CERN-TH.5051/88
UGVA-DPT 1988/05-575

ON THE RENORMALIZATION OF TWO-DIMENSIONAL CHIRAL MODELS

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ABSTRACT

A general method for the analysis of renormalizability and conformal invariance of two-dimensional chiral theories is applied to the Wess-Zumino-Witten model. The method, which is algebraic in character, does not rely on the existence of an invariant regularization. It is only based on very general properties of perturbative field theory, such as locality and power-counting.

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⁺⁾ Supported in part by the Swiss National Science Foundation.

1. The aim of this paper is to present a perturbative approach to the two-dimensional conformal field theories based on a general method, which is in particular independent of the regularization scheme. This method will be illustrated by the study of the chiral Wess-Zumino-Witten (WZW) model [1].

The recent remarkable development in the study of the two-dimensional conformal field theories is essentially based on a purely algebraic approach [2]. The connection of the algebraic results with local quantum field theory [3], and in particular the construction of states and operators in terms of the local field operators and of the vacuum of the theory is, however, often obscure. It is in this sense that the perturbative analysis turns out to be useful, since perturbation theory leads directly to the construction of a system of Ward identities reproducing at the local level the algebra defining the model. In particular it is easy to characterize in perturbation theory the conserved currents and energy-momentum tensor from which the generators of the Kac-Moody [4] and of the Virasoro [5] algebra are constructed.

The WZW model is a non-linear σ -model on a group manifold constrained by the requirement of a conserved system of chiral currents associated with the isometries of the group manifold. It is the chirality of the conserved currents which makes it possible to obtain the energy-momentum tensor by means of the Sugawara [6] construction, thus proving the conformal invariance of the theory. Notice that in perturbation theory the energy-momentum tensor is built directly in terms of the basic fields.

This indicates the importance of chirality in the construction of the two-dimensional conformal field theories. The situation is strictly analogous in the case of chiral extended supersymmetry [7]. It is, however, chirality which makes it difficult to find an invariant regularization for the WZW model. Indeed, the existence of an isometry group acting non-linearly on the fields would suggest the unique choice of dimensional regularization, but the presence of a two-dimensional antisymmetric tensor renders non-automatic the renormalizability of chirality. This motivates a regularization-independent approach to renormalization.

In this paper we present an extension of the method applied in [8] to discuss the renormalizability and the infrared properties of the non-linear σ -models on coset spaces. This method consists of changing the infinitesimal, non-linear isometries of the model into BRS transformations. The corresponding Slavnov identities are then used as defining conditions.

In our case the nature of the model depends on the chirality of the isometry current which is not uniquely identified by the rigid symmetry. We shall therefore

consider the BRS transformations associated with the local isometries. The corresponding Slavnov identity turns out to be anomalous already at the classical level, as is expected since the action contains a Wess-Zumino term [9].

The coefficient of the anomaly, which is unconstrained in the tree approximation, turns out to be the unique free parameter of the theory, up to field redefinitions. Requiring that the Slavnov identity maintains its anomalous classical form at the quantum level, we show that the WZW model is renormalizable. Its conformal invariance is verified showing the renormalizability up to the anomalous term determining the centre of the quantized Virasoro algebra, of a further extension of the Slavnov identity which also embodies the action of the conformal transformations on the fields and the relevant operators. The introduction of an infrared regulator mass term according to the prescriptions described in [8] is understood. The existence of a finite infrared limit of the physical correlation functions is then automatically guaranteed.

The paper is organized as follows. Section 2 contains a description of the classical model together with the tree approximation Ward identities and related algebras. Section 3 contains the analysis of renormalizability. The conformal invariance is discussed in Section 4.

2. The chiral WZW model [1] is a non-linear σ -model on a compact group manifold G . This coincides with the coset space $G \times G/G$. The isometries of the manifold can be constructed in terms of the adjoint action of G on itself and of its left multiplication. Considering the perturbative field as an element of the Lie algebra of G , we can identify:

$$g = e^{i\phi}, \quad \phi(x) = \tau_i \phi^i(x), \quad \phi^i = \langle \tau_i \phi \rangle$$

$$\text{with: } [\tau_i, \tau_j] = if_{ijk} \tau_k, \quad \langle \tau_i \tau_j \rangle = \delta_{ij} \quad (1)$$

It follows that the adjoint action of G ($\text{Ad}(G)$) is linear on ϕ , while the left multiplication acts non-linearly:

$$e^{i\phi} \rightarrow e^{i\phi_g} = g e^{i\phi}, \quad g \in G \quad (2)$$

Considering the results of [8] we shall take for granted that the theory remains covariant under $\text{Ad}(G)$ at the quantum level. We shall therefore limit our analysis to the left action of G .

