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DILEPTON CHARGE ASYMMETRY IN $B-\bar{B}$ DECAYS

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A B S T R A C T

The "partonic" calculation of the charge asymmetry of like-sign lepton pairs coming from $B_d-\bar{B}_d$ decays as a result of mixing is reconsidered. Several possible corrections do not change the prediction in an important way. It is unlikely, in the minimal standard model, that the absolute value of the asymmetry is larger than 4×10^{-3} .

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1. - INTRODUCTION

The discovery of $B-\bar{B}$ mixing by the ARGUS collaboration [1], following a previous indication by UA1 [2], has brought renewed attention to the prospects for finding CP violating effects in B meson decays. An effect of CP violation in the mass matrix, which has been discussed for a long time [3], would be the charge asymmetry in the like-sign dileptons coming from the decay of a $B^0-\bar{B}^0$ pair:

$$\alpha = \frac{N_{++} - N_{--}}{N_{++} + N_{--}} = \frac{\text{Im}(\Gamma_{12}/M_{12})}{1 + |\Gamma_{12}/M_{12}|^2/4} \simeq \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \quad (1)$$

In Eq. (1), $\Gamma_{12}(M_{12})$ is the width (mass) transition matrix element between B^0 and $\bar{B}^0$ states. My conventions are: $|\bar{B}^0\rangle = |b\bar{d}\rangle$, $\text{CP}|\bar{B}^0\rangle = +|B^0\rangle$.

The experimental results for $B-\bar{B}$ mixing [1]

$$x_d \equiv \frac{\Delta M}{\Gamma} \simeq 2 \frac{|M_{12}|}{\Gamma} = 0.73 \pm 0.18 \quad (2)$$

can be reproduced in the standard model provided that the top quark mass is large enough (> 50 GeV) [4]. In this case, M_{12} is dominated by the box diagram contributions with t-quarks running in the internal lines

$$M_{12} \simeq - \frac{G^2 f_B^2 B M m_t^2 F(\frac{m_t}{m_W})}{12 \pi^2} \eta_M \xi_t^2 \equiv - M_0 \xi_t^2 \quad (3)$$

In Eqs. (1) and (2), the approximation $|\Gamma_{12}| \ll |M_{12}|$ has been made. In Eq. (3), G is the Fermi constant, M and f_B are the B-meson mass and decay constant, B parametrizes the uncertainty in the evaluation of the matrix element ($B = 1$ in the vacuum insertion approximation), F is a slowly varying function [5] ($F(0) = 1$, $F(1) = 3/4$, $\lim_{x \rightarrow \infty} F(x) = 1/4$), $\eta_M \approx 0.87$ is a QCD correction factor [6] and I have defined

$$\xi_i = V_{ib} V_{id}^* \quad (4)$$

In the minimal (three generations) standard model, the unitarity of the Cabibbo-Kobayashi-Maskawa matrix V entails

$$\xi_u + \xi_c + \xi_t = 0 \quad (5)$$

In order to calculate the asymmetry α , one needs an estimate of Γ_{12} and in particular of the difference between the phases of M_{12} and Γ_{12} . This has been done several years ago [7-9] in a "partonic" approach, where the real intermediate states entering the definition

$$\Gamma_{12} = \sum_n \langle B | H_w | n \rangle \langle n | H_w | \bar{B} \rangle 2\pi \delta(E_n - E_B) \quad (6)$$

are described in terms of quarks in the spirit of quark-hadron duality. One simply takes the absorptive part of the box diagrams, and corrects the result obtained for the exchange of hard gluons in the leading logarithmic approximation. One obtains in this way an estimate for the asymmetry - in the B_d case - of about 10^{-3} , with a negative sign [10].

In an interesting recent paper [11], Altomari, Wolfenstein and Bjorken (AWBj) have cast serious doubts on the validity of the above-mentioned result, since the final number in it comes out from the cancellation (to about 90%) of much larger numbers. Since these numbers should themselves be affected by uncertainties certainly larger than ten per cent - their argument goes - the final result could be wrong even by a factor ten and the sign is unpredictable. The issue is of course relevant for future B-factories: an increase of a factor ten in the asymmetry would lower the luminosity required to observe it in a given period of time by a factor hundred.

