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VACUUM DEGENERACY IN FOUR-DIMENSIONAL STRING THEORIES

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The vacuum structure of the heterotic string in ten space-time dimensions ($d = 10$) is quite simple and completely understood. The space-time supersymmetric theories come in only two versions with gauge group $E_8 \times E_8$ and $SO(32)$ respectively. This "uniqueness" is lost if one considers theories in less than ten dimensions. Attempts to investigate possible relations of string theory with elementary particle physics would require the knowledge of the vacuum structure (moduli space) of $d = 4$ string models with $N = 1$ space-time supersymmetry. In the language of conformal field theory this would amount to a classification of all $(2,0)$ or $(2,2)$ world sheet supersymmetric superconformal field theories with $6_R + 22_L$ dimensional target space, i.e., $\hat{c} = (2/3)c = 6$ for the right-handed and $c = 22$ for the left-handed co-ordinates. This classification is not yet complete. The historical development shows that the closer we look, the more models we find. Usually such models are found by the explicit construction of a four-dimensional string model in terms of a generalized compactification of the $d = 10$ heterotic string. But we do not know whether this procedure exhausts all possibilities; there might be string models which are consistent even if our present techniques do not allow us to prove this consistency.

In this talk I want to present results obtained in collaboration with A. Font, L. Ibáñez and F. Quevedo using a method that

links explicit string constructions with the techniques of supergravity field theories¹. We make use of the fact that the supersymmetric vacua of the field theory limit of $d = 4$ $N = 1$ superstring theories are all degenerate. Given a particular string theory we can then test for new "nearby" string theories by an examination of flat directions in the scalar potential of the underlying field theory. As input from string theory we need the knowledge of the Yukawa couplings (i.e., the superpotential) for any number of fields. In the language of conformal field theory, this amounts to a search for exactly marginal operators and the classification of multicritical points.

Our results indicate an enormous vacuum degeneracy for the examined models. Some of the flat directions can be identified with vacuum degeneracies that have already been observed in the exact construction of string theories². But others point to the existence of new models, which we cannot yet construct explicitly. The phenomenological prospects of these newly discovered models are quite exciting. They allow us to obtain small rank gauge groups, smooth ways to break gauge symmetries and flexibility for Yukawa couplings. Preliminary investigations indicate that they might contain realistic models with gauge group $SU(3) \times SU(2) \times U(1)$ and three generations of quarks and leptons.

But before we enter these details let us look at the "moduli space" of the $d = 4$ $N = 1$ string

models. Inspired by Ref. 3, consider Fig. 1 as a basis for discussions. We can distinguish several classes of models.

FIGURE 1

"Moduli space" of $d = 4$, $N = 1$ heterotic string theories

1. ORBIFOLD COMPACTIFICATIONS⁴⁻⁷

Orbifolds can be visualized geometrically as manifolds on which points have been identified due to the action of a discrete symmetry group. We are interested here in "flat orbifolds" that can be obtained by dividing a six-torus T^6 by a point group P . These objects are flat almost everywhere and provide a consistent background for string theory. They can give rise to models with $(2,0)$ or $(2,2)$ world sheet supersymmetry. The $(2,2)$ models correspond to the so-called standard embedding of the spin connection in the gauge group. The corresponding orbifold singularities can be blown up to give a Calabi-Yau manifold; in the diagram they constitute the overlap region of "orbifolds" and "Calabi-Yau". Asymmetric orbifolds⁷ are those models where the left- and right-handed string components live on different orbifolds. In that sense, one can regard $d = 4$ strings on orbifolds as a generalized compactification of the $d = 10$ heterotic string. The definition of "asymmetric" is

actually somewhat ill-defined since the heterotic string by itself is already left-right asymmetric.

2. COVARIANT LATTICE CONSTRUCTIONS^{8,9}

In this method the geometrical interpretation is not as transparent as in the case of orbifold compactification; the method has a closer connection to two-dimensional conformal field theory. Its basis is the construction of self-dual Lorentzian lattices using the extra bosons as well as bosonized fermions and the superconformal ghosts. The so obtained models are then modular invariant by construction. More involved, however, is the construction of the world sheet supercurrents. Again models with $(2,0)$ and $(2,2)$ world sheet supersymmetry can be found. All models have a gauge group of rank 22. It seems at the moment that these models are a subset of the models that could be obtained via orbifold compactification although this question is not completely clarified^{10,11}. One might speculate that this method gives all orbifold models with maximal rank (i.e., 22 in the case under consideration).

