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YUKAWA COUPLINGS IN $SU(3) \times SU(2) \times U(1)_Y$ ORBIFOLD MODELS

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Abstract

We analyze the Yukawa couplings of the first $SU(3) \times SU(2) \times U(1)_Y$ orbifold models recently obtained. In these models the rank is naturally lowered due to the presence of a Fayet–Iliopoulos term associated with an “anomalous” $U(1)$ in the four-dimensional theory. It is shown that the phenomenological viability of the models selects a favoured class of vacua. In particular, for the specific examples considered, some twisted fields with one moded oscillator acting, must acquire non-vanishing vev’s. However other possibilities may exist. We also find that all the baryon and lepton number violating operators can, in general, be avoided in a completely natural way.

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1. INTRODUCTION

A great effort has recently been made in order to find consistent string theories in four dimensions with interesting phenomenological prospects. The compactification of the heterotic string on orbifolds[1] has proved to be a fruitful way to construct promising models[2–8] (other attempts at model building can be found in refs.[9,10]). In this sense it has been shown by Ibáñez et al.[3] that the use of the Z_3 orbifold with Wilson lines[1,2] as the compactified space is an efficient way to obtain supersymmetric models with gauge group $[SU(3) \times SU(2) \times U(1)^5] \times [\text{hidden sector}]$ and three generations of particles. The $U(1)$ charges and the mechanism for anomaly cancellation in these models have been analyzed in ref.[6] where it was proposed to use the Fayet–Iliopoulos terms associated with the “anomalous” $U(1)$ ’s[11–14] for breaking the gauge group further. This last point was also discussed in ref.[15]. It is important to note that the structure of the Fayet–Iliopoulos term is determined by the form of the “anomalous” $U(1)$ which is unambiguous. It is precisely the presence of this anomalous D -term which makes the $SU(3) \times SU(2) \times U(1)^n$ “vacuum” unstable, thus leading necessarily to a gauge symmetry breakdown. The most promising models obtained so far[7,8] have been built up in this way and have as outstanding characteristics:

- $N=1$ supersymmetry
- Gauge group: $SU(3) \times SU(2) \times U(1)_Y$ or $SU(3) \times SU(2) \times U(1)_Y \times U(1)$
- Three families of particles with the correct representations under $SU(3) \times SU(2) \times U(1)_Y$
plus some extra particles
- A true hidden sector

Despite these beautiful features, it is clear that a more detailed analysis of these models is necessary. In particular it is important to determine whether or not they have the correct Yukawa couplings and to check that they pass certain phenomenological tests such as, for example, those related to the existence of baryon and lepton number violating operators. The aim of this article is to analyze these points.

2. THE MODELS

Let us review, briefly, how these models are constructed (for more details see refs.[3,6–8]). The starting point for all of them is the compactification of the heterotic string on a Z_3 orbifold with two Wilson lines. The shift v^I associated with the twist θ and the two Wilson lines a_1^I, a_3^I associated with two translations e_1, e_3 are:

$$\begin{aligned} V^I &= \frac{1}{3} (11112000)(20000000) \\ a_1^I &= \frac{1}{3} (00000002)(01100000) \\ a_3^I &= \frac{1}{3} (11121011)(11000000) \end{aligned} \quad (1)$$

This particular compactification was presented in ref.[3]. Its characteristics before the Fayet–Iliopoulos breaking are[3,6]:

Gauge group:

$$\left[SU(3) \times SU(2) \times U(1)^5 \right] \times \left[SO(10) \times U(1)^3 \right]^1 \quad (2)$$

“Anomalous” combination of $U(1)$ ’s:

$$X = \frac{1}{\sqrt{3}} (U_6 - U_7 + U_8) \quad (3)$$

Combination of non – anomalous $U(1)$ ’s used in refs.[7,8] for the hypercharge:

$$Y = \frac{\sqrt{3}}{3} U_1 - \frac{\sqrt{2}}{2} U_2 - U_3 + U_4 \quad (4)$$

The spectrum of massless particles is given in Table 1. Other choices for the embedding, leading also to $[SU(3) \times SU(2) \times U(1)^5] \times [\text{hidden sector}]$ as an unbroken gauge group, have been presented in refs.[3,5], but either the Fayet–Iliopoulos terms for them have not been studied yet or they lead to unrealistic gauge groups[6]. It has been known for some time[12–14] that if the four – dimensional gauge group contains a factor $U(1)$ “anomalous” (in our case that of eq.(3)), a Fayet–Iliopoulos (F – I) D – term associated with it is generated in string loop perturbation theory. The form of such a D – term is