The parity-conserving part of the classical action is:

$$\Gamma_P = -\frac{k}{8\pi} \int d^2x \langle g^{-1} \partial_\mu g g^{-1} \partial_\nu g \rangle \eta^{\mu\nu} \quad (3)$$

Its uniqueness, up to field redefinitions, in much the same way as that of the rest of the classical action, will be one of the results of the following analysis.

Witten's strategy to ensure the chirality of the isometry current is to introduce the Wess-Zumino term [9]:

$$\Gamma_{WZ} = \frac{k}{4\pi} \int d^2x \int_0^1 dt \epsilon^{\mu\nu} \langle \bar{g}^{-1} \partial_\mu \bar{g} \bar{g}^{-1} \partial_\nu \bar{g} \rangle \quad (4)$$

where $\bar{g}(t, x)$ interpolates between the identity at $t = 0$ and $g(x)$ at $t = 1$. Notice that the parity-conserving and the Wess-Zumino terms have been chosen with the same coefficient k . This will turn out to be the inverse of the perturbative coupling constant. With this choice the theory becomes chiral. Indeed under an arbitrary change of the quantum field the total action varies as:

$$\delta(\Gamma_P + \Gamma_{WZ}) = \frac{k}{2\pi} \int d^2z \langle \delta g g^{-1} \bar{\partial}(g g^{-1}) \rangle \quad (5)$$

where we have introduced the light-cone co-ordinates:

$$\begin{aligned} z &= \frac{1}{\sqrt{2}}(x^0 + x^1) & , & & \partial &= \partial_z \\ \bar{z} &= \frac{1}{\sqrt{2}}(x^0 - x^1) & , & & \bar{\partial} &= \partial_{\bar{z}} \end{aligned} \quad (6)$$

One can read directly from (5) the existence of the conserved, chiral current:

$$J = i \frac{k}{2\pi} \partial g g^{-1} \quad , \quad \bar{\partial} J = 0 \quad (7)$$

To identify this current at the quantum level, we introduce its local source α , thus enlarging the action to:

$$\Gamma_{(\alpha)} \equiv \Gamma_P + \Gamma_{WZ} + \int d^2z \langle \alpha J \rangle \quad (8)$$

In the standard situation this action would be identified by its invariance under the chiral "gauge" transformations:

$$\begin{aligned} \delta_\lambda g &= i\lambda g \\ \delta_\lambda \alpha &= \bar{\partial}\lambda + i[\lambda, \alpha] \end{aligned} \quad (9)$$

with λ an infinitesimal, local parameter. But this is not true in our case due to the presence of the Wess-Zumino term (4). Indeed, one has:

$$\delta_\lambda \Gamma_{(\alpha)} = -\frac{k}{2\pi} \int d^2 z \langle \alpha \partial \lambda \rangle \quad (10)$$

which means that the invariance under the "gauge" transformations (9) is broken already at the classical level by the anomalous term appearing in (10). Nevertheless, since the right-hand side of (10) is independent of the quantum fields, it is meaningful to try to preserve this local, anomalous invariance at the quantum level and to use it as a characterization of the model. In particular this will fix the value of the inverse coupling constant k , thus determining completely, up to field redefinitions, the classical action.

In analogy with any two-dimensional, non-linear σ -model, the action (8) is left invariant by the conformal transformations:

$$\begin{aligned} \delta_\epsilon g &= \epsilon(z) \partial g \\ \delta_\epsilon \alpha &= \epsilon(z) \partial \alpha \end{aligned} \quad (11)$$

where the infinitesimal parameter ϵ does not depend on $\bar{z}$. We have already recalled the remarkable fact that in the present case, owing to the chirality of the conserved current (7), the the conformal invariance is expected to persist at the quantum level.

It is our purpose to translate the conformal invariance into a local Ward identity which, in turn, will be connected with the Virasoro algebra of the theory. Therefore we introduce the conserved energy-momentum tensor. This, being symmetric and traceless, has only two independent components which, in the light-cone co-ordinates, are identified with $T = T_{zz}$ and $\bar{T} = T_{\bar{z}\bar{z}}$. In our model T is:

$$T = -\frac{k}{8\pi} \langle \partial g g^{-1} \partial g g^{-1} \rangle \quad , \quad \bar{\partial} T = 0 \quad (12)$$

It generates the transformation (11). We shall not consider explicitly the other component $\bar{T}$ generating the transformations involving the variable $\bar{z}$. In much the same way as the conserved current $\bar{J}$ of chirality opposite to that of (7), $\bar{T}$ is not essential to characterize the model.

