CERN-TH.5054/88

MAD/TH/88-14

COSMOLOGICAL STRING THEORIES AND DISCRETE INFLATION

I. Antoniadis^{*)}, C. Bachas^{*)+}, John Ellis

CERN - Geneva

and

D.V. Nanopoulos

Department of Physics, University of Wisconsin
Madison, WI 53706, U.S.A.

ABSTRACT

We catalogue string theories which can be interpreted as Robertson-Walker cosmologies. In the absence of dynamic matter, we find only two discrete classes of such theories: static Einstein universes and linearly expanding Milne universes. These correspond to conformal field theories on group manifolds, with the sizes of the Einstein universes related to the level of the Kac-Moody algebra, and Milne universes also offering a geometrical interpretation of a Goddard-Kent-Olive coset construction. String loop effects or non-perturbative phenomena can generate a non-trivial dilaton potential, in which case string theory admits an inflationary cosmology whose Hubble constant is related to the gauge coupling. Presently known conformal field theories permit only a discrete set of values for the Hubble constant, all of which are $O(1)$ in Planck units.

^{*)}On leave of absence from CPT, Ecole Polytechnique, 91128 Palaiseau, France.

⁺⁾Present address: I.H.E.S., Bures-sur-Yvette, 91440 Palaiseau, France.

String theories are not yet classified. Many, if not all, of them can be described by specifying the space parametrized by two-dimensional conformal fields on the world sheet. In order for a string theory to describe contemporary microphysics, this space must include a four-dimensional Minkowski subspace. Several different ways to construct such four-dimensional string theories are now known [1,2], using different parametrizations of the space of other "internal" fields. However, in order for a string theory to describe cosmology, the "external" subspace must be described by some Robertson-Walker metric. No explicit cosmological string theories are yet known. In addition to casting light on such traditional cosmological problems as the age, size and flatness of the universe, the large time limits of cosmological string theories might only include a subset of the available four-dimensional string theories, thereby providing an important constraint on microphysical theories.

In this paper, we initiate the construction of cosmological string theories. We first look for solutions to the non-linear σ -model equations for the graviton, dilaton and antisymmetric tensor field [3] which take the Robertson-Walker form. We then show that these Robertson-Walker spaces can be interpreted as conformal field theories capable of being embedded in consistent string theories. In general, these Robertson-Walker models make contributions to the central charge c which are less than four: $c_{RW} = 4 - \delta c$, where we call $\delta c > 0$ the central charge deficit. Such solutions can therefore be embedded in consistent string theories only if the "internal" space has an equal and opposite central charge deficit, so that the total central charge is 26 (for a bosonic string) or 15 (for a superstring)*). All known conformal theories are members of discrete series [4-7] whose central charges c_n approach integers (or half-integers in the supersymmetric case) from below: $c_n - [c_n] = 1 - O(1/n)$ as $n \rightarrow \infty$. As we show later in the paper, one cannot obtain a theory with arbitrarily small positive central charge deficit as a combination of such theories with negative central charge deficits. This means that the central charge of the Robertson-Walker sector can only take a discrete set of values which are bounded above by some number $c_{max} < 4$. Except in special cases, the central charge of the "internal" subspace is neither integer nor half-integer, and so it cannot be parametrized by free fermions or bosons.

We find three classes of cosmological string solutions, two of which hold in the absence of a potential for the dilaton field. Of these, one is a class of static Einstein universes [8] which can be regarded as $SO(3)$ group manifolds [5] whose (high) level determines their (large) radii. The second class consists of Milne universes [8] whose scale factors increase linearly with time. These can be inter

*) For pedagogical reasons, in what follows we restrict ourselves to the non-supersymmetric case.

preted either as the direct products of SO(3) Wess-Zumino-Witten models and one-boson conformal field theories with a background charge [9], or as Goddard-Kent-Olive coset constructions [6]. A dilaton potential may be generated by string loop effects [10,11,12] or by non-perturbative phenomena [13]. In the presence of such a potential, there is a third class of cosmological string theories exhibiting inflation in the presence of a constant dilaton field [10]. As a result of the properties of the central charge deficit mentioned earlier, the Hubble constants in these inflationary solutions are all $O(1)$ in Planck units. This does not necessarily imply that density perturbations $\delta\rho/\rho$ are large, since only a discrete set of dilaton field values is allowed. The entire universe may jump simultaneously from one level to another, much like an electron in a Bohr atom, with the ground state being flat space.