$$D^{(a)} = -e^{-\varphi} \left[e^{-2\varphi} C^{(a)} + \sum_i Q_i^{(a)} \eta_i^* \eta_i \right] \quad (5)$$

where η_i are the scalar fields with charges $Q_i^{(a)}$ under the anomalous $U(1)$, φ is the dilaton and

$$C^{(a)} = \frac{9}{192\pi^2} \sum_i Q_i^{(a)} = \frac{24g}{192\pi^2} \left(\frac{\sqrt{54}}{2} \right) \quad (6)$$

It is necessary, in order to have a supersymmetric vacuum solution, to give vev's to particular scalar fields, say χ_k , in such a way that both anomalous and non-anomalous D -terms are cancelled. Moreover, no F -terms different from zero must be generated. These conditions are summarized in the following equations

$$\sum_k Q_k^{(a)} |\langle \chi_k \rangle|^2 = -e^{-2\varphi} C^{(a)} \quad (7)$$

$$\sum_k Q_k^{(j)} |\langle \chi_k \rangle|^2 = 0 \quad j = 1, \dots, 7 \quad (8)$$

$$W = \frac{\partial W}{\partial \eta_i} = 0 \quad (9)$$

The solutions to (7), (8) and (9) which preserve the physical hypercharge of eq.(4) obtained so far [7,8] are of the form:

$$\begin{aligned} |\langle \ell_1 \rangle|^2 &= 2 |\langle \ell_8 \rangle|^2 = 4\lambda a \\ |\langle \ell_2 \rangle|^2 &= |\langle \ell_3 \rangle|^2 = (2+2\lambda)a \\ |\langle \ell_4 \rangle|^2 &= 2 |\langle \ell_5 \rangle|^2 = (8-4\lambda)a \\ |\langle \ell_6 \rangle|^2 &= 4a \\ |\langle \ell_7 \rangle|^2 &= |\langle \ell_9 \rangle|^2 = (2-2\lambda)a \end{aligned} \quad (10)$$

with

$$a = \frac{\sqrt{3} g e^{-2\varphi}}{192 \pi^2} \left(\frac{\sqrt{54}}{2} \right) \quad 0 \leq \lambda \leq 1 \quad (11)$$

The ℓ fields are indicated in Table 1. This set of vev's leads to different unbroken gauge groups depending on the value of λ . There are three possibilities:

$$0 < \lambda < 1 \quad (\text{Vacuum 1}) \quad [SU(3) \times SU(2) \times U(1)_Y] \times [SO(10)]' \quad (12.a)$$

$$\lambda = 0 \quad (\text{Vacuum 2}) \quad [SU(3) \times SU(2) \times U(1)_Y \times U(1)_{\tilde{Z}}] \times [SO(10)]' \quad (12.b)$$

$$\lambda = 1 \quad (\text{Vacuum 3}) \quad [SU(3) \times SU(2) \times U(1)_Y \times U(1)_{\tilde{Z}'}] \times [SO(10)]' \quad (12.c)$$

where Y is defined in (4) and

$$\begin{aligned} \tilde{Z} &= 9\sqrt{3} U_1 - 5\sqrt{2} U_2 + 7(U_3 - U_4) - 17(U_5 - U_6 + U_8) \\ \tilde{Z}' &= 9\sqrt{3} U_1 - 5\sqrt{2} U_2 + 7(U_3 - U_4) + 17(U_5 - 2U_6 - 2U_7) \end{aligned} \quad (13)$$

Notice that there is a continuous set of solutions with $SU(3) \times SU(2) \times U(1)_Y$ as an unbroken gauge group. Before going on, it is worth stressing that there are some ambiguities related to the assignment of the physical states and the form of the $U(1)$'s combination for the physical hypercharge. There are, in fact, many possible choices. The advantage of the definition (4) for Y is that there are many singlets with $Y=0$. This gives a higher freedom at the time to look for vacuum solutions preserving the physical hypercharge. Even maintaining this definition for Y , there are several possible assignments for the physical states. Three of them are given in Table 2. We will consider these possibilities through the next sections (the three assignments are interesting since they will illustrate different features).

