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**YUKAWA COUPLINGS IN DEGENERATE ORBIFOLDS:
TOWARDS A REALISTIC $SU(3) \times SU(2) \times U(1)$ SUPERSTRING**

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ABSTRACT

We discuss the construction of $SU(3) \times SU(2) \times U(1)$ three generation superstrings through the "degenerate orbifolds" recently described by the authors. These are lower rank models continuously connected to rank 16 or 22 models through flat directions in the potential of the scalar fields. The structure of Yukawa couplings is carefully investigated and special attention is paid to non-renormalizable interactions in determining the flat directions and the induced cubic couplings that are forbidden in the original model. The importance of twisted oscillator modes and moduli to this effect is explained. One specific example is presented in detail and its phenomenological consequences such as quark and lepton masses, proton stability and neutrino masses are discussed. In this example there are built-in "stringy" symmetries that protect Higgs doublets from getting large tree-level masses.

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1. Introduction

The space of classical vacua for four-dimensional strings has been extensively studied during the last years, due especially to the large proliferation of constructions admitting chiral fermions in the massless spectrum [1,2]. By now it is conventional wisdom that these vacua correspond to two-dimensional conformal field theories (CFTs). In principle, a candidate CFT will have a set of marginal operators and varying the associated parameters one can generate other vacua continuously connected to the starting point. However, in practice it is difficult to identify these marginal operators, except perhaps in (2,2) models [2]. Thus, in most of the explicit four-dimensional string models only one representative of a class of continuously connected vacua has been analyzed in detail. These representatives usually correspond to so-called multicritical points in the space of continuous parameters and have gauge groups of rank sixteen or twenty-two.

In orbifold models, it has been possible to give a construction [3] in which one can consider classes of continuously related models rather than only one representative. The continuous parameters (besides the moduli) correspond to Wilson lines on the underlying torus. Recently [4], a more general class of continuously connected models, degenerate orbifolds, has been discussed in terms of flat directions in the potential of the massless fields in the twisted and untwisted sectors of (2,2) and (0,2) orbifolds. It was found that generically the models in a continuous class have lower rank symmetries than the original (multicritical) models and substantially reduced matter content. This makes degenerate orbifolds more suitable for obtaining realistic phenomenology *. For this purpose, a good starting point is the class of three-generation models with $SU(3) \times SU(2) \times U(1)^n$ gauge group whose systematic construction, based on Z_3 orbifolds with discrete Wilson lines [7], was given in references [8] and [9].

In this note we describe how a large class of $SU(3) \times SU(2) \times U(1)_Y$ models with three families can be continuously connected to a given $SU(3) \times SU(2) \times U(1)^n$ orbifold through flat directions in the potential of the Y -singlet fields in the original massless spectrum. Our results could be interpreted as implying that in the space of four-dimensional classical string vacua there are candidate points corresponding to the supersymmetric Standard Model.

The resulting pattern of Yukawa couplings among the observable fields is undoubtedly the first test that any potentially realistic model should pass. We will see that in the lower rank models there is a much richer structure than in the original starting point. We prove that in order to generate phenomenologically interesting

* For other approaches see references [5] and [6]

effective couplings, it is necessary to give non-vanishing VEVs to twisted oscillator modes. The rôle of orbifold moduli to this effect is also explained. An interesting feature of these models is the existence of built-in symmetries that prevent the Higgs doublets from acquiring very large tree-level masses. Other symmetries leading to proton stability are present along certain flat directions. We will first explain the generalities of the models. Then, we will describe one specific example in detail and discuss its phenomenological properties.

2. Flat Directions

We now describe the guidelines for searching for flat directions breaking the unwanted $U(1)$'s in orbifold models with gauge group $SU(3) \times SU(2) \times U(1)^n$. Although this class of models has been constructed so far only for the Z_3 orbifold, we make the analysis general enough as to account for any Z_N orbifold. Our generic notation for the massless matter fields is as follows.

Untwisted sector

$$\begin{aligned} U^i & \text{ untwisted charged matter} \\ M^{i\bar{j}} & \text{ orbifold moduli} \end{aligned}$$

Here $i = 1, 2, 3$. In general the U^i are $SU(3) \times SU(2)$ non-singlets. The $M^{i\bar{j}}$ are gauge singlets.

Twisted sector

$$\begin{aligned} T & \text{ twisted } SU(3) \times SU(2) \text{ non-singlets} \\ S & \text{ twisted } SU(3) \times SU(2) \text{ singlets} \\ Y^{\bar{j}} & \text{ twisted } SU(3) \times SU(2) \text{ singlets with oscillators} \end{aligned}$$

All the twisted fields carry an extra index indicating to which fixed point (or combination thereof) they are attached. For Z_N orbifolds with $N > 3$, there are also twisted states with several oscillators acting on the vacuum, i.e $Y^{\bar{j}\bar{k}\dots}$.

A knowledge of the charges of all fields under all $U(1)$'s is required. We can then find the combination [10] that gives the anomalous $U(1)$ always present in these models. The anomaly is cancelled by a 4-dimensional Green-Schwarz [11] term through the usual mechanism [12]. It is also necessary to determine a combination of $U(1)$'s that can be identified with the physical hypercharge, Y . In general, there might be many combinations giving the correct Y assignments for the candidates for quarks and leptons.

The natural question is thus whether it is possible to maintain supersymmetry while giving arbitrary VEVs to a set of the $SU(3) \times SU(2)$ singlets having $Y = 0$. We will now consider the general conditions for vanishing D and F -terms.

Let us denote by Σ_a all singlets with $Y = 0$. The anomalous $U(1)$ is denoted by X and the remaining non-anomalous $U(1)$'s by Q_α , $\alpha = 1, \dots, n-2$. The D-term for the Q_α is given by

$$D^\alpha = \sum_a q_a^\alpha |\Sigma_a|^2 \quad (1)$$

For the anomalous $U(1)$ there appears a Fayet-Illiopoulos (F-I) term [13]. Then, D^X is given by

$$D^X = \sum_a q_a^X |\Sigma_a|^2 + \frac{g}{192\pi^2} \text{Tr} X \quad (2)$$

A sufficient condition for having $\langle D^\alpha \rangle = 0$ while $\langle \Sigma_a \rangle \neq 0$, is the existence of a function $J(\Sigma_a)$ invariant under all the Q_α [14]. In order to have $\langle D^X \rangle = 0$ as well, J must be charged under X , with a charge having sign opposite to that of $\text{Tr} X$. In general there can be many such functions J leading to different flat directions.