We first examine the equations [3] for the four-dimensional graviton and anti-symmetric tensor fields, to leading order in the σ -model expansion. We express them in terms of the physical metric $g_{\mu\nu}$, which is related to the σ -model metric $G_{\mu\nu}$ through a rescaling: $g_{\mu\nu} = e^\phi G_{\mu\nu}$, where our normalization for the dilaton field ϕ is such that the string loop expansion parameter is e^ϕ . The equations read:

$$\beta_{\mu\nu}^G = 0 \Rightarrow R_{\mu\nu} = \frac{1}{2} D_\mu \phi D_\nu \phi + \frac{1}{2} g_{\mu\nu} \Box \phi + \frac{1}{4} e^{-2\phi} H_{\mu\nu}^2 \quad (1a)$$

$$\beta_{\mu\nu}^B = 0 \Rightarrow D_\lambda (e^{-2\phi} H_{\mu\nu}^\lambda) = 0 \quad (1b)$$

Here $H_{\lambda\mu\nu} \equiv 3D_{[\lambda} B_{\mu\nu]}$ is the field strength of the antisymmetric tensor, the signature of the metric is $(-+++)$, and our convention for the Ricci tensor is $R_{\mu\nu} = \partial_\lambda \Gamma_{\mu\nu}^\lambda - \partial_\mu \Gamma_{\lambda\nu}^\lambda + \dots$ in terms of the connection $\Gamma_{\mu\nu}^\lambda$. Equation (1b) is solved by the duality transformation:

$$H_{\mu\nu}^\lambda = e^{2\phi} \epsilon^\lambda_{\mu\nu\rho} D^\rho b \quad (2)$$

where b can be identified as an axion field. Equation (1a) then becomes:

$$R_{\mu\nu} = \frac{1}{2} D_\mu \phi D_\nu \phi + \frac{1}{2} g_{\mu\nu} \Box \phi + \frac{1}{2} e^{2\phi} [D_\mu b D_\nu b - g_{\mu\nu} (\Box b)^2] \quad (3a)$$

while the Jacobi identity for the antisymmetric tensor field $D_{[\rho} H_{\lambda\mu\nu]} = 0$ imposes the condition:

$$\Box b + 2 D_\lambda b D^\lambda \phi = 0 \quad (3b)$$

Finally, we must impose the dilaton field equation which is equivalent to demanding the cancellation of the conformal anomaly, or equivalently that the total central charge be $c = 26$. We will allow for the possibility that the remaining "internal" conformal fields on the world sheet, other than the four space-time co-ordinates, have a central charge $c_{\text{int}} = 22 + \delta c$, so that the dilaton equation becomes:

$$\delta c = -3e^{-\phi} \left[-R + \square\phi + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial b)^2 \right] \quad (4)$$

in units where the Regge slope $\alpha' = 1$. That the right-hand side of this equation is indeed a constant follows from Eqs. (3a,b) and the Bianchi identity: $\nabla^\mu R_{\mu\nu} = \frac{1}{2}\nabla_\nu R$. Equations (3a,b) and (4) can all be obtained as linear combinations of the equations of motion of the four-dimensional action:

$$\int d^4x \sqrt{-g} \left\{ R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial b)^2 - \frac{1}{3}e^{\phi}\delta c \right\} \quad (5)$$

The last term in the above Lagrangian, due to the "central charge deficit" δc , can be thought of as a non-vanishing potential provided by the "internal" fields of the theory.

