a site-bond percolation model in which the occupation probability of the sites, and the probability of links between neighbouring sites, are represented by the overall properties of the system, i.e. baryon density and temperature. In this approach one can study the critical temperature and density corresponding to the formation of a percolation cluster as well as the distribution of quarks in the clusters. We found that, within the percolation approaches, the two complementary aspects of the mixed phase, depending whether we start from the quark-gluon plasma itself or from a gas of nucleons and pions are rather compatible .

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Table 1

Critical densities and temperatures of the mixed phase in the aggregation model, for different bag radii R_b . The densities are calculated with respect to normal nuclear matter density, taken here to be $\rho_0 = 0.15 \text{ fm}^{-3}$. The column $\rho_c^0(50\%) / \rho_0$ refers to the conditions at which a percolation cluster containing about 50% of the total number of quarks and antiquarks is formed, while $\rho_c^0(100\%) / \rho_0$ refers to the limiting case in which there is only one cluster left. The critical temperature (at zero baryon density) for a 50% cluster is noted $T_c^0(50\%)$. If the bag radius is allowed to vary with the temperature, the corresponding critical temperature is noted $T_c^{0*}(50\%)$.

R_b (fm)	$\rho_c^0(50\%) / \rho_0$	$T_c^0(50\%)$ (MeV)	$T_c^{0*}(50\%)$ (MeV)	$\rho_c^0(100\%) / \rho_0$
0.5	4.0	265	185	8.1
0.65	1.8	215	145	3.7
0.8	1.0	175	115	2.0

Table 2

Probability distribution of clusters of 6 quarks (P_{6q}) or more than 6 quarks ($P_{n>6q}$) at zero temperature and normal nuclear density, for different bag radii. The increase of the cluster radius, as compared with the free bag radius, is denoted by δR_b .

R_b (fm)	$\delta R_b/R_b$ (%)	P_{6q} %	$P_{n>6q}$ %
0.5	11	15.5	10
0.65	30	16.5	26
0.8	46	16	23

Table 3

Critical densities, ρ_c^0 (50%), and temperature, T_c^0 (50%), of the mixed phase evaluated using a site-bond percolation calculation on a cubic lattice, for different bag constants B. The critical quantities ρ_c^0/ρ_0 and temperature T_c^0 are calculated at $T=0$ and $\rho=0$ respectively in the MIT bag model, as indicated in fig.1.

$B^{1/4}$ (MeV)	$\rho_c^0(50\%)/\rho_0$	$T_c^0(50\%)$ (MeV)	ρ_c^0/ρ_0	T_c^0 (MeV)
245	4.7	160	8.1	170
190	2.2	120	3.7	130
155	1.2	100	2.0	105

Figure captions

Figure 1 : Phase diagram of the hadronic matter and the quark-gluon plasma, in the MIT bag model. The densities are expressed with respect to normal nuclear matter density, taken here to be $\rho_0 = 0.15 \text{ nucleon/fm}^3$.

Figure 2 : Mixed phase in a simple aggregation model, for three different bag radii. The transition curves refer to the formation of a percolation cluster containing 50% of the total number of quarks and antiquarks.

Figure 3 : Formation of a percolation cluster of 50%, 70%, 90% and 100% of the total number of quarks and antiquarks. The limiting case (100%) means that there is only one cluster left.

Figure 4 : Same as fig.2, for two different radii. The dashed lines represent the transition curves when the bag radius is allowed to increase with the temperature, according to eq.(14).

Figure 5 : Site-bond percolation diagram, for a cubic lattice of $7 \times 7 \times 7$ sites. The variable p is related to the occupation probability of the sites, while q refers to the probability that a link between two adjacent sites is formed.

Figure 6 : Multiplicities of clusters for $q=0.6$, and p around the percolation transition ($p_c = 0.6$). The number of particles in the clusters is given by N . The multiplicities are normalized to $\sum_N N n(N) = p$.

Figure 7 : Mixed phase in a site-bond percolation mechanism in the MIT bag model for $B^{1/4}=245 \text{ MeV}$. The thick solid line is that of fig.1, while the dashed line represents the mixed phase in which a cluster containing 50% of the total number of sites (i.e. quarks and antiquarks) still exists. See text for the meaning of the labels I, J, K and L. The lines of equal- q (or equal energy density) and the isentropic ($s=c^{te}$) are also indicated in this figure (thin solid lines).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Site-bond percolation

Fig. 6

Fig. 7