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TRI-PP-88-4
Jan 1988

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Random phase approximation for light nuclei based on fully relativistic Hartree-Fock calculations

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ABSTRACT

The particle-hole spectra of light nuclei are examined in the self-consistent random phase approximation (RPA) based on fully relativistic Hartree and Hartree-Fock models for the nuclear ground state. The particle-hole interaction is completely prescribed by the ground state calculation. It includes σ , ω , ρ and π meson exchanges, with σ and ω parameters adjusted to fit the bulk properties of nuclear matter. Differences between Hartree (no exchange) and Hartree-Fock (with exchange) predictions for the spectra are discussed.

(submitted to Physical Review C)

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CM-P00068238

1. INTRODUCTION

In recent years relativistic models¹ for the ground state properties of nuclei have been pursued with some success. Calculations for finite nuclei have been performed in the Hartree or mean-field approximation² with an interaction based on the exchange of σ , ω and ρ mesons.

The couplings of the σ and ω mesons are determined by fitting the saturation density (1.30 fm.⁻³) and binding energy (15.75 MeV) of nuclear matter, while the mass of the phenomenological σ is fixed by the r.m.s. charge radius of ⁴⁰Ca. The model gives a reasonably good description of the bulk properties of finite nuclei, viz. the neutron and proton density distributions, r.m.s. radii, energy levels and spin-orbit splittings.

Recently Blunden and Iqbal^{3,4} have extended the above model to include exchange terms (i.e. the relativistic Hartree-Fock approximation). In addition to the above mesons, a pseudo-vector pion interaction is included. (The pion does not contribute in the Hartree approximation.) The sigma and omega meson couplings are again determined from nuclear matter, while the rho and pion couplings are taken from experiment. Although the effect of the Fock or exchange terms is large, the final results have gross features similar those of the Hartree calculations. This is probably not too surprising since both calculations saturate nuclear matter at the same place and give similar results for the bulk properties (at normal nuclear densities).

It is of interest to see how well these relativistic models do in reproducing the excited state properties of finite nuclei. Since there are no additional parameters introduced, this constitutes a severe test of the interactions as well as the underlying theory.

Another consideration is that the pion will play a more important role in excited states than it does in the ground state. In particular, the unnatural parity (0⁻, 1⁺, 2⁻, 3⁺...) isovector modes are highly sensitive to the nature of the pionic interaction. The Hartree-Fock coupling constants are (at least partially) constrained by the pion, whereas this is not true for the Hartree coupling constants. Therefore, even though the ground state properties are similar in the two theories, the excited state spectra may be very different.

Properties of collective states were examined in a relativistic semi-classical approximation by Horowitz and Walecka.⁵ They found that the experimental systematics of the collective vibrational modes (giant resonances) could be reproduced in their

model. Furnstahl⁶ has considered the negative parity states in ¹⁶O in a microscopic RPA calculation based on the Hartree approximation. He found that a reasonable excitation spectrum could be obtained with a pseudo-vector pion coupling but not with a pseudo-scalar coupling. One problem reported in his results was that the spurious $1^-, T = 0$ state did not occur at zero energy, as is expected in a self-consistent model where the wave functions of the ground and excited states are determined from the same underlying Hamiltonian. We shall examine this further in section III.

The organization of this paper is as follows. In section II we formulate the RPA equations of motion, and give formulae for the particle-hole matrix elements of the relativistic interaction. The meson parameters determined from the Hartree and Hartree-Fock calculations are reproduced.

In section III we present results for the excited state spectra in both Hartree and Hartree-Fock approximations. We discuss in detail the strength distribution of the giant dipole ($1^-, T = 1$) resonance, as well as the spurious $1^-, T = 0$ state. Section IV contains discussion of the major results and comments on possible modifications to the theory.

II. FORMULATION

Excited states $|\Psi^J\rangle$ of multipolarity J are formed as superpositions of particle-hole states

$$|\Psi^J\rangle = \sum_{p,h} \left(X_{ph}^J a_p^\dagger a_h - Y_{ph}^J a_h^\dagger a_p \right) |0\rangle. \quad (1)$$

Here $|0\rangle$ is the exact Hartree-Fock ground state, which is itself described as an anti-symmetrized Slater determinant

$$|0\rangle = \prod_{i=1}^A a_i^\dagger |vac\rangle. \quad (2)$$

$a_p^\dagger$ and a_h are creation and annihilation operators for positive energy single particle states above and below the Fermi surface. X_{ph}^J and Y_{ph}^J are the amplitudes for creating a particle-hole pair and for annihilating a particle-hole pair already present in the ground state. Negative energy states are excluded from the present description (ie. particle-anti-particle states). The implications of this omission are discussed later on in Sec. IV.