We have looked for solutions of these field equations (3a,b) and (4), with a Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t) \cdot \tilde{g}_{ij} dx^i dx^j \quad (6)$$

and time-dependent dilaton and axion fields: $b = b(t)$, $\phi = \phi(t)$. In Eqs (6) $\tilde{g}_{ij}$ is a maximally symmetric metric of a three-dimensional space, $a(t)$ is the scale factor, and we define as usual the Hubble expansion rate: $H(t) \equiv \dot{a}(t)/a(t)$. With this ansatz, the axion equation (3b) is solved by

$$b = \beta e^{-2\phi}/a^3 \quad (7)$$

where β is an integration constant. The central charge deficit is

$$\delta c = 6e^{-\phi} \left[\dot{H} + 3H^2 + \frac{2K}{a^2} \right] \quad (8)$$

where K is the constant curvature of the maximally symmetric three-space. Assuming $\delta c \neq 0$, Eq. (8) can be inverted to solve also for the dilaton:

$$\phi = \ln \left\{ \frac{6}{\delta c} \left(\dot{H} + 3H^2 + \frac{2K}{a^2} \right) \right\} \quad (9)$$

Finally, Eq. (3a) has a priori two independent components, one for $\mu = \nu = t$:

$$-6(\dot{H} + H^2) = \ddot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 \quad (10a)$$

and one for $\mu, \nu = i, j$:

$$-2\left(\dot{H} + 3H^2 + \frac{2\kappa}{a^2}\right) = \ddot{\phi} + 3H\dot{\phi} - \frac{\beta^2}{a^6} e^{2\phi} \quad (10b)$$

but if we replace ϕ and b from (7) and (9) we see that only one of Eqs. (10a,b) is independent. The reason is that the dilaton field equation was really an identity, up to a constant. Taking the difference of (10a,b) we thus obtain our final equation:

$$\left(\frac{\ddot{H} + 6H\dot{H} - \frac{4\kappa}{a^2} H}{\dot{H} + 3H^2 + \frac{2\kappa}{a^2}} \right)^2 = -4\dot{H} + \frac{4\kappa}{a^2} - \frac{(\delta_c)^2 \beta^2}{36 a^6} \cdot \frac{1}{(\dot{H} + 3H^2 + \frac{2\kappa}{a^2})^2} \quad (11)$$

This can in principle be solved to yield the Hubble expansion rate of the Universe, in a string theory without space or time-dependent background fields.

It is easy to check that Eq. (11) has only two asymptotic solutions:

(A) A vanishing expansion rate $H = 0$, i.e., a constant scale factor which we may reabsorb in the space-curvature K , so that we set $a(t) = 1$. It follows then easily from the equations that the dilaton is also constant: $\phi = \phi_0$, that $\dot{b} = \beta e^{-2\phi_0}$, and that the space curvature obeys $K = (1/4)\beta^2$ and is therefore non-negative. The four-dimensional curvature is $R = 6K$ and the central charge deficit is:

$$\delta_c = 12 e^{-\phi_0} K \quad (12)$$

This solution is a static Einstein universe with non-negative curvature. Such Einstein universes are known to be perfect-fluid solutions with positive cosmological constant [8].

(B) A linearly expanding scale factor which we may choose to be $a(t) = t$ by reabsorbing the proportionality constant into the three-space curvature. It follows that $H = 1/t$, while the dilaton and axion fields are given by:

$$\phi = -\ln t^2 + \lambda, \quad \dot{b} = 2e^{-\lambda} \sqrt{K} \cdot t \quad (13)$$

Here λ is an integration constant, and the space curvature K must again be non-negative. Note that both here and in the Einstein universe K can be made negative

if we allow an imaginary axion field. The four-dimensional curvature is $R = 6(1+K)/t^2$ and the central charge deficit:

$$\delta c = 12e^{-\lambda}(1+K) \quad (14)$$

Such a solution is referred to in the literature [8] as a Milne universe.