Introducing the source μ for T through:

$$\Gamma_{(\alpha\mu)} = \Gamma_{(\alpha)} + \int d^2 z \mu T \quad (13)$$

we obtain a new action invariant under the local transformations:

$$\delta_\epsilon \mu = \epsilon \partial \mu - \partial \epsilon \mu - 2 \bar{\partial} \epsilon \quad (14)$$

and (11). Now ϵ is a function of both variables z and $\bar{z}$.

Notice that in order to preserve the anomalous invariance (10) under the transformations (9) after the introduction of the μ -dependent term in the action (13), one has to modify the infinitesimal variation of α according to

$$\delta_\lambda \alpha = \bar{\partial} \lambda + i[\lambda, \alpha] + \frac{1}{2} \mu \partial \lambda \quad (15)$$

Now (10) can be translated in terms of the tree approximation Green functional $Z(\alpha, \mu)$, which generates the correlation functions of the operators J and T , giving the Ward identity:

$$\left(-\bar{\partial} \frac{\delta}{i\delta\alpha} + i[\alpha, \frac{\delta}{i\delta\alpha}] - \frac{1}{2} \partial \left(\mu \frac{\delta}{i\delta\alpha} \right) \right) Z = \frac{k}{2\pi} \partial \alpha Z \quad (16)$$

In much the same way the invariance of the action under the transformations (11) and (14) leads to:

$$\left(2\bar{\partial} \frac{\delta}{i\delta\mu} + \partial \mu \frac{\delta}{i\delta\mu} + \partial \left(\mu \frac{\delta}{i\delta\mu} \right) + \partial \alpha \frac{\delta}{i\delta\alpha} \right) Z = 0 \quad (17)$$

(16) and (17) imply the conservation of the operators J and T respectively (that is, they depend only on the variable z), and the commutation relations [10]:

$$\begin{aligned} [J^i(z), J^j(z')] &= i f^{ijk} \delta(z - z') J^k(z) + \frac{ik}{2\pi} \delta^{ij} \delta'(z - z') \\ [T(z), J^i(z')] &= \frac{i}{2} \delta'(z - z') J^i(z) \\ [T(z), T(z')] &= \frac{i}{2} \delta'(z - z') (T(z) + T(z')) \end{aligned} \quad (18)$$

The Kac-Moody and Virasoro algebras of the model:

$$\begin{aligned} [J_m^i, J_n^j] &= i f^{ijk} J_{m+n}^k + km \delta^{ij} \delta_{m+n,0} \\ [L_m, L_n] &= (m - n) L_{m+n} \quad , \quad [L_m, J_n^i] = -n J_{m+n}^i \end{aligned} \quad (19)$$

are deduced from (18) with the definitions^{*}):

$$\begin{aligned} J_n^i &= \int_{-\infty}^{+\infty} dz h_n(z) J^i(z) \quad , \quad h_n(z) = \left(\frac{1+iz}{1-iz} \right)^n \\ L_n &= \int_{-\infty}^{+\infty} dz f_n(z) T(z) \quad , \quad f_n(z) = (1+z^2) h_n(z) \end{aligned} \quad (20)$$

Notice that in (19) the centre of the Virasoro algebra vanishes, while k , that of the Kac-Moody algebra, is a free, unquantized parameter. This is due to the lack of unitarity of the tree approximation. We are going to see that the radiative corrections induce a non-vanishing centre for the Virasoro algebra. Concerning k , it will remain unquantized since the exact unitarity is never reached in perturbation theory. Furthermore, one should remember that in the perturbative limit k goes to infinity.

3. We now discuss the possibility of maintaining at the quantum level the conservation of the chiral current J , given in the classical limit by (7). This conservation is expressed by the anomalous invariance of the theory under the quantum extension of the "gauge" transformations (9) (in this section we shall not discuss the conformal invariance and hence we shall take $\mu = 0$).

