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A PROPAGATOR FOR FERMIONIC STRING

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A PROPAGATOR FOR FERMIONIC STRING

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Abstract

Proceeding from path integral for open fermionic string the expression for propagator is obtained. The result is compared with that of operator calculations for Green function of fermionic string. In the case of pointlike boundary conditions the expression for propagator can be written in the form of Kallen-Lehmann representation with increasing spectral density.

The computation of bosonic string propagator [1] by means of Polyakov path integral [2] has given the possibility to connect the most effective method of calculations in first-quantized string theory with the field theory of strings, to study divergences and causality properties of string theory. The idea was developed in some papers on the subject [3-5] but all of them are devoted to the bosonic string theory. The purpose of this letter is to calculate the propagator of open fermionic string.

The starting point is the action of two-dimensional supergravity [6] ($2\pi\alpha' = 1$):

$$S = \int d\tau d\sigma \sqrt{g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu x^M \partial_\nu x^M - \frac{i}{2} \bar{\Psi}^M \gamma^\mu \partial_\mu \Psi^M - i \bar{\chi}_\mu \gamma^\nu \gamma^\mu \Psi^M \partial_\nu x^M - \frac{1}{4} (\bar{\Psi}^M \Psi^M) (\bar{\chi}_\mu \gamma^\nu \gamma^\mu \chi_\nu) \right] \quad (1)$$

interacting with D ($M=1, \dots, D$) supermultiplets of matter. The action (1) is invariant under local reparametrizations and local supersymmetry.

Choosing different boundary conditions we obtain either bosonic (Neveu-Schwarz string) or fermionic (Ramond string) spectrum.

Let us consider the last case, corresponding in limit to the spin-1/2 particle [7]. To carry out the limiting procedure, it should be pointed out, that due to super Weyl invariance in (1) we can choose the gravitino, satisfying the following relations:

$\gamma^\mu \chi_\mu = 0$. Then, contracting the world-sheet into the world-line one can require for fermions $\psi_1(\sigma, \tau) = \pm \psi_2(\sigma, \tau)$ ($\alpha = 1, 2$ - two-dimensional spinor indices) only in Ramond sector (because the boundary conditions in Ramond sector are $\psi_1 = \pm \psi_2$ when $\sigma = 0$ or $\sigma = 1$ with the same sign). Choosing the two dimensional metric in the form: $ds^2 = e^2(\sigma, \tau) d\tau^2 + d\sigma^2$ and taking the limit when the width of world-sheet tends to zero, we get (after the re-

definition the variables by some constant factors) the fermionic particle or one dimensional supergravity action:

$$\int d\tau \frac{1}{2} \left[e^{-1} \dot{x}^M \dot{x}^M - i \psi^M \dot{\psi}^M - i e^{-1} \dot{\chi}^M \psi^M \right] \quad (2)$$

The propagator corresponding to (2) is the Green function of massless Dirac particle [9,10]. It should be possible to single out Dirac particle propagator from the propagator of Ramond string.

The result of the calculation of the ^{open} bosonic string propagator has the form (in critical dimension D=26):

$$\begin{aligned} & \int_{x_1(\sigma)}^{x_2(\sigma)} dx \exp \left[- \int d\sigma \sqrt{g} \frac{1}{2} g^{\mu\nu} \partial_\mu x^M \partial_\nu x^M \right] = \\ & = \int_0^{\frac{2\pi}{T}} \frac{dT}{T^{13}} e^{\pi T} \prod_{n=1}^{\infty} (1 - e^{-2nT})^{-12} \exp \left[- \frac{(x_2^i - x_1^i)^2}{2T} - S_{cl}(x_1^i, x_2^i) \right] \quad (3) \end{aligned}$$

For pointlike boundary conditions (when $x_n^{2,1} = 0$ for $n=1,2,\dots$) the expression (3) can be written as the sum of propagating scalar string modes:

$$\int \frac{d^{26}p}{(2\pi)^{26}} e^{ip(x_2^i - x_1^i)} \sum_{N=0}^{\infty} \frac{C_N}{p^2 + m_N^2} =$$

$$= \int \frac{d^{26}p}{(2\pi)^{26}} \exp[ip\Delta x] \left(\frac{1}{p^2 - 1/\alpha'} + 12 \frac{1}{p^2 + 1/\alpha'} + \dots \right) \quad (4)$$

The right hand side of (4) is the sum of scalar particle propagators - tachyon ($m^2 = -1/\alpha'$) and so on with corresponding weight factors. We can obtain the similar expression for some simple boundary conditions in the case of open fermionic string.

