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ABSOLUTE AND HIGH-PRECISION MEASUREMENTS OF PARTICLE BEAM PARAMETERS
AT CERN ANTIPROTON STORAGE RING LEAR*
USING SPECTRAL ANALYSIS WITH CORRECTION ALGORITHMS

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* LEAR = Low Energy Antiproton storage Ring

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1. INTRODUCTION

During their revolution in the machine, each particle performs a betatron oscillation with a non-integer number of betatron periods per turn (called tune Q). The horizontal and vertical tune, and any linear combination of these tunes should be non-integer. If \( nQ_H + mQ_V = p \) (\( n, m, p \) integer), the amplitude of the oscillation can grow indefinitely so that the particles will hit the vacuum chamber and be lost. It is therefore of particular importance to measure the tunes with high precision and to have information on some resonances \( (nQ_H + mQ_V = p) \).

By having the particles bunched (i.e. compressed into a bunch shorter than the circumference of the machine) and using electrostatic pick-up electrodes [1], it is possible to get information about the position of the beam and about the betatron oscillation of the bunch on each turn.

By kicking the whole bunch (i.e. giving it an oscillation amplitude of some nm) and using spectral analysis, we are able to find the fractional part of the tune with a precision better than \( 10^{-4} \) even with a lot of noise and damping of the oscillation or decaying of the signal. The precision on the oscillation phase is around \( \pm 0.5^\circ \).

Method used to reach these precisions utilizes a bunch-synchronized data acquisition system and correction algorithms for FFT analysis.

2. ACQUISITION SYSTEM

A pulse train which is synchronized with the bunch centre is electronically generated [2]. The synchronization is automatic and essentially independent of changes in revolution frequency and in length of the bunch. This pulse train is used to trigger the acquisition of changes in radial beam position after an automatic subtraction of the residual closed orbit at the time of the measurement. The bunch length is comprised between 40 and 400 nanoseconds.
3. THE MATHEMATICAL TREATMENT

The use of Discrete Fourier Transform (DFT) presents a number of major handicaps if one wishes to make selective and absolute measurements of the characteristics other than the frequency (phase, amplitude, damping) of the betatron oscillations.

The mathematical treatment of the raw data uses a spectral analysis (FFT) combined with mathematical algorithms and iterative methods. This technique and the algorithms which it uses are derived from the analysis of errors introduced by the Fourier transform when applied to the measurements of betatron oscillations (frequency, phase, amplitude and damping factor). The method employs windowing, interpolation and modulation algorithms and gives a much greater accuracy to the results than would normally be achieved using a single FFT with a limited number of samples [3-5].

3.1 Sources of the errors

The physical process is sampled at the revolution frequency \( T_0 = 1/f_{rev} \). It is causal (triggered at an arbitrary time \( t = 0 \) corresponding to the turn \( n = 0 \)) and has an indefinite duration (\( n \to \infty \)). This can be represented in time domain by:

\[
y_{\infty}(nT_0) = \sum_{n=0}^{\infty} y_n(nT_0) \tag{1}
\]

The spectrum of this signal is continuous and periodic \((= f_{rev})\) and correspond to the true spectrum in the limits of Shannon's theory (for frequency \(< f_{rev}/2 \)). It could be produced by a hypothetical DFT with an infinite number of samples separated by \( T_0 \):

\[
y_{\infty}(f') = \overline{Y_{\infty}(f)} e^{j \frac{2\pi}{T_0} f'} \tag{2}
\]

where \( Y_{\infty}(f) \) is the continuous and periodic true spectrum, \( \overline{Y_{\infty}(f)} \) the true modulus, \( \overline{Y_{\infty}(f)} \) the true phase and with \( 0 \leq f' < f_{rev} \).
and $j = \sqrt{-1}$.

In practice the DFT is made with a limited number of samples $N$ corresponding to the original process $y_n$ seen during a time limited $(N.T_s)$ window $w(t)$.