Notice that the condition $\langle D^X \rangle = 0$ forces some Σ_a to have non-vanishing VEVs. In this sense, the F-I term is the source of gauge breaking, the gauge coupling, g , will vanish unless some other fields get a VEV. This usually happens in the “degenerate orbifolds” discussed in ref. [4]. Notice also that $\langle D^X \rangle = 0$ fixes the scale of some VEVs to be

$$\langle |\Sigma_a|^2 \rangle \sim \frac{g}{192\pi^2} \quad (3)$$

Taking $g \sim 1$ gives $\langle \Sigma_a \rangle \sim 10^{16}$ GeV. Not all the VEVs are fixed by (3) however. There can be invariant functions, say for example $\Sigma_1 \Sigma_2 \Sigma_3$, that also happen to be invariant under X . In addition to VEVs satisfying (3) we can then have $\langle \Sigma_1 \rangle \sim \langle \Sigma_2 \rangle \sim \langle \Sigma_3 \rangle = v$, arbitrary, without spoiling the vanishing of the D -terms already achieved.

In $N = 1$ supergravity, supersymmetric vacua with zero cosmological constant must have

$$\begin{aligned} \langle W \rangle &= 0 \\ \langle F \rangle &= \left\langle \frac{\partial W}{\partial \phi} \right\rangle = 0 \end{aligned} \quad (4)$$

In order to find the VEVs for which conditions (4) are satisfied we must first determine the superpotential W .

Terms ϕ^{n+3} in W can be seen directly by computing scattering amplitudes of two fermions and $n+1$ scalars. We then want to investigate correlation functions of the form [4,15,16]

$$\langle V_{-\frac{1}{2}}(z_1, \bar{z}_1) V_{-\frac{1}{2}}(z_2, \bar{z}_2) V_{-1}(w_1, \bar{w}_1) V_0(w_2, \bar{w}_2) \cdots V_0(w_{n+1}, \bar{w}_{n+1}) \rangle \quad (5)$$

where the vertex operators correspond to the fields in a given coupling. We refer the reader to ref. [4] for the explicit expressions for the vertex operators for twisted and untwisted fields.

Many vertex correlations, and thus the corresponding couplings, vanish as a consequence of selection rules implied by the orbifold CFT [4,15,16]. Here we just present these rules briefly.

- 1) Point and space group invariance. For example, in Z_3 orbifolds, only couplings of the type $U^i T^{3m}$ are allowed by point group invariance. Space group invariance imposes conditions on which fixed points the twisted fields must be located at.
- 2) $\vec{H}$ -conservation. The $SO(10)$ lattice momentum of bosonized (right) NSR fermions and fermionic twists must be conserved. The corresponding statement for the $E_8 \times E_8$ lattice is just the requirement of gauge invariance. In Z_3 orbifolds, $\vec{H}$ -conservation implies that correlations of type (5) always involve a factor $(\partial X^1 \partial X^2 \partial X^3)^m$ when there are more than three twisted fields.
- 3) Invariance under phase rotations of the internal coordinates. This invariance restricts the number of derivatives ∂X^i , $\partial \bar{X}^i$, $\bar{\partial} \bar{X}^i$, $\bar{\partial} X^i$, entering in a given correlation. For instance, in Z_3 orbifolds, a combination $(\partial X^i)^m (\partial \bar{X}^i)^n (\bar{\partial} \bar{X}^i)^p (\bar{\partial} X^i)^q$ is allowed only if $m + q - n - p = 0 \bmod 3$.
- 4) Correlations such as (5) with $n \geq 1$ (which involve just ϕ , not ϕ^*) always turn out to be proportional to ∂X_{cl}^i . Roughly speaking, these derivatives are proportional to the distance, r , between the fixed points the twisted fields sit at. Then, such non-renormalizable correlations vanish when all twisted fields are located at the same fixed point because in this case $\partial X_{cl}^i \sim r^i = 0$, for all i . Actually, if the six-dimensional orbifold lattice factorizes into three two-dimensional ones, we have $\partial X_{cl}^i = 0$ provided all twisted fields sit at the same fixed point in the i^{th} sublattice.

The strategy to find the wanted flat directions is now clear. We first determine a set of VEVs for the fields Σ_a such that eqs. (1) and (2) are satisfied. We then check, using the selection rules above, that for these VEVs we have $\langle W \rangle = \langle F \rangle = 0$.

We now wish to point out several facts that facilitate the analysis of F -terms. In general, the function $J(\Sigma)$ that guarantees the vanishing of the D -terms can be written as

$$J(\Sigma) = J_0(\Sigma) I(\Sigma) \quad (6)$$

where J_0 cancels the F-I term and I is invariant under all the $U(1)$'s, including the anomalous one. I basically does not contribute to the D -terms. The existence of invariants I involving the Σ fields with non-zero VEV is by itself a dangerous F -term, because I is a gauge allowed coupling. To guarantee F -flatness, we could just require that all flat directions should not have invariants such that $I(\langle \Sigma \rangle) \neq 0$. However, this is far too restrictive and in fact, we will see in the next section the importance of certain invariants in generating effective couplings. Fortunately,

the orbifold selection rules can efficiently forbid the existence of invariants in the superpotential. In general, invariants will satisfy rules 1 and 2 but not 3. For example, the coupling

$$S_a S_b Y^{\bar{j}} \quad (7)$$

always vanishes because the corresponding correlation is proportional to $\bar{\partial} \bar{X}^j$, which is not invariant under phase rotations of the j -th coordinate. A coupling such as (7), satisfying selection rules 1 and 2 but not 3, is called an invariant block. Invariant blocks can be put together to form a coupling allowed by all selection rules. For example, the cubic coupling

$$(S_a S_b Y^{\bar{1}})(S_c Y^{\bar{2}} Y^{\bar{3}}) \quad (8)$$

is allowed by rule 3 since it is proportional to $(\partial X^1 \partial X^2 \partial X^3) (\bar{\partial} \bar{X}^1 \bar{\partial} \bar{X}^2 \bar{\partial} \bar{X}^3)$.

