BOSONIC REALIZATION OF $N = 2$ SUPER-CONFORMAL ALGEBRAS

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ABSTRACT

We present a simple bosonic realization of the discrete series unitary representations of the two-dimensional $N = 2$ super-conformal algebras. This is obtained by considering index k embeddings of $SU(2)$ into other Lie algebras. For the simplest such embedding one can obtain the complete set of highest weight states as combinations of momentum states of the bosons. This realization is used to analyse the $k = 4$ model in detail. We also discuss how the techniques presented here can be used to understand, in terms of orbifolds, many of the string theories assembled via representations of $N = 2$ super-conformal discrete series.

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One of the more successful methods of analysing the discrete series representations of $N = 2$ super-conformal algebras (SCA's) [1] [2] is to exploit their close relationship to higher level $SU(2)$ Kac-Moody current algebras. The usual method of establishing this relationship is to use para-fermions [3] to convert two of the Kac-Moody currents into the supercurrents [4] [5]. Our initial intention here is to obtain directly a bosonic realization of the supercurrents and, as a by-product, obtain a bosonic realization of para-fermions.

The central charge of the $N = 2$ discrete series is $\frac{3k}{k+2}$, $k = 1, 2, \dots$ and this is also precisely the central charge of the Virasoro algebra obtained by the Sugawara construction from a level k $SU(2)$ Kac-Moody current algebra. The simplest way of obtaining the level k $SU(2)$ Kac-Moody algebra is to embed diagonally $SU(2)$ in k copies of itself. That is, one takes:

$$\begin{aligned} J^\pm(z) &= \sum_{j=1}^k \exp(i\sqrt{2} X^j(z)) \\ J^3(z) &= i \sum_{j=1}^k \partial X^j(z), \end{aligned} \tag{1}$$

where X^j , $j = 1, 2, \dots, k$, are the bosonic fields describing the k copies of $SU(2)$. The Sugawara energy-momentum tensor obtained from this is:

$$T(z) = -\frac{1}{(k+2)} \left\{ \sum_{j=1}^k (\partial X^j(z))^2 - \frac{1}{2} (J^3(z))^2 - \sum_{\substack{i,j=1 \\ i \neq j}}^k \exp(i\sqrt{2} (X^i(z) - X^j(z))) \right\}. \tag{2}$$

Introduce the (canonically normalized) bosonic field:

$$\phi(z) = \frac{1}{\sqrt{k}} \sum_{j=1}^k X^j(z) \tag{3}$$

and define:

$$G^\pm(z) = \frac{1}{\sqrt{k+2}} \sum_{j=1}^k \exp(\pm i\sqrt{2} (X^j(z) + a\phi(z))), \tag{4}$$

where $a = \frac{1}{\sqrt{2k}}(-\sqrt{2} + \sqrt{k+2})$. The choice of a has been made so that the exponentials in (4) have the form $\exp(i\vec{t}_j \cdot \vec{X})$ with $(\vec{t}_j)^2 = 3$. The required current is given by re-scaling

$J^3(z)$:

$$J(z) = \frac{1}{\sqrt{k+2}} J^3(z) = i\sqrt{\frac{k}{k+2}} \partial\phi(z). \quad (5)$$

It is then elementary to verify that $J(z)$, $G^\pm(z)$ and $T(z)$ form an $N = 2$ SCA with $c = \frac{3k}{k+2}$.

One can relate the foregoing description to that obtained from the para-fermions by observing that the basic para-fermion field $\psi_1(z)$ [3][4][5], has a bosonic realization:^{*}

$$\psi_1(z) = \frac{1}{\sqrt{k}} \sum_{j=1}^k \exp(i\sqrt{2} (X^j(z) - \frac{1}{\sqrt{k}}\phi(z))). \quad (6)$$

The virtue of the bosonic realization is that one finds a very simple description of the highest weight states of both the $N = 2$ SCA's and the para-fermionic system. We will concentrate on the $N = 2$ SCA's in the sequel, but similar analysis can be made for para-fermions.