Defining the integration measure for fermionic string path integral one should pay attention to the integration over modular and supermodular parameters [10,11]. The remaining integrations give the volume of symmetry group of the action (1) (in critical dimension D=10), which can be singled out. In critical dimension there is no dynamical contribution of Weyl and super Weyl modes

[2], however the dependence upon the boundary values of corresponding ghost fields can be present. This problem will not be considered in this letter. In the case of simple (pointlike) boundary conditions the propagator can be defined in an invariant way.

Let us turn to the calculation of integration measure following [12]. In a Wess-Zumino gauge the action can be written in the following form (after the field redefinition):

$$S = \int d^2\sigma \left[\partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + \chi \bar{\psi} \bar{\partial} X + \right. \\ \left. + \bar{\chi} \psi \partial X + \frac{1}{2} \psi \bar{\psi} \chi \bar{\chi} \right] \quad (5)$$

(the sum over $M=1, \dots, D$ is omitted, for example $\partial X \bar{\partial} X = \partial X^M \bar{\partial} X^M$; $\psi = \psi_1$, $\bar{\psi} = \bar{\psi}_2$ etc.). The variation of supergravity fields can be written as ($\delta h_{\mu}^a = e_{\mu}^a \delta e_{\mu}^a$):

$$\delta h_{\bar{z}}^z = \bar{\partial} \xi - \frac{1}{2} \bar{\chi} \epsilon + \eta \delta T \\ \delta \bar{\chi} = -\xi \partial \bar{\chi} + \frac{1}{2} \bar{\chi} \partial \xi + \bar{\partial} \epsilon + \bar{\rho} \delta \bar{z} \quad (6)$$

and the expressions obtained from (6) by interchange of barred and unbarred symbols. The final terms in the right hand side of (6) correspond to the variation in the "modular" and "supermodular" directions - these variations cannot be written in the form of reparametrization ($(-1,0)$ -differential ξ) and supersymmetry ($(-1/2,0)$ -differential ϵ). In (6) it is supposed that these variations can be represented by variation of one real modular parameter T and one supermodular Grassman parameter ξ , when $\eta = \text{const}$, $\bar{\rho} = \text{const}$.

Then, the measure is

$$Dh D\chi D\bar{h} D\bar{\chi} \sim \text{sdet } \Delta [\text{Vol}] dT d\xi \quad (7)$$

where $\text{sdet } \Delta$ (the Berezinian of transformation) can be written as the ghost $(c, \bar{c}, \gamma, \bar{\gamma})$ and antighost $(b, \bar{b}, \beta, \bar{\beta})$ path integral:

$$\text{sdet } \Delta = \int db Dc D\bar{b} D\bar{c} D\beta D\gamma D\bar{\beta} D\bar{\gamma} \langle \eta, b \rangle \delta(\langle \rho, \beta \rangle) \times \\ \times \exp \left\{ - \int d^2\sigma [b\bar{\partial}c + \beta\bar{\partial}\gamma + \bar{b}\partial\bar{c} + \bar{\beta}\partial\bar{\gamma} + \chi G_{gh} + \bar{\chi} \bar{G}_{gh}] \right\}$$

where

$$\langle \eta, b \rangle = \int d^2\sigma (\eta b + \bar{\eta} \bar{b}) \quad (8)$$

$$\langle \rho, \beta \rangle = \int d^2\sigma (\bar{\rho}\beta + \rho\bar{\beta})$$

$\chi = \rho\gamma$, G_{gh} - ghost current

Then, taking $\eta = \text{const}$, $\rho = \text{const}$, for propagator we obtain:

$$\int dT dz \int db Dc D\bar{b} D\bar{c} D\beta D\gamma D\bar{\beta} D\bar{\gamma} D\chi D\psi D\bar{\psi} \langle \eta, b \rangle \times \\ \times \delta(\langle \rho, \beta \rangle) \exp \left\{ - \int d^2\sigma [\partial\chi \bar{\partial}\chi + \psi \bar{\partial}\psi + \bar{\psi} \partial\bar{\psi} + \right. \\ \left. + b\bar{\partial}c + \bar{b}\partial\bar{c} + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} + \rho\gamma (\bar{\psi} \bar{\partial}\chi + \frac{1}{2} \bar{b}\bar{\gamma} + \right. \\ \left. + \frac{1}{2} \bar{\beta} \partial\bar{c}) + \bar{\rho}\gamma (\psi \partial\chi + \frac{1}{2} b\gamma + \frac{1}{2} \beta \partial c)] \right\} \quad (9)$$

Let us integrate in (9) over supermodular parameter and over antighost zero modes (boundary conditions ensure that ghosts have no zero modes). To integrate over antighost zero modes the measure should be written in the form:

$$db D\bar{b} = db_0 D\bar{b} D'\bar{b} , D\beta D\bar{\beta} = d\beta_0 D'\beta D'\bar{\beta} \quad (10)$$

