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CONFORMAL INVARIANCE AND STRING EFFECTIVE ACTIONS

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ABSTRACT

A new approach to the off-shell effective action in string theories is presented. Our method, illustrated for the tachyon background, elucidates the relation between conformal invariance and equations of motion for the background. For a particular choice of the pole subtraction, we derive a differential equation for the effective action which is similar to the one proposed by Zamolodchikov for the central charge.

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1. INTRODUCTION

The effective action is an important concept in field theory expected to be useful for dynamical questions such as the determination of the ground state of the theory. In the last several years, a lot of elaborate work has been done towards understanding how to extend this concept to the case of string theory. We have acquired plenty of knowledge concerning, for example, equations of motion for the fields corresponding to each string excitation mode.

In our opinion, however, there are still conceptual as well as technical problems which have not yet been fully discussed. In the following, we would like to present a new approach to the effective action, giving a new insight into these problems. The present work is based on our collaboration¹ with G. Veneziano.

2. NON-LINEAR SIGMA MODEL AND S-MATRIX APPROACHES

Let us begin with a quick survey of two approaches to string theory effective actions which are commonly used in the literature, thereby pointing out several aspects that have not yet been fully discussed.

The first one is the so-called non-linear sigma model approach, which usually starts with the Polyakov-type string action² with appropriate background fields³⁻⁸. In the case of the closed bosonic string, the action takes the following form (for the massless background)

$$S = 1/2\pi\alpha' \int d^2z \left\{ \frac{1}{2} \sqrt{g} g^{\mu\nu} G_{\mu\nu}(X) \partial_m X^\mu \partial_n X^\nu + \frac{1}{2} \epsilon^{\mu\nu} B_{\mu\nu}(X) \partial_m X^\mu \partial_n X^\nu + \alpha' \sqrt{g} R^{(2)} \Phi(X) \right\}, \quad (2.1)$$

where $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ are background fields corresponding to the massless modes of the string (graviton, antisymmetric tensor and dilaton) and are functions of the string variables X^μ ($\mu = 1, \dots, D = 26$).

The principle to determine the background fields is that of conformal invariance, i.e., the vanishing β -function conditions. The β -functions associated with $G_{\mu\nu}$, $B_{\mu\nu}$ and Φ are calculated in a rather conventional way⁹ and we get a set of equations for the background fields.

The second approach, which is also popular in the literature, is the S-matrix approach. The

S-matrix is an on-shell concept in the sense that external particles are put on the mass shell, and in string theories we are given on-shell scattering amplitudes. By looking at the scattering amplitudes we may be able to guess the action from which they are derived.

Perhaps we may proceed more systematically in the following way. Suppose we can extend the string S-matrix off-shell. Then we may define a functional which generates connected (off-shell) Green's functions for, say, string massless modes. In exactly the same way as in the usual field theories, this generating functional is converted by the Legendre transformation into another functional generating the one-particle-irreducible parts, namely, the effective action Γ . The effective action will be a functional of the classical fields corresponding to the string modes, graviton ($\bar{G}_{\mu\nu}$), antisymmetric tensor ($\bar{B}_{\mu\nu}$), dilaton ($\bar{\phi}$), and so on, i.e., $\Gamma = \Gamma[\bar{G}_{\mu\nu}, \bar{B}_{\mu\nu}, \bar{\phi}, \dots]$. [We use a bar to distinguish classical fields from the sources appearing in Eq. (2.1).] The equations of motion for these fields are obtained by the stability conditions

$$\delta\Gamma/\delta\bar{G}_{\mu\nu} = \delta\Gamma/\delta\bar{B}_{\mu\nu} = \delta\Gamma/\delta\bar{\phi} = 0. \quad (2.2)$$

The questions that we would like to ask ourselves in the present work are the following.

- (I) What is the relation between the non-linear sigma model and S-matrix approaches?

We have two conceptually different kinds of equations, the vanishing β -function condition and the stability condition (2.2). The β -function is basically a two-dimensional concept, while the effective action is a D-dimensional space-time concept. The relation between the

two is not quite obvious, and we have to find a link connecting the two.