The transformations (9) being non-linear, we translate them into the system of BRS transformations:

$$\begin{aligned} s e^{i\phi} &= i c e^{i\phi} \quad , \quad s \alpha = \bar{\partial} c + i[c, \alpha] \quad , \quad s c = i c^2 \\ s^2 &= 0 \end{aligned} \quad (21)$$

where c is an anticommuting, local, unquantized field with value in the Lie algebra of G . We also introduce the source γ_1 for the BRS variation of ϕ^1 . The anomalous invariance condition (10) becomes an anomalous Slavnov identity:

$$\begin{aligned} S(\Gamma) &\equiv \int d^2 z \left(\frac{\delta \Gamma}{\delta \gamma} \frac{\delta \Gamma}{\delta \phi} + (\bar{\partial} c + i[c, \alpha]) \frac{\delta \Gamma}{\delta \alpha} + i c^2 \frac{\delta \Gamma}{\delta c} \right) \\ &= -\frac{k}{2\pi} \int d^2 z \langle \alpha \partial c \rangle \end{aligned} \quad (22)$$

^{*}) Notice that the test functions for the local operators J and T appearing in (20) are admissible under the very weak assumption that the generators of the restricted conformal group $SL(2, R)$ and of the rigid isometries are defined. This is in fact true in perturbation theory.

which we want to preserve at the quantum level in terms of the vertex functional Γ .

Both the uniqueness of the classical action and the possible quantum breakings to the Slavnov identity are controlled by the nilpotent, linearized Slavnov operator:

$$D = \int d^2z \left(\frac{\delta \Gamma_{cl}}{\delta \gamma} \frac{\delta}{\delta \phi} + \frac{\delta \Gamma_{cl}}{\delta \phi} \frac{\delta}{\delta \gamma} + (\bar{\partial}c + i[c, \alpha]) \frac{\delta}{\delta \alpha} + ic^2 \frac{\delta}{\delta c} \right) \quad (23)$$

Indeed we are going to show that:

- a) the general solution ($\Gamma_{cl} + \Gamma'$) of the Slavnov identity in a neighbourhood of Γ_{cl} , which satisfies automatically: $D\Gamma' = 0$, turns out to be $\Gamma' = D\Gamma$; it is therefore equivalent to Γ_{cl} through a field redefinition;
- b) a quantum breaking Δ to the Slavnov identity, which automatically satisfies $D\Delta = 0$, turns out to have the general form $\Delta = D\Lambda$, which means that it can be reabsorbed by the introduction of finite counterterms.

Here above, Γ' , Φ , Δ and Λ are local functionals of dimension two. Assigning ghost number 1 and -1 to c and γ respectively, Γ' and Δ have ghost number 0, while Λ and Γ have respectively ghost number +1 and -1. The operator D increases the ghost number by 1.

We have thus to verify that the cohomology of D restricted to the integrated, local functionals of dimension 2 and ghost number 0 and 1 is trivial:

$$D\omega = 0 \quad \Rightarrow \quad \omega = D\hat{\omega} \quad (24)$$

Remember that ω and $\hat{\omega}$ have to be invariant under $\text{Ad}(G)$.

As shown in [8], to prove (24) it is sufficient to show that the cohomology of D_0 , the linear approximation to D , is trivial. D_0 acts on the fields as:

$$\begin{aligned} D_0\phi &= c, \quad D_0c = 0, \quad D_0\alpha = \bar{\partial}c \\ D_0\gamma &= \frac{k}{2\pi} \partial(\alpha - \bar{\partial}\phi) \equiv \partial\beta \\ D_0^2 &= 0 \end{aligned} \quad (25)$$

where we have introduced $\beta = k/2\pi \partial(\alpha - \bar{\partial}\phi)$. Thus we have to solve

$$D_0\omega = 0 \quad (26)$$

Writing a generic integrated, Lorentz invariant, local functional of dimension 2 as:

$$\begin{aligned} \omega = \int d^2 z \{ & \gamma_i E^i + \bar{\partial} \phi^i [\partial \phi^j A_{ij} + \partial c^j (B + C)_{ij}] \\ & + \bar{\partial} c^i [\partial \phi^j (-B + C)_{ij} - 2 \partial c^j D_{ij}] \\ & + \beta^i [\partial \phi^j G_{ij} + \partial c^j H_{ij}] \} \end{aligned} \quad (27)$$

The coefficients A_{ij} , B_{ij} , C_{ij} , D_{ij} , E^i , G_{ij} and H_{ij} are dimensionless, formal power series in the variables ϕ and c . We introduce the differential operator:

$$d = \int d^2 z c^i \frac{\delta}{\delta \phi^i} \quad ; \quad d^2 = 0 \quad (28)$$

Considering the γ -dependent term in (27), we notice that (26) implies that $dE^i = 0$ and thence $E^i = dE^i$. Indeed the integration constant vanishes due to the invariance under $\text{Ad}(G)$. This implies the triviality of the first term in ω .