In this paper, I will reconsider the "partonic" calculation in some detail, paying attention to several different possible corrections, not all considered in Refs. [8,11], in order to try to assess the stability of the result for the asymmetry. It turns out that although the "large numbers" are fairly uncertain (and so is therefore the modulus of Γ_{12} and the difference in the widths, $\Gamma_L - \Gamma_S$), those particular differences that enter in the calculation of the asymmetry are remarkably stable under variations of the parameters (Λ_{QCD} , m_b , ...). This, in my opinion, shows that the criticism of AWBj is maybe too drastic and that it is difficult to envisage an asymmetry at the level of 1%, in the minimal standard model.

2. - GENERAL FORMULAE FOR Γ_{12} AND a

Neglecting for a moment the exchange of gluons, the transition width Γ_{12} can be written, for the partonic approach, in the form [12]

$$\Gamma_{12} = \frac{G^2}{4M} \sum_{i,j=u,c} \xi_i \xi_j T_{ij}^{\mu\nu}(K) \langle B | \bar{d}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \bar{d}_\beta \gamma_\nu (1-\gamma_5) b_\alpha | \bar{B} \rangle \quad (7)$$

with

$$T_{ij}^{\mu\nu}(K) = \frac{1}{6\pi} \omega\left(1, \frac{m_i^2}{K^2}, \frac{m_j^2}{K^2}\right) \left[\left(K^2 - \frac{\Sigma^2}{2} - \frac{\Delta^4}{2K^2}\right) g^{\mu\nu} - \left(1 + \frac{\Sigma^2}{K^2} - 2 \frac{\Delta^4}{K^4}\right) K^\mu K^\nu \right]$$

$$w(a, b, c) = (a^2 + b^2 + c^2 - 2ab - 2ac - 2bc)^{1/2}$$

$$\Sigma^2 = m_i^2 + m_j^2$$

$$\Delta^4 = (m_i^2 - m_j^2)^2 \quad (8)$$

In Eq. (7), K is the external momentum running in the loop of the box diagram, α and β are colour indices to be summed over, and I have neglected irrelevant terms proportional to m_c^2/m_W^2 [13].

The matrix element has two different contributions, corresponding to the "annihilation" (more exactly, W -exchange) and the "spectator" (b -quark decay) diagrams

$$\langle B | \bar{d}_\alpha \gamma_\mu (1-\gamma_5) b_\beta \bar{d}_\beta \gamma_\nu (1-\gamma_5) b_\alpha | \bar{B} \rangle \equiv M_{\mu\nu}^{(a)} + M_{\mu\nu}^{(s)}$$

$$M_{\mu\nu}^{(a)} = \frac{2}{N} f_B^2 B_2 P_\mu P_\nu$$

$$M_{\mu\nu}^{(s)} = f_B^2 (B_1 M^2 g_{\mu\nu} - 2 B_2 P_\mu P_\nu) \quad (9)$$

In Eq. (9), $N (=3)$ is the number of colours, P is the meson four-momentum and f_B its decay constant. B_1 and B_2 parametrize the uncertainty in the matrix element evaluation and are equal to one in the vacuum insertion approximation.

It should be noted that actually the external momentum K has different values for the two contributions. In the "annihilation" case $K_\mu^{(a)} = P_\mu$, while for the "spectator" diagram $K_\mu^{(s)} = (p_b - p_d)_\mu \approx P_\mu m_b/M$. The difference between the two is usually neglected; I will introduce a parameter $y = (K^{(s)})^2/M^2$ and consider the effect of its variation later on.