So far we have shown that the Einstein and Milne universes are solutions, in fact, the only asymptotic ones, of the σ -model equations to leading order. To prove that they remain solutions to all orders we must identify the corresponding conformal theories on the world sheet. For the static Einstein universe this is particularly easy: it corresponds to a Wess-Zumino-Witten model on an $O(3)$ group manifold [5], together with a time co-ordinate which is a free world-sheet field. The central charge is

$$c = 1 + \frac{3k}{k+1} = 4 - \frac{6}{k+1} \quad (15)$$

so that the central charge deficit (which can be thought of as the σ -model expansion parameter) is $\delta c = 6/(k+1) = 12e^{-\lambda}K + \text{higher order}$. Here k is the level of the $O(3)$ Kac-Moody algebra which must be integer, as can be seen either from topological considerations or from a unitarity and modular-invariance analysis [5]. Thus δc is quantized; we will soon argue that, furthermore, it can only take a finite number of values, when one tries to construct a consistent theory by adding internal fields with $c_{\text{int}} = 22 + \delta c$.

To find the conformal theory that corresponds to the Milne universe, we must go back to the σ -model metric $G_{\mu\nu} = e^{\phi} g_{\mu\nu}$. Using Eq. (13) and changing time co-ordinates to $\tau = e^{\frac{\lambda}{2}} \log t$ we find:

$$G_{\mu\nu} dx^{\mu} dx^{\nu} = -(d\tau)^2 + e^{\lambda} \tilde{g}_{ij} dx^i dx^j \quad (16a)$$

and

$$\phi = -2e^{-\lambda/2} \tau + \lambda \quad (16b)$$

Thus in this metric the time co-ordinate decouples while the dilaton is linear in it. The corresponding conformal field theory is then again a Wess-Zumino-Witten model with curvature $e^{-\lambda}K$, plus a time co-ordinate which is a free field but with a charge at infinity $Q = e^{-\lambda/2}$ [9]. This is described by the Lagrangian:

$$\mathcal{L}^{(2)} = -(\partial\tau)^2 - Q\tau R^{(2)} + \mathcal{L}_{\text{WZW}}(O(3)) \quad (17)$$

with $R^{(2)}$ the world-sheet curvature. The central charge is:

$$C = 1 - 12Q^2 + C_{WZW}$$

which precisely reproduces the two terms we found in Eq. (14). Note that the contribution of the background charge Q to the anomaly is negative, because τ is a world-sheet field with negative norm. Note also that the WZW curvature $e^{-\lambda} K$ must, as before, be quantized. Whether $e^{-\lambda}$ and K are separately quantized can only be decided by an analysis of modular invariance and unitarity. Note however that if we naively demand that a Wick rotation $\tau \rightarrow i\tau$ produces a unitary model, then the background charge is indeed constrained [9,7] to take one of the values of the discrete minimal series [4], i.e., $Q = 1/\sqrt{2m(m+1)}$ with $m = 3, 4, \dots$. Finally, we should point out that the Einstein universe can be thought of as a special case of (17) with $Q = 0$.

It is at first astonishing that the simple conformal theories (17) exhaust all possible Robertson-Walker type solutions of the string equations, at least asymptotically. There is at least one other series of conformal theories which look like candidates for Robertson-Walker cosmologies: these are the Goddard-Kent-Olive (GKO) models [6] with $G/H = O(1,4)/O(1,3)$ or, possibly, various Wick rotations of these groups. These models have an energy-momentum tensor:

$$L_0 \propto \sum_{O(1,4)} :J^2: - \rho \sum_{O(1,3)} :J^2: \quad (18)$$

where $\rho = (k'+3)/(k'+2)$, with k' the level of both the $O(1,4)$ and the embedded $O(1,3)$ Kac-Moody algebras of the currents J . The central charge is:

$$C = \frac{10k'}{k'+3} - \frac{6k'}{k'+2} = 4 - 6 \cdot \frac{3k'+4}{(k'+2)(k'+3)} \quad (19)$$

If we were dealing with compact groups, k' would be a positive integer, and $1 < \rho \leq 4/3$; but in our case the allowed range of parameters cannot be decided a priori without a careful analysis of unitarity.