3. YUKAWA COUPLINGS BEFORE THE FAYET–ILIPOULOS BREAKING

We study here the allowed Yukawa couplings previous to taking into account the impact of the $F-I$ breaking. These come restricted by several considerations[16,17,15]. From the point group selection rule, cubic couplings between twisted (T) and untwisted (U) states are forbidden. The consideration of the whole space group selection rule imposes further restrictions on couplings involving twisted fields. Furthermore, there are constraints for couplings involving untwisted matter coming from the six-dimensional space transformation properties.

Let us start by considering the Yukawa couplings responsible for the quark and lepton masses: QH_1d^c , QH_2u^c , LH_1e^c . We first analyze Assignment 1 (see Table 2). Since Q belongs to the U-sector and d^c to the T-sector, the coupling

$$Q H_1 d^c \quad (14)$$

is forbidden by the point group selection rule (or, alternatively, by gauge invariance). On the contrary it can be checked that the other two Yukawa couplings

$$Q H_2 u^c, \quad L H_1 e^c \quad (15)$$

are allowed by all the selection rules. The leptonic coupling of eq.(15) can be used to illustrate how a family mass hierarchy could be generated in a natural way[18]: L , H_1 and e^c come in three copies corresponding to the three equivalent fixed points with shift $v^I - a_1^I - a_3^I$. If for some reason there is only one physical Higgs H_1 , associated with one fixed point (say 1), the relevant couplings for lepton masses are

$$(L)_1 (H_1)_1 (e^c)_1, (L)_2 (H_1)_1 (e^c)_3, (L)_3 (H_1)_1 (e^c)_2 \quad (16)$$

The other six couplings $L(H_1)_1 e^c$ are forbidden by the space group selection rule. The last two couplings of (16) get a suppression factor $\exp(-ad^2)$ [16,17], where d is the distance between two of the three fixed points and a is a positive constant. In this way we obtain two "light" and one "heavy"

leptonic families[6]. For the other two assignments of Table 2, it is easy to check that the only coupling allowed by all the selection rules is QH_2u^c . It would not be correct to infer from the preceding results that Assignments 1, 2 and 3 do not work, the reason being that the $F-I$ breaking has the potential to generate some of the previously forbidden couplings, as will be seen below.

Let us analyze now baryon and lepton number violating operators. The possible couplings of this kind involving just the standard particles are:

$$QLd^c, \quad u^cd^cd^c, \quad LLe^c \quad (17)$$

It is easy to check that all of them are forbidden (for the assignments considered here) by $U(1)$ invariance. Although this result is beautiful it is important to keep in mind that it may be changed by the subsequent $F-I$ breaking. However, if one of the couplings of eq.(17) were present at this stage, the $F-I$ breaking could not rescue. This justifies in general the study of Yukawa couplings before the $F-I$ breaking: some possibilities can be already ruled out. There are further dangerous operators involving extra colour triplets D 's and $\bar{D}$'s:

$$\begin{aligned} &QQD, \quad QL\bar{D}, \quad u^cd^c\bar{D}, \quad e^cu^cD \\ &u^cu^c\bar{D}, \quad d^cd^c\bar{D}, \quad e^cd^cD \\ &u^c\bar{D}\bar{D}, \quad d^c\bar{D}\bar{D}, \quad e^c\bar{D}D \end{aligned} \quad (18)$$

It is amusing to verify that all of them are forbidden again by $U(1)$ conservation except $f_7f_5f_4$ which corresponds to $e^c\bar{D}D$ and is harmless. However, if some of the dangerous couplings of eq.(18) were present this would not be enough to rule out the corresponding assignment. The reason is that the D 's or $\bar{D}$'s involved in such couplings can become massive (and hence harmless) through the $F-I$ breaking process. This possibility will be exemplified in the next section.