The transition strength between the ground and excited states is given by

$$\begin{aligned} B(J) &= \left| \langle \Psi^J | T^J | 0 \rangle \right|^2 = \sum_{p,h} \left| \langle h^{-1} p; J | T^J | 0 \rangle X_{ph}^J + (-1)^J \langle 0 | T^J | h^{-1} p; J \rangle Y_{ph}^J \right|^2 \\ &= \sum_{p,h} \left| \langle p | T^J | h \rangle \left(X_{ph}^J + (-1)^k Y_{ph}^J \right) \right|^2. \end{aligned} \quad (3)$$

The phase $(-1)^k$ is related to the hermiticity of the transition operator T^J via

$$\langle p | T^J | h \rangle = (-1)^{h-p+k} \langle h | T^J | p \rangle^*, \quad (4)$$

where the convention of Edmonds⁷ has been used for the reduced matrix element. The notation $h - p$ in the exponent is shorthand for $j_h - j_p$.

In the present relativistic calculation, the operators T^J will have the structure of 2×2 matrices constructed from the Lorentz-invariant matrices 1 , γ^μ and γ_5 . The two-component relativistic wave functions have the form²

$$|p\rangle \equiv \begin{pmatrix} {}^2G_{\frac{1}{2}}(\vec{r}) \Phi_{nm}(\vec{r}) \\ -F_p(\vec{r}) \Phi_{-nm}(\vec{r}) \end{pmatrix} \equiv \begin{pmatrix} {}^2\phi_p(r) \\ -\phi_p(r) \end{pmatrix}, \quad (5)$$

where $\kappa = (\ell - j)(2j + 1)$. A tilde is used to distinguish lower component quantities from those of the upper component.

Using the ansatz of Eq. (1) for the wave functions and linearizing the equations of motion leads to the familiar RPA eigenvalue problem⁸

$$\begin{pmatrix} A & B \\ -A & -B \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}, \quad (6)$$

with

$$A_{1,2}^{J,T} = (\varepsilon_{p_1} - \varepsilon_{h_1}) \delta_{h_1, h_2} \delta_{p_1, p_2} + \langle h_1^{-1} p_1; J, T | V | h_2^{-1} p_2; J, T \rangle_{dir+exch}, \quad (7)$$

and

$$B_{1,2}^{J,T} = (-1)^{h_2-p_2} \langle h_1^{-1} p_1; J, T | V | p_2^{-1} h_2; J, T \rangle_{dir+exch}. \quad (8)$$

The two-body matrix elements are reduced with respect to angular momentum J and isospin T . In this paper we will ignore the Coulomb interaction, and work with states of good isospin.

The direct and exchange terms of $A^{J,T}$ and $B^{J,T}$ are depicted graphically in Fig. 1. The exchange matrix element may be rewritten in terms of direct particle-hole matrix elements by angular momentum recoupling. Also, by schematically writing

the interaction V in terms of an isoscalar piece V_0 and an isovector piece $V_1 \tau_1 \cdot \tau_2$, the isospin dependence may be factored out, so that

$$\begin{aligned} \langle h_1^{-1} p_1; J, T | V | h_2^{-1} p_2; J, T \rangle_{dir+exch} = & (2V_0 \delta_{T,0} + 2V_1 \delta_{T,1}) \langle h_1^{-1} p_1; J | V | h_2^{-1} p_2; J \rangle_{dir} \\ & + \sum_{J'} (-1)^{h_2 - p_1 + J' + J} (2J' + 1) \begin{Bmatrix} h_1 & h_2 & J' \\ p_2 & p_1 & J \end{Bmatrix} \\ & \times ((V_0 + 3V_1) \delta_{T,0} + (V_0 - V_1) \delta_{T,1}) \langle h_1^{-1} h_2; J | V | p_1^{-1} p_2; J' \rangle_{dir}. \end{aligned} \quad (9)$$

The first term corresponds to the direct particle-hole interaction A_{dir} of Fig. 1a, while the second corresponds to the exchange term A_{exch} of Fig. 1b.

