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ASPECTS OF THE FLIPPED UNIFICATION OF STRONG, WEAK
AND ELECTROMAGNETIC INTERACTIONS

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ABSTRACT

We explore phenomenological aspects of a recently-proposed flipped $SU(5) \times U(1)$ supersymmetric GUT which incorporates an economical and natural mechanism for splitting Higgs doublets and triplets, and can be derived from string theory. Using experimental values of $\sin^2 \theta_W$ and the strong QCD coupling, we estimate the grand unification scale M_G , where the strong and weak coupling strengths are equal, and the superunification scale M_{SU} , where all couplings are equal. We find typical values of $M_G \sim 10^{15}$ to 10^{17} GeV, with M_{SU} somewhat higher and close to the value suggested by string models. We discuss different mechanisms for baryon decay, finding that the dominant one is gauge boson exchange giving rise to $p \rightarrow e^+ \pi^0$, $\bar{\nu} \pi^+$ and $n \rightarrow e^+ \pi^-$, $\bar{\nu} \pi^0$ with partial lifetimes $\sim 10^{35 \pm 2}$ y. We show that a large GUT symmetry-breaking scale M_G is naturally generated by radiative corrections to the effective potential if a small amount $\sim m_W$ of soft supersymmetry breaking is generated dynamically at a large scale. We analyze the low-energy effective theory obtained using the renormalization group equations, demonstrating that electroweak symmetry breaking is obtained if $m_t \sim 60$ to 90 GeV. We analyze the spectrum of sparticles, with particular attention to neutralinos.

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1. - INTRODUCTION

It is generally agreed that the Standard Model is unsatisfactory in at least three major respects: it does not unify the strong, weak and electromagnetic interactions, it does not explain the proliferation of quark and lepton flavours, and the hierarchy $m_W/M_P \ll 1$ is neither explained nor even natural. An attractive approach to the unification problem was to grand-unify the strong, weak and electromagnetic interactions in a simple non-Abelian group [1]. However, early GUTs accentuated the hierarchy problem by introducing a new grand unification scale M_G , close to, but apparently significantly below, the Planck scale M_P . It was difficult to see how the dynamics prevented this large scale from communicating via radiative corrections with m_W , and thereby destroying the GUT hierarchy [2]. This problem was solved in principle by supersymmetry, which ensured that radiative corrections to m_W were naturally small [3]. However, there remained the more profound problem of generating a small value of m_W in the first place.

In conventional GUTs, this required splitting the members of GUT Higgs supermultiplets so that their triplet members were very heavy, thus avoiding rapid proton decay, whilst their doublet partners were very light ($m \lesssim m_W$), and thus available to break the electroweak gauge symmetry. Models which solved this doublet-triplet splitting problem in a natural way were quite cumbersome, containing several large Higgs representations [4]. Moreover, even if light Higgs doublets were obtained, there remained the problem of exploiting them to generate dynamically a suitable electroweak gauge symmetry breaking scale.

An attractive solution to this problem was dimensional transmutation, based on the logarithmically slow evolution of the soft supersymmetry breaking parameters with the renormalization scale [5]. Electroweak symmetry breaking with $m_W \sim 80$ GeV could be triggered by the renormalization effects associated with a t quark of comparable mass.

Nowadays, the appropriate framework for resolving all the problems of the Standard Model seems to be string theory, which also promises unification within a consistent quantum theory of gravity. In early models of string unification based on Calabi-Yau compactification from ten to four dimensions, GUT gauge groups which were subgroups of E_6 were available [6], but there were no scalar fields in the adjoint representations generally needed to break the GUT symmetry down to the Standard Model. On the other hand, string models without simple GUT groups in four dimensions were also available [6], which did not need the adjoint Higgses and could, in principle, avoid baryon-number-violating interactions for triplet Higgs fields. Dimensional transmutation has been used successfully to generate a small electroweak scale in such ten-dimensional string models, again with $m_t/m_W \sim 1$ [7,8].

Recently, it has been realized that there are many more string theories which can be formulated directly in four dimensions [9,10], and offer a wider choice of gauge groups. However, these theories do not contain Higgs fields in adjoint representations, and so cannot be used to obtain conventional GUTs. However, there is just one GUT known which does not need adjoint (or larger) Higgses to break its gauge symmetry down to the Standard Model. This GUT is flipped $SU(5) \times U(1)$ [11], which only requires $\underline{10} + \overline{\underline{10}}$ and $\underline{5} + \overline{\underline{5}}$ Higgs representations.

The particle content of this model is as follows: it has three generations of matter fields:

$$F_i = (\underline{10}, \frac{1}{2}) , \quad \bar{F}_i = (\overline{\underline{5}}, -\frac{3}{2}) , \quad l_i^c = (\underline{1}, \frac{5}{2}) \quad (i=1,2,3) \quad (1.1)$$

a conjugate pair of five-dimensional Higgses to break $SU(2)_L \times U(1)_Y$:

$$h = (\underline{5}, -1) , \quad \bar{h} = (\overline{\underline{5}}, 1) \quad (1.2)$$

and another conjugate pair of ten-dimensional Higgses to break $SU(5) \times U(1) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$H = (\underline{10}, \frac{1}{2}) , \quad \bar{H} = (\overline{\underline{10}}, -\frac{1}{2}) \quad (1.3)$$

The minimal version of the model contains in addition four singlet fields ϕ_m . The superpotential of the model is

$$\begin{aligned} W = & \lambda_1^i F_i F_j h + \lambda_2^i F_i \bar{F}_j \bar{h} + \lambda_3^i \bar{F}_i l_j^c h \\ & + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\ & + \lambda_6^{im} F_i \bar{H} \phi_m + \lambda_7^m h \bar{h} \phi_m + \lambda_8^{mnp} \phi_m \phi_n \phi_p \end{aligned} \quad (1.4)$$

This model has several remarkable features [11]. (i) It contains a natural doublet-triplet splitting mechanism, whereby the large v.e.v.'s V and $\bar{V}$ of v^c and $\bar{v}^c$ -like components of the H and $\bar{H}$ (1.3) give Dirac mass terms to the dangerous colour triplet Higgses $h_3 \in h$ and $\bar{h}_3 \in \bar{h}$, coupling them to states in the H and $\bar{H}$ multiplets with the quantum numbers of d^c and $\bar{d}^c$ respectively:

$$m_{d_H^c h_3}^c = \lambda_4 V , \quad m_{\bar{d}_H^c \bar{h}_3}^c = \lambda_5 \bar{V} \quad (1.5)$$

(ii) It has a seesaw mass matrix for the neutrinos of the form

$$(\nu_i, \nu_i^c, \phi_m) \begin{pmatrix} 0 & m_u^{ij} & 0 \\ m_u^{ji} & 0 & \lambda_6^{im} \bar{V} \\ 0 & \lambda_6^{mi} \bar{V} & Q(m_w) \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_i^c \\ \phi_m \end{pmatrix} \quad (1.6)$$

which gives light eigenstates $\nu_i + O(m_u/M_G)\phi_m$ with masses $O(m_w^3/M_G^2)$. (iii) It is unique among models whose potentials have flat directions in having strong coupling effects that prevent H and $\bar{H}$ from becoming stuck at the origin until $T \lesssim m_w$, and subsequently releasing excessive entropy when the GUT symmetry-breaking phase transition is completed^{*)} [12].

Moreover, flipped $SU(5) \times U(1)$ can be derived from string theory in either ten [15] or four dimensions [16,17]. In the former case, an "Identikit" list of the topological requirements on a compactification manifold has been provided [15]. In the latter case, explicit examples have been given [16,17] of four-dimensional string models based on flipped $SU(5) \times U(1)$ which are formulated in terms of general boundary conditions on world-sheet fermions [10]. In one example, the light particles are almost exactly those of the minimal flipped $SU(5) \times U(1)$ model (1.1)-(1.3), but not all the desired superpotential terms (1.4) are present - in particular, charge $+2/3$ quark mass terms are absent [16]. In another example, there are additional $\underline{10}+\overline{\underline{10}}$ and $\underline{5}+\overline{\underline{5}}$ Higgs fields, whose additional Yukawa couplings enable a more realistic quark mass matrix to be obtained [17].

In view of its innate attractions, as well as this connection with string theory, supersymmetric flipped $SU(5) \times U(1)$ is a highly-qualified candidate to be the theory of everything below the Planck scale. Accordingly, in this paper we explore in some detail its phenomenological aspects, as has been done previously for conventional $SU(5)$ [18], supersymmetric $SU(5)$ [19], and a superstring-inspired subgroup of E_6 [7].

Our strategy is to start in Section 2 from what is known of the gauge interactions at present energies, using them and the renormalization group equations to deduce preferred ranges for the grand unification scale M_G at which the strong and $SU(2)$ electroweak couplings become equal, and for the superunification scale M_{SU} at which the $SU(5)$ and $U(1)$ couplings become equal. Gratifyingly, generic input values of $\sin^2\theta_w$ and the strong coupling yield $M_G \lesssim M_{SU}$, with $M_G \sim 10^{16 \pm 1}$ GeV and M_{SU} quite close to the string unification scale [20], especially in the non-minimal model with additional Higgs representations [17].

^{*)} The model also appears capable of adequate baryogenesis in the early Universe [13], and has interesting predictions for astrophysical dark matter [14].

Next, in Section 3, we analyze proton decay in the $SU(5) \times U(1)$ model, showing that dimension-six and -five operators due to Higgs exchange are insignificant compared to dimension-six operators due to gauge boson exchange. We predict

$$\tau(p \rightarrow e^+ \pi^0) = 2 \tau(p \rightarrow \bar{\nu} \pi^+) = 2 \tau(n \rightarrow e^+ \pi^-) = \tau(n \rightarrow \bar{\nu} \pi^0) \quad (1.7)$$

with lifetimes of order $10^{35 \pm 2}_y$.

In Section 4 we analyze dynamics near the superunification and grand unification scales, showing in particular that radiative corrections to the effective potential break $SU(5) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)$ at a large scale M_G close to the scale where the dynamics generates soft supersymmetry breaking. In particular, we show that this occurs even when the dominant source of supersymmetry breaking in the observable sector is a gaugino mass $m_{\frac{1}{2}} \sim m_W \ll M_G$ [21].

In Section 5 we integrate the renormalization group equations for the soft supersymmetry breaking parameters and Yukawa couplings down to low energies. We demonstrate that electroweak gauge symmetry breaking occurs through dimensional transmutation for suitable values of the t quark and other Yukawa couplings. The suitable range of t quark masses is quite narrow:

$$m_t = (60 \text{ to } 90) \text{ GeV} \quad (1.8)$$

once phenomenological constraints on other particles are taken into account. We also make predictions for sparticle masses, discussing in particular predictions for the lightest neutralino (i.e., gaugino/Higgsino mixture).