4. YUKAWA COUPLINGS AFTER THE FAYET – ILIOPOULOS BREAKING

Let us study now how the previous results are modified when one takes into account the F – I breaking. The first step is to derive the massless spectrum for the specific vacuum under consideration. If the set of physical particles, according to one of the possible assignments, does not entirely survive the process (i.e. some of the physical particles acquire masses) the corresponding assignment cannot be used for that vacuum. The effective mass terms come from operators of the type:

$$(\chi \dots \chi)\xi \quad , \quad (\chi \dots \chi)\xi\xi \quad (19)$$

where χ means any field acquiring a vacuum expectation value (vev) and ξ any field. (Of course higher order operators must be taken into account here.) The calculation of the massless spectrum for the vacua of eq.(12) was performed in refs.[7,8]. The results are shown in Table 3. It is worth noticing that these spectra are quite close to the standard model one. In particular there are no dangerous colour triplets. Notice also that some of the massless fields are combinations of states coming from different twisted sectors, so it is no longer possible to say, in general, which sector a particle belongs to. With Tables 2 and 3 one can check that Assignment 2 survives for the three vacua under consideration, Assignment 1 survives for vacua 1 and 2 only, and Assignment 3 does not survive in any case. As stated above, this does not mean that Assignment 3 is useless: it could survive for other vacuum states. However, it would be a phenomenal task to study all the possibilities (i.e. all the possible combinations for the hypercharge, all the possible assignments of the physical particles and all the possible vacua). Therefore, it is highly desirable to have available criteria able to discard models before going into a more detailed analysis. One example of this was given in the previous section (the consideration of baryon and lepton number violating operators before the F – I breaking). Another example will be given below. It is important to note that for Assignments 1 and 2 some physical particles appear (after the breaking) combined with other states, so the assignment has been "changed" by the breaking process. (This new assignment would not have any sense before considering the breaking since particles with different quantum numbers do not mix.)

Let us study whether new Yukawa couplings have been generated for Assignments 1 and 2. There are two fundamental sources of new Yukawa couplings. First, since some of the “old” physical particles have combined with other ones (e.g. $d^c \rightarrow ad^c + bf_{29} + cf_{55}$ for Vacuum 1; see Tables 2 and 3), the “new” particles may have more couplings allowed. Second, an effective coupling of massless fields, say $\xi_1 \xi_2 \xi_3$, can be generated by a higher order operator

$$(\chi \dots \chi) \xi_1 \xi_2 \xi_3 \quad (20)$$

where, as in (19), χ means any field acquiring a vev. (Of course one has to consider here the “new” massless fields again.) It is, however, quite remarkable that no new Yukawa couplings are generated at all for these particular vacua (this is even true for the couplings of eq.(18), although the D and $\bar{D}$ fields have already acquired a high mass). Recall that the situation was completely different for the effective mass terms. As a simple example of how the systematic analysis has been realized let us consider the coupling LLe^c for Assignment 2. In this assignment this coupling corresponds to $f_{71}f_{71}f_{74}$ (see Table 2). Since the f_{71} and f_{74} fields are not mixed after the breaking for the three vacua under consideration (see Table 3), we have to analyze possible couplings of the form:

$$(\chi \dots \chi) f_{71} f_{71} f_{74} \quad (21)$$

On the other hand, the Q_6 charge of $f_{71}f_{71}f_{74}$ is -12 (see Table 1), but all the χ fields at hand (i.e. $l_1, \dots, l_9$) have $Q_6 \leq 0$, so the operators (21) are always forbidden by Q_6 conservation. Remarkably enough, one of the few operators of type (20) allowed on grounds of gauge invariance is

$$l_1 f_1 f_{73} f_{29} \quad (22)$$

This operator would generate the previously forbidden coupling $QH_1 d^c$, precisely the only one that is missing for Assignment 1 (see Tables 2 and 3) in order to have a realistic model. Unfortunately, this operator is also forbidden because the Q field belongs to the U -sector and hence it carries a holomorphic i index ($i = 1, 2, 3$). Each two-dimensional sublattice of the 6-torus has a Z_3 symmetry:

$x_i \rightarrow e^{2\pi i/3} x_i$ which is left unbroken by the construction of the orbifold[16,17]. The coupling (22) is not invariant under these Z_3 symmetries. In fact, all the relevant gauge invariant couplings of type (20) are forbidden in this way.