To find the representations of the level k $SU(2)$ Kac-Moody algebra, one can tensor together k copies of level one representations. In general, this will result in infinitely many copies of the level k representations as well as their corresponding highest weight states. By the same token we will obtain infinitely many copies of the representations of $N = 2$ SCA's by considering the Hilbert space of the bosons introduced above, with their momenta lying on an appropriately chosen lattice. These momenta are chosen so that $T(z)$ has no branch cuts when acting on $|\vec{p}\rangle$, while $G^\pm(z)$ are required to have no branch cuts in the Neveu-Schwarz (NS) sector, and have $z^{1/2}$ branch cuts in the Ramond (R) sector. Let $\vec{X}$ denote the vector $(X^1, X^2, \dots, X^k)$, and introduce an orthonormal basis $\vec{e}_j$ such that $\vec{e}_j \cdot \vec{X} = X^j$, define $\vec{u} = \sum_{j=1}^k \vec{e}_j$, then from (4) one finds that:

$$\vec{p} = \frac{\bar{q}}{k\sqrt{k+2}} \vec{u} + \frac{1}{\sqrt{2}} \left\{ \sum_{j=1}^k a_j \vec{e}_j - \frac{1}{k} \left(\sum_{j=1}^k a_j \right) \vec{u} \right\}, \quad (7)$$

* Moreover, one can represent $\psi_l(z)$ by $\psi_l(z) = A_l \sum_{\vec{w}_l} \exp(i\sqrt{2} \vec{w}_l \cdot \vec{X})$ where $\vec{w}_l$ are all the weights of the rank l anti-symmetric tensor of $SU(k)$. This use of the weights of $SU(k)$ will be described more fully later in this letter.

where $\bar{q}$ and a_j are integers, and for the NS sector

$$\bar{q} \equiv \sum_{j=1}^k a_j \pmod{k} \quad (8)$$

while for the R sector

$$\bar{q} \equiv \left\{ \sum_{j=1}^k a_j \right\} + \frac{k}{2} \pmod{k} \quad (9)$$

It is interesting to observe that the second term in (7) has the form $\frac{1}{\sqrt{2}}\vec{w}$, where $\vec{w}$ is a weight of $SU(k)$. This is a consequence of the fact that the last term in $T(z)$ (see (2)) has the form $\exp(i\sqrt{2}\vec{r}\cdot\vec{X})$ where $\vec{r}$ is a root of $SU(k)$. This $SU(k)$ can be viewed as permuting the original k copies of $SU(2)$. Indeed if one views $(SU(2))^k$ as diagonally embedded in $SU(2k)$, and considers the index k , $SU(2)$ subgroup defined in (1) then this $SU(2)$ subgroup of $SU(2k)$ commutes with precisely the $SU(k)$. Moreover, this $SU(k)$ is of index two and hence defines a Kac-Moody algebra of level two. The central charge of the corresponding Sugawara tensor is $\frac{2(k^2-1)}{k+2}$, which is $(2k-1) - \frac{3k}{k+2}$. As a consequence the Sugawara tensor of this $SU(2)$ and this $SU(k)$ add to that of the $2k-1$ bosons in the maximal torus of $SU(2k)$. This observation would be extremely useful in decomposing the level one representations of $SU(2k)$ into level k representations of $SU(2)$, or level k representations of $N=2$ SCA's. Moreover it implies that given any highest weight states of the level k $SU(2)$ or SCA, one can get infinitely many copies of it by acting with any element of the $SU(k)$ Kac-Moody group. Rather than embarking on such a general classification we will content ourselves with finding a particularly simple set of highest weight states.