Finally, in Section 6 we summarize the current status of supersymmetric flipped $SU(5) \times U(1)$, and discuss what remains to be done to demonstrate its acceptability as the theory of everything below the Planck mass.

2. - COUPLING CONSTANT UNIFICATION

In this section we use the renormalization group equations for the gauge couplings to investigate whether flipped $SU(5) \times U(1)$ GUT models are compatible with the observed values of α_3 and of $\sin^2 \theta_W$, and, if so, to compute the expected grand unification scale M_G at which $\alpha_3 = \alpha_2 = \alpha_5$ and $25/\alpha_Y = 1/\alpha_5 + 24/\alpha_1$, and superuni-

fication scale M_{SU} at which $\alpha_5 = \alpha_1 = \alpha_U^*$). We will study both the minimal $SU(5) \times U(1)$ model [11] with the particle content listed in Section 1, and also a non-minimal variant [17] derived from four-dimensional string. There are two differences in our analysis from classical analyses of GUT renormalization group equations. One is that we use as one input the value of $\sin^2\theta_W$ extracted recently [22] from a global analysis of low-energy neutral currents, m_W and m_Z :

$$\sin^2\theta_W = 0.228 \pm 0.004 \quad (2.1)$$

The other is that we make explicit comparison with the value of M_{SU} expected in string theories. It has recently been shown [20] that if one integrates out the heavy string degrees of freedom, then the gauge coupling constants in the low-energy effective field theory should approach each other in such a way that they would become equal at a renormalization scale which has some model-dependence, but is expected to be close to

$$M_U = g_U \times 5.3 \times 10^{17} \text{ GeV} \quad (2.2)$$

where g_U is the unified gauge coupling at the same scale. Ideally, this scale should be the same as the M_{SU} emerging from the renormalization group equations.

The two-loop approximations to the renormalization group equations for the five gauge couplings are

$$\frac{dg_i}{dt} = \frac{g_i^3}{16\pi^2} \left(b_i + \frac{1}{16\pi^2} \sum_j b_{ij} g_j^2 \right) \quad (2.3)$$

where

*) Henceforth, we normalize all gauge coupling strengths: $\text{Tr}(T^2) = 2$ for a complete generation, and denote the extra $U(1)$ coupling strength by α_1 . In this convention, the $U(1)$ charge assignments (1.1) to (1.3) are multiplied by $1/\sqrt{10}$.

$$\begin{aligned}
 b_{i=3,2,\gamma} &= - \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} N_G + \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{3}{10} \end{pmatrix} N_2 + \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{5} \end{pmatrix} N_3 + \begin{pmatrix} 1 \\ \frac{3}{2} \\ \frac{1}{10} \end{pmatrix} N_{32} \\
 b_{ij} &= - \begin{pmatrix} 54 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{68}{3} & 3 & \frac{1}{5} \\ 8 & 14 & \frac{2}{5} \\ \frac{88}{15} & \frac{6}{5} & \frac{38}{15} \end{pmatrix} N_G \\
 &+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{7}{2} & \frac{3}{10} \\ 0 & \frac{9}{10} & \frac{9}{50} \end{pmatrix} N_2 + \begin{pmatrix} \frac{17}{3} & 0 & \frac{2}{15} \\ 0 & 0 & 0 \\ \frac{16}{15} & 0 & \frac{4}{75} \end{pmatrix} N_3 + \begin{pmatrix} \frac{34}{3} & 3 & \frac{1}{15} \\ 8 & \frac{21}{2} & \frac{1}{10} \\ \frac{8}{15} & \frac{3}{10} & \frac{1}{150} \end{pmatrix} N_{32}
 \end{aligned} \tag{2.4a}$$

and

$$b_{i=5,1} = - \begin{pmatrix} 15 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} N_G + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} N_5 + \begin{pmatrix} \frac{3}{2} \\ \frac{1}{4} \end{pmatrix} N_{10}$$

$$b_{ij} = - \begin{pmatrix} 150 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{232}{5} & \frac{3}{5} \\ \frac{72}{5} & \frac{13}{5} \end{pmatrix} N_G + \begin{pmatrix} \frac{49}{5} & \frac{1}{5} \\ \frac{24}{5} & \frac{1}{5} \end{pmatrix} N_5 + \begin{pmatrix} \frac{183}{5} & \frac{3}{20} \\ \frac{18}{5} & \frac{1}{40} \end{pmatrix} N_{10} \tag{2.4b}$$

In addition to the gauge contribution $g_i^3 g_j^2$ to the two-loop RGE's (2.3), there are also contributions of order $g_i^3 \lambda^2$, where λ is a generic Yukawa coupling. In practice, the only Yukawa coupling likely to be relevant is that giving mass to the top quark. We find a posteriori in Section 5 that electroweak symmetry breaking only occurs for relatively small $m_t \sim 60-90$ GeV, and that the top-quark Yukawa contributes negligibly to the evolution of the gauge couplings. We have allowed in Eqs. (2.4) for the possibility that there might be a non-minimal set of light states including N_G generations and $N_{2(3,32,5,10)}$ Higgs doublets (triplets, doublet-triplets, pentaplets, decuplets) in the effective low-energy theory. Explicit string-derived non-minimal models of the type that we discuss later contain additional Higgs fields with intermediate masses $m_I: m_W \ll m_I \ll M_G$ which can be taken into account correctly by changing the N_i appropriately at the intermediate scale. To simplify the treatment of threshold effects in our analysis of coupling constant unification, we assume that all supersymmetric partners of Standard Model particles have masses equal to m_W . In addition, all $SU(5) \times U(1)$ models contain particles with masses of order λM_G such as right-handed neutrinos. We assume that their Yukawa couplings are $\sim g_5$ so that their masses are exactly

M_G except when stated otherwise.

Figure 1a shows the values of M_G and M_{SU} estimated in the minimal model [11] for different input values of $\sin^2\theta_W$ and $\alpha_3(m_W)$. Whereas M_G depends on both $\sin^2\theta_W$ and $\alpha_3(m_W)$, in the minimal model with $N_2 = N_5 = N_{10} = 2$, $N_3 = N_{32} = 0$, M_{SU} is only sensitive to $\alpha_3(m_W)^*$. Indicated on the vertical axis of Fig. 1a are the central experimental value of $\sin^2\theta_W$ and its $\pm$ one standard deviation range [22]. Indicated on the horizontal axis are $\alpha_3(m_W) = 0.131^{+0.018}_{-0.014}$ corresponding in the one-loop approximation to $\Lambda_{\overline{MS}} = 0.2^{+0.2}_{-0.1}$ GeV in QCD with four flavours. This is a range consistent with experiment, but we believe that the experimental errors may in fact be larger. We have only retained in Fig. 1a the domain of input parameters where $M_G < M_{SU}$ as required by the physics of unification. Since we calculate that a typical value of α_U is $\sim 1/24$, the values of M_{SU} that we find without adjusting particle masses in this model are somewhat smaller than the string estimate (2.2), unless we simultaneously take $\sin^2\theta_W < 0.224$ and $\alpha_3(m_W) > 0.15$. In fact, large values of $\alpha_3(m_W)$ can be avoided by adjusting particle masses, since the α_3 -dependence of M_G , M_{SU} is sensitive to $M_{d_H^c h_3}$ and $M_{\overline{d}_H^c \overline{h}_3}$, which depend on λ_4 and λ_5 respectively. For example, if we fix $M_{d_H^c h_3}$, $M_{\overline{d}_H^c \overline{h}_3} = 10^{15}$ GeV, we obtain consistency with (2.2) for more central values of $\alpha_3(m_W)$, but this still requires $\sin^2\theta_W < 0.224$. Such inputs are not completely excluded by experiment.

Figure 2 shows the values of M_G and M_{SU} estimated in a non-minimal $SU(5) \times U(1)$ model [17] derived from the fermionic formulation of strings in four dimensions. This model contains one extra set of Higgs fields, which we denote by $h', \bar{h}', H', \bar{H}'$. The colour triplets $h'_3, \bar{h}'_3$ acquire intermediate-scale masses $\sim M_G^2/M_P$ through a seesaw coupling to an additional set of massive $H'', \bar{H}''$ fields [17]. All components in the $H', \bar{H}'$ except for the $v_H^c, \bar{v}_H^c$, also acquire masses $\sim M_G^2/M_P$ as a result of non-renormalizable terms in the superpotential. The intermediate-scale triplets $h'_3, \bar{h}'_3$ counteract the effects in the RGE's of the extra light Higgs doublets $h'_2, \bar{h}'_2$, while the $H', \bar{H}'$ serve to raise M_{SU} relative to M_G . For simplicity, we have set $M_{H', \bar{H}'} = M_{h'_3, \bar{h}'_3} = M_{d_H^c h_3} = M_{\overline{d}_H^c \overline{h}_3} = 10^{12}$ GeV. Even with this simple choice, we observe in Fig. 2 that values of M_{SU} compatible with the estimate (2.2) are obtained for plausible values of $\alpha_3(m_W)$, $\sin^2\theta_W$ and M_G .

*) This is exactly true to one loop in any $SU(5) \times U(1)$ model in which the Higgs representations with mass $< M_G$ satisfy $N_2 - N_3 - N_{32} = 6 - 2N_{10}$.

3. - BARYON DECAY

As in other supersymmetric GUTs, there are three mechanisms for baryon decay which should be reconsidered in the flipped $SU(5) \times U(1)$ GUT. These are: (a) gauge boson exchange giving dimension-six operators [18], (b) Higgs exchange giving dimension-six operators, and (c) Higgsino exchange giving dimension-five operators [23]. We will first argue that mechanisms (b) and (c) are much less important than mechanism (a), and then analyze the latter in more detail.