3. FREE FERMIONIC CONSTRUCTIONS¹²⁻¹⁴

The method uses the existence of "bosonization" in two-dimensional field theories. All the extradimensional $6_R + 22_L$ bosons are fermionized. Then various twisted boundary conditions are imposed on these fermions satisfying the constraints of modular invariance and world sheet supersymmetry. Again these models are believed to be contained in the class obtained by orbifold compactification. For details consult the talk of Antoniadis at this Conference.

4. TENSOR PRODUCTS OF $c < 3$ MODELS^{15,16}

This method uses superconformal models from the discrete series with central charge $c < 3$ as building blocks for the construction of space-

time supersymmetric string models. The models are tensored to give the correct conformal anomaly and further constraints from modular invariance have to be satisfied. Up to now the constructed models all have $(2,2)$ world sheet supersymmetry. Although this method is not very geometrical in its construction, it has been conjectured that this method gives models equivalent to conformal field theories on smooth Calabi-Yau manifolds¹⁷. It seems, however, more likely that also those models correspond to orbifolds, as has been demonstrated in some simple examples¹⁸, leading to the conjecture that the conformal field theories on some orbifolds could be equivalent to those of some smooth Calabi-Yau manifolds. While this interesting conjecture might be correct, we are still far from being able to prove it. The blob in Fig. 1 would have to be readjusted once this situation is clarified.

5. CALABI-YAU MANIFOLDS^{19,20}

The considerations of strings moving on this class of Ricci flat Kähler manifolds with $SU(3)$ holonomy has been at the origin of many phenomenologically inspired investigation. Using the standard embedding of the spin connection in the gauge group this is believed to be a consistent string background with $(2,2)$ world sheet supersymmetry. We are not yet able to construct explicitly the full string theory on such a smooth Calabi-Yau background and have to rely on topological arguments that determine the number and properties of the massless states. In that sense it is not proved beyond any doubt that this background provides an exact solution for string theory. The most convincing argument in this direction is given by the fact that string models on Calabi-Yau manifolds could be iteratively constructed through a "blowing-up" procedure²¹ from $(2,2)$ -models on orbifold backgrounds. It would also be interesting to

investigate whether one could formulate string theory on asymmetric Calabi-Yau manifolds. Details of the construction and a description of the properties of Calabi-Yau manifolds can be found in the talk of Lütken at this Conference.

6. DEGENERATE CALABI-YAU MODELS²²⁻²⁴

By this we mean strings on smooth manifolds that lead to models with $(2,0)$ -world sheet supersymmetry which are continuously connected to the $(2,2)$ -models given above. The geometrical construction of such manifolds seems to be quite complicated^{22,23}. In iterative procedure to obtain such models is again provided via the construction of orbifolds and a distortion of some parameters as discussed by Cvetic²⁴.

7. DEGENERATE ORBIFOLDS¹

Here one starts with any orbifold and examines the flat directions of the potential of the massless states with all the parameters of this potential computed in the underlying string theory. This leads to new models both in the case of symmetric and asymmetric orbifolds. Some of the models can be given a geometrical interpretation as blown up versions for $(2,0)$ manifolds. The models can be constructed only perturbatively in the vacuum expectation values of massless fields, but they are believed to be exact solutions, in the same way as Calabi-Yau models. Given the central rôle of orbifolds in the construction of lower-dimensional string models one might conjecture that the class of degenerate orbifolds eventually covers all of the moduli space of $d = 4$ $N = 1$ string models, although Fig. 1 does not yet indicate this.

As discussed above, the models in classes 1 to 4 can be explicitly constructed while those in classes 5-7 can only be obtained iteratively in the background values of some massless fields. It is not so surprising that these models in classes 6 and 7 exist, what is

surprising is the huge number of such theories. Even if we cannot construct these models explicitly, we are not allowed to ignore their existence. Such continuous adjustable parameters are also welcome²⁵ in the discussion of the phenomenological prospects of four-dimensional string theories (for a detailed discussion of these issues consult the talk of Ross at this Conference). The method used in cases 6 and 7 allows us to construct a full class of continuously connected models. Certain topological properties like a chiral structure of the fermion representation cannot be altered in this procedure. Whenever a gauge group is broken in such a way this will happen in a smooth way exhibiting a Higgs mechanism. Examples of such smooth transitions had been found already earlier in the case of torus compactification²⁶ as well as in the case of orbifolds and Wilson lines². It seems to us that such smooth transitions are often related to mechanisms that allow us to lower the rank of the gauge group.