Consider the operators $G^\pm(z)$ defined in (4) acting on a state $|\vec{p}\rangle$, with $|\vec{p}\rangle$ given by (7). For this to represent a highest weight state in the NS sector, $G^\pm(z)|\vec{p}\rangle$ can diverge, at worst, as z^{-1} as $z \rightarrow 0$, while in the R sector it can diverge, at worst, as $z^{-3/2}$. This means that the integers $\bar{q}$ and a_j in (7) must satisfy:

$$\begin{aligned} |a_j + s| &\leq 1 && \text{(NS sector)} \\ |a_j + s| &\leq \frac{3}{2} && \text{(R sector)} \end{aligned} \quad (10)$$

where s is the integer or half integer (see (8) and (9)) given by:

$$s = \frac{1}{k} \left(\bar{q} - \sum_{j=1}^k a_j \right). \quad (11)$$

Observe that our description of $\vec{p}$ in terms of a_j is over-parametrized in that replacing a_j by $a_j + s$ does not change $\vec{p}$. Hence we are free to choose the parameter s above. We therefore take $s = 0$ in the NS sector and $s = -1/2$ in the R sector, and thus $a_j \in \{0, \pm 1\}$ for the NS sector and $a_j \in \{-1, 0, 1, 2\}$ for the R sector.

It is trivially true that $J_n |\vec{p}\rangle = 0$, $n \geq 1$, and in the NS sector all of the foregoing conditions also guarantee that $L_n |\vec{p}\rangle = 0$, $n \geq 1$. In the R sector $L_1 |\vec{p}\rangle = 0$ yields the extra condition that $|a_i - a_j| \leq 2$, and so we have two choices $a_j \in \{0, \pm 1\}$ or $a_j \in \{0, 1, 2\}$. The distinction between these two sets is that the corresponding states $|\vec{p}\rangle$ are annihilated by G_0^- and G_0^+ respectively. Another completely equivalent way of describing these Ramond states is to simply take $a_j \in \{0, \pm 1\}$ but allow s to take the values $\pm 1/2$, and this will be adopted in the following.

It is elementary to see that the $U(1)$ charge, J_0 , of these states is given by:

$$J_0 |\vec{p}\rangle = \frac{\bar{q}}{k+2} |\vec{p}\rangle. \quad (12)$$

However a single state $|\vec{p}\rangle$ will not in general be an eigenstate of L_0 . To select a combination of states $|\vec{p}\rangle$ that is an eigenstate of L_0 we define the symmetrized momentum states:

$$|S(\vec{p})\rangle = \sum_{perm.} |\vec{p}\rangle \quad (13)$$

where $\sum_{perm.}$ denotes the sum over all permutations of the entries in the vector $\vec{p}$.[★] Thus for the highest weight states the symmetrized momentum states are distinguished only by the total number of a_j 's taking values 0, +1 and -1. Let l be the total number of non-zero

★ It is amusing to note that this sum is essentially over the action of the Weyl group of $SU(k)$ on $|\vec{p}\rangle$.

values of a_j , i.e. $l = \sum_{j=1}^k |a_j|$, and let $q = \sum_{j=1}^k a_j$. Label the corresponding symmetrized highest weight state by $|l, q\rangle$. It is a straightforward calculation to show that:

$$\begin{aligned} L_0 |l, q\rangle &= \left\{ \frac{1}{4(k+2)} [l(l+2) - q^2] + \frac{\bar{q}^2 - q^2}{2k(k+2)} \right\} |l, q\rangle \\ &= \left\{ \frac{1}{4(k+2)} [l(l+2) - (q - 2s)^2] + \frac{s^2}{2} \right\} |l, q\rangle. \end{aligned} \quad (14)$$

The second identity in (14) follows from $\bar{q} - q = ks$, which is a consequence of (11). Since l is the total number of non-vanishing a_j , it takes all integer values between 0 and k . Since q is the difference between the numbers of positive (+1) and negative (−1) a_j 's, it clearly takes the values $-l, -l+2, \dots, l-2, l$. Finally, recall that $s = 0$ in the NS sector and $s = \pm 1/2$ in the R sector. It follows directly from (11) and (12) that:

$$\begin{aligned} J_0 |l, q\rangle &= \left(\frac{q + ks}{k+2} \right) |l, q\rangle \\ &= \left(\frac{q - 2s}{k+2} + s \right) |l, q\rangle \end{aligned} \quad (15)$$