We will be comparing the results of calculations based on both Hartree and Hartree-Fock models of the nuclear ground state. Consistency demands that the prescription for evaluating two-body matrix elements of the excited states should be the same as that used to construct the ground state. In other words, with the Hartree basis one should only take direct particle-hole matrix elements, while for the Hartree-Fock basis both direct and exchange matrix elements should be used.

The two-body matrix elements $\langle h_1^{-1} p_1; J | V | h_2^{-1} p_2; J \rangle_{dir}$ are easily evaluated in terms of products of one-body matrix elements. A multipole decomposition of $V(r)$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, yields in general

$$V(r) = \sum_J M^J(r_1, \sigma_1) \cdot M^J(r_2, \sigma_2), \quad (10)$$

where $M^J(r_1, \sigma_1)$ is a 2×2 matrix whose elements are spherical tensors of rank J formed from the position and spin coordinates of particle 1. Then

$$\begin{aligned} \langle h_1^{-1} p_1; J | V | h_2^{-1} p_2; J \rangle_{dir} = \\ \frac{(-1)^J}{2J+1} \langle h_1^{-1} p_1; J || M^J(r_1, \sigma_1) || 0 \rangle \langle 0 || M^J(r_2, \sigma_2) || h_2^{-1} p_2; J \rangle. \end{aligned} \quad (11)$$

For the current problem, the interaction in momentum space is given by the exchange of mesons in the form

$$\begin{aligned} V(q) = & \frac{-g_\sigma^2}{q^2 + m_\sigma^2} 1^{(1)} 1^{(2)} + \left[\frac{g_\omega^2}{q^2 + m_\omega^2} + \frac{g_\rho^2}{q^2 + m_\rho^2} \tau_1 \cdot \tau_2 \right] \gamma^{(1)} \cdot \gamma^{(2)} \\ & - \frac{g_\pi^2}{4M^2} \frac{\gamma_\delta^{(1)} \gamma_\mu^{(1)} \gamma_\delta^{(2)} \gamma_\nu^{(2)} q^\nu}{q^2 + m_\pi^2} \tau_1 \cdot \tau_2. \end{aligned} \quad (12)$$

Retardation effects are small for low-lying excitations, so we have set $q_0 = 0$. We have also used the following conventions:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}; \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (13)$$

There are two important features of this interaction to point out. First, we have not included a tensor coupling for the ρ . Secondly there is no contact (or Landau-Migdal) term for the π exchange, as is commonly added to non-relativistic calculations. One of our aims in the present calculation is to see how well the bare interaction does without the addition of phenomenological terms. We shall comment on this further in Sec. IV.

The coordinate space interaction $V(r)$ is related to $V(q)$ by the Fourier transform

$$V(r) = \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q} \cdot \mathbf{r}} V(q) \quad (14)$$

The multipole decomposition may be performed in either coordinate space or momentum space. We prefer the latter for the present problem, since the resulting one-body spherical tensors of Eq. (11) turn out to be the same ones needed to evaluate the one-body matrix elements of Eq. (3). Details are given in Appendix A, where the explicit form of the multipole expansions are given for the various types of interactions.

Table 1 gives the Hartree and Hartree-Fock coupling constants used in the present calculation. Both parameter sets saturate nuclear matter at 1.30 fm.⁻¹ and 15.75 MeV in their respective approximations. The unperturbed basis states are obtained from the numerical Dirac-Hartree-Fock code of Blunden and Iqbal.⁴ The continuum has been discretized similarly to Ref. 6 by imposing a boundary condition that the lower wave function component $F(r_{max}) = 0$ at $r_{max} = 12$ fm. and restricting all integrations to within this sphere. This choice of r_{max} places the $d_{3/2}$ resonance in ¹⁷O at about the right energy, yet has a negligible effect on the bound states. Other boundary conditions were found to give similar unperturbed spectra. The basis has been truncated at three major shell spacings (between 30-50 MeV), with which we were able to obtain reasonable convergence.

In the following we will somewhat loosely refer to "Hartree" as the result of using direct interactions with the Hartree basis and parameter set, and "Hartree-Fock" as the results of using both direct and exchange interactions with the Hartree-Fock basis and parameter set.

III. RESULTS

It is well to keep in mind that different multipole excitations are sensitive to different pieces of the residual interaction. Although exchange terms mix the contributions of all the mesons into the particle-hole interaction, the dominant direct

terms are highly selective. Therefore the Hartree based spectra will strongly reflect the features of the various mesons in the interaction.