The resulting spectrum is discrete and periodic; each line is separated by $\Delta f = \frac{f_{\text{rev}}}{N}$. One can express the spectrum given by the DFT, by the convolution:

\[
\mathcal{F}_N (F_k) \ast \mathcal{F}_N (F) = \frac{f_{\text{rev}}}{2} \int_{-f_{\text{rev}}/2}^{f_{\text{rev}}/2} \mathcal{F}_N (F) \times \mathcal{W}(F_k - F) \text{ d}F
\]

with $F_k = k\Delta f$, $k$ is an integer; for the principal period $0 \leq k \leq N-1$.

Each component $\mathcal{F}_N (F_k)$ of the spectrum given by the DFT of $N$ samples should be considered as a continuous vector summation of all the true vector components $\mathcal{Y}_n (f)$ of a period which are distributed on each line by a modulation phenomena with all components of the spectrum of the window.

The values of interest are the true modulus $|\mathcal{Y}_n (f)|$ and phase $\angle \mathcal{Y}_n (f)$. In this way we can see the errors introduced by the DFT. To show that each complex component $\mathcal{F}_N (F_k)$ of the DFT represents the vector resulting from the window distribution phenomena of the true spectrum on each line $F_k$, we write:

\[
\mathcal{F}_N (F_k) = \mathcal{N}_w [\mathcal{Y}_n - k]
\]

3.2 Correction of the errors

Practically the true process can be written as the sum of a principal damped oscillation (frequency = $q_{\text{rev}}f_{\text{rev}}$) and some perturbative terms: an other component of damped oscillation (frequency = $q_{\text{rev}}f_{\text{rev}}$)
and a supposed random noise. We have \( q_H, q_V < 0.5 \).

\[
y_{n}(n) = \sum_{n=0}^{n_m} \left[ h(n) + v(n) + b(n) \right] \quad \text{with}
\]

\[
h(n) = M_H e^{-n/\delta_H} \cos(2\pi q_H n + \phi_H)
\]

\[
v(n) = M_V e^{-n/\delta_V} \cos(2\pi q_V n + \phi_V)
\]

\[
b(n) = M_b r(n)
\]

where: \( M_H \) and \( M_V \) are the initial amplitudes (for \( n = 0 \)); \( \delta_H, \delta_V \) are the damping constants; \( M_b \) = noise amplitude, \( r(n) \) = random function such that \(-1 \leq r(n) \leq 1\). We want to measure \( q_H, \phi_H, \delta_H, M_H \) and we assume that \( f_{rev} = 1/T_s = 1 \). Generally \( q_H \) and \( q_V \) are non rational and they are comprised between two lines ( \( k_H \) and \( k_H + 1 \) for \( q_H \) and \( k_V \) and \( k_V + 1 \) for \( q_V \) ) of the DFT. The analysis has shown that it is interesting to consider two types of windows, rectangular \( W_r(t) \) and sine \( W_s(t) \):

\[
W_r(t) = \begin{cases} 
1, & \text{for } 0 < t < NT_s \\
0, & \text{elsewhere}
\end{cases}
\]

\[
W_s(t) = \begin{cases} 
\sin \frac{\pi t}{NT_s}, & \text{for } 0 < t < NT_s \\
0, & \text{elsewhere}
\end{cases}
\]

The modulus of the two lines \( k_H \) and \( k_H + 1 \) of a DFT made on \( N \) samples of the true signal (5), comes from the vector composition of the true spectrum of (5) distributed on the lines \( k_H \) and \( k_H + 1 \):

\[
Y_N(k_H) = (D_w[H \rightarrow k_H] + D_w[V \rightarrow k_H] + D_w[B \rightarrow k_H])
\]

\[
Y_N(k_H + 1) = (D_w[H \rightarrow (k_H + 1)] + D_w[V \rightarrow (k_H + 1)] + D_w[B \rightarrow (k_H + 1)])
\]
3.2.1 Frequency measurement with an analytical interpolation method

If the damping factor $N/\delta_H$ is zero, we have shown that [3-5]:

with the rectangular window:

$$q_H = \frac{1}{N} \left[ k_H + \frac{D_W[H_{\infty}\to(k_H+1)]}{D_W[H_{\infty}\to k_H] + D_W[H_{\infty}\to(k_H+1)]} \right]$$  \hspace{1cm} (8)

with the sine window:

$$q_H = \frac{1}{N} \left[ k_H + \frac{2D_W[H_{\infty}\to(k_H+1)]}{D_W[H_{\infty}\to k_H] + D_W[H_{\infty}\to(k_H+1)]} - \frac{1}{2} \right]$$  \hspace{1cm} (9)