The question that arises is what is the geometric interpretation of these algebraic constructions. To answer this question we note that in string theory the equation of motion of a scalar is given by the zeroth-moment Virasoro gauge condition. Thus, if a geometric background corresponding to these models exists, we should have:

$$L_0 \propto e^{-\phi} \square \quad (20)$$

where here the covariant box is calculated with the rescaled metric in four dimensions. Note that for non-trivial dilaton backgrounds, the factor $e^{-\phi}$ is necessary in order that the operator on the right-hand side of (20) yields a c-number mass when acting on physical states.

On a scalar, i.e., a highest weight of the Kac-Moody algebra, L_0 acts as the $O(1,4)$ Casimir minus ρ times the $O(1,3)$ Casimir. Therefore, in some co-ordinate system we must have

$$\begin{aligned} L_0 &\propto \frac{1}{2} (1-\rho) J_{\mu\nu}^2 + J_{4\mu}^2 \\ &= (\pm R^2 - \rho x^2) \partial^2 - (1-\rho)(x \cdot \partial)^2 + (2\rho-3)(x \cdot \partial) \end{aligned} \quad (21)$$

where $J_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$ and $J_{4\mu} = x_4 \partial_\mu$ are the generators of 5d rotations, restricted to the hyperboloid:

$$x^2 + x_4^2 = \pm R^2 \quad (22)$$

We equate the operator (21) with $e^{-\phi} \square$, calculated for the most general $O(1,3)$ -symmetric ansatz: $\phi = \phi(x^2)$ and $g_{\mu\nu} = A(x^2)\eta_{\mu\nu} + B(x^2)x_\mu x_\nu$. The result is:

$$g_{\mu\nu} = |\pm R^2 - \rho x^2|^{\frac{1}{2}} \left(\eta_{\mu\nu} + \frac{(1-\rho)}{(R^2 - x^2)} x_\mu x_\nu \right) \quad (23a)$$

$$\phi = -\frac{3}{2} \ln |\pm R^2 - \rho x^2| + \lambda + \ln(9/4) \quad (23b)$$

where we have absorbed all proportionality factors in the constant piece of the dilaton which we wrote as $\ln(9/4) + \lambda$ for later convenience. We must still verify that the above metric and dilaton satisfy the conformal invariance conditions, i.e., the tree-level string equations of motion (3). The answer is that they do, provided we set the radius of the hyperboloid $R = 0$, and we include an axion field:

$$-\frac{1}{\rho} \frac{d\lambda}{dx^2} = i \frac{4}{9} e^{-\lambda} \sqrt{\rho} (\rho x^2)^{\frac{1}{2}} \quad (24)$$

Note that $x^2 = -x_4^2$ is always negative. It follows easily that the metric

(23a) has the right signature: $(-+++)$ only if $\rho > 0$, in which case the axion field is purely imaginary. Thus, a priori this solution is unphysical: what is, however, interesting is that it is not a new solution. The change of co-ordinates:

$$-\rho x^2 = \left(\frac{3}{2}t\right)^{4/3} \quad (25)$$

transforms the metric (23a) for $R = 0$ into the Milne universe with constant negative space-curvature $K = -4/9\rho$, and the axion and dilaton into those given by Eq. (13). This same change of co-ordinates transforms the physical Milne solution (with K positive) to the metric:

$$g_{\mu\nu} = \left| -\rho x^2 \right|^{1/2} \cdot \left(\delta_{\mu\nu} - (1-\rho) \frac{x_\mu x_\nu}{x^2} \right) \quad (26)$$

which has the right signature since ρ is now negative. The scalar field equation in this metric and the corresponding dilaton background is the zeroth-moment Virasoro condition for the GKO construction $O(1,4)/O(4)$. Since the axion turns out to be, in this case, real, we may conjecture that this is the unitary GKO construction, but this needs further analysis.