In general a multipole excitation of a given spin-parity and isospin will receive a direct contribution from those mesons which carry the same spin-parity and isospin quantum numbers. For example, the σ ($0^+, T = 0$) meson will only contribute to natural parity ($0^+, 1^-, 2^+, 3^- \dots$) $T = 0$ excitations (electric multipoles). The ω ($1^-, T = 0$) meson has both a "Coulombic" interaction which contributes to natural parity excitations, and a weaker "magnetic" interaction which contributes to both natural parity and unnatural parity ($1^+, 2^-, 3^+ \dots$ but not 0^-) excitations (magnetic multipoles). The ρ ($1^-, T = 1$) behaves similarly to the ω , but for $T = 1$ modes. The π ($0^-, T = 1$) only affects the unnatural parity $T = 1$ modes (including the 0^-).

A. ^{16}O

With these comments in mind, we shall begin by examining the negative parity states of ^{16}O , shown in Fig. 2. In the first column is shown the unperturbed energies of the Hartree basis. The unperturbed Hartree-Fock energies are quite similar and have therefore not been drawn. The next two columns are the Hartree and Hartree-Fock predictions for the spectra. Only those levels which have a non-negligible transition strength to the ground state are shown. The fourth column is a selection from the compilation of Ref. 9 of those experimental levels which show predominantly 1p-1h character. Clearly not all levels have been shown.

There are only minor differences between the Hartree and Hartree-Fock $T = 0$ spectra. Both predict a strongly collective 3^- as the first (negative parity) excited state, with the positions of most of the remaining levels being comparable. One exception is the second 0^- level, which lies at its unperturbed level in the Hartree calculation (recall that the ω can contribute to all unnatural parity excitations except the 0^-) but lies some 4 MeV lower in the Hartree-Fock calculation. The 4^- level also does somewhat better in this case. The overall agreement with the experimental systematics is modest.

Differences are more marked for the $T = 1$ excitations. The Hartree is too attractive, particularly for the unnatural parity states, which are strongly influenced by the pion interaction. Indeed, one notices that for these cases the lowest-lying $T = 1$ states are below the corresponding $T = 0$ states, in strong disagreement with experiment. For example, the first $2^-, T = 0$ occurs at 9.0 MeV while the first $2^-, T = 1$

occurs at 7.7 MeV. Similarly, the $0^-, T = 0$ lies at 11.3 MeV and the $0^-, T = 1$ at 9.7 MeV; the $4^-, T = 0$ occurs at 16.4 MeV and the $4^-, T = 1$ at 14.7 MeV.

The Hartree-Fock interaction is repulsive, pushing the levels above their unperturbed values. The correct ordering of the $T = 0$ unnatural parity levels below the $T = 1$ ones is now seen. The low-lying cluster of $0^-, 1^-, 2^-$ and 3^- levels is reproduced, although it lies about 2 MeV below experiment. The rest of the spectrum is also in reasonably good agreement.

In Fig. 3 we have shown the Hartree and Hartree-Fock predictions for the position and strength of the giant dipole ($1^-, T = 1$) resonance. The two-component dipole transition operator to be used in Eq. 3 has the form $T^{1M} = \frac{1}{2} \tau r Y_{1M}(\hat{r}) \gamma^0$. Also shown for comparison is the experimental $^{15}\text{N}(p, \gamma)^{16}\text{O}$ cross section, taken from Ref. 10, which is predominantly dipole in character. The Hartree levels occur at essentially their unperturbed positions, while the Hartree-Fock pushes the centroid up by about 4 MeV. The position of the dominant configuration is then about right, although the splitting with the next strongest configuration is somewhat too large.

A few remarks about the spurious $1^-, T = 0$ state are in order. In any self-consistent model using the same underlying Hamiltonian to generate the ground state and the excited states, there should occur a spurious excitation mode corresponding to a translation of the whole system. In the RPA, this means there will be a $1^-, T = 0$ excitation which occurs at zero energy and which carries 100% of the dipole transition strength.⁸ For both Hartree and Hartree-Fock we found a collective state at about 3 MeV. It carries over 99% of the total strength, and can therefore be clearly identified with the spurious state. To get convergence toward zero energy would require enlarging the basis considerably.

To illustrate the effect of self-consistency, we also took the Hartree coupling constants and looked at the effect of including both direct and exchange interactions. We then obtained similar results to Ref. 6, namely that there are two states near 10 MeV which carry about 10% and 75% of the strength, with the second excited state being the stronger one. The concerns about the spurious state reported in Ref. 6 are therefore resolved if the correct self-consistent couplings are used for each model.