Furthermore, many couplings forbidden by rule 3 are allowed if one adds moduli to the given coupling. VEVs for moduli correspond to deformations of the shape of the orbifold. For the Z_3 orbifold, a general analysis can be carried out, indicating that only couplings proportional to $(\partial X^1 \partial X^2 \partial X^3)^m$, $m \neq 0 \bmod 3$ or proportional to $(\partial X^i)^{3m+2} (\partial X^j)^{3n+1} (\partial X^k)^{3l}$ can be made allowed by adding enough moduli $M^{i\bar{j}}$. For example, the coupling $(S_a S_b Y^{\bar{1}})^2 (S_c S_d Y^{\bar{2}})$ does not satisfy rule 3, but

$$(S_a S_b Y^{\bar{1}})^2 (S_c S_d Y^{\bar{2}}) M^{3\bar{2}} \quad (9)$$

is an allowed coupling.

We might worry that the presence of non-renormalizable couplings such as (8) and (9) will make it impossible to have vanishing F -terms. However, non-renormalizable couplings can vanish as a consequence of selection rule 4. For example, both (8) and (9) vanish when all the twisted fields are located at the same fixed point or at fixed points with the same coordinate in one of the sublattices when the six-torus factorizes. We can thus see the general structure of flat directions. Either they do not have invariant blocks or the fields in the invariant blocks are located at the same fixed points.

3. Yukawa Couplings

After finding a set of VEVs achieving the desired breaking of $U(1)$'s, we must work out the resulting massless spectrum of chiral particles. Many of the original massless states disappear either because they are swallowed by gauge bosons or because they acquire superlarge masses through Yukawa couplings with the fields having VEVs. The relevant Yukawa couplings can be of the form $S_a T T$ or $U^i U^j U^k$ if there are singlet VEVs in the untwisted sector. Since fields with zero VEV do

not have to be located at the same fixed points as fields with non-zero VEV we can also have non-renormalizable Yukawa couplings that survive rule 4. In general we can have couplings such as $(S_a Y^j) U^j T$ that respects selection rule 3, or such as $(S_1^{p_1} \dots S_L^{p_L} Y_1^{r_1} \dots Y_M^{r_M}) TT$ that in general does not respect rule 3.

A coupling that satisfies rule 3 can always be generated from one that does not by multiplying by invariant blocks and/or moduli. In the previous section (see the discussion before eq. (9)) we explained which kind of couplings, in the Z_3 orbifold, can be made compatible with rule 3 by inserting moduli. For the invariant blocks there are two possibilities. If the block is of type $S^{3n}, n \neq 3k$ it will contribute a term $(\partial X^1 \partial X^2 \partial X^3)^n$ to the given correlation so that its effect can be reproduced with moduli. If the block contains twisted oscillator modes it will contribute extra factors of $\bar{\partial} \bar{X}^j$ to the correlation, thus making it possible to satisfy rule 3. An untwisted field would have a similar effect.

Once we have found the massless fields after the gauge breaking, we can look for the effective cubic couplings among these “observable” fields. These couplings encode many phenomenological properties of the model : quark and lepton masses (through couplings with the candidate Higgses), baryon and lepton number violation, flavor structure, and others. We classify the observable trilinear couplings into renormalizable and non-renormalizable as we now explain.

Renormalizable

These are couplings involving just three observable fields. Point group invariance restricts them to be of two types.

i) $U^i U^j U^k$. These couplings are constrained by $\vec{H}$ -conservation to be of the form

$$\eta \varepsilon_{ijk} U^i U^j U^k \quad (10)$$

Therefore, there are no self-couplings among untwisted fields. If one of the U^i fields is a Higgs, this implies that the induced mass matrix will have one zero and two degenerate eigenvalues. The coefficient η is some numerical factor of order 1 (it can be determined from factorization of four-point amplitudes).

ii) $T_a T_b T_c$. These couplings are of two types. The three twisted fields can be either at the same fixed point or at three different fixed points satisfying the space group selection rule. In the first case the coupling is of order λ , with λ a numerical factor of order 1. In the second case, the coupling is roughly given by $\lambda e^{-c|r|^2}$, where r is the distance between fixed points and c is again a numerical factor of order 1 [15].

Point group invariance forbids couplings mixing untwisted and twisted fields at the renormalizable level, except for even order orbifolds in which interactions UTT are allowed for twisted sectors of order 2.

Non-renormalizable

These couplings involve three observable fields and a string of the Σ_a with non-zero VEV. The insertion of Σ_a 's can produce observable couplings that would be forbidden otherwise. We can ask the following question. Given a coupling (like UTT), which is not allowed by the selection rules, is it possible to generate it from non-renormalizable interactions?

It can be seen that couplings $U^i U^j U^k$ that are not of form (10) cannot be generated from any non-renormalizable coupling. The reason is that terms $U^l, l > 3$ vanish because in this case also $\partial X_{cl} = 0$ and couplings $U^3 I(\Sigma)$, where I is an invariant block, will not lead to effective couplings if the direction is flat. Therefore, the untwisted sector couplings are exact at the renormalizable level and given by (10). However, couplings mixing untwisted and twisted fields can be induced by non-renormalizable couplings. For example, $U^i TTY^{\bar{i}}$ can give an effective $U^i TT$ coupling if $\langle Y^{\bar{i}} \rangle \neq 0$. Also, couplings of type $ii)$ above which are forbidden by space group invariance can in principle be induced, as well as UUT couplings. The general procedure to obtain any of these couplings is as follows.