Let us finally note the following apparent puzzle: the conventional Milne universe suffers from the initial singularity problem, as at $t = 0$ the scale factor $a(t)$ vanishes while the curvature blows up. However, there is at first sight no such singularity in string theory, at least in the underlying two-dimensional conformal theory (17) where $t = 0$ is transferred to the limit $\tau \rightarrow \pm\infty$. This is reminiscent of the fact that internal curvature singularities appear in orbifold compactifications of strings, which are illegal for field theories.

In the previous analysis, we have considered only vacuum solutions, in the sense that the universe is empty of ordinary time-dependent matter. The effect of the latter on our results can be studied by the addition of a matter action S_M to our initial action (5). It follows that

$$D^\nu T_{\mu\nu}^M = \frac{1}{2} \frac{\delta S_M}{\delta \phi} D_\mu \phi \quad (27)$$

and thus the matter energy-momentum tensor $T_{\mu\nu}^M = \delta S_M / \sqrt{-g} \delta g^{\mu\nu}$ is conserved by itself if the dilaton is not coupled to it. Furthermore, Eqs. (3a) and (4) are modified to become

$$R_{\mu\nu} = R_{\mu\nu}^{old} + T_{\mu\nu}^M - \frac{1}{2} g_{\mu\nu} T_\lambda^{\lambda} + \frac{1}{2} g_{\mu\nu} \frac{\delta S_M}{\delta \phi} \quad (28a)$$

$$S_C = S_C^{old} + 3e^{-\phi} \left(T_\lambda^{\lambda} - \frac{\delta S_M}{\delta \phi} \right) \quad (28b)$$

where $R_{\mu\nu}^{\text{old}}$ and δc^{old} are given by the expressions in the right-hand side of Eqs. (3a) and (4). Assuming for $T_{\mu\nu}^M$ a perfect fluid form, characterized by an energy density $\rho(t)$ and pressure $p(t)$, and taking $e^{-\phi}(\delta S_M/\delta\phi)$ to be proportional to some linear combination of ρ , p corresponding to the matter potential (e.g., $\rho-p$ for scalars), one finds that the previous asymptotic solutions change to more familiar ones, namely for large t $a(t) \sim t^{\frac{1}{2}}$ or $t^{2/3}$ for radiation ($p=1/3\rho$) or matter ($p=0$) dominated universes, respectively. The value (28b) of the central charge, and hence the nature of the corresponding conformal field theory, needs to be investigated on a case-by-case basis.

Up to now we have examined the four-dimensional string equations of motion at tree level in the topological expansion, and have found some interesting non-trivial cosmological solutions but no inflationary ones. However, it is known [10,11] that string loop effects may modify the σ -model β -functions and, in particular, can lead to a non-trivial dilaton potential^{*}). In space-time supersymmetric string theories such modifications can occur in perturbation theory only in the case of an anomalous $U(1)$ gauge factor [12]. In our context, such a dilaton potential $\delta V(\phi)$ should be included in the Lagrangian (5) and would modify Eqs. (3a) and (4) to become

$$R_{\mu\nu} = R_{\mu\nu}^{\text{old}} + \frac{1}{2} g_{\mu\nu} [\delta V(\phi) - \delta V'(\phi)] \quad (29a)$$

$$\delta c = \delta c^{\text{old}} - 3e^{-\phi} [2\delta V(\phi) - \delta V'(\phi)] \quad (29b)$$

where $\delta V'$ denotes differentiation with respect to ϕ . It is easy to check that Eqs. (29a) and (3b) imply that δc is a c-number for an arbitrary potential $\delta V(\phi)$. In perturbation theory, $\delta V(\phi)$ can be expressed as a power series in the string loop expansion e^{ϕ} , namely $\delta V(\phi) = \sum_{n \geq 1} \alpha_n e^{(n+1)\phi}$. The new equations now admit as solutions de Sitter space, which is an exponentially expanding universe (H constant), for constant background values of the dilaton ϕ and axion b :

$$R = 12H^2 = 2 [\delta V(\phi) - \delta V'(\phi)] \quad (30a)$$

$$\delta c = -3\delta V'(\phi) e^{-\phi} \quad (30b)$$

^{*}) Non-perturbative phenomena may also generate a non-trivial dilaton potential.