This last argument allows us to discard the three vacua analyzed here for any assignment of the physical states. In order to generate the coupling $QH_1 d^c$ it is necessary to give vev's to $N_L = 1/3$ (one moded oscillator acting) fields, which could compensate the non-trivial transformation of Q under the previously mentioned symmetries. This is a good criterion to rule out possible vacua before analyzing them. It must be noticed that untwisted fields with non-vanishing vev's can be useful as well, but in the case at hand there are no singlets in the untwisted sector. Finally, the orbifold moduli carry also holomorphic indices, but they cannot, by themselves, solve this particular problem¹.

5. CONCLUSIONS

We have analyzed the Yukawa couplings of the first $SU(3) \times SU(2) \times U(1)_Y (\times U(1))$ orbifold models recently obtained[7,8]. It has been found that pretty remarkable features occur. For example, all the baryon and lepton number violating operators are forbidden in a completely natural way. However, for the specific cases considered here the coupling responsible for the d quark mass, $QH_1 d^c$, is forbidden by stringy selection rules. Nevertheless it is clear that this is not a general fact and it has been pointed out that, in order to generate this coupling, some twisted fields with one moded oscillator acting must acquire non-vanishing vev's, which is a good criterion to look for phenomenologically promising vacua (in the present example there are 9×3 fields of this kind with vanishing hypercharge, see Table 1). In fact, there is an enormous degeneration of possible vacua, even for a specific choice of Wilson-lines (in this case, for example, there is a large number of singlets with vanishing hypercharge, that can be used to explore other $SU(3) \times SU(2) \times U(1)_Y$ vacua). Moreover the mechanism used in these examples for lowering the rank[6,15] is applicable for a large class of models[3], to be precise those with $SU(3) \times SU(2) \times U(1)^n$ as gauge group after the breaking due to the presence of Wilson-lines. This makes it extremely important to look for other phenomenological criteria which could discard most (not all!) of the models before going into a more detailed analysis.

¹ We thank F. Quevedo for this observation.

P.S. After this work was completed, we received a preprint by A. Font, L.E. Ibáñez, H.P. Nilles and F. Quevedo[19] where a different set of $SU(3) \times SU(2) \times U(1)_Y$ vacua is analyzed. The authors also discuss general features of Yukawa couplings, arriving at similar conclusions to those presented in this paper.

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TABLE CAPTIONS

Table 1

U(1) charges of the model (U or T(T^{*}) in column M indicates if the matter is untwisted or twisted (twisted with one moded oscillator acting)) multiplied by $6N_i$, where $N_1=\sqrt{3}$, $N_2=\sqrt{2}$, $N_3=8=1$. n_1 and n_3 denote the different twisted sectors. Representations under the non-Abelian unbroken group $G \times G'$ are also given (1 means a singlet state). The fields are labelled in column f.

Table 2

Three possible assignments of the physical particles with the hypercharge defined in the text.

Table 3

Massless spectrum for the three vacua considered in the text. The notation $(a,...,b)_{1,2}$ refers to two orthogonal combinations of the fields $a,...,b$.

