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Mass-Splittings in the Baryon Octet and the Nucleon σ -Term in Lattice QCD*

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Abstract

We calculate in the valence approximation the F and D parameters governing the mass-splittings in the baryon octet. The use of an improved fermion action enables us to go down in quark mass to the $SU(3)$ symmetric point. The F parameter turns out to be smaller by a factor of 2 compared to experiment. The D parameter on the other hand shows a strong dependence on the quark mass. This is evidence that the $\Sigma - \Lambda$ mass-splitting is not a first order $SU(3)$ flavour symmetry breaking effect. We also estimate the contribution of the valence quarks to the nucleon σ -term.

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The program of calculating static hadron properties via the simulation of lattice QCD has been intensely pursued in recent years. Following the standard determination of hadron masses from 2-point correlators, the natural next step is to measure 3-point correlation functions and to extract hadronic matrix elements therefrom. The matrix elements of scalar densities provide a simple example of this type. They tell us how the hadron masses shift as the bare quark mass is changed. In case of the baryon octet, there are three such parameters characterising the mass shifts. The F and D parameters control the strengths of the flavour $SU(3)$ breaking effects within the baryon octet, while the nucleon σ -term measures the effect of changing the mean quark mass. σ appears in the analysis of pion-nucleon scattering data as well, and has been a subject of much controversy lately [1].

In a recent paper [2], Maiani, Martinelli, Paciello and Taglienti have performed a first computation of F , D and σ within the lattice QCD framework with Wilson fermions. Their results look fairly encouraging as far as the D/F ratio is concerned, whereas the absolute size of the individual terms differs by a factor of two from experiment. It should be mentioned, however, that these first estimates were obtained at rather large quark masses, corresponding to the large pion to rho mass-ratios (0.89, 0.84, 0.73).

It has turned out that with the standard Wilson action, it is very difficult to get away from the static limit into the chiral domain. This situation can be improved by using effective fermion actions which are derived from block-diagonalization [3]. We found in our previous spectrum calculations [4] that in the blocking schemes with scale 2 and $\sqrt{3}$ geometries one can approach the chiral limit as close as $M_\pi/M_\rho = 0.45$. This motivated us to extend the computations of ref.[2] to smaller quark masses.

Throughout, we assume exact isospin symmetry and follow the notation of ref.[2]. To begin with, let us recall the mass term in the Hamiltonian :

$$H_m = \bar{m} \bar{\Psi} \mathbf{1} \Psi - \frac{m_s - m}{\sqrt{3}} \bar{\Psi} \lambda_8 \Psi, \quad m = m_u = m_d, \quad \bar{m} = \frac{2m + m_s}{3}. \quad (1)$$

Assuming first order effects of the $SU(3)$ flavour symmetry breaking, the mass-splittings within the baryon octet are conventionally parameterized in terms of two reduced matrix elements (evaluated in the $SU(3)$ symmetric limit), F and D :

$$\langle B_1 | \bar{\Psi} \lambda_i \Psi | B_2 \rangle = F \text{Tr}(B_1^\dagger [\lambda_i, B_2]) + D \text{Tr}(B_1^\dagger \{\lambda_i, B_2\}). \quad (2)$$

The quantities F and D are related to the matrix elements of scalar densities according to

$$\begin{aligned} F &= (A - C)/2, \quad D = (A - 2B + C)/2, \\ A &= \langle p | \bar{u}u | p \rangle, \quad B = \langle p | \bar{d}d | p \rangle, \quad C = \langle p | \bar{s}s | p \rangle. \end{aligned} \quad (3)$$

The nucleon σ -term can be written in terms of these matrix elements as

$$\sigma = m(A + B) = m(3F - D + 2C) = \sigma_{val} + 2mC. \quad (4)$$

From the masses of the baryon octet, one estimates

$$D/F \approx -0.32, \quad \frac{\sigma_{val} M_N}{M_\pi^2} = \frac{3M_N(M_8 - M_N)}{2(M_K^2 - M_\pi^2)} \approx 1.32, \quad (5)$$

where M_8 is the average mass of the baryon octet.

Following ref.[2], we extract the matrix element of a scalar density sandwiched between proton states from

$$R_q(t) = \frac{\sum_{\vec{x}} \langle \mathcal{P}(\vec{x}, t) | \sum_z \bar{q}q(z) | \mathcal{P}(\vec{0}, 0) \rangle}{\sum_{\vec{x}} \langle \mathcal{P}(\vec{x}, t) | \mathcal{P}(\vec{0}, 0) \rangle} \xrightarrow{t \text{ large}} \text{const.} + \langle p | \bar{q}q | p \rangle t . \quad (6)$$

Here the l.h.s. denotes the ratio between a 3-point correlation function and the proton propagator, whereas $\mathcal{P}$ denotes the proton operator. The two contractions that occur in the 3-point correlation function are shown in fig. 1. Only the first graph given there is relevant for the computation of the non-singlet quantities F and D . The second graph contributes to σ , but we drop it ($C = 0$) in the spirit of the valence approximation and arrive at σ_{val} [§].

The present computation is based on the 47 quenched configurations at $\beta = 6.3$ on a $24^3 \times 48$ lattice that we used previously to estimate the hadron spectrum [4]. The background gauge fields were blocked twice with $\sqrt{3}$ geometry. The quark propagators were evaluated on the resulting blocked configurations of size $8^3 \times 16$ with the improved fermionic action and the finite-size reduction technique described in ref.[4]. The 3-point correlations appearing in eq.(6) were computed with the method explained in ref.[2]. For the numerical inversions, we used the conjugate residual and (when necessary) the conjugate gradient algorithms. The values of the hopping parameter κ and the number of inversions at each of them are compiled in table 1. These values were selected in order to cover the range $M_\pi/M_\rho \in [0.45, 0.98]$.