We have also shown that the perturbative distributions:

$$D_W[V_{\infty}\to k_H], \quad D_W[B_{\infty}\to k_H]$$  \hspace{1cm} (10)

are decreasing if $N$ is increasing and if we use the sine window. With a good choice of $N$ it is possible to neglect these distributions and we have in this case:

$$\frac{V_N(k_H)}{N} \approx \frac{D_W[H_{\infty}\to k_H]}{D_W[H_{\infty}\to k_H]}$$  \hspace{1cm} (11)

$$\frac{V_N(k_H+1)}{N} \approx \frac{D_W[H_{\infty}\to(k_H+1)]}{D_W[H_{\infty}\to(k_H+1)]}$$

Hence with the sine window:

$$q_H \approx \frac{1}{N} \left[ k_H + \frac{2D_W(k_H+1)}{V_N(k_H) + V_N(k_H+1)} - \frac{1}{2} \right]$$  \hspace{1cm} (12)

This interpolation formula assumes that $N/\delta_H = 0$. If $N/\delta_H$ is non-zero, the maximum possible normalized error is
\[ |N(d_{\delta H})|_{\text{max}} = 0.1875 \pi^2 (1 + e^{-N/\delta H}) \]
\[ \ldots \] \[ \ldots \times \left\{ \frac{3}{\sqrt{9\pi^2 + \left( \frac{N}{\delta H} \right)^2}} \left[ \frac{\pi^2}{\pi^2 + \left( \frac{N}{\delta H} \right)^2} \right] \right\} \]
\[ \ldots \ldots \frac{1}{\pi^2 + \left( \frac{N}{\delta H} \right)^2} \] \[ (13) \]

Hence if \( N/\delta H \leq 1 \), \( N(d_{\delta H}) \mid_{\text{max}} = 10^{-2} \) and for \( N = 256 \), \( (d_{\delta H}) \mid_{\text{max}} \leq 3.9 \times 10^{-5} \). The minimum error (equal to zero) introduced by the interpolation when the damping factor \( N/\delta H \) is non-zero, occurs when the true frequency \( \delta H \) correspond exactly to the middle of the interval \( k_H, k_H + 1 \). Hence, when the damping factor is not negligible \( (N/\delta H > 1) \) we combine the analytic interpolation method with an iterative convergent algorithm which displaces frequency (by an adequate nodulation) to be measured at the middle of the interval between two consecutive lines of the DFT. In this method, the evaluation of the value of \( \delta_H \) is made with a "moving FFT". The residual error is caused by the distribution of the noise and other parasitic signals which have been neglected. The maximum possible noise error is:

\[ |d_{\delta H}|_{\text{max}} \approx \frac{4 \cdot \frac{N}{\delta H} \sqrt{4\pi^2 + \left( \frac{N}{\delta H} \right)^2}}{(\pi^2 N)(N/\delta H) - \delta_H} \cdot \frac{M_b}{M_H} \] \[ (14) \]

Hence if \( N = 256 \), \( \delta_H = 100 \) and \( M_b/M_H = 10 \% \), we have \( (d_{\delta H}) \mid_{\text{max}} = 1.86 \times 10^{-4} \).

The analytical interpolation method gives a possible decrease of frequency error by a factor of 10 to 1000. It is important because the accuracy of the other measurements depends directly on the accuracy of the frequency measurement.
3.2.2 Phase measurement

The method uses the fact that if the frequency is known we can, by an adequate modulation, displace the spectral component such that it coincides with one of the line of the DFT. In this case, by neglecting the distribution of the other components, we can obtain with the DFT a spectrum which resembles the true spectrum of the displaced component.

3.2.3 Modulus measurements

Let $\mathcal{Y}_N(k_H)$ and $\mathcal{Y}_N(k_H + 1)$ be the modulus of the two lines given by the DFT when the true frequency was displaced to the middle of the interval between two lines (frequency measurement) we have:

$$
M_H \approx \frac{\sqrt{4\pi^2 + \left(\frac{N}{N_H}\right)^2}}{\pi(1 - \alpha) N/\delta_H} N
$$

(15)

The value of the damping factor $N/\delta_H$ is obtained as shown in paragraph 3.2.4.