f	M	n_1	n_3	G or G'	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Y
1	U	-	-	(3,2)	-6	-6	0	0	0	0	0	0	1
2	U	-	-	(1,2)	0	6	0	0	0	0	0	0	-1
3	U	-	-	(3,1)	6	0	0	-6	0	0	0	0	-1
4	U	-	-	16'	0	0	0	0	0	3	-3	3	0
5	T	0	0	(3,1)	0	4	0	0	0	4	0	0	0
6	T	0	0	(1,2)	6	-2	-3	-3	4	0	0	0	0
7	T	0	0	1	-3	-2	-3	-3	4	0	0	0	-2/3
8	T	0	0	1	-3	-2	-3	3	4	0	0	0	2
9	T	0	0	1	-3	-2	3	3	4	0	0	0	2
10	T	0	0	1	-3	-2	3	3	-3	4	0	0	2
11	T	1	0	1	6	4	0	0	-2	-2	-4	2	1/2
12	T	1	0	1	6	4	0	0	-2	-2	-4	2	-1/2
13	T	1	0	1	6	4	0	0	-2	-2	-4	2	1
14	T	1	0	1	-3	-2	-3	-3	1	-2	-4	2	0
15	T	1	0	1	-3	-2	-3	1	-2	-4	2	0	0
16	T	1	0	1	-3	-2	-3	1	-2	-4	2	0	-1
17	T	1	0	1	-3	-2	-3	1	-2	-4	2	0	0
18	T	1	0	1	-3	-2	3	3	1	-2	-4	0	1/3
19	T	1	0	1	-3	-2	3	3	1	-2	-4	0	-1/2
20	T	-1	0	1	6	4	0	0	2	-2	-4	0	-1
21	T	-1	0	1	6	4	0	0	2	-2	-4	0	0
22	T	-1	0	1	6	4	0	0	2	-2	-4	0	1
23	T	-1	0	1	-3	-2	-3	3	-1	-2	-4	1	0
24	T	-1	0	1	-3	-2	-3	3	-1	-2	-4	1	-2/3
25	T	-1	0	1	-3	-2	-3	3	-1	-2	-4	1	2/3
26	T	-1	0	1	-3	-2	-3	3	-1	-2	-4	1	-1/2
27	T	-1	0	1	-3	-2	3	3	-1	-2	-4	1	1/2
28	T	-1	0	1	-3	-2	3	3	-1	-2	-4	1	0
29	T	0	1	(3,1)	0	0	0	2	2	0	-4	0	1
30	T	0	1	(1,2)	3	0	-3	-1	-1	0	-4	0	-1
31	T	0	1	1	-6	0	0	-4	2	0	-4	0	0
32	T	0	1	1	-6	0	0	-4	2	0	-4	0	0
33	T	0	1	1	3	-6	3	-1	-1	0	-4	0	0
34	T	0	1	1	3	6	3	-1	-1	0	-4	0	-1/2
35	T	0	-1	(3,1)	-3	2	-3	1	1	2	-2	0	-2
36	T	0	-1	(3,1)	3	2	3	1	1	2	-2	0	-4
37	T	0	-1	(1,2)	0	2	0	4	-2	2	-2	0	-1/2
38	T	0	-1	(1,2)	0	2	0	-2	4	2	-2	0	1

TABLE 1

state	As.1	As.2	As.3
Q	f ₁	f ₁	f ₁
u ^c	f ₃	f ₃	f ₃
d ^c	f ₃₅	f ₃₅	f ₃₅
L	f ₇₂	f ₇₁	f ₇₂
e ^c	f ₇₄	f ₇₄	f ₂₃
H ₁	f ₇₃	f ₇₂	f ₇₃
H ₂	f ₂	f ₂	f ₂

TABLE 2

state	G or G'	Y
VACUUM 1		
1	(3,2)	1/6
3	($\bar{3}$,1)	-2/3
(29,35,55)	($\bar{3}$,1)	1/3
2, 6	(1,2)	1/2
71, (38,56,73), (63,48,72)	(1,2)	-1/2
24, 74	1	1
(8,25,39,42,49,59,66,75,76)	1	1
27, (9,28)	1	-1
22, 44, 53, 70	1	0
(7,10,13,16,19,33,50,69)	1	0
(1 ₁ ,1 ₂ ,1 ₃ ,1 ₄ ,1 ₅ ,1 ₆ ,1 ₇ ,1 ₈ ,1 ₉) _{1,2}	1	0
4	(16)'	0
VACUUM 2		
1	(3,2)	1/6
3	($\bar{3}$,1)	-2/3
(29,35,55)	($\bar{3}$,1)	1/3
2, 6	(1,2)	1/2
71, (38,56,73), (63,48,72)	(1,2)	-1/2
24, 74, (8,39,59,75) _{1,2}	1	1
9, 27, (26,31,41,52,57,67)	1	-1
12, 22, 44, 53, 58, 70, (16,19,50)	1	0
(1 ₂ ,1 ₃ ,1 ₄ ,1 ₅ ,1 ₆ ,1 ₇ ,1 ₉)	1	0
4	(16)'	0
VACUUM 3		
1	(3,2)	1/6
3	($\bar{3}$,1)	-2/3
(35,55)	($\bar{3}$,1)	1/3
2, 6	(1,2)	1/2
71, (38,56), (48,72)	(1,2)	-1/2
27, (9,28)	1	-1
24, 74, (39,42,49,59,76)	1	1
22, 44, 51, 53, 60, 69, 70	1	0
(33,50), (7,10,13,16,19)	1	0
(1 ₁ ,1 ₂ ,1 ₃ ,1 ₄ ,1 ₅ ,1 ₆ ,1 ₈)	1	0
4	(16)'	0

TABLE 3

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