In figs. 2 and 3, we show the “raw data” for the ratios R_u and $R_u - 2R_d$ for κ values of 0.1345 and 0.1360. The figures demonstrate the quality of the signal as given by the linear behaviour of the data in the ranges :

$$\begin{aligned} 4 \leq t \leq 7 & \quad \text{for the proton} \\ 11 \leq t \leq 13 & \quad \text{for the } N^*(\tfrac{1}{2}^-) \end{aligned}$$

Our errors were estimated with the jackknife method. The matrix elements of the scalar densities in both the $\tfrac{1}{2}^+$ and $\tfrac{1}{2}^-$ octets (cf. eq.(3)) are extracted from the associated slopes. They are shown in fig. 4 as a function of m_q . Note that the behaviour of F and D is distinctly different in the regions of large and small quark masses. Both F and D increase in absolute magnitude at smaller quark masses, D much more rapidly than F . The D/F ratio is plotted in fig. 5 as a function of M_π/M_ρ together with the previous results of ref.[2]. For comparison we have also included the quark model prediction [5] (with m_{eff} being the constituent quark mass) :

$$\frac{D}{F} = -\frac{3x}{2+x} , \quad x = \left(\frac{223 \text{ MeV}}{m_{eff}} \right)^3 . \quad (7)$$

[§] The discrepancy between σ extracted from pion-nucleon scattering data and σ_{val} of eq.(5) leaves open the possibility that the contribution we have dropped, $\sigma - \sigma_{val}$, may be sizeable.

The figures show that our signals are good enough to bring us down to the $SU(3)$ symmetric point $\bar{\kappa}^{\S}$. Though D turns out to be of the correct size, its rapid variation in the range $m_q < m_s$ implies that the $\Sigma - \Lambda$ mass-splitting cannot be treated as a first order $SU(3)$ flavour symmetry breaking effect. A realistic estimate of this mass-splitting would therefore necessitate combining quark propagators of unequal masses. F and consequently the $\Xi - N$ mass-splitting can be estimated, and they are too small by about a factor of two. The estimate of D/F is also off by roughly the same amount compared to the number quoted in eq.(5).

Fig. 4 also indicates that the D and F matrix elements in the $\frac{1}{2}^-$ octet are smaller than their counterparts in the $\frac{1}{2}^+$ octet. However, the quality of the data is not good enough to allow for a prediction at the $SU(3)$ symmetric point. For further reference, we have included in table 1 the measured $N^*(\frac{1}{2}^-)$ masses, determined from fits in the range $10 \leq t \leq 12$. The M_{N^*}/M_N mass-ratio extrapolates to approximately the correct value.

Finally, we show in fig. 6 our results for the nucleon σ -term plotted in the combination $\sigma_{val}M_N/M_\pi^2$, which is more suitable for extrapolation to the chiral limit. Our data suggest a rise at small quark masses towards the value derived from experiment. Assuming $(A+B)$ to be approximately independent of the quark mass close to the chiral limit, σ_{val} can also be obtained from the slope of M_N versus $1/\kappa$. Our results in the range $0.1358 \leq \kappa \leq 0.1362$ [4] yield $(A+B) \sim 14$ or $\sigma_{val}M_N/M_\pi^2 \sim 1.1$.

Both in figs. 5 and 6, we note a substantial decrease in the statistical errors with respect to the ones of ref.[2]. We stress that only a minor part of this error reduction is due to the present increase in statistics and the finite-size reduction technique. The major improvement comes from a better overlap between our blocked proton operators and the physical proton wave-function (by a factor of the order 100).

To summarize, we have calculated F , D and σ_{val} down to the point $M_\pi/M_\rho = 0.45$. Regarding F and D , this value is small enough to avoid the nuisance of extrapolations from large quark masses. *Ceterum censemus*, the simulation should be repeated with dynamical quarks.

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[§] We estimated $\bar{\kappa}$ in two ways : using π , ρ and K masses gives $\bar{\kappa} = 0.13617$, while π , N and K masses produce $\bar{\kappa} = 0.13605$. Our value for the critical hopping parameter is $\kappa_c = 0.13650$.

κ	0.1270	0.1290	0.1345	0.13525	0.1360	0.1362
No. of props	5	5	46	46	94	94
$M_{N^*} a$	3.57(11)	3.24(15)	2.13(5)	2.01(4)	1.90(27)	—
M_{N^*}/M_N	1.19(5)	1.23(8)	1.44(3)	1.58(5)	1.88(27)	—

Table 1 : The no. of quark propagators at various κ values in our data sample. Also listed are the $N^*(\frac{1}{2}^-)$ masses in lattice units and the M_{N^*}/M_N mass-ratios.

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Figure Captions

- [1] The two different contractions of the 3-point correlation function appearing in the scalar density matrix element of a baryon. The second diagram is understood to mean only the connected part of the correlation function.
- [2] The ratios R_u and $R_u - 2R_d$ at $\kappa = 0.1345$. The two fits correspond to the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ baryon octet states.
- [3] Same as fig. 2 but at $\kappa = 0.1360$.
- [4] The F and D matrix elements of the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ baryon octets as functions of the lattice bare quark mass $m_q = \frac{1}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_c})$. The arrow marks the position of the $SU(3)$ symmetric point.
- [5] The D/F ratio as a function of M_π/M_ρ . The insert shows the same data on a log scale. The arrow marks the position of the $SU(3)$ symmetric point.
- [6] The value of $\sigma_{val} M_N/M_\pi^2$ as a function of M_π/M_ρ . The result in eq.(5) is plotted at the $SU(2)$ symmetric point.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6