- (II) How do we extend the string scattering amplitudes to the case of off-shell in the S-matrix approach?

It is known that the off-shell extrapolation of the scattering amplitudes in string theories is ambiguous. We have to make clear what properties we do wish to keep in defining off-shell amplitudes.

In the following we will give a new approach to the string effective action, which is, in a sense, a combined approach of the two methods. We will start with the Polyakov path integral with background fields, and after going off-shell in a particular way, we derive the effective action. We will study the conformal factor dependence of the effective action, giving a link between the two methods. As to the off-shell extension, we make full use of old¹⁰ as well as recent^{11,12} results.

3. TACHYON BACKGROUND

Throughout this paper, we deal with the closed bosonic string. To make our ideas as clear as possible we consider only the tachyon background, but extension to other background fields will go along the same lines. For the time being, we consider the sphere as a two-dimensional world surface of the string. Extension to the string-loop case will be given in Section 9.

The partition function in the presence of the (Fourier-transformed) tachyon background $\phi^{(0)}(p)$ is written as a path integral

$$Z = \int \frac{Dx^{\mu} Dq^{mn}}{\Omega(\text{gauge})} \exp\{-S_0 + \int d^D p \phi^{(0)}(p) V^{(0)}(p)\}, \quad (3.1)$$

where S_0 is the free Euclidean string action

$$S_0 = 1/4\pi\alpha' \int d^2z \sqrt{g} g^{mn} \delta_{\mu\nu} \partial_m X^\mu \partial_n X^\nu \quad (3.2)$$

and $V^{(0)}(p)$ is the tachyon emission vertex

$$V^{(0)}(p) = \int d^2z \sqrt{g} \exp(ip \cdot X). \quad (3.3)$$

The superscript zero in $\phi^{(0)}(p)$ and $V^{(0)}(p)$ indicates that they are unrenormalized quantities. The gauge volume $\Omega(\text{gauge})$ is the one associated with the two-dimensional diffeomorphism and conformal invariance. The two-dimensional metric for the sphere may be taken as conformally flat

$$ds^2 = e^{\sigma(z)} dz d\bar{z}, \quad (3.4)$$

where $\sigma(z)$ is the conformal factor.

Let us note that the partition function (3.1) may be related to the generating functional of connected Green's functions for tachyons. This can be seen by noting that the n (> 3) tachyon amplitudes are expressed as

$$\begin{aligned} \delta^n Z / \delta \phi^{(0)}(p_1) \cdots \delta \phi^{(0)}(p_n) \\ = \langle V^{(0)}(p_1) \cdots V^{(0)}(p_n) \rangle. \end{aligned} \quad (3.5)$$

Once we have the generating functional of the connected Green's functions we can, in the same way as in the usual field theories, derive the effective action straightforwardly by applying the Legendre transformation.

Before doing so, however, the following problems must be cleared up.

(i) The scattering amplitudes appearing in Eq. (3.5) are the on-shell amplitudes and

we have to extend them somehow to the off-shell case before applying the Legendre transformation.

(ii) The amplitudes in Eq. (3.5) entail two-dimensional short-distance divergences. We have to specify how to absorb the divergences into the renormalized tachyon background field $\phi(p)$.

(iii) The scattering amplitudes in (3.5) are given a definite meaning for $n > 3$, but not for $n = 2$ because of the $SL(2, C)$ invariance. We have to supplement the two-point function and then proceed to apply the Legendre transformation.

We will discuss these three points in the following sections.

4. OFF-SHELL AMPLITUDES

The off-shell extension of the scattering amplitudes is known to be ambiguous and is an interesting subject in its own right. The following consideration, however, gives general restrictions on the way to go off-shell.

When the external states are on-shell, even after fixing the two-dimensional diffeomorphism and Weyl invariance, the usual scattering amplitudes possess an $SL(2, C)$ invariance. We factor out the (infinite) invariance volume associated with this symmetry by fixing three of the positions of external particles. In order to get a smooth off-shell extrapolation starting from the path integral method, it appears to us that the $SL(2, C)$ invariance and its fixing procedure is a crucial element that should be preserved for the off-shell amplitudes as well.