Let us now consider the terms linear in β ; (26) gives $G_{ij} = dH_{ij}$ showing that also the last two terms in ω are trivial.

For the remaining coefficients A_{ij} , B_{ij} , C_{ij} and D_{ij} (26) writes:

$$\begin{aligned} (a) \quad dA_{ij} &= 2A_{[ij]}^0 \\ (b) \quad A_{ij} - dC_{ij} &= A_{[ij]}^0 + D_{[ij]}^0 \\ (c) \quad dB_{ij} &= D_{(ij)}^0 - A_{(ij)}^0 \\ (d) \quad B_{ij} - dD_{ij} &= D_{(ij)}^0 \end{aligned} \quad (29)$$

where we have put $F_{,i} = \partial_{\phi} F$ and $F_{,\tilde{i}} = \partial_{c^i} F$, and (ij) or $[ij]$ mean symmetrization or antisymmetrization with respect to the indices i and j . The terms appearing on the right-hand side of (29) involving the functions A_i^0 and D_i^0 represent contributions which vanish after space-time integration.

The cohomological triviality of ω ($\omega = D_0 \omega$) is written in the analogous fashion:

$$\begin{aligned}
 (a) \quad A_{ij} &= d\hat{A}_{ij} + 2\hat{A}_{[i,j]}^0 \\
 (b) \quad C_{ij} &= \hat{A}_{ij} - d\hat{C}_{ij} + \hat{A}_{[i,j]}^0 \\
 (c) \quad B_{ij} &= -d\hat{B}_{ij} + \hat{A}_{(i,j)}^0 \\
 (d) \quad D_{ij} &= \hat{B}_{ij} - d\hat{D}_{ij}
 \end{aligned} \tag{30}$$

Combining (29c) and (29d), we get the constraint:

$$A_i^0 = dD_i^0 + X_{,i} \tag{31}$$

for some function X of ϕ and c . Then comparing (29a) and (29b) yields, after substitution of (31):

$$2X_{,[ij]} + d(X_{,ij}) \equiv (dX)_{,ij} = 0 \tag{32}$$

whose general solution is:

$$X = Y(\phi) + d\hat{X} \tag{33}$$

Now setting:

$$\hat{A}_i^0 = D_i^0 - \hat{X}_{,i} \tag{34}$$

and substituting (31), (33) and (34) into (29b) gives:

$$A_{ij} - 2\hat{A}_{[i,j]}^0 = d(C_{ij} - \hat{A}_{[i,j]}^0) \tag{35}$$

which is equivalent to (30a) and (30b).

In much the same way from (29d), we get (30c) and (30d). We have thus shown that the whole cohomology of D_0 and hence of D vanishes [that is (24)].

Therefore we have proved that the theory is renormalizable and that it has only one physical parameter, the centre of the Kac-Moody algebra.

4. Now it remains to verify that the renormalized WZW model is conformal invariant, the vertex functional being left invariant by the transformations (11) and (14). One should, however, remember that, introducing the source μ for the energy-momentum tensor, the chiral transformation (9) has to be modified according to (15).

The transformations (11) and (14) act linearly on the fields. However, since the conformal invariance is a consequence of the chiral one, it is convenient to embed it into the BRS transformations. This requires the introduction of an anticommuting local parameter p . We consider the extended BRS transformations:

$$\begin{aligned}
 S e^{i\phi} &= s e^{i\phi} + p \partial e^{i\phi} \\
 S \alpha &= s \alpha + \mu \partial c + p \partial \alpha \\
 S c &= s c + p \partial c \\
 S \mu &= p \partial \mu - \partial p \mu - 2 \bar{\partial} p \\
 S p &= p \partial p \\
 S^2 &= 0
 \end{aligned}
 \tag{36}$$

and we modify accordingly the Slavnov identity:

$$\begin{aligned}
 \mathbf{S} Z &\equiv \int d^2 z \left\{ \left\langle -J \frac{\delta}{\delta \gamma} + S \alpha \frac{\delta}{\delta \alpha} + S c \frac{\delta}{\delta c} \right\rangle + S \mu \frac{\delta}{\delta \mu} + S p \frac{\delta}{\delta p} \right\} Z \\
 &= -i \frac{k}{2\pi} \int d^2 z \langle \alpha \partial c \rangle Z
 \end{aligned}
 \tag{37}$$