In the minimal $SU(5) \times U(1)$ model there is just one conjugate pair of Higgs triplets h_3 and antitriplets $\bar{h}_3$ which acquire large Dirac masses through couplings to an antitriplet d_H^c in the single $\underline{10}$ of Higgses H and to a triplet $\bar{d}_H^c$ in the single $\bar{\underline{10}}$ of Higgses $\bar{H}$:

$$m_3 \equiv m_{d_H^c h_3} = \frac{\lambda_4}{g_5} M_G, \quad \bar{m}_3 \equiv m_{\bar{d}_H^c \bar{h}_3} = \frac{\lambda_5}{g_5} M_G \quad (3.1)$$

These masses depend on the a priori unknown couplings λ_4 and λ_5 , and could well be as large as $M_G \sim 10^{16}$ GeV. Exchange of the scalar component of the first Dirac eigenstate in (3.1) gives $\Delta B \neq 0$ dimension-six operators of the form

$$\frac{1}{m_3^2} \left\{ \lambda_1^{ij} (F_i F_j) [\lambda_3^{kl} (\bar{F}_k \bar{F}_l)]^* + h.c. \right\} \quad (3.2a)$$

where $(AB) \equiv \epsilon^{\alpha\beta} \phi_{A\alpha} \phi_{B\beta}$ with ϕ_A , etc., the fermionic components of the supermultiplets A , etc., and α, β spinor indices. Exchange of the second Dirac eigenstate in (3.1) gives $\Delta B \neq 0$ dimension-six operators of the form

$$\frac{1}{m_3^2} \lambda_2^{ij} (F_i F_j) [\lambda_2^{kl} (F_k F_l)]^* \quad (3.2b)$$

Recalling that the Yukawa coupling matrices

$$\lambda_1^{ij} = m_d^{ij}/v, \quad \lambda_2^{ij} = m_u^{ij}/v, \quad \lambda_3^{ij} = m_l^{ij}/v \quad (3.3)$$

where $v, \bar{v} = \langle 0 | h, \bar{h} | 0 \rangle^*$, we see that the proton decay amplitudes generated by the effective Higgs exchange interactions (3.2) are suppressed relative to those mediated via the corresponding gauge boson exchanges by factors

*) Note that in our flipped $SU(5) \times U(1)$ notation, h^0 has hypercharge $-\frac{1}{2}$ and gives mass to the b quark whereas $\bar{h}^0$ has hypercharge $+\frac{1}{2}$ and gives mass to the t quark.

$$\frac{g_5^2}{\lambda_4^2} \left(m_d m_{e,\mu} / v^2 \right) \quad (3.4a)$$

$$\frac{g_5^2}{\lambda_5^2} m_u^2 / v^2 \quad (3.4b)$$

Since we know $m_{d,e,\mu,u} \ll v, \bar{v}$, but have no reason to expect $\lambda_{4,5} \ll g_5$, we neglect the dimension-six Higgs boson exchange mechanism (b) relative to the gauge boson exchange mechanism (a).

Next we discuss the dimension-five Higgsino exchange mechanism (c) [23]. This mechanism gives an effective non-renormalizable superpotential term containing three quark and one lepton superfields with a coefficient

$$C_5 \sim \lambda_{1,2,3}^2 \frac{m_{h_3 \bar{h}_3}}{m_{3, \bar{3}}^2} \quad (3.5)$$

where $m_{h_3 \bar{h}_3}$ is a Higgs triplet mixing term. This is just $\lambda_7 \langle \phi \rangle \sim m_W$ in the minimal flipped $SU(5) \times U(1)$ model [17], whereas, as recalled above in Eq. (3.1), h_3 and $\bar{h}_3$ have large Dirac mass couplings $\sim M_G$ to d_H^c and $\bar{d}_H^c$ respectively. In the non-minimal flipped $SU(5) \times U(1)$ model [17] derived from the fermionic formulation of strings in four dimensions, there are two conjugate pairs of $\underline{5} + \bar{\underline{5}}$ Higgs supermultiplets: $h, \bar{h}$ and $h', \bar{h}'$. Although the $h', \bar{h}'$ have an apparently dangerous triplet mixing term $M_T h'_3 \bar{h}'_3$, only h and $\bar{h}$ couple to quarks and leptons, and there is no dangerous $h_3 \bar{h}'_3$ coupling. Therefore the dimension-five mechanism (c) is also negligible in this non-minimal $SU(5) \times U(1)$ model.

We now study the dominant gauge boson exchange mechanism (a). It gives a dimension-six operator of the same form as that found in minimal non-supersymmetric $SU(5)$ [18], but with the quarks and leptons flipped: $(d, -u) \leftrightarrow (u, d)$, $d^c \leftrightarrow u^c$, $(e, -\nu) \leftrightarrow (\nu, e)$ and $e^c \leftrightarrow \nu^c$, where ν^c is taken to be superheavy. The effective baryon decay operator, neglecting Cabibbo mixing, is therefore

$$\mathcal{L}^{56} = A \frac{g_5^2}{2M_G^2} \left[-d_R^\mu d_L u_R^\nu \tau_L + d_R^\mu u_L u_R^\nu \tau_L^- \right] \quad (3.6)$$

where A is a standard renormalization factor and τ_L^- is an unknown linear combination of e_L , μ_L and τ_L mass eigenstate fields. This ambiguity results from the fact that in flipped $SU(5) \times U(1)$, the λ_3^{ij} Yukawa couplings (1.4) responsible for the charged lepton mass matrix are unrelated a priori to the couplings λ_2^{ij} responsible

for the up-quark mass matrix. This opens up the possibility of unusual baryon decay modes like $p \rightarrow \mu^+ \pi^0$ and $n \rightarrow \mu^+ \pi^-$. One might expect, however, that whatever mechanism is responsible for generating hierarchical quark and lepton masses would relate the light u-quark eigenstate to the electron, so that the less conventional decay modes $p \rightarrow \mu^+ \pi^0$, $n \rightarrow \mu^+ \pi^-$, etc., would be suppressed by small mixing angles. Such is in fact the case for the non-minimal $SU(5) \times U(1)$ model [17] derived from the four-dimensional string. In the following analysis, we will assume that the λ_L^- in (3.6) is predominantly an e_L^- , though we emphasize that it is possible that $p \rightarrow \mu^+ \pi^0$, $n \rightarrow \mu^+ \pi^-$, etc., could occur and even dominate over $p \rightarrow e^+ \pi^0$.

With the choice of inputs in Section 2, the renormalization factor $A(m_W) \approx 3.4$. Using standard current algebra estimates [24], we find

$$\Gamma(p \rightarrow e^+ \pi^0) = \alpha^2 \left(\frac{A^2 g_s^4}{32 M_G^4} \right) \frac{m_p^2}{2\pi f_\pi^2} (1+g_A)^2 \left(1 - \frac{m_\pi^2}{m_p^2} \right)^2 \quad (3.7)$$

where α is defined by

$$\langle 0 | \epsilon_{ijk} (d_{iR} u_{jR}) u_{kL} | p \rangle = \alpha u_L(p) \quad (3.8)$$

and $f_\pi = 139$ MeV, $g_A = 1.24$. We also have

$$\begin{aligned} \Gamma(p \rightarrow \bar{\nu} \pi^+) &= 2 \Gamma(p \rightarrow e^+ \pi^0), \quad \Gamma(n \rightarrow e^+ \pi^-) = 2 \Gamma(p \rightarrow e^+ \pi^0), \\ \Gamma(n \rightarrow \bar{\nu} \pi^0) &= \Gamma(p \rightarrow e^+ \pi^0) \end{aligned} \quad (3.9)$$

Note that the ratios of these partial decay rates are different from those in conventional $SU(5)$. The most uncertain low-energy particle physics parameter in the rate (3.7) is the matrix element α (3.8). Lattice calculations of this quantity are now becoming available. We will use a first lattice estimate of α [25] that falls between previous estimates based on QCD sum rules [26] and baryon wave functions at short distances [24]:

$$\alpha(m_W) = 1.3 \times 10^{-2} \text{ GeV}^3 \quad (3.10)$$

We then evaluate (3.7) to obtain

$$\tau(p \rightarrow e^+ \pi^0) = 3.3 \times 10^{31} \left(\frac{M_G}{10^{15} \text{ GeV}} \right)^4 \left(\frac{0.042}{\alpha_s} \right)^2 \text{ y} \quad (3.11)$$

and similarly for the other pionic decay modes using the relations (3.9). Current algebra can also be used [24] to estimate the rates for $p \rightarrow e^+ \eta$ and $n \rightarrow \bar{\nu} \eta$ decays:

$$\Gamma(p \rightarrow e^+ \eta) = \frac{1}{3} \left(\frac{1+D-3F}{1+D+F} \right)^2 \left(\frac{1-m_\eta^2/m_p^2}{1-m_\pi^2/m_p^2} \right)^2 \Gamma(p \rightarrow e^+ \pi^0) = \Gamma(n \rightarrow \bar{\nu} \eta) \quad (3.12)$$

where $D = 0.76$ and $F = 0.48$ may be taken from a Cabibbo fit to hyperon decays. With these values, the $p \rightarrow e^+ \eta$ amplitude is anomalously small and Eq. (3.12) gives $\tau(p \rightarrow e^+ \eta) \gtrsim 10^2 \tau(p \rightarrow e^+ \pi^0)$. However, we cannot expect the current algebra analysis to be exactly correct, and an alternative way of estimating $\tau(p \rightarrow e^+ \eta)/\tau(p \rightarrow e^+ \pi^0)$ using non-relativistic SU(6) gives a much smaller ratio [27], so we prefer to quote

$$\tau(p \rightarrow e^+ \eta), \tau(n \rightarrow \bar{\nu} \eta) \gtrsim 10 \tau(p \rightarrow e^+ \pi^0) \quad (3.13)$$

It is also possible to use non-relativistic SU(6) to estimate $p \rightarrow e^+ \rho^0$, $e^+ \omega$, $\bar{\nu} \rho^+$ and $n \rightarrow e^+ \rho^-$, $\bar{\nu} \rho^0$, $\bar{\nu} \omega$. However, we do not think that these estimates are as reliable as that for $p \rightarrow e^+ \pi^0$ (3.11), which would in any case give more stringent constraints on the GUT parameters.

If we compare the theoretical prediction for $\tau(p \rightarrow e^+ \pi^0)$ (3.11) with the experimental lower limit [28]

$$\tau(p \rightarrow e^+ \pi^0) > 2.5 \times 10^{32} \text{ y} \quad (3.14)$$

we conclude

$$\left(\frac{M_G}{10^{15} \text{ GeV}} \right) \left(\frac{0.042}{\alpha_5} \right)^{1/2} \gtrsim 1.7 \quad (3.15)$$

Since α_5 is close to 0.042 in the domain of interest, one can interpret (3.15) as effectively a lower bound $M_G \gtrsim 1.7 \times 10^{15} \text{ GeV}$. This can be compared with the theoretical values of M_G inferred from the renormalization group analysis of Section 2 and shown in Fig. 1, namely

$$2 \times 10^{15} \text{ GeV} \lesssim M_G \lesssim 5 \times 10^{16} \text{ GeV} \quad (3.16)$$

We see that only a small domain of parameter space is excluded by the $\tau(p \rightarrow e^+ \pi^0)$ constraint (3.15). In general, we would expect on the basis of (3.11) and (3.16) that $\tau(p \rightarrow e^+ \pi^0) \lesssim 2 \times 10^{38} \text{ y}$, with the most likely range being

$$10^{33} \text{ y} \lesssim \tau_p \lesssim 10^{37} \text{ y} \quad (3.17)$$

if one takes a plausible range of M_G and allows for other p decay modes.