Let us denote observable (twisted or untwisted) fields by $\mathcal{O}$. To look for effective couplings $\mathcal{O}^3$, we first search for operators of the type $\mathcal{O}^3 \Sigma^p$ that are invariant under all $U(1)$'s. In general, these operators will not be allowed by selection rule 3 but one can generate allowed couplings by multiplying by invariant blocks or moduli as discussed at the beginning of the section. Once we have an operator allowed by all selection rules we substitute the Σ 's by their VEVs and see whether the effective coupling exists. Notice that non-renormalizable couplings will vanish when all the twisted fields are located at the same fixed point. When the coupling is different from zero, the coefficient h in $h\mathcal{O}^3$ will be roughly given by

$$h \sim \lambda e^{-c|r|^2} \left(\frac{\langle \Sigma \rangle}{M_P} \right)^p \quad (11)$$

4. Example

To illustrate the ideas discussed above, we will now consider a class of degenerate orbifolds based on one of the $SU(3) \times SU(2) \times U(1)^n$ models built in ref. [8]. We will closely follow their notation. The model consists of a Z_3 orbifold with gauge shift $v = \frac{1}{3}(1, 1, 1, 1, 2, 0, 0, 0) \times (1, 1, 0, \dots, 0)'$. There are two discrete Wilson lines with $a_1 = a_2 = \frac{1}{3}(0, 0, 0, \dots, 2) \times (0, 1, 1, \dots, 0)'$ and $a_3 = a_4 = \frac{1}{3}(1, 1, 1, 2, 1, 0, 0, 0) \times (1, 1, 0, \dots, 0)'$. The 27 twisted sectors split into 9 different sectors labelled by (n_1, n_3) , $n_1, n_3 = 0, 1, -1$. The gauge group of the model is $SU(3) \times SU(2) \times U(1)^8 \times SO(10)$, and the massless spectrum was given in ref. [8] (3 generations plus extra exotic particles). The $U(1)$ charges are defined in Table 1. One combination of the

$U(1)$'s may be identified with the standard hypercharge but there is some freedom in that assignment. A convenient choice that has the $U(1)$ generator completely contained in the first E_8 is $6Y = \frac{1}{3}Q_1 - \frac{1}{2}Q_2 + Q_4$ [10].

The charged massless spectrum of the model is given in Table 2 where all gauge quantum numbers are also shown. There are three copies of each of the states in the table. For the twisted matter this is because for each of the nine sectors one can vary the fixed points in the third sub-lattice, $n_5 = 0, 1, -1$. For the untwisted states the factor 3 is due to the existence of three complex compactified dimensions *. The states Y_1, Y_2, Y_3 have an extra multiplicity of three because they have oscillators $\bar{\alpha}_{-\frac{1}{3}}^i, i = 1, 2, 3$ acting on the vacuum. Notice that only the field $16'$ carries $SO(10)$ quantum numbers while the other fields are neutral.

Let us now look for flat directions breaking all $U(1)$'s except Y . To have vanishing D^α -terms we must construct a function I of the $SU(3) \times SU(2) \times U(1)_Y$ singlets that is invariant under all the Q_α . In general we can write

$$I = S_1^a S_2^b S_3^c Y_1^d Y_2^f Y_3^g S_4^h S_5^k S_6^l S_7^m S_8^n S_9^p S_{10}^q S_{11}^r \quad (12)$$

Invariance under the Q_α gives relations among the exponents. Since all these singlets have $Q_3 = 0$ and $Y = 0$, there are only five independent equations. Using the charges given in Table 2 we find

$$\begin{aligned} m &= b - c + h \\ n &= -a + 3c + f + g - 3h - l \\ p &= a + h - c - k \\ q &= \frac{1}{2}(-a + b + 2c - d + f + 2g - h - k - l) \\ r &= \frac{1}{2}(-a + b - d + f + h + k + l) \end{aligned} \quad (13)$$

For the anomalous $U(1)$, X , we find that I has a charge $q_X = 36(h - c)$. To have vanishing D^X we must have $q_X < 0$, since $\text{tr } Q_X > 0$ in the model. This then implies $c > 0$ and $c > h$. Since all the exponents in (12) must be positive, relations (13) imply $a > 0$ and $b > 0$ necessarily. This means that we will always need VEV's for S_1 , S_2 and S_3 . This fact is remarkable because as we will see shortly these are precisely the fields whose VEVs give masses to all unwanted color triplets and many other extra particles. Using equations (12) and (13), it is simple to find flat directions breaking all the extra $U(1)$'s and with vanishing D -terms.

* In this class of orbifold models the appearance of three quark-lepton generations is closely connected to the existence of three complex internal dimensions.

In this class of models there are invariant blocks of order three, namely

$$\begin{array}{llll} Y_1 S_3 S_4 & Y_1 S_2 S_7 & Y_2 S_1 S_9 & Y_2 S_6 S_{11} \\ Y_3 S_1 S_5 & Y_3 S_8 S_{10} & Y_1 Y_3 S_6 & Y_1 Y_2 S_8 \end{array} \quad (14)$$

In fact, these are the only cubic couplings among the hypercharge-singlets. There are also higher order invariant blocks which we do not display here for brevity. Giving VEVs to more than one field in any of these blocks may generate dangerous susy-breaking F -terms. Although the couplings in (14) are forbidden by selection rule 3, products of powers of them (sometimes supplemented by moduli insertions) are allowed and care should be taken not to ruin flatness. One way to solve this problem is to allow at most one of the fields in each invariant to have a VEV. However, this has the shortcoming that some non-renormalizable Yukawa couplings will not be generated, which, as we shall see, may be phenomenologically problematic. An alternative is to make full use of selection rule 4 and assume that all singlets acquiring VEVs sit at fixed points with identical coordinate in the third sub-lattice, e.g. $n_5 = 0$. Then, all possible couplings among these singlets vanish. This is certainly the simplest way to pick a field direction that is *flat to all orders* in the VEVs of the fields. Notice also that flat directions of this type are always stable under arbitrary deformations (moduli insertions) of the underlying $SU(3)^3$ lattice of the orbifold.

We will now select a subclass of vacua by selecting the class of flat directions implied by the invariant

$$(S_1 S_2 S_3 S_8^2 S_{10})^\beta (Y_3 S_8 S_{10})^\gamma (Y_3 S_1 S_5)^\delta (Y_1 S_3 S_4)^\rho (Y_1 S_2 S_7)^\sigma \quad (15)$$

The first parenthesis indicates that the fields inside get VEVs to cancel the F-I term. The other four terms are invariant under X and indicate that we can give further VEVs to the fields inside without any contribution to the D -terms. We have added them in order to have invariant blocks available and also to give VEVs to some fields that will be useful later on. Notice that by satisfying $\langle D^X \rangle = 0$ one fixes the value of the gauge coupling constant g in terms of the VEV of one of the fields in the first parenthesis. Then, there are four free parameters left in this flat direction, corresponding to the last four terms in (15). In particular, one can put all VEVs $\sim M_P$ if one wishes, leading to an effective $SU(3) \times SU(2) \times U(1)_Y$ model below the Planck scale.