Another property that we would like to have off the mass shell is the factorization of the scattering amplitudes. If the factorization

were not fulfilled, there would be little hope of getting a consistent effective action.

The old¹⁰ as well as recent¹¹ investigations show that, in fact, there exists an off-shell extension of scattering amplitudes such that, while the conformal invariance is spoiled, one can still maintain the $SL(2,C)$ invariance. As to the factorization, we have verified explicitly that the off-shell tachyonic amplitudes presented in Ref. 11 factorize at the tachyon pole. These two properties rest on the fact that the amplitudes in Ref. 11 contain the conformal factor σ which depends on a certain fixed, but otherwise arbitrary, point z_0 . By using their amplitudes, one can study the relation between the requirement of the conformal invariance and the equations of motion for the backgrounds.

At the moment, it is not quite clear where and how to place the off-shell extension in the Polyakov framework in a coherent fashion. In the Polyakov method, the path integral over the conformal mode is rather trivial at $D = 26$, while in our case it is not so, because of the off-shellness. We will forget about the conformal mode path integral and will study the conformal factor dependence of the integrand.

Let us conclude this section by summarizing our basic assumptions, which are indispensable for our programme to work well.

Assumptions

There exists an off-shell extension of the scattering amplitudes such that:

- (A1) $SL(2,C)$ invariance is preserved;
- (A2) the amplitudes satisfy the factorization property.

As far as these assumptions are satisfied, the following arguments will not depend on any further details of the prescription of the off-shell extension.

5. RENORMALIZATION

Let us go on to the renormalization of the two-dimensional short-distance infinities. The renormalization has been extensively discussed in the literature in the framework of non-linear sigma models. In the following we will take an equivalent but seemingly different procedure.

The n -point tachyon scattering amplitudes (3.5) are expressed as an integral of

$$\exp\{i\pi\alpha' \sum p_i \cdot p_j N(z_i, z_j)\} \quad (5.1)$$

over the positions $\{z_i\}$ of the vertices. Here $N(z_i, z_j)$ is the Neumann function on the sphere. The divergence comes from the $i = j$ terms in the sum in Eq. (5.1) because of the logarithmic singularity of the Neumann function. The structure of divergence is not changed in the off-shell amplitudes given in Ref. 11.

The divergence may be regularized by introducing a cut-off in the Neumann function. Namely, we should use the following regularized form of the Neumann function:

$$N(z, z') = \frac{1}{4\pi} \ln\{|z - z'|^2 + e^{-\sigma} a^2\}, \quad (5.2)$$

where a is an invariant cut-off and σ is the conformal factor. This regularization renders the $i=j$ terms in the form $\exp\{\alpha'/4 \sum (p_i)^2 \ln(a^2 e^{-\sigma})\}$.

This cut-off-dependent factor is of the factorized form with respect to each external leg and may be absorbed into $\phi^{(0)}(p_i)$. We are thus led to define the renormalized tachyon background field $\phi(p)$ by

$$\phi(p) = (a^2)^{\alpha' p^2/4} \phi^{(0)}(p). \quad (5.3)$$

Note that the relation (5.3) includes all-order effects in α' , which is in contrast with the non-linear sigma model approach. The generating functional of the connected Green's functions turns out to be a finite functional in terms of the renormalized field (5.3).

6. LEGENDRE TRANSFORMATION

The generating functional (3.1) is now expanded in power series of the renormalized field $\phi(p)$

$$Z = T[\phi] = \sum \frac{1}{n!} \int \prod_{i=1}^n d^D p_i \prod_{i=1}^n \phi(p_i) \times T_n(p_1, \dots, p_n), \quad (6.1)$$

where the (renormalized) off-shell tachyon scattering amplitudes $T_n(p_1, \dots, p_n)$ are to be understood as those of Refs. 11 and 12. Since we have laid aside the conformal mode path integral, these amplitudes contain the conformal factor explicitly.