In the non-minimal $SU(5) \times U(1)$ model [17] derived from the four-dimensional string, we find $\alpha_5(M_G) \sim 0.07$ for our choice of parameters $M_{H^c \bar{H}} = M_{h_3^c \bar{h}_3} = M_{d_{H^c h_3}} = M_{\bar{d}_{H^c h_3}} = 10^{12}$ GeV. Hence the $\tau(p \rightarrow e^+ \pi^0)$ constraint (3.15) implies that $M_G \gtrsim 2.2 \times 10^{15}$ GeV in this case, which is again compatible with our estimates (Fig. 2).

We conclude that both the minimal flipped $SU(5) \times U(1)$ model and its non-minimal extension offer hope that proton decay can be observed in the foreseeable future. Its distinctive branching ratios (3.9) would distinguish $SU(5) \times U(1)$ from other, less appealing GUTs.

4. - DYNAMICAL GENERATION OF THE GRAND UNIFICATION SCALE

In this section we discuss the first stage of GUT symmetry breaking close to the superunification scale at which $SU(5) \times U(1)$ is broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$. The possibility of a large GUT symmetry-breaking scale M_G is offered by the fact that the superpotential (1.4) gives rise to a tree-level supersymmetric effective potential with a D- and F-flat direction corresponding to $\langle 0 | \nu_H^c | 0 \rangle \equiv V = \langle 0 | \bar{\nu}_H^c | 0 \rangle \equiv \bar{V}$. The flatness of the potential along this direction is in general broken by soft supersymmetry-breaking terms. It is not yet known how supersymmetry is broken in string theory, nor how this breaking of supersymmetry first manifests itself in the observed sector of the theory. Our analysis of GUT symmetry breaking is based on two assumptions about soft supersymmetry breaking that are consistent with our (lack of) knowledge on these issues. We assume that: (a) the dominant source of supersymmetry breaking in the observable sector is a uniform gaugino mass $m_{\frac{1}{2}}$ - as advocated previously [21], and (b) supersymmetry breaking is generated dynamically at a scale M_{SX} equal to or somewhat below the superunification scale M_{SU} - for example by gaugino condensation [29]. Moreover, we will not attempt to determine dynamically the overall magnitude of supersymmetry breaking in the observable sector, but assume that $m_{\frac{1}{2}} \ll M_G, M_{SU}$.

Since the supersymmetric tree-level potential is flat in the direction $V = \bar{V}$, and by assumption (a) there are no soft supersymmetry-breaking scalar masses at the tree level, a non-trivial potential capable of determining M_G can only appear through the radiative corrections to the effective potential, which take the general form

$$\Delta \mathcal{V} = \frac{1}{64\pi^2} \text{Str} \left[M^4 \left(\ln \frac{M^2}{Q^2} \right) \right] \quad (4.1)$$

where M is the mass matrix and Q is an arbitrary renormalization scale. The heavy ($\sim M_G$) mass eigenstate fields which give a contribution $\sim m_W^2 M_G^2$ to ΔV (4.1) are:

j = 1: 12 heavy charged vector bosons (X_i, Y_i) with

$$m_X^2 = g_s^2 |V|^2 \quad (4.2a)$$

and one neutral heavy vector boson V_0 with

$$m_{V_0}^2 = \left(\frac{g_1^2}{10} + \frac{12g_s^2}{5} \right) |V|^2 \quad (4.2b)$$

j = 1/2: 24 spartners of the (X_i, Y_i) and $(u_{H_i}, d_{H_i}, \bar{u}_{\bar{H}_i}, \bar{d}_{\bar{H}_i})$ with

$$m_{\tilde{X}}^2 = g_s^2 |V|^2 \pm \mu_s g_s |V| + \frac{\mu_s^2}{2} \quad (4.3a)$$

one spartner of the V_0 and one combination of neutral components of H and $\bar{H}$ with:

$$m_N^2 = \left[\frac{g_1^2}{10} + \frac{12g_s^2}{5} \right] |V|^2 \pm \frac{g_1^2 \mu_1 + 24g_s^2 \mu_s |V|}{\sqrt{10(g_1^2 + 24g_s^2)}} + \left(\frac{\mu_1^2 + \mu_s^2}{2} \right) - \frac{(24g_s^2 \mu_1 + g_1^2 \mu_s)^2}{2(g_1^2 + 24g_s^2)^2} \quad (4.3b)$$

$2N_g$ states (ν_i^c, ϕ_i) with

$$m_6^2 = |\lambda_6 V|^2 \quad (4.3c)$$

six Dirac d_H^c/h_3 states with

$$m_{h_3}^2 = |\lambda_4 V|^2 \quad (4.3d)$$

and six Dirac $\bar{d}_H^c/\bar{h}_3$ states with

$$m_{\bar{h}_3}^2 = |\lambda_5 V|^2 \quad (4.3e)$$

j = 0: 12 scalars with the same quantum numbers as the vectors (X_i, Y_i) with

$$m_{X_0}^2 = g_s^2 |V|^2 + \frac{m_{10}^2 + m_{\bar{10}}^2}{2} \quad (4.4a)$$

one neutral scalar with

$$m_{\tilde{N}}^2 = \left(\frac{g_1^2}{10} + \frac{12g_5^2}{5} \right) |V|^2 + \frac{m_{10}^2 + m_{\bar{10}}^2}{2} \quad (4.4b)$$

$4N_g$ spartners of (ν_i^c, ϕ_i) with

$$m_{\tilde{\nu}_6}^2 = |\lambda_6 V|^2 \pm |A_6 \lambda_6 V| + \frac{m_F^2 + m_{\phi}^2}{2} \quad (4.4c)$$

12 charge $|1/3|$ scalars with

$$m_{\tilde{h}_3}^2 = |\lambda_4 V|^2 \pm |A_4 \lambda_4 V| + \frac{m_{10}^2 + m_5^2}{2} \quad (4.4d)$$

12 charge $|1/3|$ scalars with

$$m_{\tilde{h}_3}^2 = |\lambda_5 V|^2 \pm |A_5 \lambda_5 V| + \frac{m_{10}^2 + m_{\bar{5}}^2}{2} \quad (4.4e)$$

In the above, we have denoted the $SU(5)$ and $U(1)$ gaugino masses by μ_5 and μ_1 respectively, and have included for completeness the soft supersymmetry-breaking scalar masses of the $F, \phi, H, \bar{H}, h$ and $\bar{h}$ supermultiplets (1.1)-(1.3) denoted by $m_F^2, m_{\phi}^2, m_{10}^2, m_{\bar{10}}^2, m_5^2$ and $m_{\bar{5}}^2$ respectively, and the soft supersymmetry-breaking trilinear coefficients multiplying $\lambda_{4,5,6}$ terms in the superpotential (1.4) denoted by $A_{4,5,6}$.

Noting that the mass terms in (4.2)-(4.4) can be written in the generic form

$$m^2 = \lambda^2 |V|^2 \pm \alpha V + \beta^2 \quad (4.5)$$

where $\lambda = O(1)$, α and $\beta = O(m_{\frac{1}{2}}) \ll M_G$ and $V \sim M_G$, we can rewrite (4.1) as

$$\Delta \mathcal{V} = \frac{V^2}{64\pi^2} \sum_j (-1)^{2j} (2j+1) \left[\lambda^2 \beta^2 + \frac{3}{2} \alpha^2 + (2\lambda^2 \beta^2 + \alpha^2) \ln \lambda^2 \frac{V^2}{Q^2} \right] \quad (4.6)$$

Substituting into (4.6), the individual expressions (4.2)-(4.4), we obtain

$$\begin{aligned} \Delta \mathcal{V} = & \frac{V^2}{64\pi^2} \left\{ \ln \frac{V^2}{Q^2} \left[-\frac{(24)^2 g_5^2 \mu_5^2}{5} - \frac{4g_1^2 \mu_1^2}{5} + \left(\frac{g_1^2}{10} + \frac{72g_5^2}{5} \right) (m_{10}^2 + m_{\bar{10}}^2) \right. \right. \\ & + 4 \sum_i \lambda_6^2 (A_{6i}^2 + m_{F_i}^2 + m_{\phi_i}^2) + 12\lambda_4^2 (A_4^2 + m_{10}^2 + m_5^2) + 12\lambda_5^2 (A_5^2 + m_{10}^2 + m_{\bar{5}}^2) \left. \right] \\ & - 96g_5^2 \mu_5^2 - \frac{4(g_1^2 \mu_1^2 + 24g_5^2 \mu_5^2)}{5} + \frac{48g_1^2 g_5^2 (\mu_1 - \mu_5)^2}{5(g_1^2 + 24g_5^2)} + \left(\frac{g_1^2}{20} + \frac{72g_5^2}{10} \right) (m_{10}^2 + m_{\bar{10}}^2) \end{aligned} \quad (4.7)$$

$$\begin{aligned}
& + 2 \sum_i \lambda_6^{i^2} (3A_{6i}^2 + m_{F_i}^2 + m_{\phi_i}^2) + 6\lambda_4^2 (3A_4^2 + m_{10}^2 + m_5^2) + 6\lambda_5^2 (3A_5^2 + m_{10}^2 + m_5^2) \\
& - 96g_5^2 \mu_5^2 \ln g_5^2 - \frac{4}{5} (g_1^2 \mu_1^2 + 24g_5^2 \mu_5^2) \ln \left(\frac{g_1^2}{10} + \frac{12g_5^2}{5} \right) \\
& + 12g_5^2 (m_{10}^2 + m_{10}^2) \ln g_5^2 + \left(\frac{g_1^2}{10} + \frac{12g_5^2}{5} \right) (m_{10}^2 + m_{10}^2) \ln \left(\frac{g_1^2}{10} + \frac{12g_5^2}{5} \right) \\
& + 4 \sum_i \lambda_6^{i^2} (A_{6i}^2 + m_{F_i}^2 + m_{\phi_i}^2) \ln \lambda_6^{i^2} + 12\lambda_4^2 (A_4^2 + m_{10}^2 + m_5^2) \ln \lambda_4^2 \\
& + 12 \lambda_5^2 (A_5^2 + m_{10}^2 + m_5^2) \ln \lambda_5^2 \} \tag{4.7}
\end{aligned}$$

cont.