Therefore, by restricting ourselves to symmetrized momentum states, we have found precisely one copy of highest weight states of all $N = 2$ discrete unitary series [2]. It is equally straightforward to do the same for the para-fermion system. Again one obtains all highest weight states using our method, and the canonical identifications [3] $\Phi_m^l \equiv \Phi_{m+2k}^l \equiv \Phi_{k+m}^{k-l}$ are manifest for the bosonic representation of Φ_q^l .

One of the uses of the realization given above is to help understand the relationship between string constructions of [6] [7] and earlier techniques using orbifolds [8] or lattices [9]. As an example we consider the $k = 4, N = 2$ SCA, with central charge, $c = 2$.

One starts with the currents of $(SU(2))^4$, but it is convenient to view these as being embedded in the $SO(8)$ Lie algebra[†]. We take, therefore (suppressing cocycle factors

[†] In particular we replace $\sqrt{2}X^{2i-1}$ by $X^{2i-1} + X^{2i}$ and $\sqrt{2}X^{2i}$ by $X^{2i-1} - X^{2i}$.

throughout):

$$\begin{aligned} J^\pm(z) &= \exp(\pm i(X^1 + X^2)) + \exp(\pm i(X^1 - X^2)) \\ &\quad + \exp(\pm i(X^3 + X^4)) + \exp(\pm i(X^3 - X^4)) \\ J^3(z) &= i\sqrt{2} (\partial X^1 + \partial X^3) \end{aligned} \quad (16)$$

and hence:

$$\begin{aligned} G^\pm(z) &= \frac{1}{\sqrt{6}} \left\{ \exp(\pm i[(1+a)X^1 + X^2 + aX^3]) + \exp(\pm i[(1+a)X^1 - X^2 + aX^3]) \right. \\ &\quad \left. + \exp(\pm i[aX^1 + (1+a)X^3 + X^4]) + \exp(\pm i[aX^1 + (1+a)X^3 - X^4]) \right\}, \end{aligned} \quad (17)$$

where $a = -1/2 + \sqrt{3}/2$. The Sugawara tensor is given by

$$\begin{aligned} T(z) &= -\frac{1}{6} \left\{ \sum_{j=1}^4 (\partial X^j)^2 + (\partial X^1 + \partial X^3)^2 \right\} \\ &\quad + \frac{1}{6} \left\{ \exp(2iX^2) + \exp(2iX^4) - \exp(i(X^1 + X^2 - X^3 - X^4)) \right. \\ &\quad - \exp(i(X^1 + X^2 - X^3 + X^4)) - \exp(i(X^1 - X^2 - X^3 - X^4)) \\ &\quad \left. - \exp(i(X^1 - X^2 - X^3 + X^4)) + \text{hermitian conjugate} \right\} \end{aligned} \quad (18)$$

In terms of the bosons describing $J^\pm(z)$ in (16) highest weight states are now described by momenta of the form:

$$\begin{aligned} \vec{p} &= \frac{1}{4\sqrt{3}} \bar{q} (\vec{e}_1 + \vec{e}_3) + \frac{1}{2} \left\{ (a_1 + a_2)\vec{e}_1 + (a_1 - a_2)\vec{e}_2 + (a_3 + a_4)\vec{e}_3 + (a_3 - a_4)\vec{e}_4 \right. \\ &\quad \left. - \frac{1}{2}(a_1 + a_2 + a_3 + a_4)(\vec{e}_1 + \vec{e}_3) \right\} \end{aligned} \quad (19)$$