We can write, using Eq. (5),

$$\begin{aligned} \Gamma_{12} &= \frac{G^2 f_B^2 B M^3}{8 \pi} \left(\xi_u^2 X_{uu} + \xi_c^2 X_{cc} + 2 \xi_u \xi_c X_{uc} \right) = \\ &= \frac{G^2 f_B^2 B M^3}{8 \pi} \left[\xi_u^2 (X_{uu} - X_{uc}) + \xi_c^2 (X_{cc} - X_{uc}) + \xi_t^2 X_{uc} \right] \end{aligned} \quad (10)$$

From Eqs. (1) and (3) one obtains

$$a = - \frac{G^2 f_B^2 B M^3}{8 \pi M_0} \left[\text{Im} \left(\frac{\xi_u^2}{\xi_t^2} \right) (X_{uu} - X_{uc}) + \text{Im} \left(\frac{\xi_c^2}{\xi_t^2} \right) (X_{cc} - X_{uc}) \right] \quad (11)$$

The previous relation may be put in the following more convenient form [11], using the Wolfenstein [14] parametrization of the KM matrix [$\xi_u \approx A\lambda^3(\rho - i\eta)$, $\xi_c \approx -A\lambda^3$, $\xi_t \approx A\lambda^3(1 - \rho + i\eta)$]

$$a = - \frac{G_F^2 f_B^2 B M^3}{4\pi M_0} \frac{\eta}{(1-\rho)^2 + \eta^2} \left\{ X_{uu} - X_{uc} - \frac{1-\rho}{(1-\rho)^2 + \eta^2} (X_{uu} + X_{cc} - 2 X_{uc}) \right\} \quad (12)$$

In the limit of equal charm and up quark masses, $X_{uu} = X_{uc} = X_{cc}$ and the asymmetry vanishes, as it must, since the CP violating phase in the KM matrix can be rotated away if two quarks are degenerate in mass.

Neglecting QCD corrections, assuming $B_1 = B_2 = B$ for the time being and neglecting the mass of the up quark, one has ($x = m_c^2/M^2$, $y \approx m_b^2/M^2$)

$$\begin{aligned} X_{uu}^{(bare)} &= y \\ X_{cc}^{(bare)} &= y \sqrt{1 - 4 \frac{x}{y}} - \frac{2}{N} x \sqrt{1 - 4x} \\ X_{uc}^{(bare)} &= y \left(1 + \frac{x}{y}\right) \left(1 - \frac{x}{y}\right)^2 - \frac{1}{N} x (1-x)^2 \end{aligned} \quad (13)$$

In Eq. (13), the "annihilation" contributions (terms multiplied by $1/N$) have the usual helicity suppression factor, and are therefore neglected in X_{uu} . If we make an expansion in x (not a particularly good one for X_{cc} , numerically, since the series for $\sqrt{1-4x}$ ceases to converge at $x = 0.25$), we have

$$\begin{aligned} X_{uu}^{(bare)} - X_{uc}^{(bare)} &= x \left(1 + \frac{1}{N}\right) + O(x^2) \\ X_{cc}^{(bare)} - X_{uc}^{(bare)} &= -x \left(1 + \frac{1}{N}\right) + O(x^2) \end{aligned} \quad (14)$$

Therefore the dominant $O(x)$ term in the asymmetry would not be affected by the uncertainty due to the value of y . Since, however, $x \approx 0.1$ and $y \approx 1$, there is a very large cancellation, as noted by AWBj. It is therefore important to consider all the possible corrections that can modify the $O(x)$ terms in Γ_{12} or generate new such terms.

3. - QCD CORRECTIONS

The hard gluon exchange QCD corrections, for virtual momenta $m_b^2 \lesssim p^2 \lesssim m_W^2$, resummed to all orders in the leading logarithmic approximation (LLA), have been first calculated in Refs. [8,9]. Neglecting [8] the mixing with "Penguin" operators [15] one obtains