It is straightforward to check that $\mathcal{V}_T = \mathcal{V}_0 + \Delta \mathcal{V}$; $\mathcal{V}_0 = (m_{10}^2 |V|^2 + m_{10}^2 |\bar{V}|^2) |V|=|\bar{V}|$ is Q-independent to one loop, taking into account the Q-dependence of $m_{10,10}^2$ (A.22,23) and the wave-function renormalization of the $v_H^c, \bar{v}_H^c$ fields appearing in $\mathcal{V}_0$. The functional form of the effective potential $\mathcal{V}_T(Q = |V|)$ is plotted in Fig. 3a, where one observes a global^{*} minimum at the scale $V \sim M_{SX}$. Indeed, if we take $Q = M_{SX}$ and initial conditions according to our assumption (a) above, namely

$$\left. \begin{aligned}
\mu_5 = \mu_1 = m_{1/2} \neq 0, \\
m_F^2 = m_{\phi}^2 = m_{10}^2 = m_{10}^2 = m_5^2 = m_5^2 = 0, \\
A_4 = A_5 = A_6 = 0
\end{aligned} \right\} \tag{4.8}$$

and assuming that M_{SX} is sufficiently close to M_{SU} that we can disregard any evolution in g_5 relative to g_1 , we find

$$\mathcal{V}_T = \Delta \mathcal{V} \approx \frac{\bar{g} V^2 m_{1/2}^2}{64\pi^2} \left[-116 - 20 \ln \frac{5g^2}{2} - 96 \ln g^2 - 116 \ln \frac{V^2}{M_{SX}^2} \right] \tag{4.9}$$

In the neighbourhood of $V \sim M_{SX}$, it has the approximate form

^{*}) Figure 3 also reveals a barrier at $V \sim 0.5 M_{SX}$, which could in principle prevent $v_H^c, \bar{v}_H^c$ from attaining its global minimum at M_{SX} . We find this possibility unlikely, since we have previously shown [13] that the temperature of the $v_H^c, \bar{v}_H^c$ field following inflation must be $> \sqrt{m_{WGC} M_G}$ to prevent excessive entropy generation due to coherent field oscillations, in which case the $v_H^c, \bar{v}_H^c$ would fluctuate over this potential barrier.

$$\Delta \mathcal{V} \approx -0.053 V^2 m_{1/2}^2 \quad (4.10)$$

We do not expect the formulae (4.7), (4.9) and (4.10) to be appropriate for $V \gtrsim M_{SX}$ where the supersymmetry breaking disappears according to assumption (b) above. Instead, we expect the effective potential for $V \gtrsim M_{SX}$ to have the qualitative form shown as a dotted line in Fig. 3b. We therefore conclude that the grand unification scale M_G is fixed dynamically to be $\sim M_{SX}^*$.

Although the detailed form of the one-loop effective potential (4.9) depends on the specific model studied here, the qualitative form of Fig. 3 and this mechanism for the dynamical determination of M_G are valid in a wide class of GUT models. Indeed, our assumption (a) that the dominant source of supersymmetry breaking in the observable sector is a universal gaugino mass $m_{1/2}$ is not essential, for we find that radiative GUT breaking also occurs if $m_{1/2} = m_0 = 0$, $A \neq 0$; or if $m_{1/2} = 0$, $A = m_0 \neq 0$. Our key assumption is assumption (b), that dynamical supersymmetry breaking switches on at a high scale M_{SX} at or somewhat below M_{SU} . Our result suggests that this would be a desirable feature for any proposed mechanism of supersymmetry breaking in string models.

Finally, we note that in the non-minimal superstring $SU(5) \times U(1)$ model [17], hierarchical fermion mass matrices are generated by the ratio V/M_{SU} :

$$m_f = m_f^{(0)} + \sum_{n=1}^{\infty} m_f^{(n)} \left(V/M_{SU} \right)^n \quad (4.11)$$

where $m_f^{(0)} = 0(m_W)$ since all the non-zero Yukawa couplings are $O(1)$ at the superunification scale in four-dimensional string models [31]. Therefore in this scenario one wants

$$V \sim M_{SX} \sim M_{SU} \times 10^{-2 \pm 1} \quad (4.12)$$

in order to obtain a realistic fermion mass spectrum, which is highly consistent with our analysis of coupling constant unification shown in Fig. 2.

*) These results differ significantly from those of a recent analysis [30] in which the full one-loop corrections to the effective potential (4.7) were not included.

5. - LOW-ENERGY PHENOMENOLOGY

Having established in Section 4 that GUT symmetry breaking is likely to occur at a scale M_G in the range favoured by the renormalization group analysis of gauge couplings in Section 2, we now explore the resulting low-energy phenomenology of the model. We integrate down to low energies the coupled renormalization group equations for the soft supersymmetry-breaking parameters μ_i , m_j^2 and A_k listed in Appendix B, using as boundary conditions at M_{SX} the values

$$\mu_Y = \mu_Z = \mu_3 = \mu_5 = \mu_1 = m_{1/2} ; m_j^2 = 0 ; A_k = 0 \quad (5.1)$$

Included in the RGE's are the λ_7 , λ_8 and the t- and b-quark Yukawa couplings λ_2^t , λ_1^b . The effective low-energy theory therefore contains five parameters ($m_{1/2}$, λ_7 , λ_8 , λ_2^t , λ_1^b). We seek solutions in which h_2 , $\bar{h}_2$ and ϕ_0 acquire v.e.v.'s denoted by v , $\bar{v}$ and x respectively^{*}). Two combinations of the input parameters are fixed by requiring that the physical values of m_W and m_b are obtained, leaving a three-parameter space to be searched. We impose on the solutions the following experimental requirements: (a) all unseen charged particles have masses $\gtrsim 23$ GeV [32], (b) squark masses $m_{\tilde{q}} \gtrsim 47$ GeV and the gluino mass $m_{\tilde{g}} \gtrsim 50$ GeV [33].

In our numerical analysis, we terminate the renormalization group scaling of the low-energy parameter at a scale Q^2 set by the mass of the heaviest supersymmetric particle, i.e., at $Q^2 = m_{\tilde{g}}^2$, at which point the supersymmetric RGE's (B.1)-(B.16) are no longer applicable, and then analyse the resulting low-energy potential at the tree level.

The form of the neutral low-energy potential is

$$\begin{aligned} \mathcal{V} = & \frac{1}{8} (g_2^2 + \frac{3}{5} g_Y^2) (h^2 - \bar{h}^2)^2 + \Lambda_7^2 \Phi^2 (h^2 + \bar{h}^2) + \Lambda_7^2 h^2 \bar{h}^2 + 9 \Lambda_8^2 \Phi^4 \\ & + m_h^2 h^2 + m_{\bar{h}}^2 \bar{h}^2 + m_\Phi^2 \Phi^2 + 6 \Lambda_7 \Lambda_8 h \bar{h} \Phi^2 \cos(\hat{j} - 2\hat{\phi} + \hat{\lambda}_7 - \hat{\lambda}_8) \\ & + 2 \Lambda_7 A_7 h \bar{h} \Phi \cos(\hat{j} + \hat{\phi} + \hat{\lambda}_7) + 2 \Lambda_8 A_8 \Phi^3 \cos(3\hat{\phi} + \hat{\lambda}_8) \end{aligned} \quad (5.2)$$

where we have introduced positive real parameters h , $\bar{h}$, Φ , Λ_7 , Λ_8 , $\hat{h}$, $\hat{\bar{h}}$, $\hat{\phi}$, $\hat{\lambda}_7$, $\hat{\lambda}_8$, $\hat{j}$ defined by

^{*}) Here ϕ_0 is the linear combination of singlet fields ϕ_m that has no λ_6 coupling (1.4) and hence does not acquire a large mass through the seesaw mechanism (1.6).

$$\left. \begin{aligned} h^0 &= h e^{i\hat{h}}, \quad \bar{h}^0 = \bar{h} e^{i\hat{h}}, \quad \phi^0 \equiv \phi e^{i\hat{\phi}} \\ \lambda_7 &= \lambda_7 e^{i\hat{\lambda}_7}, \quad \lambda_8 = \lambda_8 e^{i\hat{\lambda}_8}, \quad \hat{j} = \hat{h} + \hat{h} \end{aligned} \right\} \quad (5.3)$$

For regions of parameter space $\lambda_2^t, \lambda_7, \lambda_8$ in which A_7, A_8 acquire opposite signs under renormalization group scaling, the phases $\hat{h}, \hat{h}, \hat{\phi}$ can be chosen such that all three terms in (5.2) involving $\hat{h}, \hat{h}, \hat{\phi}$ are simultaneously minimized, i.e.,

$$\hat{j} = \pi - \hat{\lambda}_7 + \hat{\lambda}_8/3, \quad \hat{\phi} = -\hat{\lambda}_8/3 \quad (A_7 > 0, A_8 < 0) \quad (5.4a)$$

or

$$\hat{j} = \pi - \hat{\lambda}_7 + \hat{\lambda}_8/3, \quad \hat{\phi} = \pi - \hat{\lambda}_8/3 \quad (A_7 < 0, A_8 > 0) \quad (5.4b)$$

It is clear that one of these choices corresponds to the global minimum of $\mathcal{V}$. If, however, A_7 and A_8 have the same sign, as occurs in certain regions of parameter space, then $\hat{h}, \hat{h}, \hat{\phi}$ can be picked so that any two of the three terms involving phases are minimized while the third is maximized. These choices correspond to extrema or to saddle points of $\mathcal{V}$ with respect to the phases, but not necessarily to the global minimum, which could in principle occur for some other, non-trivial choice of $\hat{h}, \hat{h}, \hat{\phi}$. We have searched for such spontaneous CP-violating global minima and concluded that they do not exist for our set of initial conditions $m_{\frac{1}{2}} \neq 0$, $A = m_0^2 = 0$, and $0 < |\lambda_2^t, \lambda_7, \lambda_8| \lesssim 4$. Moreover, we have found that acceptable phenomenology (in particular $m_{\tilde{\nu}}^2 > 0$, $m_{\tilde{e}_R} > 23$ GeV) restricts our set of solutions to those of the form (5.4b), with $A_7 < 0$ and $A_8 > 0$.

Since observable quantities such as m_t are relatively insensitive to λ_8 , it is most useful to view the space of solutions in the (λ_2^t, λ_7) plane. To assess the sensitivity of the solutions to variations in input parameters $\alpha_3, \sin^2\theta_W$, etc., we will plot our results for two different values of the GUT scale: one "LOGUT" scenario $M_G = 2 \times 10^{15}$ GeV ($\alpha_3 \approx 0.11$) which is marginally consistent with the constraint (3.15) from $\tau(p \rightarrow e^+\pi^0)$, and one "HIGUT" scenario with $M_G = 5 \times 10^{16}$ GeV ($\alpha_3 \approx 0.14$) which is close to the upper limit $M_G = M_{SU}$ of Fig. 1a. Allowed regions of the (λ_2^t, λ_7) parameter space for both of these scenarios are shown in Figs. 4 and 5, which respectively plot λ_2^t, λ_7 renormalized at $Q^2 = M_G^2$, and λ_2^t, λ_7 renormalized at $Q^2 = m_g^2$, our low-energy cut-off. Solid curves correspond to boundaries within which three non-zero v.e.v.'s $v, \bar{v}, x$ are achieved. The saw-tooth lines result from the phenomenological constraints $m_{\tilde{\nu}}^2 > 0$, $m_{\tilde{e}_R} > 23$ GeV. Superimposed on Fig. 5 are contours of constant gaugino masses $m_{\frac{1}{2}}$, which sets the scale for all other low-energy dimensionful parameters.