The renormalizable couplings of the model (see Table 3) give masses $\sim M_P$ to most of the particles that are non-chiral under $SU(3) \times SU(2) \times U(1)_Y$. Thus,

- i) All extra color triplets, $u_2, D_1, D_2, D_3, D_4, \bar{D}_1, \bar{D}_2$ and a linear combination of d_1 and d_2 , get large masses due to the VEVs for S_1, S_2, S_3 , as we already anticipated.

- ii) All extra pairs of $SU(2)$ -singlet leptons also get large masses, except one linear combination of the l_a which remains massless providing for the observed right-handed leptons (recall the extra multiplicity of 3).
- iii) All exotic $Y = \pm \frac{1}{2}$ leptons ($\bar{A}_b, A_b$ fields) disappear from the low-energy spectrum.
- iv) Three $SU(2)$ doublets (one linear combination of G_1 and G_4 , one linear combination of G_2 and G_3 and the field G_5) and two $SU(2)$ antidoublets ($\bar{G}_1$ and $\bar{G}_2$) remain massless at this level. They all come in three copies so that one has three families of usual left-handed leptons plus six pairs of doublet-antidoublet Higgses.

It is remarkable that this model, already from the outset, admits no extra massless color triplets nor $SU(2)$ -singlet leptons but necessarily has prospective Higgs doublets. Notice that this is an $SU(3) \times SU(2) \times U(1)$ superstring, not a GUT, and problems such as “doublet-triplet splitting” make no sense, they were never together to begin with. Besides, as we have seen, all extra triplets acquire large masses. The relevant question, which we will address shortly, is that of proton decay induced by explicit baryon and lepton number violating terms.

Since only a $SU(3) \times SU(2) \times U(1)_Y$ is unbroken now, one may worry that, after including non-renormalizable Yukawa couplings, all Higgs doublets will get large masses spoiling the symmetry breaking at the weak scale. Again, it seems miraculous that some “stringy zeros” (essentially due to selection rule 4 and/or $\vec{H}$ -conservation) forbid this to happen. We have made a systematic analysis of all the non-renormalizable Yukawas that could give masses to doublet-antidoublet pairs. There are two types,

- a) Couplings not requiring the presence of invariant blocks. These are

$$\bar{G}_1^i G_3 \langle S_2 Y_3 \rangle \quad ; \quad \bar{G}_1^i G_2 \langle S_3 Y_3 \rangle \quad i = 1, 2, 3 \quad (16)$$

Since $\langle S_2 \rangle$, $\langle S_3 \rangle$ and $\langle Y_3 \rangle$ sit at the same fixed point in the 3^{rd} sublattice ($n_5 = 0$) and also due to the particular space group properties of the fields, the only non-vanishing coupling according to rule 4 is

$$\bar{G}_1^1 G_3^{(0)} \langle S_2 Y_3 \rangle \quad (17)$$

where (0) refers to $n_5 = 0$.

- b) Couplings requiring invariant blocks. That is the case of

$$\bar{G}_2 G_5 \langle Y_1 \rangle \quad (18)$$

This coupling is forbidden by selection rule 3 as we have explained. However, the coupling

$$\bar{G}_2 G_5 \langle Y_1 (Y_1 S_3 S_4)^2 M^{1\bar{3}} M^{2\bar{3}} \rangle \quad (19)$$

is allowed. Thus, for generic shapes of the orbifold lattice, there is an effective mass term $\bar{G}_2 G_5$. Notice however that the coupling vanishes for $\bar{G}_2^{(0)}$ and $G_5^{(0)}$ as consequence of rule 4. Thus, $\bar{G}_2^{(1,-1)}$ and $G_5^{(1,-1)}$ will gain masses but not $\bar{G}_2^{(0)}$ nor $G_5^{(0)}$.

Altogether, the couplings discussed above give masses to three pairs of Higgses while there remain three massless pairs ($\bar{G}_1^2$, $\bar{G}_1^3$, $\bar{G}_2^{(0)}$, $G_5^{(0)}$ and a linear combination of G_2 and G_3 in two copies, $n_5 = 1, -1$). Thus, we have seen that, contrary to expectations, purely stringy properties (e.g. rule 4 cannot be understood as a regular field theory symmetry [17]) protect some of the doublets from disappearing from the low-energy spectrum.

The model below the string scale is essentially a supersymmetric standard model with three generations and three sets of Higgs doublets. There are also some massless hypercharge-singlets. Most of the latter get masses through non-renormalizable couplings involving products of powers of the invariant blocks in (14). In principle, other flat directions could give masses to some of the remnant Higgses but the stringy selection rules tend to protect them. Finally, there is an $SO(10) \times U(1)_3$ hidden sector.

Let us now consider the Yukawa couplings among the quark, lepton and Higgs fields which appear at low energies.

u -quark Yukawas. They have couplings already at the non-renormalizable level, namely

$$Q_L^i u_1^j \bar{G}_1^k, \quad i \neq j \neq k \quad (20)$$

However, only $\bar{G}_1^2$ and $\bar{G}_1^3$ are massless and are the candidates to acquire VEVs at the weak scale. Once that happens there is one massless (up) quark and two massive ones. The prospective charm and top quarks are degenerate at this level, but this may be modified by the effect of one-loop wave function mixing [18] between the different quark families.

d -quark Yukawas. Point group invariance forbids renormalizable couplings of this type because Q_L is in the untwisted sector and the right-handed d -quarks and Higgses are in the twisted sector. Thus, d -quark masses are absent in flat directions without invariant blocks. In the generic directions given by (15) they will be present in general. To simplify the discussion, let us assume $\langle S_2 \rangle = \langle S_3 \rangle$ (or $\rho = \sigma$). In this case the right-handed quarks are given by the linear combination $d_R = d_1 - d_2$, $n_5 = 0, 1, -1$, and the relevant Higgses by $H = G_2 - G_3$. The following couplings are allowed by the space group and gauge symmetries,

$$\begin{aligned} Q_L^i d_1 G_2 \langle S_8 \rangle, \quad Q_L^i d_2 G_3 \langle S_8 \rangle \\ Q_L^i d_1 G_3 \langle Y_1 S_2 S_4 S_8 \rangle, \quad Q_L d_2 G_2 \langle Y_1 S_3 S_7 S_8 \rangle \end{aligned} \quad (21)$$

They can be multiplied by an invariant block and moduli fields so that they are not forbidden by rule 3. Then, the physically relevant coupling h in $hQ_L^i d_R H$ is proportional to the VEVs :

$$h(Q_L^i d_R H) \sim \langle 2S_8 - Y_1 S_8(S_2 S_4 + S_3 S_7) \rangle \quad (22)$$

which are in general non-vanishing. Notice however, that $d_R^{(0)}$ does not couple due to selection rule 4. Once $H, n_5 = 1, -1$ acquires a VEV, one of the quarks (bottom) gets a mass and two remain massless at this level.