Define the following level one, $SO(8)$ Kac-Moody currents:

$$\begin{aligned} K^1(z) &= \frac{1}{\sqrt{2}} \left\{ \exp[i(X^2 + X^4)] + \exp[-i(X^2 + X^4)] \right\} \\ K^2(z) &= \frac{1}{\sqrt{2}} \left\{ \exp[i(X^2 - X^4)] + \exp[-i(X^2 - X^4)] \right\} \\ K^3(z) &= \frac{1}{\sqrt{2}} \left\{ \exp[i(X^1 - X^3)] + \exp[-i(X^1 - X^3)] \right\} \end{aligned} \quad (20)$$

The currents $K^1(z) - K^2(z)$ and $K^1(z) + K^2(z) - 2K^3(z)$ commute with each other and with $J^3(z)$, $J^\pm(z)$ and $G^\pm(z)$. Since the level one $SO(8)$ current algebra has $c = 4$, the

required Hilbert space of the $k = 4$, SCA must be, in some sense, the complement of the Hilbert space defined by $K^1(z) - K^2(z)$ and $K^1(z) + K^2(z) + 2K^3(z)$. Therefore define a canonically normalized current, $K(z)$, that is orthogonal to the two currents above:

$$K(z) = \frac{1}{\sqrt{3}}[K^1(z) + K^2(z) + K^3(z)]. \quad (21)$$

One can check from (19) that this current is in fact the vertex operator of the highest weight $l = 4$, $q = 0$ symmetrized momentum state^{*}. Moreover the Sugawara energy momentum tensor can be written as:

$$T(z) = \frac{3}{4}J(z)J(z) + \frac{1}{2}K(z)K(z) \quad (22)$$

where $J(z) = \frac{1}{\sqrt{6}}J^3(z)$ is the $U(1)$ current of the $N = 2$ SCA. Taking the operator product expansion of $K(z)$ with $G^\pm(w)$ one obtains:

$$K(z) G^\pm(w) = \pm \sqrt{\frac{3}{2}} \frac{1}{z-w} G'^\pm(w) \quad (23)$$

where $G'^\pm(z)$ is given by (17) but with X^1 and X^3 interchanged. Define

$$\begin{aligned} F_1^\pm(z) &= \frac{1}{\sqrt{2}}[G^\pm(z) + G'^\pm(z)] \\ F_2^\pm(z) &= \frac{1}{\sqrt{2}}[G^\pm(z) - G'^\pm(z)] \end{aligned} \quad (24)$$

and introduce (canonically normalized) bosons ϕ_1 and ϕ_2 such that

$$\begin{aligned} \partial\phi_1(z) &= i \left[\frac{\sqrt{3}}{2} J(z) + \frac{1}{\sqrt{2}} K(z) \right] \\ \partial\phi_2(z) &= i \left[\frac{\sqrt{3}}{2} J(z) - \frac{1}{\sqrt{2}} K(z) \right] \end{aligned} \quad (25)$$

One finds that

$$\partial\phi_a(z) F_b^\pm(w) = \pm i\sqrt{3} \frac{\delta_{ab}}{z-w} F_b^\pm(w). \quad (26)$$

(with no summation over the index b). Thus we can identify $F_a^\pm(z)$ with $\exp(i\sqrt{3} \phi_a(z))$

^{*} We would like to thank A.N. Schellekens for pointing out to us that the second current of the model must correspond to this highest weight state.

and one also sees from (22) that

$$T(z) = -\frac{1}{2}[(\partial\phi_1(z))^2 + (\partial\phi_2(z))^2]. \quad (27)$$

Thus we have completely ‘rebosonized’ the $k = 4$ model into two copies of the usual realization of $k = 1$ theory [2].