We see that the Hubble constant during inflation is determined by the dilaton field value, which is also the string loop expansion parameter and fixes the four-dimensional gauge coupling constant.

We have seen that the Robertson-Walker asymptotic solutions to the one-loop non-linear σ -model equations can be interpreted as conformal field theories. In the cases of the Einstein universes and of the spatial co-ordinates of the Milne universes, their central charges are known to take discrete values, but this is not known for the time co-ordinates of the Milne universes, nor for the non-compact GKO construction. We now consider the combination of one of these cosmological conformal field theories with another "internal" space to give a total central charge $c = 26$. We will argue that in fact only a finite set of Robertson-Walker conformal field theories with central charges < 4 can be combined with known "internal" conformal field theories to have a total central charge which is integer.

Our argument is based on the observation that all known unitary and crossing-symmetric conformal field theories fall into discrete series [4-7] whose central charges approach from below accumulation points with integer or half-integer values: $c_n = c_\infty - O(1/n)$, $c_\infty \in \mathbb{Z}/2$. The "internal" space must be a direct product of such "building-block" theories with a total central charge $c = 22 + \delta c$. Since all building-blocks have central charges $> \frac{1}{2}$, the possible "internal" spaces contain only a finite number of building blocks. It is evident that there is a discrete infinity of such "internal" spaces, whose central charges also have integer and half-integer accumulation points. Moreover, if a subsequence of internal spaces has central charges which approach one of these accumulation points, they do so from below. This means that arbitrarily small values of the central charge deficit δc are not allowed. In particular, only finite subsets of the discrete series of Robertson-Walker conformal field theories found above are allowed. As for the Robertson-Walker theories which were not known a priori to be discrete, the discreteness of the central charges of the known "internal" conformal field theories tells us a posteriori that they must also be discrete. Since the four-dimensional curvature $R = \frac{1}{2}(\delta c)e^\phi$ for both the Einstein and Milne universes, this means that the curvature is quantized in terms of the string coupling constant.

The fact that the set of central charges of consistent cosmological string theories is discrete also means that there is only a discrete set of values of the Hubble expansion rate H in inflationary cosmologies. Moreover, the value of the effective cosmological constant during the inflationary phase cannot be an arbitrarily small fraction of one (in Planck units). We are then led to the following "Bohr atom" picture of an inflationary universe. It may start by inflating with one of the larger discrete values of the Hubble expansion rate. It may then cascade down by quantum jumps through a discrete sequence of consistent cosmologi-

cal string theories with lower values of the inflationary expansion rate. Eventually it makes the quantum jump to a Robertson-Walker space with zero cosmological constant, and inflation ends. Although the cosmological constant is always $O(1)$ while it is non-zero, this need not necessarily imply that density perturbations $\delta\rho/\rho$ are large. These are normally calculated from fluctuations in some continuous "inflaton" field corresponding to continuous perturbations in the effective potential, whereas here only a discrete set of potential values is allowed.

We have demonstrated the existence of cosmological string theories corresponding to static Einstein, linearly-expanding Milne, and inflationary universes. A novel feature is that all of these cosmologies are quantized: the size of an Einstein universe is related to the level of the corresponding $SO(3)$ Kac-Moody algebra, as is the three-space curvature of a Milne universe, while the quantization of the expansion rates of Milne and inflationary universes is enforced by the discrete central charges of the known unitary conformal field theories. It seems quite likely that the history of the universe includes jumps from one string model to another, characterized by reshuffles of the central charge between different conformal subtheories. Thus it seems that to make the connection between contemporary microphysics and the very early universe, one needs to understand whether all the different string theories are just different manifestations (or ground states) of an underlying metatheory, and how Nature chooses which manifestation (or ground state) to use for physics. Cosmological density perturbations $\delta\rho/\rho$ may turn out to be physical signatures of this process of choice, and hence of more than astrophysical interest.