Values of the ratios $|v/\bar{v}|$ and $|x/\bar{v}|$ are given in Figs. 6a and 6b, where the

relative signs $v/\bar{v} < 0$, $x/\bar{v} > 0$ are fixed by Eq. (5.4b)*). Although these ratios are rather parameter-dependent, we see that $v/\bar{v} \ll 1$ when $m_{\frac{1}{2}} \gtrsim 100$ GeV, corresponding to λ_1^b/λ_2^t considerably larger than m_b/m_t . This is why the b-quark contribution to the RGE's (B.1)-(B.16) was not neglected, as has often been the case in other analyses. We find that including λ_1^b significantly increases the range of parameters λ_2^t , λ_7 , λ_8 in which acceptable electroweak breaking occurs. Since we expect all the Yukawa couplings to be $O(1)$ [31], the result $v/\bar{v} \ll 1$ provides a natural explanation for the observed ratio $m_b/m_t \ll 1$. This line of argument also suggests that $m_{\frac{1}{2}}$ cannot be very small (see Fig. 6a).

With regard to the particle spectrum, we find that the minimal $SU(5) \times U(1)$ model is quite specific in its predictions. In particular, we find that the top-quark mass is rather tightly constrained. Figure 7 plots the physical top quark mass $m_t(Q^2 = m_t^2)$ as a function of $m_{\frac{1}{2}}$. We observe relatively little variation in m_t over a wide range of $m_{\frac{1}{2}}$. Although the predictions for the two GUT scenarios defined above differ by about 10 GeV, we find

$$60 \text{ GeV} < m_t \lesssim 90 \text{ GeV} \quad (5.5)$$

throughout the allowed domain of parameter space.

It is also worth noting that the non-minimal superstring-derived [17] version of the flipped $SU(5) \times U(1)$ model has a broken $SO(10)$ symmetry which ensures that $m_b = m_\tau$ [18] at the superunification scale. In the minimal model [11], renormalization effects would then yield $m_b \sim 5$ to 6 GeV for the physical bottom quark mass as seen in the Table. The numerical value of m_b is not greatly affected by the values of λ_1^b , λ_2^t and λ_7 that appear in the renormalization group equation (B.1) for λ_1^b , as long as these are kept within the phenomenologically preferred domain discussed above. Lower (higher) values of m_b correspond to lower (higher) values of $\alpha_3(m_W)$ although the values of m_b do not vary greatly. The renormalization of m_b/m_t is somewhat larger in the non-minimal $SU(5) \times U(1)$ model [17], where the precise value of m_b is somewhat sensitive to M_{SU}/M_G and α_G .

Figure 8 plots the masses of the various supersymmetric particles in units of $m_{\frac{1}{2}}$. We find that even with D-terms the predictions are quite specific. Notable exceptions are the third-generation $\tilde{t}$ and $\tilde{b}$ squarks (not shown). This is because

*) Again, we note that in our flipped notation, $h^0(\bar{h}^0)$ has hypercharge $-\frac{1}{2}$ ($+\frac{1}{2}$) and gives mass to the bottom (top) quark.

the $\tilde{t}_L, \tilde{t}_R$ and $\tilde{b}_L, \tilde{b}_R$ mass matrices contain off-diagonal entries proportional to λ_2^t, λ_1^b , respectively, that can be important:

$$(\tilde{t}_L, \tilde{t}_R) \begin{pmatrix} m_Q^2 + |\lambda_2^t \bar{v}|^2 + \left(\frac{g_Y^2}{20} - \frac{g_2^2}{4}\right)(\bar{v}^2 - v^2) & \lambda_2^t \lambda_7^* \chi^* v^* + \lambda_2^t A_2 \bar{v} \\ \lambda_2^{t*} \lambda_7 \chi v + \lambda_2^{t*} A_2^* \bar{v}^* & m_U^2 + |\lambda_2^t \bar{v}|^2 - \frac{g_Y^2}{5}(\bar{v}^2 - v^2) \end{pmatrix} \begin{pmatrix} \tilde{t}_L^* \\ \tilde{t}_R^* \end{pmatrix} \quad (5.6a)$$

$$(\tilde{b}_L, \tilde{b}_R) \begin{pmatrix} m_Q^2 + |\lambda_1^b v|^2 + \left(\frac{g_Y^2}{20} + \frac{g_2^2}{4}\right)(\bar{v}^2 - v^2) & -\lambda_1^b \lambda_7^* \chi^* \bar{v}^* - \lambda_1^b A_1 v \\ -\lambda_1^{b*} \lambda_7 \chi \bar{v} - \lambda_1^{b*} A_1^* v^* & m_D^2 + |\lambda_1^b v|^2 + \frac{g_Y^2}{10}(\bar{v}^2 - v^2) \end{pmatrix} \begin{pmatrix} \tilde{b}_L^* \\ \tilde{b}_R^* \end{pmatrix} \quad (5.6b)$$

The off-diagonal elements split the physical mass eigenstates on either side of m_Q by amounts that tend to be large and highly sensitive to the input parameters $\lambda_2^t, \lambda_7, \lambda_8$. As a consequence, the third-generation squark masses are difficult to predict, and there are isolated regions of parameter space for which the constraint $m_{\tilde{q}} \gtrsim 47$ GeV becomes the principal phenomenological constraint.

Finally, we consider the lightest combination of neutral gauginos and Higgsinos, which we denote by χ . Its mass and composition are governed by the following 5×5 mixing matrix for the $(\tilde{w}^3, \tilde{B}, i\tilde{h}^0, i\tilde{h}^0, i\tilde{\phi}_0)$ fields:

$$\begin{pmatrix} \mu_2 & 0 & \frac{g_2 v^*}{\sqrt{2}} & -\frac{g_2 \bar{v}^*}{\sqrt{2}} & 0 \\ 0 & \mu_Y & -\frac{\sqrt{3}}{\sqrt{10}} g_Y v^* & \frac{\sqrt{3}}{\sqrt{10}} g_Y \bar{v}^* & 0 \\ \frac{g_2 v^*}{\sqrt{2}} & -\frac{\sqrt{3}}{\sqrt{10}} g_Y v^* & 0 & -\lambda_7 x & -\lambda_7 \bar{v} \\ -\frac{g_2 \bar{v}^*}{\sqrt{2}} & \frac{\sqrt{3}}{\sqrt{10}} g_Y \bar{v}^* & -\lambda_7 x & 0 & -\lambda_7 v \\ 0 & 0 & -\lambda_7 \bar{v} & -\lambda_7 v & -6\lambda_8 x \end{pmatrix}$$

(5.7)

The eigenvalues are shown as functions of $m_{\frac{1}{2}}$ in Fig. 9, where the principal content of each eigenstate is also indicated except where large, model-dependent mixing prevents such an identification. Here $\tilde{Z}_+$ and $\tilde{Z}_-$ are the orthogonal combinations $\tilde{Z} \pm i\tilde{A}$, with $\tilde{A} \equiv (v \tilde{h}^0 - \bar{v} \tilde{h}^0)/\sqrt{|v|^2 + |\bar{v}|^2}$. The lightest neutralino χ is predominantly a symmetric Higgsino $\tilde{S} \equiv (v \tilde{h}^0 + \bar{v} \tilde{h}^0)/\sqrt{|v|^2 + |\bar{v}|^2}$. Its mass m_χ is always less than $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$, $m_{\tilde{q}}$ and $m_{\tilde{g}}$, so that $\tilde{e} \rightarrow e\chi$, $\tilde{q} \rightarrow q\chi$ and $\tilde{g} \rightarrow q\bar{q}\chi$ decays are always kinematically allowed. As discussed in Ref. [14], there may not be any R-parity in the minimal supersymmetric flipped $SU(5) \times U(1)$ model, and the $\tilde{h}^0$, $\tilde{h}^0$, $\tilde{\phi}_0$ may have small mixings of order m_W/M_G with the v_L . In this case, in contrast to most supersymmetric models, the χ is unstable and has as dominant decay modes $\chi \rightarrow f\bar{f}v_L$. However, the lifetime for these decays is at least 1s, so the χ would not have visible decays in any laboratory experiment.

Note that if $m_{\frac{1}{2}} \lesssim 80$ GeV, the $\tilde{v}$ can be the lightest supersymmetric particle (see Fig. 8). In this case, the $\tilde{v}$ would have similar cosmological properties to the χ , decaying into $f\bar{f}h^0$ with a lifetime $\gtrsim 1$ s. In either case, stable dark matter could be provided by the $\tilde{\Phi}$, the supersymmetric partner of the Φ field whose v.e.v. breaks $SU(5) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$, which has a lifetime $\sim (10^{28}-10^{34})$ years [14]. In non-minimal $SU(5) \times U(1)$ models derived from the string, R-parity may be conserved^{*}).

We conclude that the minimal flipped $SU(5) \times U(1)$ model makes rather definite predictions for the top-quark mass and the supersymmetric spectrum, including the

^{*}) In this case, the χ is stable and provides a natural dark matter candidate with $\Omega_\chi \sim 1$, modulo potential problems with the repopulation of χ 's by Φ decays subsequent to $\chi\chi$ annihilation [14].

light neutralino sector^{*)}. Experimentalists should be encouraged that the flipped $SU(5) \times U(1)$ model suggests that many as yet unseen particles may be accessible to forthcoming rounds of experiments.

6. - SUMMARY AND OUTLOOK

In this paper we have explored the phenomenology of the supersymmetric flipped $SU(5) \times U(1)$ GUT [11], including both the minimal model [11] and a variant [17] derived from the fermionic formulation of string in four dimensions. We have estimated the grand unification scale M_G , where $SU(5)$ is broken, to be $M_G \sim 10^{16 \pm 1}$ GeV in the minimal model, and the superunification scale M_{SU} , where the $SU(5)$ and $U(1)$ couplings are equal, to be between M_G and 10^{17} GeV. The values of M_G and M_{SU} are more variable in the string-inspired variant, for which values of M_{SU} equal to the likely string scale (2.2) are plausible. We have shown that the dominant mechanism for baryon decay is gauge boson exchange, which makes characteristic predictions for the ratios of partial pionic decay rates. We have verified that plausible assumptions on the pattern of soft supersymmetry breaking in the observable sector lead to an early breaking of $SU(5)$ at a scale $M_G \sim M_{SX}$, the scale at which supersymmetry is dynamically broken. Finally, we have explored the phenomenology of the resulting low-energy model, showing that electroweak symmetry breaking can occur with the correct value of m_W for acceptable and accessible values of m_t , and exploring the associated spectrum of detectable supersymmetric particles.