3.2.4 Damping measurement

For the damping measurement it is necessary to increase the sensitivity by using the rectangular window and choose $N$ such that we have $N/\delta_H \geq 2$. We displace the spectral component $\mathcal{Y}_H$ on one of the lines of the DFT. If this line is $k_H$, we have:

$$
\delta_H \approx \frac{N}{2\pi} \sqrt{\left[\frac{2\mathcal{Y}_N(k_H)}{\mathcal{Y}_N(k_H - 1) + \mathcal{Y}_N(k_H + 1)}\right]^2 - 1}
$$

(16)
4. BEAM MEASUREMENTS

The unperturbed movement of a particle in a storage ring can be written as [6]

\[ z = \sqrt{2\beta_z(s)J_0} \cos(\mu_z(s) + \phi_0) \quad (17) \]

where \( z \) stands for a particle's position in the horizontal or vertical plane. \( \beta_z(s) \) and \( \mu_z(s) \) are called betatron functions and phase advance at location \( s \) along the trajectory, and are given by the ion optical properties of the storage ring. \( J \) is an invariant of the motion given by the initial conditions of a particle.

4.1 Tune and phase advances

The measurements are of particular interest for the knowledge of the machine working point \((Q_x, Q_y)\) and also to correct beam trajectory misteering during the injection process.

If we use two horizontal (or vertical) pick-ups at different places we can measure the phase advance between these points, compare it with theoretical values and eventually find focusing errors.

4.2 Perturbations

An error of the electromagnetic guiding- and focusing fields in a storage ring gives a perturbation to the movement of particles. The possible effects are:

i. Tune shifts as a function of the amplitude of oscillation. To measure these, kicks of increasing force are applied to the whole beam and the tune change are measured [7].

ii. excitation of resonances along certain lines \( Q_x + Q_y \) in the tune diagram. The perturbations can act in one plane (\( nQ_x = \) integer or \( nQ_y = \) integer) or generate coupling between the two transverse planes. Using a Hamiltonian formulation and canonical transformations it is possible to find the perturbed motion of a particle [8,9]. Consequently the
additional frequencies appearing in the spectra of transverse oscillations can be related to particular resonance lines. The amplitudes and the phases of the perturbation can be extracted from the spectra, see Figure 1. The beam was kicked in the H plane. From the oscillation in the H plane (a) we can find the tune \( Q_H \) of this plane and the phase of oscillation. Due to coupling two peaks appear in the Fourier spectrum of V oscillation (b). One corresponding to V tune \( Q_V \), the other to H tune that indicates a skew quadrupole perturbation i.e. extraction of the closest resonance line \( Q_H + Q_V \approx 5 \).

From these spectra the phase of a correction was found. After only three iterations this resonance was compensated. Using this information, correction elements can be devised and powered to compensate the perturbation.

![Figure 1: Measurement of linear coupling](image-url)
4.3 Phase-space

In the special case where a resonance line is used to extract the beam it is of particular interest to measure the behaviour of particles. To reconstruct the normalized phase space at the observed point, a possible way exists to find the derivative of the position of the particle ($\frac{dz}{ds}$). This derivative is computed using Fourier transform and Lanczos factors [10], see Figure 2. For extraction at LEAR we use the resonance $3Q_H = 7$ excited by normal sextupolar field. Fig A shows the recorded oscillations at a working point close to resonance line. Fig B shows the spectrum where 2 peaks appear, one for the tune $Q_H$ and one at $2Q_H$ indicating a resonance of type $3Q_H$. Fig C shows the computed derivative of $x$ and Fig D the normalized phase space (dots). It shows a rounded regular triangle. The stability limit given by the regular triangle is drawn in plain line. The result in phase space is then compared to the theoretical set-up [11].

![Figure 2: Reconstruction of phase space](image)
4.4 Simulation

These Fourier methods are also used to analyse the motion of particle from simulation. By using accelerator "modelling" programs we can simulate the behaviour of a particle (or a pseudo-beam of particles) and analyse the oscillations. It permits us to predict the necessary corrections elements that will be installed in the machine.
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