The functional (6.1) needs further clarification with respect to the $n = 2$ kinetic term in the sum. This term is not well defined in our framework, where the $SL(2, \mathbb{C})$ invariance holds off-shell: we are not able to fix the $SL(2, \mathbb{C})$ invariance completely for the $n = 2$ term. Under these circumstances, one has to introduce an ad hoc procedure giving a certain form to the kinetic term.

The most naive choice for the $n = 2$ term will be

$$-1/2 \int d^D p \phi(-p) (p^2 + m^2) \phi(p), \quad (6.2)$$

where $m^2 = -4/\alpha'$ is the tachyon mass. It is, however, not really necessary for the following discussions to give such an explicit form. We would rather keep our consideration as general as possible, taking the following kinetic term

$$T[\phi] = 1/2 \int d^D p \phi(-p) \Pi(p^2) \phi(p) + T'[\phi], \quad (6.3)$$

where $T'[\phi]$ contains all the sum starting from $n = 3$. The kinetic factor $\Pi(p^2)$ behaves like $\Pi(p^2) \sim -(p^2 + m^2)$ near the pole $p^2 = -m^2$, and its inverse (the propagator) does not have any other pole. This is all we need to characterize the kinetic term.

We are now in a position to apply the Legendre transformation to get the effective action. An important remark to be made here is that the amplitudes T_n in Eq. (6.1) are truncated ones (T stands for truncated). The propagator $\Pi^{-1}(p^2)$ is not attached to the external legs. In order that $T[\phi]$ should be the usual generating functional of the connected Green's functions, $W[J]$, say, one has to put the source function $J(p)$ equal to

$$J(p) = \Pi(p^2) \phi(p). \quad (6.4)$$

Under this identification, we are allowed to set

$$T[\phi] = W[J]. \quad (6.5)$$

A similar observation has also been made in Ref. 13.

Once we have Eqs. (6.4) and (6.5), the Legendre transformation goes straightforwardly. The effective action is given by

$$\Gamma[\bar{\phi}] = W[J] - \int d^D p \bar{\phi}(-p) J(p), \quad (6.6)$$

where the classical field $\bar{\phi}$ has been defined by

$$\bar{\phi}(-p) = \delta W[J] / \delta J(p). \quad (6.7)$$

The effective action (6.6) gives us a one-particle-irreducible part with respect to the tachyon and generates back $T[\phi]$ from tree diagrams with the propagator $\Pi^{-1}(p^2)$. Note, in passing, that Eq. (6.7) provides us with a field redefinition, expressing $\bar{\phi}$ in terms of ϕ . This is a non-linear, non-local field redefinition.

7. EQUATION OF MOTION

The equation of motion is obtained through minimization of the effective action, which turns out to be the vanishing of the source function

$$\delta \Gamma[\bar{\phi}] / \delta \bar{\phi}(-p) = -J(p) = -\Pi(p^2)\phi(p) = 0. \quad (7.1)$$

If we choose $\Pi(p^2) = -(p^2 + m^2)$, then Eq. (7.1) tells us that $\phi(p)$ should obey the free Klein-Gordon equation

$$(p^2 + m^2)\phi(p) = 0. \quad (7.2)$$

Equation (7.1) may be re-expressed in terms of $\bar{\phi}$. Recall that Eq. (6.7) provides us with a relation between ϕ and $\bar{\phi}$, and we can go back and forth between these two field variables. More explicitly, the relation becomes

$$\begin{aligned} \bar{\phi}(p) = \phi(p) + \sum_{n=2}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n d^D p_i \prod_{i=1}^n \bar{\phi}(p_i) \\ \times T_{n+1}(p_1, \dots, p_n, -p) \Pi(p^2)^{-1}. \end{aligned} \quad (7.3)$$

One may use iteration to express ϕ in terms of $\bar{\phi}$, and Eq. (7.1) is transformed into

$$\begin{aligned} 0 = \Pi(p^2)\bar{\phi}(p) - \sum_{n=2}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n d^D p_i \prod_{i=1}^n \bar{\phi}(p_i) \\ \times T_{n+1}(p_1, \dots, p_n, -p) \\ + \dots \end{aligned} \quad (7.4)$$

The ellipsis in the last line of Eq. (7.4) stands for terms containing more than one T_n .