It has been shown in other work [16,17] how supersymmetric flipped $SU(5) \times U(1)$ may be obtained from string theory, and the cosmology of the minimal model and some variants have also been explored [12,13,14]. As noted during the course of this paper, the results of our phenomenological calculations are particularly favourable for our improved string model [17]. It has a value of M_{SU} which is naturally of order the string estimate (2.2), and the fact that $M_G < M_{SU}$ provides a natural mechanism [17] for the generation of hierarchical fermion masses.

We now point to a few outstanding issues. We are well on the way to deriving [17] from string theory a completely satisfying $SU(5) \times U(1)$ model with all the desired Yukawa couplings. Even when this is done, it would remain to demonstrate that the gauge and Yukawa couplings fell in the ranges allowed by the phenomenological analysis of this paper. Moreover, we have made certain assumptions about the magnitude of soft supersymmetry breaking, its form, and the scale M_{SX} at which it originates. These are not implausible, given our present level of understanding of string dynamics, but are as yet unproven. These are a few of the steps needed

*) Detailed predictions of the model for Higgs bosons are discussed in Ref. [34].

to verify that supersymmetric flipped $SU(5) \times U(1)$ is a viable theory of everything below the Planck scale.

Even if this were established, there would remain other, more fundamental questions. Are there any other viable low-energy models, and if so, what are they? How does string theory select one of these from among the myriad of different possible consistent models, the vast majority of which are phenomenologically unacceptable? The end of theoretical physics is not yet in sight.

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A P P E N D I X A

ONE-LOOP RENORMALIZATION GROUP EQUATIONS FOR YUKAWA COUPLINGS
AND SOFT SUPERSYMMETRY BREAKING PARAMETERS FOR $Q^2 > M_G^2$

For the superpotential

$$\begin{aligned}
 W = & \frac{\lambda_1^b}{8} \epsilon^{\alpha\beta\gamma\delta\epsilon} F_{\alpha\beta} F_{\gamma\delta} h_\epsilon + \lambda_2^t F_{\alpha\beta} \bar{f}^\alpha \bar{h}^\beta \\
 & + \frac{\lambda_4}{8} \epsilon^{\alpha\beta\gamma\delta\epsilon} H_{\alpha\beta} H_{\gamma\delta} h_\epsilon + \frac{\lambda_5}{8} \epsilon^{\alpha\beta\gamma\delta\epsilon} \bar{H}^{\alpha\beta} \bar{H}^{\gamma\delta} \bar{h}_\epsilon \\
 & + \frac{\lambda_6^i}{2} F_{i\alpha\beta} \bar{H}^{\alpha\beta} \phi_i + \lambda_7 h_\alpha \bar{h}^\alpha \phi_0 + \lambda_8^0 \phi_0^3 + \lambda_8^i \phi_i \phi_0^2
 \end{aligned}$$

The relevant renormalization group equations are

$$\frac{d\lambda_1^b}{dt} = \frac{\lambda_1^b}{32\pi^2} \left[18\lambda_1^{b^2} + 8\lambda_2^{t^2} + 6\lambda_4^2 + 4\lambda_6^{3^2} + 2\lambda_7^2 - \frac{192}{5}g_5^2 - \frac{3}{5}g_1^2 \right] \quad (A.1)$$

$$\frac{d\lambda_2^t}{dt} = \frac{\lambda_2^t}{32\pi^2} \left[6\lambda_1^{b^2} + 20\lambda_2^{t^2} + 6\lambda_5^2 + 2\lambda_6^{3^2} + 2\lambda_7^2 - \frac{168}{5}g_5^2 - \frac{7}{5}g_1^2 \right] \quad (A.2)$$

$$\frac{d\lambda_4}{dt} = \frac{\lambda_4}{32\pi^2} \left[6\lambda_1^{b^2} + 18\lambda_4^2 + 2\lambda_7^2 - \frac{192}{5}g_5^2 - \frac{3}{5}g_1^2 \right] \quad (A.3)$$

$$\frac{d\lambda_5}{dt} = \frac{\lambda_5}{32\pi^2} \left[8\lambda_2^{t^2} + 18\lambda_5^2 + \sum_i 4\lambda_6^{i^2} + 2\lambda_7^2 - \frac{192}{5}g_5^2 - \frac{3}{5}g_1^2 \right] \quad (A.4)$$

$$\frac{d\lambda_6^i}{dt} = \frac{\lambda_6^i}{32\pi^2} \left[6\lambda_1^{b^2} \delta_{3i} + 4\lambda_2^{t^2} \delta_{3i} + 6\lambda_5^2 + 22\lambda_6^{i^2} + \sum_i 2\lambda_6^{i^2} + 4\lambda_8^{i^2} - \frac{144}{5}g_5^2 - \frac{1}{5}g_1^2 \right] \quad (A.5)$$

$$\frac{d\lambda_7}{dt} = \frac{\lambda_7}{32\pi^2} \left[6\lambda_1^{b^2} + 8\lambda_2^{t^2} + 6\lambda_4^2 + 6\lambda_5^2 + 14\lambda_7^2 + 36\lambda_8^{0^2} + \sum_i 8\lambda_8^{i^2} - \frac{96}{5}g_5^2 - \frac{4}{5}g_1^2 \right] \quad (A.6)$$

$$\frac{d\lambda_8^0}{dt} = \frac{\lambda_8^0}{32\pi^2} \left[30\lambda_7^2 + 108\lambda_8^{0^2} + \sum_i 24\lambda_8^{i^2} \right] \quad (A.7)$$

$$\frac{d\lambda_8^i}{dt} = \frac{\lambda_8^i}{32\pi^2} \left[72\lambda_8^{0^2} + 4\lambda_8^{i^2} + \sum_i 16\lambda_8^{i^2} + 20\lambda_6^{i^2} \right] \quad (A.8)$$

$$\frac{dA_b}{dt} = \frac{1}{16\pi^2} \left[18\lambda_1^{b^2} A_b + 8\lambda_2^{t^2} A_t + 6\lambda_4^2 A_4 + 4\lambda_6^{3^2} A_6^3 + 2\lambda_7^2 A_7 + \frac{192}{5} g_5^2 \mu_5 + \frac{3}{5} g_1^2 \mu_1 \right] \quad (A.9)$$

$$\frac{dA_t}{dt} = \frac{1}{16\pi^2} \left[6\lambda_1^{b^2} A_b + 20\lambda_2^{t^2} A_t + 6\lambda_5^2 A_5 + 2\lambda_6^{3^2} A_6^3 + 2\lambda_7^2 A_7 + \frac{168}{5} g_5^2 \mu_5 + \frac{7}{5} g_1^2 \mu_1 \right] \quad (A.10)$$

$$\frac{dA_4}{dt} = \frac{1}{16\pi^2} \left[6\lambda_1^{b^2} A_b + 18\lambda_4^2 A_4 + 2\lambda_7^2 A_7 + \frac{192}{5} g_5^2 \mu_5 + \frac{3}{5} g_1^2 \mu_1 \right] \quad (A.11)$$

$$\frac{dA_5}{dt} = \frac{1}{16\pi^2} \left[8\lambda_2^{t^2} A_t + 18\lambda_5^2 A_5 + \sum_i 4\lambda_6^{i^2} A_6^i + 2\lambda_7^2 A_7 + \frac{192}{5} g_5^2 \mu_5 + \frac{3}{5} g_1^2 \mu_1 \right] \quad (A.12)$$

$$\frac{dA_6^i}{dt} = \frac{1}{16\pi^2} \left[6\lambda_1^{b^2} A_b \delta_{3i} + 4\lambda_2^{t^2} A_t \delta_{3i} + 6\lambda_5^2 A_5 + 22\lambda_6^{i^2} A_6^i + \sum_i 2\lambda_6^{i^2} A_6^i + 4\lambda_8^{i^2} A_8^i + \frac{144}{5} g_5^2 \mu_5 + \frac{1}{5} g_1^2 \mu_1 \right] \quad (A.13)$$

$$\frac{dA_7}{dt} = \frac{1}{16\pi^2} \left[6\lambda_1^{b^2} A_b + 8\lambda_2^{t^2} A_t + 6\lambda_4^2 A_4 + 6\lambda_5^2 A_5 + 14\lambda_7^2 A_7 + 36\lambda_8^{o^2} A_8^o + \sum_i 8\lambda_8^{i^2} A_8^i + \frac{96}{5} g_5^2 \mu_5 + \frac{4}{5} g_1^2 \mu_1 \right] \quad (A.14)$$

$$\frac{dA_8^o}{dt} = \frac{1}{16\pi^2} \left[30\lambda_7^2 A_7 + 108\lambda_8^{o^2} A_8^o + \sum_i 24\lambda_8^{i^2} A_8^i \right] \quad (A.15)$$

$$\frac{dA_8^i}{dt} = \frac{1}{16\pi^2} \left[72\lambda_8^{o^2} A_8^o + 4\lambda_8^{i^2} A_8^i + \sum_i 16\lambda_8^{i^2} A_8^i + 20\lambda_6^{i^2} A_6^i \right] \quad (A.16)$$