In contrast to this smooth mechanism of supersymmetry breakdown string theories can also exhibit abrupt transitions. They occur whenever some background value is quantized and models for different discrete values can look fundamentally different, like having, for example, a different chiral structure. A gauge symmetry breakdown of this type will not exhibit a Higgs mechanism; in general some of the gauge bosons are just projected from the massless spectrum. Such an abrupt breakdown can, of course, not be revealed through an examination of flat directions in the potential. Nonetheless one needs such abrupt transitions to obtain phenomenologically acceptable models. For example, the $d = 10$, $N = 1$, $E_8 \times E_8$ model is continuously connected to the $d = 4$, $N = 4$, $E_8 \times E_8$ model obtained by torus compactification of the six extra dimensions. Such a model does not allow chiral fermions and any model with chiral

fermions cannot be continuously connected to the above model. On the other hand, we would argue that the breakdown of the $SU(2) \times U(1)$ group of weak interactions should proceed in a smooth way²⁷. As a strategy to construct a realistic model one would therefore start with the discussions of a string model with desired "topological" properties, e.g., the correct number of generations of quarks and leptons. To our knowledge, there is no smooth way to change such a chiral structure like the number of generations. A word of caution has to be added here since string theory allows some "singular" smooth ways to change chirality by letting the radius of a compact dimension go to zero or infinity²⁸. We exclude this possibility from our discussion. Having found now such representative models with the desired "topological" structure we then have to examine the flat directions in the potential and identify the full class of continuously connected models. After breakdown of supersymmetry we would then expect the selection of a unique vacuum only within such a connected class. This is the recipe to follow. Take an explicitly constructed string model and consider non-vanishing vacuum expectation values (vev's) for the massless states that satisfy

$$W = F = D = 0 \quad (1)$$

Here W represents the superpotential and $F(D)$ the auxiliary fields of chiral and gauge supermultiplets

$$F_i = \frac{\partial W}{\partial \phi_i} \quad (2)$$

and

$$D_\alpha = \phi_i^* T_{\alpha j}^i \phi^j \quad (3)$$

where $(T_\alpha)_j^i$ represent the generators of the gauge group. It is advisable to first satisfy

the equation for the D-terms. Methods for a solution of these equations have been given in the literature²⁹ and I shall not repeat them here. A complication arises in the case of an anomalous $U(1)$ -group where the Green-Schwarz anomaly cancellation terms induce a Fayet-Iliopoulos term for this anomalous $U(1)$. This then necessarily leads to the appearance of vev's for some of the scalar fields that have non-trivial charges under this $U(1)$. In all known cases such a term, however, does not induce supersymmetry breakdown nor does it alter the discussion of the flat directions. In general one can easily satisfy $D_\alpha = 0$ and finds in fact many solutions. More tricky is the investigation of the equations $W = 0$ and $F_i = 0$ which requires the calculation of the Yukawa couplings and this is also the place where the information about the full string theory enters. It is therefore important to start this procedure with an explicit string model where all these couplings can be computed exactly. Observe that it is not enough to just compute the trilinear (renormalizable) Yukawa couplings but one must have a control over all couplings with an arbitrary number of external fields. The scattering amplitudes for the physical states of strings propagating on orbifolds can be computed using the tools of conformal field theory³⁰. Flat directions in the potential require the absence of certain Yukawa couplings in the superpotential. These are in one-to-one correspondence with the vanishing of certain correlation functions in the presence of exactly marginal operators in the corresponding conformal field theory. The details of such an investigation can become quite complicated and for a thorough discussion we refer to Ref. 1. Let us here just consider a simple example to illustrate the procedure. We choose this to be the Z_3 -orbifold with non-standard embedding and gauge group $SU(9) \times O(14) \times U(1)$. The charged matter