The zero mode of (super) antighost corresponds to the single super (modular) transformation. Then, from (9) we obtain:

$$\int_0^\infty \frac{dT}{T} \frac{\det \Delta_{-1}}{\sqrt{T}} \left(\frac{\det \Delta_{-1/2}}{\sqrt{T}} \right)^{-1} (\det -\square_0)^5 (\det \Delta_{1/2})^5 \times \\ \times \int d^2\sigma (\bar{\psi}_{cl} \bar{\partial}\chi_{cl} + \psi_{cl} \partial\chi_{cl}) \exp \left[- \frac{(X_0^i - X_0^i)^2}{2T} - S_{cl}(X_n^i, X_n^i) \right] \quad (11)$$

$$X_{i,f}(\sigma) = X_0^{i,f} + \sum_{n=1}^{\infty} X_n^{i,f} \cos \pi n \sigma$$

where we do not pay attention to the "classical" ghost and anti-ghost contribution. In connection with (11) we should bear in mind that fermion determinants are calculated taking antiperiodic boundary condition in $\tau = \sigma_2$ direction. Only the antiperiodic boundary conditions give the correct string spectrum and the correct result for scattering amplitude when introducing external field. But it should be stressed that zero mode of supersymmetry transformation satisfy Dirichlet boundary conditions (different boundary conditions on zero and non-zero modes were suggested in [13]). Calculating determinants in (11), one has:

$$\begin{aligned} & \frac{\det \Delta_{-1}}{\sqrt{T}} \left(\frac{\det \Delta_{-1/2}}{\sqrt{T}} \right)^{-1} (\det -\square_0)^{-5} (\det \Delta_{1/2})^5 = \\ & = \eta(\tau) [\Omega(\tau)]^{-1} [\eta(\tau)]^{-5} [\Omega(\tau)]^5 = \\ & = T^{-5} \prod_{n=1}^{\infty} \left(\frac{1+e^{-\pi n \tau}}{1-e^{-\pi n \tau}} \right)^4, \text{ where} \end{aligned} \quad (12)$$

$$\eta(\tau) = e^{-\frac{\pi T}{12}} \prod_{n=1}^{\infty} (1 - e^{-2\pi n \tau}), \quad \Omega(\tau) = e^{-\frac{\pi T}{12}} \prod_{n=1}^{\infty} (1 + e^{-\pi n \tau})^2$$

Let us study the general formula (11).

In the case of pointlike boundary conditions ($X_i(\sigma) = X_0^i$, $X_f(\sigma) = X_0^f$) the result for the propagator will be:

$$\begin{aligned} & \int_0^{\infty} \frac{dT}{T^5} \prod_{n=1}^{\infty} \left(\frac{1+e^{-\pi n \tau}}{1-e^{-\pi n \tau}} \right)^4 \psi_0^M \frac{(X_0^f - X_0^i)^M}{T} \exp \left[-\frac{(X_0^f - X_0^i)^2}{2T} \right] \sim \\ & \sim \int \frac{d^{10}p}{(2\pi)^{10}} e^{ip\Delta X} \not\propto \sum_{N=0}^{\infty} \frac{C_N^i}{p^2 + m_N^2} = \end{aligned} \quad (13)$$

$$= \int \frac{d^{26}p}{(2\pi)^{26}} e^{ip\Delta X} \not{p} \left[\frac{1}{p^2} + 8 \frac{1}{p^2 + 1/\alpha'} + \right. \\ \left. + 40 \frac{1}{p^2 + 2/\alpha'} + 160 \frac{1}{p^2 + 3/\alpha'} + 552 \frac{1}{p^2 + 4/\alpha'} + \dots \right]$$

One can obtain (when $\alpha' \rightarrow 0$) the massless Dirac particle propagator from (13). Massive terms correspond to the propagation of fields, satisfying some unlocal equations of motion.

The expression (13) can be obtained in operator formalism.

The propagator of Ramond string in light-cone gauge is:

$$G = F_0 \int_0^{\infty} dT \exp(-TL_0), \text{ where}$$

$$F_0 = \frac{1}{\sqrt{2}} \not{p} + \frac{1}{\sqrt{\alpha'}} \sum_{n \neq 0} \alpha_{-n}^i \psi_n^i \quad (14)$$

$$L_0 = p^2 + \frac{1}{\alpha'} N, \quad N = \sum_{n=1}^{\infty} (\alpha_{-n}^i \alpha_n^i + n \psi_{-n}^i \psi_n^i)$$