Introducing the following notation:

$$\begin{aligned}
 F_b &\equiv \lambda_1^2 (2m_{F_3}^2 + m_h^2 + A_b^2) & F_t &\equiv \lambda_2^2 (m_{F_3}^2 + m_{\tilde{F}_3}^2 + m_h^2 + A_t^2) \\
 F_4 &\equiv \lambda_4^2 (2m_H^2 + m_h^2 + A_4^2) & F_s &\equiv \lambda_5^2 (2m_H^2 + m_h^2 + A_s^2) \\
 F_b^i &\equiv \lambda_6^2 (m_{F_i}^2 + m_H^2 + m_{\phi_i}^2 + A_b^{i2}) & F_7 &\equiv \lambda_7^2 (m_{\phi_0}^2 + m_h^2 + m_H^2 + A_7^2) \\
 F_8^0 &\equiv \lambda_8^2 (3m_{\phi_0}^2 + A_8^{02}) & F_8^i &\equiv \lambda_8^2 (m_{\phi_i}^2 + 2m_{\phi_0}^2 + A_8^{i2})
 \end{aligned}$$

we also have

$$\frac{dm_{F_i}^2}{dt} = \frac{1}{16\pi^2} \left[6F_b \delta_{3i} + 4F_t \delta_{3i} + 2F_6^i - \frac{144}{5} g_s^2 \mu_s^2 - \frac{1}{5} g_1^2 \mu_1^2 \right] \quad (A.17)$$

$$\frac{dm_{\tilde{F}_i}^2}{dt} = \frac{1}{16\pi^2} \left[8F_t \delta_{3i} - \frac{96}{5} g_s^2 \mu_s^2 - \frac{9}{5} g_1^2 \mu_1^2 \right] \quad (A.18)$$

$$\frac{dm_{\tilde{L}_i}^2}{dt} = \frac{1}{16\pi^2} \left[-5 g_1^2 \mu_1^2 \right] \quad (A.19)$$

$$\frac{dm_h^2}{dt} = \frac{1}{16\pi^2} \left[6F_b + 6F_4 + 2F_7 - \frac{96}{5} g_s^2 \mu_s^2 - \frac{4}{5} g_1^2 \mu_1^2 \right] \quad (A.20)$$

$$\frac{dm_H^2}{dt} = \frac{1}{16\pi^2} \left[8F_t + 6F_s + 2F_7 - \frac{96}{5} g_s^2 \mu_s^2 - \frac{4}{5} g_1^2 \mu_1^2 \right] \quad (A.21)$$

$$\frac{dm_{\tilde{H}}^2}{dt} = \frac{1}{16\pi^2} \left[6F_4 - \frac{144}{5} g_s^2 \mu_s^2 - \frac{1}{5} g_1^2 \mu_1^2 \right] \quad (A.22)$$

$$\frac{dm_{\tilde{H}}^2}{dt} = \frac{1}{16\pi^2} \left[6F_s + \sum_i 2F_6^i - \frac{144}{5} g_s^2 \mu_s^2 - \frac{1}{5} g_1^2 \mu_1^2 \right] \quad (A.23)$$

$$\frac{dm_{\phi_0}^2}{dt} = \frac{1}{16\pi^2} \left[10F_7 + 36F_8^0 + \sum_i 8F_8^i \right] \quad (A.24)$$

$$\frac{dm_{\phi_i}^2}{dt} = \frac{1}{16\pi^2} \left[20F_6^i + 4F_8^i \right] \quad (A.25)$$

A P P E N D I X B

RENORMALIZATION GROUP EQUATIONS FOR LOW-ENERGY YUKAWA COUPLINGS AND SOFT
SUPERSYMMETRY-BREAKING PARAMETERS

For the superpotential $W = \lambda_1^b b^c (b h^0 - t h^-) + \lambda_2^t t^c (t \bar{h}^0 - b \bar{h}^+) + \lambda_7 (h^0 \bar{h}^0 - h^- \bar{h}^+) \phi + \lambda_8 \phi^3$,
the relevant renormalization group equations are:

$$\frac{d\lambda_1^b}{dt} = \frac{\lambda_1^b}{8\pi^2} \left[3\lambda_1^{b^2} + \frac{1}{2}\lambda_2^{t^2} + \frac{1}{2}\lambda_7^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{3}{5}\left(\frac{7}{18}\right)g_Y^2 \right] \quad (B.1)$$

$$\frac{d\lambda_2^t}{dt} = \frac{\lambda_2^t}{8\pi^2} \left[\frac{1}{2}\lambda_1^{b^2} + 3\lambda_2^{t^2} + \frac{1}{2}\lambda_7^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{3}{5}\left(\frac{13}{18}\right)g_Y^2 \right] \quad (B.2)$$

$$\frac{d\lambda_7}{dt} = \frac{\lambda_7}{8\pi^2} \left[\frac{3}{2}\lambda_1^{b^2} + \frac{3}{2}\lambda_2^{t^2} + 2\lambda_7^2 + 9\lambda_8^2 - \frac{3}{2}g_2^2 - \frac{3}{5}\left(\frac{1}{2}\right)g_Y^2 \right] \quad (B.3)$$

$$\frac{d\lambda_8}{dt} = \frac{\lambda_8}{8\pi^2} \left[3\lambda_7^2 + 27\lambda_8^2 \right] \quad (B.4)$$

$$\frac{dA_b}{dt} = \frac{1}{8\pi^2} \left[6\lambda_1^{b^2} A_b + \lambda_2^{t^2} A_t + \lambda_7^2 A_7 + \frac{16}{3}g_3^2 \mu_3 + 3g_2^2 \mu_2 + \frac{3}{5}\left(\frac{7}{9}\right)g_Y^2 \mu_Y \right] \quad (B.5)$$

$$\frac{dA_t}{dt} = \frac{1}{8\pi^2} \left[\lambda_1^{b^2} A_b + 6\lambda_2^{t^2} A_t + \lambda_7^2 A_7 + \frac{16}{3}g_3^2 \mu_3 + 3g_2^2 \mu_2 + \frac{3}{5}\left(\frac{13}{9}\right)g_Y^2 \mu_Y \right] \quad (B.6)$$

$$\frac{dA_7}{dt} = \frac{1}{8\pi^2} \left[3\lambda_1^{b^2} A_b + 3\lambda_2^{t^2} A_t + 4\lambda_7^2 A_7 + 18\lambda_8^2 A_8 + 3g_2^2 \mu_2 + \frac{3}{5}g_Y^2 \mu_Y \right] \quad (B.7)$$

$$\frac{dA_8}{dt} = \frac{1}{8\pi^2} \left[6\lambda_7^2 A_7 + 54\lambda_8^2 A_8 \right] \quad (B.8)$$

Introducing the following notation:

$$\begin{aligned} F_b &\equiv \lambda_1^{b^2} (m_{D_3}^2 + m_{Q_3}^2 + m_h^2 + A_b^2) & F_t &\equiv \lambda_2^{t^2} (m_{U_3}^2 + m_{Q_3}^2 + m_h^2 + A_t^2) \\ F_7 &\equiv \lambda_7^2 (m_h^2 + m_{\bar{h}}^2 + m_\phi^2 + A_7^2) & F_8 &\equiv \lambda_8^2 (3m_\phi^2 + A_8^2) \end{aligned}$$

we also have

$$\frac{dm_\phi^2}{dt} = \frac{1}{8\pi^2} \left[2F_7 + 18F_8 \right] \quad (B.9)$$

$$\frac{dm_h^2}{dt} = \frac{1}{8\pi^2} \left[3F_b + F_7 - 3g_2^2 \mu_2^2 - \frac{3}{5}g_Y^2 \mu_Y^2 \right] \quad (B.10)$$

$$\frac{dm_{\bar{h}}^2}{dt} = \frac{1}{8\pi^2} \left[3F_t + F_7 - 3g_2^2 \mu_2^2 - \frac{3}{5}g_Y^2 \mu_Y^2 \right] \quad (B.11)$$

$$\frac{dm_{\phi_i}^2}{dt} = \frac{1}{8\pi^2} \left[F_b \delta_{3i} + F_t \delta_{3i} - \frac{16}{3} g_3^2 \mu_3^2 - 3 g_2^2 \mu_2^2 - \frac{3}{5} \left(\frac{1}{9} \right) g_Y^2 \mu_Y^2 \right] \quad (\text{B.12})$$

$$\frac{dm_{\phi_i^c}^2}{dt} = \frac{1}{8\pi^2} \left[2 F_b \delta_{3i} - \frac{16}{3} g_3^2 \mu_3^2 - \frac{3}{5} \left(\frac{4}{9} \right) g_Y^2 \mu_Y^2 \right] \quad (\text{B.13})$$

$$\frac{dm_{\psi_i^c}^2}{dt} = \frac{1}{8\pi^2} \left[2 F_t \delta_{3i} - \frac{16}{3} g_3^2 \mu_3^2 - \frac{3}{5} \left(\frac{16}{9} \right) g_Y^2 \mu_Y^2 \right] \quad (\text{B.14})$$

$$\frac{dm_{L_i}^2}{dt} = \frac{1}{8\pi^2} \left[-3 g_2^2 \mu_2^2 - \frac{3}{5} g_Y^2 \mu_Y^2 \right] \quad (\text{B.15})$$

$$\frac{dm_{\bar{L}_i^c}^2}{dt} = \frac{1}{8\pi^2} \left[-\frac{3}{5} (4) g_Y^2 \mu_Y^2 \right] \quad (\text{B.16})$$

Table

Physical values of m_b (in GeV) assuming $m_b = m_\tau$ at the scale M_{SU}

$\Lambda_{\overline{MS}}$ (4 flavours) (GeV)	no Yukawa couplings	max. Yukawa couplings
0.4	6.3	6.0
0.2	5.4	5.2
0.1	4.8	4.6

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FIGURE CAPTIONS

- Fig. 1 : Values of M_G and M_{SU} in the minimal $SU(5) \times U(1)$ model. The error bars indicate ± 1 standard deviation values for the parameters $\alpha_3(m_W)$, $\sin^2 \theta_W$.
- Fig. 2 : Values of M_G and M_{SU} in a non-minimal $SU(5) \times U(1)$ model derived from four-dimensional string.
- Fig. 3 : The one-loop effective potential $\mathcal{V}_T = \mathcal{V}_0 + \Delta \mathcal{V}$ as a function of $\langle v_H^c, \bar{v}_H^c \rangle$. Figure (a) shows a global minimum at M_{SX} where soft supersymmetry breaking switches on; (b) illustrates the anticipated form of the potential at scales $Q^2 > M_{SX}^2$.
- Fig. 4 : Regions of $\lambda_2^t(M_G)$, $\lambda_7(M_G)$ parameter space for which acceptable electroweak symmetry breaking and low-energy phenomenology occur: (a) for the LOGUT scenario $M_G = 2 \times 10^{15}$ GeV; (b) for the HIGUT senario $M_G = 5 \times 10^{16}$ GeV.
- Fig. 5 : As in Fig. 4, but with λ_2^t , λ_7 renormalized at a low-energy scale $Q^2 = m_{\tilde{g}}^2$. Superimposed on the regions are contours of constant gaugino mass $m_{\frac{1}{2}}$, which sets the scale for all low-energy dimensionful parameters.
- Fig. 6 : Values of the ratios $|v/\bar{v}|$ (a) and $|x/\bar{v}|$ (b) as functions of $m_{\frac{1}{2}}$ in the minimal $SU(5) \times U(1)$ model.
- Fig. 7 : Values of m_t as a function of $m_{\frac{1}{2}}$ in the LOGUT scenario $M_G = 2 \times 10^{15}$ GeV and in the HIGUT scenario $M_G = 5 \times 10^{16}$ GeV.
- Fig. 8 : Values of $m_{\tilde{g}}$, $m_{\tilde{q}}$, $m_{\tilde{e}_L}$, $m_{\tilde{e}_R}$ and $m_{\tilde{\nu}}$ as functions of $m_{\frac{1}{2}}$ in the minimal $SU(5) \times U(1)$ model.
- Fig. 9 : Neutralino mass eigenvalues and approximate eigenvectors as functions of $m_{\frac{1}{2}}$ in the minimal $SU(5) \times U(1)$ model.

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6a

Fig. 6b

Fig. 7

Fig. 8

Fig. 9