Lepton Yukawas. Here the situation is somewhat more involved, since working out in detail (including non-renormalizable terms) which are the candidates for right-handed leptons, one finds a generic linear combination $l_R = c_a l_a$ (in three copies, $n_5 = 0, 1, -1$). To simplify matters, we further specify the flat direction by taking $\langle S_4 \rangle = -\langle S_7 \rangle$. Also we make use of the free parameters in eq. (15) to set $\langle S_1 \rangle, \langle S_4 \rangle, \langle S_7 \rangle \ll 1$. Under these circumstances, the three physical right-handed leptons are given by the combinations $l_R = l_1 - l_3$. The only renormalizable couplings are

$$H^{(1)} G_5^{(0)} l_R^{(-1)} \quad , \quad H^{(-1)} G_5^{(0)} l_R^{(1)} \quad (23)$$

whereas non-renormalizable couplings either vanish or are suppressed. This suggests the identification $G_5^{(0)} = (\tau_L, \nu_\tau)$ so that the τ lepton has a mass at this level. There are still three massless doublet leptons, $L = G_1 - G_4, n_5 = 0, 1, -1$, and one anti-doublet $\bar{G}_2^{(0)}$. We have checked that there are one-loop $\Delta I_w = 0$ masses for the pair $\bar{G}_2^{(0)} L^{(0)}$ once supersymmetry is broken so that only two leptons (presumably e, μ) will remain massless.

It is important to remark at this point that lepton number violating couplings (e.g. $LLl_R, G_5^{(0)} Ll_R, HHl_R$, etc.) are all suppressed due to the hierarchy of VEVs stated above. In fact, in the limit in which $\langle S_1 \rangle, \langle S_4 \rangle, \langle S_7 \rangle \rightarrow 0$, an extra $U(1)$ gauge boson remains massless. The corresponding generator is given by

$$Q_{LN} = -Q_1 + Q_2 - Q_4 + Q_5 - Q_6 + Q_7 \quad (24)$$

One can check that $Q_{LN}(Q_L, d_R, u_R, \bar{H}, H) = 0$, whereas $Q_{LN}(G_5, L) = 6$ and $Q_{LN}(l_R) = -6$, so that this is a sort of gauged lepton-number $U(1)$ symmetry which suppresses lepton-number violating processes by powers of $(M_{LN}/M_P)^n$. Notice that this gauge boson may have a very large mass up to a few orders of magnitude below M_P and still those processes are efficiently suppressed*.

* On the other hand, it could well be the case that this gauge boson remained light up to energies just above the Z_0 mass. Notice that this Z'_0 couples only to leptons and thus the present experimental limits on its mass would be very weak.

Baryon-number violating Yukawas. The following coupling satisfies space-group and gauge invariance

$$u_1 d_1 d_2 \langle S_1 \rangle \quad (25)$$

This coupling is allowed by all selection rules after multiplication by invariant blocks (and moduli) and leads to $\Delta B = 1$ effective interactions such as $u_R b_R s_R$. In spite of this, there is no fast proton decay because there are no quark-antiquark-lepton couplings allowed. One can easily find the following contribution to those dangerous Yukawa couplings

$$\begin{aligned} h(Q_L d_R L) &\sim \langle (S_1 S_5 + S_8 S_{10})(S_2 S_7 + S_3 S_4)I \rangle + \langle S_6 I' \rangle \\ h(Q_L G_5^{(0)} d_R) &\sim \langle (S_4 - S_7)I'' \rangle \end{aligned} \quad (26)$$

where I, I', I'' are appropriately chosen invariant blocks. The first coupling vanishes along the chosen flat direction ($\langle (S_2 S_7 + S_3 S_4) \rangle = 0, \langle S_6 \rangle = 0$). The second vanishes due to selection rule 4. Thus, along the present flat direction the proton is stable. Notice that in the limit in which an extra lepton-number gauge boson remains light the absence of couplings (26) is automatic.

Neutrino Yukawas. There are many $SU(3) \times SU(2) \times U(1)$ singlets in the model (all those in eq. (12)), all coming in three copies ($n_5 = 0, 1, -1$) except for Y_1, Y_2, Y_3 which come in nine copies. Seven linear combinations of the gauge coupling dilaton and the fourteen fields in (15) (all with $n_5 = 0$) are swallowed by the seven broken $U(1)$'s. All other hypercharge-singlets located at fixed points with $n_5 = 1, -1$ get large masses through non-renormalizable couplings induced by invariant blocks like those in (14). Those corresponding to $n_5 = 0$ do not get tree-level masses due to rule 4. However, they tend to get one-loop masses once supersymmetry is broken. In any case, all residual hypercharge-singlets are unobservable since they do not have Yukawa couplings to quarks and leptons due to the selection rules. There are couplings allowed by space group and gauge invariance. For example

$$\bar{G}_1^{2,3} G_4 Y_2^{(0)} \langle S_3 \rangle, \quad \bar{G}_1^{2,3} G_1 Y_2^{(0)} \langle S_2 \rangle, \quad \bar{G}_1^{2,3} G_5^{(0)} Y_2^{(0)} \langle Y_3 \rangle \quad (27)$$

They could induce Dirac neutrino masses once $\bar{G}_1^{2,3}$ get a VEV. However, they all vanish as a consequence of rule 4 (notice that in these couplings all twisted fields have the same coordinate in the second sublattice, $n_3 = -1$). The same happens with a possible “ μ -term” in the Higgs superpotential. Terms of the form $\bar{G}_1^{2,3} G_2 \langle Y_3 S_3 \rangle, \bar{G}_1^{2,3} G_3 \langle Y_3 S_2 \rangle$ also vanish. However, masses of that form ($H\bar{H}$) could be generated in loops once supersymmetry is broken. As a general conclusion, there are no Dirac neutrino masses and candidates for right-handed neutrinos are unobservable.