The functional derivatives of (37) with respect to c and p in the limit $p = c = 0$ give the quantum extensions of the Ward identities (16) and (17). Now the analysis of the cohomology of the linearized Slavnov operator (23) performed in the previous section should be repeated taking into account the extension (36). One should, however, notice that what we have in fact to deal with is the linear approximation D_0 to D , which is still given by (25), completed with:

$$D_0 p = 0 \quad , \quad D_0 \mu = -2 \bar{\partial} p
 \tag{38}$$

The vanishing of the cohomology of the extended D_0 would imply the full quantum validity of the Ward identity (17) and hence (19). This would imply quantum conformal invariance and Virasoro algebra with vanishing centre.

In order to analyze the extended version of (26) we have to enlarge the space of integrated local functionals ω introducing also terms depending on p and μ . Notice that, due to translation invariance, p appears only through its derivatives ∂p and $\bar{\partial} p$. Indeed translation invariance follows from the renormalizable identity:

$$T Z \equiv \int d^2 z \left\{ \frac{\delta}{\delta p} + \gamma_i \partial \frac{\delta}{\delta J_i} \right\} Z = 0
 \tag{39}$$

since the commutator of T and S in (37) gives the infinitesimal generator of translations.

Therefore the new generic, integrated local functional ω is obtained from (27) adding the contribution:

$$\int d^2 z \left\{ \partial p [\bar{\partial} \phi^i F_i(\psi, \partial^2 p) + \bar{\partial} c^i G_i(\psi, \partial^2 p) + \beta^i H_i(\psi, \partial^2 p) + \gamma_i I^i(\psi)] + \mu [K(\psi, \partial p, \partial^2 p) + \partial p \partial^3 p M(\psi) + \partial^3 p J(\psi)] - 2 \bar{\partial} p [L(\psi, \partial p, \partial^2 p) + \partial p \partial^3 p N(\psi)] \right\} \quad (40)$$

where ψ stands for ϕ and c and their derivatives with respect to z . The coefficients F_i , G_i and H_i have dimension 1, K and L have dimension 2, I^i , J , M and N are dimensionless (∂p has dimension 0). Notice that the terms proportional to $\bar{\partial} \phi \partial^2 p$, $\bar{\partial} c \partial^2 p$ and $\bar{\partial} p \partial^3 p$ have been eliminated exploiting completely the freedom of performing partial integrations. Hence, contrary to (27) the coefficients in (40) are related one-to-one with the corresponding integrated functionals. This remarkably simplifies the analysis of (26). Indeed one finds:

$$\begin{aligned} (a) \quad F_i &= dG_i & (b) \quad K &= dL & (c) \quad M &= dN \\ (d) \quad dH_i &= 0 & (e) \quad dI^i &= 0 & (f) \quad dJ &= 0 \end{aligned} \quad (41)$$

It is evident that (41a), (41b) and (41c) imply the triviality of the corresponding terms, (41d) and (41e) have the general solution $H_i = dH_i$ and $I^i = dI^i$ without integration constants due to the prescribed invariance under $\text{Ad}(G)$. The general solution of (41f) is:

$$J(\phi, c) = d\hat{J}(\phi, c) - \frac{C}{48\pi} \quad (42)$$

Here the integration constant C generates the anomalous term giving rise to a central extension of the Virasoro algebra (19) which now acquires the well-known form:

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{C}{12}m(m^2 - 1)\delta_{m+n,0} \quad (43)$$

5. We have given in this paper a complete and self-contained proof of the renormalizability and conformal invariance of the chiral WZW model. The proof is independent of the regularization procedure. It implies that in any regularization the possible singular terms appearing in the perturbative construction of the vertex functional, and hence of any Green function, can be reabsorbed into a field

redefinition. Our method can also be fruitfully applied to σ -models with extended supersymmetry [7].

The results given here have already been reached with other techniques [11] which, however, are much less explicit than ours: they depend in particular of the existence of an invariant regularization scheme and on quite delicate arguments based on tensor analysis.

ACKNOWLEDGEMENTS

The authors wish to thank A. Blasi, F. Delduc, S. Ferrara, R. Gatto and R. Stora for useful comments.

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