$$\begin{aligned}
 X_{uu}^{(1)} &= \eta^{(s)} \gamma \\
 X_{cc}^{(1)} &= \eta^{(s)} \gamma \sqrt{1-4\frac{x}{y}} - \eta^{(a)} \frac{2}{N} \times \sqrt{1-4x} \\
 X_{uc}^{(1)} &= \eta^{(s)} \gamma \left(1+\frac{x}{y}\right) \left(1-\frac{x}{y}\right)^2 - \eta^{(a)} \frac{1}{N} \times (1-x)^2
 \end{aligned} \tag{15}$$

where

$$\begin{aligned}
 \eta^{(s)} &= \frac{1}{2} \left(1+\frac{1}{N}\right) c_+^2 + \frac{1}{2} \left(1-\frac{1}{N}\right) c_-^2 = \frac{2c_+^2 + c_-^2}{3} \\
 \eta^{(a)} &= \left(\frac{N+1}{2}\right)^2 c_+^2 + \left(\frac{N-1}{2}\right)^2 c_-^2 - \frac{N^2-1}{2} c_+ c_- = (2c_+ - c_-)^2
 \end{aligned} \tag{16}$$

In Eq. (16), the usual definitions [16] have been used:

$$c_+ = (c_-)^{-\frac{1}{2}} = \left(\frac{\alpha'(\mu_b)}{\alpha'(\mu_w)} \right)^{-\frac{6}{23}} \tag{17}$$

where $\alpha'(\mu)$ is the running coupling constant in an effective theory with five active flavours and $N = 3$. With $\Lambda_4^{LL} = 0.10 \pm 0.05$ GeV, corresponding to $\alpha'(\mu_b) = 0.19 \pm 0.03$, one has $1.06 \lesssim \eta^{(s)} \lesssim 1.09$ and $0.16 \lesssim \eta^{(a)} \lesssim 0.27$. Therefore the "annihilation" contribution is strongly suppressed, while the "spectator" one is only slightly increased.

It can be noted that, if one were to follow the ideas of Bardeen, Buras and Gérard [17] and calculate Γ_{12} in the $N \rightarrow \infty$ limit, one would obtain in leading order a result corresponding to the bare formula, Eq. (13), without the "annihilation" contribution. This turns out to be a reasonable approximation for the asymmetry, as we will see later on.

One has, for $m_b = 5.1$ GeV, $m_c = 1.7$ GeV and $N = 3$

$$\begin{aligned}
 & (.129, -.138, .862) \\
 (X_{uu}^{(1)} - X_{uc}^{(1)}, X_{cc}^{(1)} - X_{uc}^{(1)}, X_{uc}^{(1)}) &= (.129, -.139, .878) \\
 & (.130, -.140, .892)
 \end{aligned} \tag{18}$$

for $\Lambda_4^{LL} = 50, 100, 150$ MeV, respectively. In the $N \rightarrow \infty$ limit

$$(X_{uu} - X_{uc}, X_{cc} - X_{uc}, X_{uc})_{N \rightarrow \infty} = (.114, -.124, .822) \tag{19}$$

I choose a rather high value for the charm mass (a value $m_c \lesssim 1.5$ GeV would be more appropriate for the running charm mass evaluated at m_b), in order to look for possible high asymmetries. The dependence on $\Lambda \equiv \Lambda_4^{LL}$ is very weak: in the following, I will give results calculated with $\Lambda = 0.1$ GeV.

The effect of "Penguin" operators was considered in Ref. [9]. It is easy to see that by inserting in the box diagram vertices one left-left and one left-right four-fermion operator, terms proportional to the product of the masses of the quarks in the internal lines are obtained. However, the coefficients of the Penguin operators are fairly small at the scale m_b and this is accordingly a small effect. The rather lengthy exact formulae are given in the Appendix. Numerically, for the same parameter values, one has

$$(X_{uu}^{(2)} - X_{uc}^{(2)}, X_{cc}^{(2)} - X_{uc}^{(2)}, X_{uc}^{(2)}) = (.128, -.138, .844) \quad (20)$$

More important, in principle, could be corrections due to hard-gluon emission by the light-quark lines, first considered for D decays [18]: they have been discussed for the case at hand in Ref. [9], in a non-relativistic model for the B meson. These terms are proportional to $\alpha_s/\pi M/E_d$, E_d being the average energy of the light-quark in the meson rest frame (constituent down-quark mass), and this factor is of the order of unity; moreover, the three-body phase space gives rise to terms $O(x \log x)$, that could change appreciably the asymmetry. The actual calculation shows, however, that their numerical coefficients are small. Therefore, when considering the correction to these terms coming from hard-gluon exchanges in the vertices, I will neglect the mixing with Penguin operators. The relevant formulae are given in Ref. [9] and reported here in the Appendix.