Some phenomenological comments are in order. First, in the specific model above only the heaviest quarks and leptons (t, b, c, τ) can acquire tree-level masses. One would expect the lightest (u, d, s, e, μ) to get masses through loop corrections, as often happens in supersymmetric models [19]. R -parity is violated in this model in an explicit way [20]. The lightest supersymmetric particles are thus not stable and decay into quarks and leptons (e.g. $\tilde{\gamma} \rightarrow uds$, $\tilde{\nu}_e \rightarrow \nu_e uds$). This very much worsens the present experimental limits on sparticle masses. The $\Delta B = 1$ interaction $u_R d_R s_R$ has been considered as a possible source of the baryon asymmetry of the universe. It may also give rise to neutron-antineutron oscillations at rates consistent with experimental limits [21].

It is worth recapitulating what is the source of each low-energy symmetry in the above model. First, notice that conformal field theory selection rules forbid many couplings in a “stringy” way, i.e., one cannot find field-theoretical four-dimensional global symmetries or R -symmetries which forbid these couplings. That is the case of interactions not allowed by selection rule 4 which, for example, protects some Higgs doublets from gaining large tree-level masses. On the other hand, the approximate lepton-number conservation of the model, as well as proton stability, are consequences of the flat direction chosen (or alternatively, of the existence of an extra unbroken $U(1)_{LN}$). It could be possible to find some model in which string selection rules (such as $\tilde{H}$ -conservation or rule 4) forbid fast proton decay. There is a trivial example of that: if we pick a flat direction without invariant blocks (e.g. one in which all hypercharge-singlets except Y_1, Y_2, Y_3 have non-zero VEVs) all $\Delta B \neq 0$ operators and $\Delta L \neq 0$ operators such as $Q_L d_R L$ vanish simply because Q_L, u_R are in the untwisted sector and d_R and leptons are in the twisted sector. However, this also forbids d -quark masses and the number of massless Higgs doublets tends to proliferate (6 pairs). We think it will be hard to find a built-in symmetry (i.e. not depending on the flat direction) guaranteeing sufficient proton stability and providing masses for the heaviest quark and leptons. If this is the case, in a generic string model there will appear R -parity violating operators like those discussed above. This would have important experimental consequences.

We must also remark that in any string model with flat directions it is of the utmost importance to have complete control of the non-renormalizable couplings (to all orders in scalar VEVs insertions). They may ruin candidate flat directions, modify completely the massless spectrum and change the quark-lepton Yukawa couplings. It is not correct to assume the existence of any particular coupling allowed by gauge invariance. We have shown many examples of how CFT rules forbid otherwise allowed couplings. Furthermore, string models preserving baryon or lepton-number symmetries (or R -parity) at the renormalizable level will in general

not conserve it once the gauge symmetry is broken to $SU(3) \times SU(2) \times U(1)$ through flat directions.

Let us finally comment that one may build many more $SU(3) \times SU(2) \times U(1)^n$ models starting from the Z_3 orbifold with two or three Wilson lines. The general recipe to construct this type of models is given in [9]. Also, different flat directions will lead to different low-energy physics. The general method outlined at the beginning of this paper should be applicable also to other Z_N orbifolds. It seems to us that a systematic investigation of the class of $SU(3) \times SU(2) \times U(1)$ degenerate orbifolds discussed in this paper is worth pursuing.

P.S. After this work was completed, we received preprints by J.A. Casas and C. Muñoz [22] in which the same model of ref. [8] is studied. The flat direction considered there belongs to the class of flat directions without the invariant blocks discussed in the text.

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TABLE 1

$U(1)$ charge generators in terms of $E_8 \times E'_8$ lattice vectors.

$$\begin{aligned}
 Q_1 &= 6 (1, 1, 1, 0, \dots, 0) \times (0, \dots, 0)' \\
 Q_2 &= 6 (0, 0, 0, 1, -1, 0, 0, 0) \times (0, \dots, 0)' \\
 Q_3 &= 6 (0, \dots, 0, 1, 0, 0) \times (0, \dots, 0)' \\
 Q_4 &= 6 (0, \dots, 0, 0, 1, 0) \times (0, \dots, 0)' \\
 Q_5 &= 6 (0, \dots, 0, 0, 0, 1) \times (0, \dots, 0)' \\
 Q_6 &= 6 (0, \dots, 0) \times (1, 1, 0, 0, \dots, 0)' \\
 Q_7 &= 6 (0, \dots, 0) \times (0, 1, 1, 0, \dots, 0)' \\
 X &= 6 (0, \dots, 0) \times (1, -1, 1, 0, \dots, 0)'
 \end{aligned}$$