B. ^{12}C

Positive parity states are usually poorly reproduced in 1p-1h RPA calculations. This is because the low-lying positive parity levels in closed shell nuclei have a significant 2p-2h or even 4p-4h character. In order to examine positive parity states we

therefore consider ^{12}C , for which the simplest basis consists of the $p_{3/2}$ and $p_{1/2}$ levels. Fig. 4 shows the results of our calculations. The unperturbed energy splitting is somewhat too large here compared with the empirical one deduced from neighboring nuclei. As in ^{16}O , the ordering of the unnatural parity 1^+ states is wrong for the Hartree case and of about the right magnitude for the Hartree-Fock. The natural parity 2^+ states have the correct ordering in the Hartree calculation, but the splitting is too small. The splitting is a little larger for the Hartree-Fock, but it still falls far short of experiment.

C. ^{40}Ca

The ^{40}Ca results concur with our previous observations about ^{16}O and ^{12}C . The $T = 0$ spectra, shown in Fig. 5, are very similar for Hartree and Hartree-Fock. The collective 3^- states are somewhat too low in energy, but the lowest $5^-, 4^-, 2^-$ and 3^- are in good agreement with experiment. Once again, the ordering of the $0^-, 2^-, 4^-$ and 6^- $T = 0$ and $T = 1$ levels is inverted for Hartree. The low-lying cluster of $4^-, 3^-, 2^-$ and 5^- $T = 1$ levels is well reproduced by the Hartree-Fock results.

The $1^-, T = 1$ modes in the giant resonance region have an energy-weighted centroid of 16.8 MeV in the Hartree case compared with 20.0 MeV for the Hartree-Fock. Experimentally the giant resonance occurs at 20.0 MeV.¹²

The spurious $1^-, T = 0$ excitation occurred at a small imaginary energy in both calculations.

IV. DISCUSSION

The most striking feature of the present results is the difference between Hartree and Hartree-Fock based calculations for the $T = 1$ spectra. The differences are most apparent for the unnatural parity modes, where the pion plays an essential role. The exchange terms considerably weaken the strongly attractive direct pion interaction. Indeed, without some mechanism to damp this interaction pion condensates would likely occur at or near nuclear matter densities.¹

In non-relativistic calculations it is usual to add a repulsive contact interaction (Landau-Migdal term) of the form $g'\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$. Based on the present Hartree-Fock results, there is no compelling case for introducing a comparable term into relativistic interactions. However, one also cannot yet rule out such a term. We believe further study is necessary before a final judgement can be made.

We return now to the implications of omitting negative energy states from the present theory. Although direct transitions to a particle-anti-particle state are ruled out by energy considerations, these states can affect quantities such as the transition strength through virtual polarization (Fig. 6). For example, in order to correctly describe magnetic moments in a relativistic model one has to incorporate polarization corrections coming from these negative energy states (so-called backflow).¹³ This polarization correction will also screen pieces of the interaction, such as the magnetic interaction of the ω . Therefore the transition strengths, and to a much lesser extent the positions, of the magnetic multipoles in our calculation are affected.

A direct extension of the basis to include these negative energy states appears unwieldy because of the enormous increase in the number of configurations it would entail. A more appropriate calculational scheme might be to divide the full Hilbert space into a small model space, such as the one we have used, and a larger residual space which includes both the high-lying particle-hole states and the anti-particle states. The RPA can then make use of the small space in the usual way, but the operators will be renormalized by the residual space (the box in Fig. 6). Renormalizations of magnetic moment operators along these lines in a local density approximation, based on nuclear matter results, have been shown to give results comparable to those of calculations which explicitly evaluate the anti-particle contribution.¹⁴

To summarize, we have examined the self-consistent RPA spectra based on a relativistic Hartree-Fock model for the nuclear ground state. With no additional parameters one is able to reproduce reasonably well the spectra of light nuclei. Hartree based calculations fail to describe the $T = 1$ spectrum, whereas the Hartree-Fock based calculations agree quite well. This difference is largely due to the strongly attractive pion interaction, which is weakened considerably by exchange terms in the Hartree-Fock case.

APPENDIX

Here we present the details of the multipole decomposition (10) of the residual interaction (12). The particle-hole matrix elements can then be obtained via Eqs. (11) and (9).