Equation (7.4) for $\bar{\phi}$ is rather complicated because of the string interaction terms, while Eq. (7.1) is of the free type. The field redefinition (6.7) gives the mechanism¹⁴ for turning the free-type equation into the fully-interacting equation of motion. For the choice $\Pi(p^2) = -(p^2 + m^2)$, Eq. (7.4) agrees with the equations of motion discussed in Refs. 13 and 15 for a different method.

8. CONFORMAL INVARIANCE

Let us move on to the relation between the conformal invariance and the equation of motion derived in Section 7. This will give us a link connecting the S-matrix approach with the non-linear sigma model method.

The conformal factor dependence of the interaction part $T'[\phi]$ in Eq. (6.3) may be extracted from the work of Ref. 11, which shows the following structure

$$\partial T' / \partial \sigma = \int d^D p K(p^2) \phi(p) \delta T' / \delta \phi(p). \quad (8.1)$$

Here we use the notation

$$K(p^2) = (-\alpha' p^2/4 = -\alpha'(p^2+m^2)/4. \quad (8.2)$$

Note that the conformal invariance of $T'[\phi]$ implies the free equation (7.2).

What is unknown to us a priori is the conformal factor dependence of the kinetic term in Eq. (6.3), i.e., $\Pi(p^2)$. We do not delve into a detailed study of this dependence. Instead, we have studied in Ref. 1 the following three conceivable cases:

$$(a) \Pi(p^2) = -(p^2+m^2), \quad \partial\Pi(p^2)/\partial\sigma = 0, \quad (8.3a)$$

$$(b) \partial\Pi(p^2)/\partial\sigma = 2K(p^2)\Pi(p^2), \quad (8.3b)$$

$$(c) \partial\Pi(p^2)/\partial\sigma = K(p^2)\Pi(p^2). \quad (8.3c)$$

Choice (a) is the most naive one, and the second is such that the conformal factor dependence of $T[\phi]$ obeys the same equation as that of $T'[\phi]$, i.e., Eq. (8.1). The third case, (c), is something in between.

Once we have Eqs. (8.1) and (8.3), it is an easy exercise to derive, through the Legendre transformation, a corresponding equation satisfied by the effective action. We have, in fact, found in Ref. 1 the following equations corresponding to the cases in Eq. (8.3):

$$(a) \partial\Gamma/\partial\sigma = -\alpha'/4 \int d^D p \delta\Gamma/\delta\bar{\phi}(p) \times \delta T'/\delta\phi(-p), \quad (8.4a)$$

$$(b) \partial\Gamma/\partial\sigma = \int d^D p K(p^2)\bar{\phi}(p)\delta\Gamma/\delta\bar{\phi}(p), \quad (8.4b)$$

$$(c) \partial\Gamma/\partial\sigma = -1/2 \int d^D p \delta\Gamma/\delta\bar{\phi}(-p) \times K(p^2)\Pi^{-1}(p^2)\delta\Gamma/\delta\bar{\phi}(p). \quad (8.4c)$$

Equation (8.4) clearly shows that, once we activate the equation of motion (7.1), then the conformal invariance of the effective action automatically follows.

It is amusing to note here that Eq. (8.4c) is of the same form as the equation derived by Zamolodchikov¹⁶ for the central charge in two-dimensional field theories. The analogy between our case and Zamolodchikov's is such that our effective action corresponds to his central charge and the change of the conformal factor in our case is analogous to the change of the renormalization point in Zamolodchikov's. The fact that in case (a) we find a simple formula¹⁶ for the β -function

$$\beta(\phi) = \frac{\alpha'}{4} \delta\Gamma[\bar{\phi}]/\delta\bar{\phi}. \quad (8.5)$$

also deserves attention.