($i=1, \dots, 8$ - the number of transverse directions, the sum over them is meant). For transition between initial and final point-like states one should compute the matrix element:

$$\langle\langle X_f, \alpha \parallel G \parallel X_i, \beta \rangle\rangle,$$

$$\parallel X, \alpha \rangle\rangle = |x\rangle \otimes \exp\left[\frac{1}{2} \sum_{n=1}^{\infty} n^{-1} \alpha_{-n}^i \alpha_n^i\right] |0\rangle \otimes \hat{\Psi} |u_\alpha\rangle \quad (15)$$

where $\hat{\Psi}$ is pure fermionic (two-dimensional) operator. We fix only a part of quantum numbers in the initial and final states, that is why we should trace over the others. For calculation it is convenient to use the eigenfunctions of fermionic number operator. Calculating (15), one has:

$$\langle\langle X_f, \alpha \parallel G \parallel X_i, \beta \rangle\rangle = (\not{p})_{\alpha\beta} \sum_{N=0}^{\infty} \frac{C_N}{p^2 + m_N^2} \quad (16)$$

where $\alpha'_N m_N^2 = N$ ($N=0,1,\dots$) - the spectrum of Ramond string. The weight number C_N is the sum of the norms of the states of N^{th} mass level. The contribution of pure fermionic states to C_N is equal to their number (all of them are normalized to unity), the contribution of boson states of each mass level:

$$\exp\left[\frac{1}{2}\sum_{n=1}^{\infty} n^{-1}\alpha_{-n}^i\alpha_{-n}^i\right]|0\rangle = \left(1 + \frac{1}{2}\alpha_{-1}^i\alpha_{-1}^i + \frac{1}{4}\alpha_{-2}^i\alpha_{-2}^i + \frac{1}{8}\alpha_{-1}^i\alpha_{-1}^i\alpha_{-1}^i\alpha_{-1}^i + \dots\right)|0\rangle \quad (17)$$

is equal to their norms which are calculated directly. As a result, for C_N one has:

$$\begin{aligned} C_0 &= 1 \\ C_1 &= 8 \quad (\psi_{-1}^i|0\rangle, i=1,\dots,8) \\ C_2 &= 40 \quad (= 8 + 28 + 4) \\ C_3 &= 160 \quad (= 8 + 64 + 56 + 32) \\ C_4 &= 552 \quad (= 8 + 64 + 28 + 224 + 70 + 4 + 10 + 32 + 112) \\ &\text{etc} \end{aligned} \quad (18)$$

i.e. the C_N numbers in (18) coincide with C'_N numbers in (13). Let us demonstrate the last equality in (18) by the table:

$\psi_{-4}^i 0\rangle$	8
$\psi_{-3}^i\psi_{-1}^j 0\rangle$	64
$\psi_{-2}^i\psi_{-2}^j 0\rangle$	28
$\psi_{-2}^i\psi_{-1}^j\psi_{-1}^k 0\rangle$	224
$\psi_{-1}^i\psi_{-1}^j\psi_{-1}^k\psi_{-1}^l 0\rangle$	70
$\frac{1}{4}\alpha_{-2}^i\alpha_{-2}^i 0\rangle$	4
$\frac{1}{8}\alpha_{-1}^i\alpha_{-1}^i\alpha_{-1}^j\alpha_{-1}^j 0\rangle$	10
$\frac{1}{2}\alpha_{-1}^i\alpha_{-1}^i\psi_{-2}^j 0\rangle$	32
$\frac{1}{2}\alpha_{-1}^i\alpha_{-1}^i\psi_{-1}^j\psi_{-1}^k 0\rangle$	112

The position of poles and residues in these points in (4), (13), (16) are gauge invariant quantities. Thus calculating a propagator by operator methods one can use Lorentz covariant expressions for F_0 and L_0 .

In conclusion let us point out that (16) can be written in the form of Kallen-Lehmann representation with spectral density

$$\rho(N) = C_N. \text{ For bosonic string in (4):}$$

$$C_N \sim \exp\left\{a\sqrt{\alpha'}\sqrt{\frac{N}{\alpha'}}\right\}, \quad a = \pi\sqrt{\frac{D-2}{3}} \quad (19)$$

For fermionic string in (13) and (16):

$$C_N \sim \exp\left\{b\sqrt{\alpha'}\sqrt{\frac{N}{\alpha'}}\right\}, \quad b = \pi\sqrt{\frac{D-2}{2}} \quad (20)$$

The expressions (19) and (20) correspond to nonlocalizable theory, when spectral density increases as a linear exponent of mass [14]. In this case Feynman Green function of string (in Minkowski space) with pointlike boundary conditions has pole singularity in spacelike domain for critical dimensions, when:

$$-(\Delta x)^2 = \ell^2 = (2\pi)^2 \alpha'$$

where ℓ is the fundamental length. From general considerations it can be shown that for general boundary conditions the domain of nonlocality will not encrease.

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