ACKNOWLEDGEMENTS

The work of D.V. Nanopoulos was supported in part by Department of Energy grant DE-AC02-76ER0081 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation. One of us, C.B., acknowledges the hospitality of CERN where part of this work was completed and thanks K. Gawedski for useful conversations.

REFERENCES

- [1] I. Antoniadis, C. Bachas, C. Kounnas and P. Windey, Phys. Lett. 171B (1986) 51;
H. Kawai, D.C. Lewellen and S.H.H. Tye, Phys. Rev. Lett. 57 (1986) 1832 and Nucl. Phys. B288 (1987) 1;
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B289 (1987) 87;
I. Antoniadis and C. Bachas, Nucl. Phys. B298 (1988) 586;
K.S. Narain, Phys. Lett. 169B (1986) 41;
W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B287 (1987) 477.
- [2] K. Bardacki, E. Rabinovici and B. Sáring, CERN preprint TH.4760/87 (1987);
G. Gepner, Princeton Univ. preprints PUPT-1056 (1987) and 1066 (1987).
- [3] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B158 (1985) 316;
C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B262 (1985) 593;
A. Sen, Phys. Rev. D32 (1985) 2102 and Phys. Rev. Lett. 55 (1985) 1846.
- [4] A. Belavin, A. Polyakov and A. Zamolodchikov, Nucl. Phys. B241 (1984) 333;
D. Friedan, Z. Qiu and S. Shenker, Phys. Rev. Lett. 52 (1984) 1575.
- [5] E. Witten, Comm. Math. Phys. 92 (1984) 451;
V. Knizhnik and A.V. Zamolodchikov, Nucl. Phys. B247 (1984) 83;
D. Gepner and E. Witten, Nucl. Phys. B278 (1986) 493.
- [6] P. Goddard, A. Kent and D. Olive, Phys. Lett. 152 (1985) 88.
- [7] For more examples, see, e.g.,
Greg Anderson and Greg Moore, IASSNS-HEP-87/69 (1987);
J. Bagger, D. Nemeschansky and S. Yankielowicz, Phys. Rev. Lett. 60 (1988) 389;
P. Goddard and A. Schwimmer, Phys. Lett. 206 (1988) 62;
J.-L. Gervais and A. Bilal, LPTENS preprint (1988);
M. Caselle and K.S. Narain, preprint in preparation and references therein.
- [8] A. Einstein, Sitz. Preuss. Akad. Wiss. 142 (1917). For an English translation, see "The Principle of Relativity" (Methuen, 1923, reprinted by Dover Publications), p. 35;
E.A. Milne, Nature 130 (1932) 9;
For reviews, see:
D. Kramer, H. Stephani, E. Herlt and M. MacCallum, "Exact Solutions of Einstein's Field Equations", ed. by E. Schmutzer, Cambridge Univ. Press (1979);
N.D. Birrell and P.C.W. Davies, "Quantum Fields in Curved Space", Cambridge Univ. Press (1982).
- [9] V. Dotsenko and V. Fateev, Nucl. Phys. B240 [FS12] (1984) 312 and B251 [FS13] (1985) 691;
J.-L. Gervais and A. Neveu, Comm. Math. Phys. 100 (1985) 15.
- [10] W. Fischler and L. Susskind, Phys. Lett. 173B (1986) 262;
W. Fischler, I. Klebanov and L. Susskind, SLAC-PUB-4298 (1987).
- [11] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys. B288 (1987) 525.
- [12] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 589.
- [13] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55.