9. EXTENSION TO THE TORUS

It is not difficult to extend our theoretical framework to the string-loop level. Let us consider the torus (one-loop) effect. One-loop scattering amplitudes have to be generalized off the mass shell; this will be fulfilled along the lines of Refs. 11 and 12. What then is the criterion we require of the one-loop off-shell amplitudes? One-loop on-shell amplitudes are given as an integral over the moduli space whose integration region is restricted to the fundamental region due to the modular invariance. It appears to us that the modular invari-

ance is a crucial property which ought to be preserved in order that we get smooth off-shell extension. The factorization is also a necessary property, for the same reason as before. We will assume hereafter that the one-loop off-shell amplitudes satisfy the modular invariance and factorization.

In addition to these assumptions, we should take into account the following difference between string theory and conventional field theories. In the latter, we usually sum up the self-energy bubbles of the one-particle-irreducible part to get the dressed propagator. In string theories, on the other hand, the separation of one-particle-reducible and irreducible parts is not straightforward on the loop level, and a simple-minded sum of the bubble diagrams does not work well. It is also to be noted that, to each finite order in the string-loop expansion, poles are always at the tree level positions.

Bearing these considerations in mind, we have put forward in Ref. 1 a modified Legendre transformation at the one-loop level. Our method works pretty well for deriving the effective action, generating functional of the one-particle-irreducible part. We have also studied in Ref. 1 the conformal factor dependence of the effective action, assuming that the torus two-point function should have the same conformal factor dependence as that of the tree. We have confirmed that, irrespective of the choices (8.3), the conformal invariance of the effective action does hold on the stability condition of the effective action. Furthermore, it is interesting to remark that, for the cases (8.3b) and (8.3c), both Eqs. (8.4b) and (8.4c) are unchanged, valid up to the torus level.

It is also amusing to see that as far as the functional $T[\phi]$ is concerned, the condition of conformal invariance (expressed in terms of ϕ) is given by the free equation (7.2). Yet the equation of motion for $\bar{\phi}$ receives string-loop

corrections, such as those to masses, couplings and so on. In fact, the mass renormalization shows up naturally in our scheme in the same way as has been worked out in the literature¹⁷. For further details, see Ref. 1.

10. SUMMARY

In the present work, we have given a new approach to the string effective action for tachyons at the tree and one-loop levels. Our starting point is the fact that the Polyakov path integral (3.1) is the generating functional of the tachyon scattering amplitudes.

Scattering amplitudes in string theories are well defined on the mass shell. We have required that the amplitudes be extended off-shell in such a way that the $SL(2, \mathbb{C})$ invariance and factorizability are preserved. The two-point function of the tachyon fields is introduced by hand in the general form (6.3). After these procedures, we have identified the Polyakov path integral as the generating functional of the connected Green's functions of tachyons.

Once we have the generating functional (6.5), it is almost straightforward to derive the (off-shell) effective action by the Legendre transformation. The effective action is constructed so that it reproduces the off-shell tachyon amplitudes with the propagator $\Pi(p^2)^{-1}$. While the original field $\phi(p)$ satisfies the free-type equation (7.1), the equation of motion for $\bar{\phi}(p)$ turns out to be a fully-interacting one (7.4).

The amplitudes in Ref. 11 contain the conformal factor and we can study the conformal factor dependence of the effective action explicitly. We have found that, irrespective of the details of the kinetic term, the conformal invariance of the effective action is always satisfied on the stable point of the effective action. This gives us a bridge between a two-dimensional notion like conformal invariance and a space-time concept such as the effective action. It is also

interesting that for a particular choice of the kinetic term, we have derived a Zamolodchikov-type equation for the effective action.

It will probably not be difficult to extend our work to the string massless modes (graviton, antisymmetric tensor and dilaton), and we can derive a similar relation between conformal invariance and equations of motion. The Zamolodchikov-type equation looks similar to the equation for the potential flow and we will be able to study the "flow" in connection with the stability of the vacuum. These subjects will be discussed elsewhere¹⁸.

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