These two models are, however, not exactly equivalent, as can be seen from highest weight states whose momenta are defined by (19). Note that all the currents $K^1(z), K^2(z), K^3(z)$ and hence $K(z)$ have local operator products with the highest weight momentum states defined using (19) if and only if $l = \sum_{j=1}^k |a_j|$ is even, and they have $z^{1/2}$ branch cuts if and only if l is odd. Thus all highest weight states with even l can be obtained in the untwisted ϕ_1, ϕ_2 Hilbert space, while highest weight states with odd l can be obtained by giving $K(z) = -\frac{i}{\sqrt{2}}(\partial\phi_1 - \partial\phi_2)$ a Z_2 twist. This Z_2 twist interchanges ϕ_1 and ϕ_2 . In other words the whole $k = 4$ model, complete with all highest weight states can be constructed from Z_2 twisted and untwisted Hilbert spaces of two bosons. This also appears to be the conclusion of the authors of [10]. A further consequence of the foregoing analysis is that the $k = 4$ para-fermion theory is essentially equivalent to a suitably twisted boson. This observation has also been made in [11].

The procedure that we have just described has a multitude of simple generalizations. To take two non-trivial examples, there is an embedding of $SU(2)_{10} \times SU(2)_2$ in $SO(8)$, where the subscripts on the $SU(2)$ denote the embedding index and hence the level of the corresponding Kac-Moody algebra. The central charges of the $k = 10$ and $k = 2$ models are $5/2$ and $3/2$ respectively, and thus there is some simple realization of, in the notation of [7], $10^1 2^1$ in terms of 4 bosons. Similarly there is an embedding of $SU(2)_8 \times (SU(2)_3)^2 \times (SU(2)_1)^2$ in $SO(16)$. One can orthogonalize, much as we did above, with respect to the level one algebras, and hence the $8^1 3^2$ model can be realized using six bosons. One must, however, add a note of caution. In neither of these embeddings do all the $SU(2)_k$ factors embed into the larger group via $(SU(2))^k$, and thus it is not clear that one will be able to identify all highest weight states. Usually one will be able to get a subset. The simplest example of this is an $SU(2)_4$ subgroup of $SU(3)$. This trivially realizes the $k = 4$ theory in terms of two bosons. But using the techniques above one only finds highest weight states with l even. The fact that one finds the highest weight states $|l, q\rangle$ with both even and

odd l using $SO(8)$ but not using $SU(3)$ is probably because the relevant twist of the bosons is an outer automorphism of $SU(3)$ but is inner in $SO(8)$. This means that the twisted sectors of the bosons can be represented by momentum shifts only for $SO(8)$. It would be interesting to find a simple technique for finding highest weight states amongst bosons twisted by outer automorphisms.

From the point of view of constructing string theories, the fact that by using some $SU(2)$ embeddings one may not be able to isolate all highest weight states may itself not be a problem. Indeed the particular modular invariant combinations of $N = 2$ SCA characters may project out precisely the troublesome representations. We find it highly suggestive that many of the $c = 6$ models appearing in [7] consist of combinations of models that can be realized by considering embeddings of $SU(2)$ groups into a rank six Lie group. For example, we showed above that the $8^1 3^2$ model has a realization using exactly six bosons. At the very least our construction shows how one can take a set of $N = 2$ characters, view them as characters of twisted bosons, and thus demonstrates how one can view the partition functions of orbifolds constructed out of such twisted bosons as simple (though possibly restricted) combinations of characters of the $N = 2$ discrete series. It remains to be seen if these models are the same as those of [7]. We are currently examining these issues.

In order to obtain an orbifold description of a combination of discrete series models with a particular integral value of c , it is certainly necessary to isolate precisely c commuting currents. Each conformal block provides one such current from the $N=2$ SCA itself, while the rest of the currents, if present, must be found amongst combinations of highest weight states. The $k = 4$ example considered above illustrates how this can happen. A cursory inspection of the $c = 6$ models listed in [7] reveals that all but a few have the six commuting currents. When one can find such currents one might also be able to find a simple bosonic description of the theory by $SU(2)$ embeddings in a rank c group.

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