TABLE 2

Charged Massless Spectrum

(n_1, n_2)	Field	Rep.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	X	$\bar{G}$
Unit.	Q_7	(3,2)	6	6	0	0	0	0	0	0	1
	u_1	(3,1)	6	0	0	-6	0	0	0	0	-4
	$\bar{G}_1$	(1,2)	0	6	0	6	0	0	0	0	3
	$16'$	1	0	0	0	0	0	0	0	9	0
(0,0)	D_1	(3,1)	0	4	0	0	0	4	0	4	-2
	$\bar{G}_2$	(1,2)	6	-2	0	0	0	4	0	4	3
	$\bar{A}_1$	1	-3	-2	-3	-3	-3	4	0	4	-3
	$\bar{A}_2$	1	-3	-2	3	-3	3	4	0	4	-3
	A_1	1	-3	-2	-3	3	3	4	0	4	3
	A_2	1	-3	-2	3	3	-3	4	0	4	3
(1,0)	S_4	1	6	4	0	0	-2	6	4	4	0
	S_5	1	6	4	0	0	-2	-6	-2	4	0
	S_6	1	6	4	0	0	-2	0	-2	-8	0
	$\bar{A}_3$	1	-3	-2	-3	-3	1	6	4	4	-3
	$\bar{A}_4$	1	-3	-2	-3	-3	1	-6	-2	4	-3
	$\bar{A}_5$	1	-3	-2	-3	-3	1	0	-2	-8	-3
(-1,0)	A_3	1	-3	-2	3	3	1	6	4	4	3
	A_4	1	-3	-2	3	3	1	-6	-2	4	3
	A_5	1	-3	-2	3	3	1	0	-2	-8	3
	S_7	1	6	4	0	0	2	2	-4	4	0
	S_8	1	6	4	0	0	2	2	2	-8	0
	S_9	1	6	4	0	0	2	-4	2	4	0
(0,1)	$\bar{A}_6$	1	-3	-2	3	-3	-1	2	-4	4	-3
	$\bar{A}_7$	1	-3	-2	3	-3	-1	2	-4	-8	-3
	$\bar{A}_8$	1	-3	-2	3	-3	-1	-4	2	4	-3
	A_6	1	-3	-2	-3	3	-1	2	-4	4	3
	A_7	1	-3	-2	-3	3	-1	2	2	-8	3
	A_8	1	-3	-2	-3	3	-1	-4	2	4	3
(0,1)	d_1	(3,1)	0	0	0	2	2	-4	-4	4	2
	F_1	(1,2)	3	0	-3	-1	-1	-4	-4	4	0
	$\bar{A}_9$	1	3	6	3	-1	-1	-4	-4	4	-3
	A_9	1	3	-6	3	-1	-1	-4	-4	4	3
	l_1	1	-6	0	0	-4	2	-4	-4	4	-6
	S_{10}	1	-6	0	0	2	-4	-4	-4	4	0

(n_1, n_2)	Field	Rep.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	X	$\bar{G}$
(1,1)	D_2	(3,1)	6	0	0	2	0	-2	0	4	4
	u_2	(3,1)	0	0	0	-4	0	-2	0	4	-4
	F_2	(1,2)	3	0	3	-1	-3	-2	0	4	0
	F_3	(1,2)	3	0	-3	-1	3	-2	0	4	0
(0,-1)	S_1	1	-6	0	0	2	0	4	0	-8	0
	$\bar{Y}_1$	1	-6	0	0	2	0	-2	0	4	0
	$\bar{A}_{10}$	1	3	6	-3	-1	-3	-2	0	4	-3
	$\bar{A}_{11}$	1	3	6	3	-1	3	-2	0	4	-3
	A_{10}	1	3	-6	3	-1	3	-2	0	4	3
	A_{11}	1	3	-6	-3	-1	-3	-2	0	4	3
(-1,-1)	d_2	(3,1)	0	0	0	2	-2	0	4	4	2
	F_4	(1,2)	3	0	3	-1	1	0	4	4	0
	$\bar{A}_{12}$	1	3	6	-3	-1	1	0	4	4	-3
	$\bar{A}_{13}$	1	3	-6	-3	-1	1	0	4	4	3
	l_2	1	-6	0	0	-4	-2	0	4	4	-6
	S_{11}	1	-6	0	0	2	4	0	4	4	0
(0,-1)	D_3	(3,1)	-3	2	-3	1	1	0	-2	4	-1
	$\bar{G}_3$	(1,2)	0	2	0	4	-2	0	-2	4	1
	G_1	(1,2)	0	2	0	-2	4	0	-2	4	-3
	S_2	1	0	-4	0	-2	-2	0	4	-8	0
	Y_2	1	0	-4	0	-2	-2	0	-2	4	0
	l_3	1	0	-4	0	4	4	0	-2	4	6
(1,-1)	$\bar{A}_{12}$	1	0	8	0	-2	-2	0	-2	4	-6
	$\bar{A}_{13}$	1	-9	2	3	1	1	0	-2	4	-3
	A_{12}	1	9	2	-3	1	1	0	-2	4	3
	D_4	(3,1)	-3	2	3	1	-1	2	2	4	-1
	$\bar{G}_4$	(1,2)	0	2	-3	1	-1	2	2	4	1
	G_2	(1,2)	0	2	0	4	2	2	2	4	3
(-1,-1)	S_3	1	0	-4	0	-2	2	-4	-4	-8	0
	Y_3	1	0	-4	0	-2	2	2	2	4	0
	l_3	1	0	-4	0	4	-4	2	2	4	6
	l_4	1	0	8	0	-2	2	2	2	4	-6
	A_{14}	1	-9	2	-3	1	-1	2	2	4	-3
	A_{15}	1	9	2	3	1	-1	2	2	4	3
(0,-1)	G_3	(1,2)	0	2	0	-2	0	-2	-6	4	-3
	G_4	(1,2)	0	2	0	-2	0	4	6	4	-3
	G_5	(1,2)	0	2	0	-2	0	-2	0	-6	-3
	l_5	1	0	-4	0	4	0	4	6	4	6
	l_6	1	0	-4	0	4	0	-2	-6	4	6
	l_5	1	0	-4	0	4	0	-2	0	-6	6

TABLE 3

Couplings of type $TT\Sigma$ for triplets, doublets and $SU(2)$ - singlet leptons.
Couplings marked with an asterisk are forbidden by selection rule 3.

$d_1 D_1 S_2$ $d_2 D_1 S_3$	$\bar{D}_1 D_3 S_2$ $\bar{D}_2 D_4 S_3$	$u_2 D_2 S_1$	
$\bar{G}_3 G_1 S_2$ $\bar{G}_3 G_4 S_3$ $*\bar{G}_3 G_5 Y_3$	$\bar{G}_4 G_2 S_3$ $\bar{G}_4 G_3 S_2$ $*\bar{G}_4 G_5 Y_2$	$F_1 F_4 S_1$ $F_2 F_3 S_1$	$*\bar{G}_2 G_5 Y_1$
$\bar{l}_1 l_2 S_8$ $\bar{l}_1 l_3 S_6$ $\bar{l}_1 l_5 S_4$	$\bar{l}_2 l_1 S_6$ $\bar{l}_2 l_4 S_8$ $\bar{l}_2 l_5 S_7$	$\bar{l}_3 l_1 S_2$ $\bar{l}_3 l_3 S_3$ $*\bar{l}_3 l_5 Y_3$	$\bar{l}_4 l_2 S_3$ $\bar{l}_4 l_4 S_2$ $*\bar{l}_4 l_5 Y_2$