in the untwisted sector consists of three copies of $(84,1)_0 + (1,14)_{-2} + (1,64)_1$ and we have one $(\bar{9},1)_{4/3}$ at each of the twenty seven fixed points in the twisted sector. The 84 of $SU(9)$ is the three-index antisymmetric tensor X^{ijk} . Consider a vev $\langle X^{123} \rangle = \langle X^{456} \rangle = \langle X^{789} \rangle = 0$ for one of the three 84's. The equality of the three vev's ensures the absence of D-terms. F-terms are not induced since the only allowed Yukawa couplings are $\epsilon_{\alpha\beta\gamma} 84_\alpha 84_\beta 84_\gamma$ excluding a self-coupling of that 84 with a non-vanishing vev. This then gives a flat direction in which the gauge group $SU(9)$ is broken to $SU(3)$ ³. This flat direction was already known from earlier investigations in explicitly constructed string models and could be understood as a background gauge field that can be varied continuously². Such a situation actually occurs in all cases where we consider exclusively vev's for the fields in the untwisted sector. This one-to-one correspondence actually reveals a limitation of the low energy field theoretic method. It is a method perturbative in the vev's of the fields and it does not allow us to find multicritical points which are far away in field space. The explicit string construction of the present model shows an enhancement of the gauge group to $SU(3)^3 \times U(1)$ ² for some large value of the background gauge field² which one cannot recover in the perturbative mechanism.

New models - actually such models that cannot yet be constructed as a complete string theory - can be found along flat directions for vev's of the twisted sector fields. In the present model this would correspond to vev's for one or several of the $\bar{9}$'s. The dangerous W-terms that might prevent flat directions are of the form

$$[84 \times \bar{9}^3 \times 14^2]^n \quad (4)$$

Giving a vev to one of the 84 's and one of the 14 's we find many flat directions for vev's of the $\bar{9}$'s provided we only allow certain combinations of fixed points to appear in this choice; for details consult Ref. 1. $SU(9)$ is broken completely with this procedure.

Generically we find many of those flat directions indicating the existence of large classes of new consistent string models, continuously connected to already known models. Some of these models had been considered previously like the blown up versions of $(2,2)$ -models²¹. Among those models that allow a nice geometrical interpretation we found new ways to blow up $(2,0)$ models. Others like the ones described above do not have a simple geometrical interpretation.

The phenomenological prospects of these models are quite exciting. They give us new parameters as in the case of Calabi-Yau models, and yet allow an explicit calculation of all couplings among the massless fields, at least perturbative in the vev's of the fields along the flat directions. Some properties of the models seem to be of particular importance. The rank of the gauge group need not be large. It seems less and less convincing that superstring models prefer additional $U(1)$'s. The breakdown of gauge symmetries appears in a smooth way, particularly suited for the breakdown of $SU(2) \times U(1)$ giving us the opportunity to parametrize a small parameter. Previously the investigation of potentially realistic orbifold models revealed a lack of flexibility and made it very difficult to have a satisfactory mechanism to generate quark and lepton masses. With the observation of the new flat directions, we find that such Yukawa couplings can be continuously varied. Usually this happens because of higher-order renormalizable couplings leading to trilinear effective couplings, once some of the fields receive a non-vanishing vev. Together

with the possibility of exponentially suppressed Yukawa couplings, we might now try to understand the quark and lepton masses.

Some time ago we constructed a large number of string models with gauge group $SU(3) \times SU(2) \times U(1)^5$ and three families of quarks and leptons³¹. These models deserve further consideration in view of our new mechanism to lower the rank of the gauge group. In fact, preliminary investigations show that model 3 of Ref. 31 is very promising. There exists a multitude of flat directions where all $U(1)$'s are broken, except for that corresponding to the correct hypercharge. Additional colour-triplets become heavy in this process. Potential problems with proton decay and neutrino masses can be solved and an examination of the fermion mass matrix is under way.

On the more theoretical side, it would be interesting to obtain a better understanding of the moduli space sketched in Fig. 1. This should, of course, include a stringy construction (non-perturbative in the vev's of the massless fields) of the large class of degenerate orbifold models which eventually might cover all of moduli space. We need a better control of those conformal field theories that might appear in string theories compactified on Calabi-Yau manifolds to finally examine the question to what extent models with different geometrical structure might lead to equivalent superconformal field theories on the world sheet.

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