Consider a 2×2 matrix M^J whose elements are spherical tensors of rank J :

$$M = \begin{pmatrix} M_{11}^J & M_{12}^J \\ M_{21}^J & M_{22}^J \end{pmatrix}. \quad (\text{A.1})$$

Then

$$\begin{aligned} \langle h^{-1}p; J \| M^J \| 0 \rangle &= (-1)^{h+p-J} \langle p \| M^J \| h \rangle \\ &= (-1)^{h+p-J} \{ \langle \phi_p \| M_{11}^J \| \phi_h \rangle - \langle \tilde{\phi}_p \| M_{22}^J \| \tilde{\phi}_h \rangle \\ &\quad + i \langle \phi_p \| M_{12}^J \| \tilde{\phi}_h \rangle + i \langle \tilde{\phi}_p \| M_{21}^J \| \phi_h \rangle \}, \end{aligned} \quad (\text{A.2})$$

where we have used Eq. (5) to reduce the two-component wave function to its one-component elements.

It should be clear that the interaction (12) can also be reduced to a sum of one-component interactions of spin-independent, spin-dependent ($\sigma_1 \cdot \sigma_2$), and pionic ($\sigma_1 \cdot \mathbf{q} \cdot \sigma_2 \cdot \mathbf{q}$) form. We therefore need only give the multipole expansions of these interactions, starting from their form in momentum space.

Expanding (14) gives

$$V(r) = \frac{2}{\pi} \int_0^\infty dq q^2 v(q) \sum_j F_j(q; r_1, r_2), \quad (\text{A.3})$$

where $v(q)$ generally is of the form $g^2/(q^2 + m^2)$. We introduce the notation

$$\begin{aligned} M_J(qr) &= j_J(qr) Y_J(\hat{r}) \\ M_{JL}(qr) &= j_L(qr) [Y_L(\hat{r}) \otimes \sigma]^J; \quad L = J-1, J, J+1. \end{aligned} \quad (\text{A.4})$$

(a) spin-independent interactions:

$$F_J(q; r_1, r_2) = M_J(qr_1) \cdot M_J(qr_2). \quad (\text{A.5})$$

(b) spin-dependent interactions $\sigma_1 \cdot \sigma_2$:

$$F_J(q; r_1, r_2) = \sum_L (-1)^{J+1-L} M_{JL}(qr_1) \cdot M_{JL}(qr_2). \quad (\text{A.6})$$

(c) pionic interactions $\sigma_1 \cdot \mathbf{q} \cdot \sigma_2 \cdot \mathbf{q}$:

$$F_J(q; r_1, r_2) = q^2 \sum_{L, L'} i^{L-L'} \alpha_{JL} \alpha_{JL'} M_{JL}(qr_1) \cdot M_{JL'}(qr_2), \quad (\text{A.7})$$

with

$$\alpha_{JL} = \begin{cases} \left(\frac{J}{2J+1} \right)^{1/2}, & \text{for } L = J-1; \\ - \left(\frac{J+1}{2J+1} \right)^{1/2}, & \text{for } L = J+1. \end{cases}$$

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FIGURES

1. Fig. 1. Direct and exchange particle-hole interactions which appear in the RPA.
2. Fig. 2. Isoscalar and isovector negative parity states in ^{16}O . Experimental levels are from Ref. 9. Not all levels are shown.
3. Fig. 3. Hartree and Hartree-Fock predictions for the strength distribution of the giant dipole mode. Also shown is the dipole-dominated $^{15}\text{N}(p,\gamma)^{16}\text{O}$ cross section, taken from Ref. 10.
4. Fig. 4. Positive parity states of ^{12}C , labelled by spin and isospin. We have only shown those states which occur within a $p_{1/2} - p_{3/2}^{-1}$ model space.
5. Fig. 5. Isoscalar and isovector negative parity states in ^{40}Ca . Experimental levels are from Ref. 11. Not all levels are shown.
6. Fig. 6. Polarization correction to external magnetic probe (solid double line) arising from negative energy states. The dashed lines represent the magnetic part of the ω or ρ interaction.

TABLE I. Hartree and Hartree-Fock parameters sets which saturate nuclear matter at 1.30 fm. $^{-1}$. The masses are 520 MeV, 783 MeV, 770 MeV and 138 MeV for the σ , ω , ρ and π mesons respectively.

	g_σ	g_ω	g_ρ	g_π
Hartree	10.47	13.80	2.426	13.45
Hartree-Fock	10.00	11.90	2.426	13.45

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8