The numerical evaluation has been made for $E_d = 330$ MeV and $\alpha_s = \alpha_s(m_c)$, since $m_c \approx (E_d M)^{1/2}$. The result is (same parameters as before)

$$(X_{uu}^{(3)} - X_{uc}^{(3)}, X_{cc}^{(3)} - X_{uc}^{(3)}, X_{uc}^{(3)}) = (.118, -.120, .799) \quad (21)$$

As one can see, these terms tend to decrease the absolute value of the asymmetry by about 10%. Their evaluation is obviously affected by a considerable uncertainty, but it seems difficult to envisage a change of the sign of the asymmetry due to these terms.

More recently, attention has been drawn [12] to the fact that it is not safe to use the vacuum insertion approximation to evaluate the matrix element in Eq. (9) with the operator normalized at a scale m_b , since this matrix element can contain (large) logarithms of the ratio m_b^2/μ^2 , μ being a typical QCD scale, and these logarithms (that have been called "hybrid" in Ref. [19]) should be resummed in the

LLA. For $m_d \ll \mu \ll m_b$, the heavy (b) quark acts essentially as a static source of colour, and the anomalous dimensions of the operators are different. Also the axial current, $\bar{d}\gamma_\mu\gamma_5 b$, has a non-zero anomalous dimension in this range of scales, and it happens, as shown in Ref. [19], that the relation

$$\langle B | \bar{d}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha \bar{d}_\beta \gamma^\mu (1-\gamma_5) b_\beta | \bar{B} \rangle = 2 \left(1 + \frac{1}{N}\right) f_B^2 M^2 B \quad (22)$$

holds in the LLA with a B factor independent of the scale μ for any $m_d \ll \mu \ll m_b$, since the anomalous dimension of the operator in the left-hand side is twice as large as the anomalous dimension of the axial current. Therefore in M_{12} , Eq. (3), these "hybrid" logarithms have no effect (on the dominant m_t^2 term).

The situation is different for Γ_{12} , Eq. (7), because $T^{\mu\nu}$ contains also a term $K^\mu K^\nu$. The relevant anomalous dimensions have been calculated in Ref. [12]. If one writes the matrix element in Eq. (9) for the operator normalized at the scale m_b

$$\begin{aligned} \langle O_{\mu\nu}(m_b) \rangle &= \langle B | \bar{d}_\alpha \gamma_\mu (1-\gamma_5) b_\alpha \bar{d}_\beta \gamma_\nu (1-\gamma_5) b_\beta | \bar{B} \rangle = \\ &= f_B^2(m_b) \left(B_2(m_b) M^2 g_{\mu\nu} - B_2(m_b) \left(1 - \frac{1}{N}\right) 2 P_\mu P_\nu \right) \end{aligned} \quad (23)$$

then one has ($B(m_b) \approx B(\mu) = B$)

$$\begin{aligned} B_1(m_b) &= B \left(\frac{1+\beta}{2} + \frac{1}{N} \frac{1-\beta}{2} \right) \\ B_2(m_b) &= B \beta \end{aligned} \quad (24)$$

In Eq. (24), I used the definitions

$$\begin{aligned} \beta &= \frac{B_2(\mu)}{B} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right)^{\frac{\Delta\gamma}{b_3}} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{\frac{\Delta\gamma}{b_4}} \\ b_f &= \frac{11N - 2f}{3} \\ \Delta\gamma &= -2 \left(1 + \frac{1}{N} \right) = -\frac{8}{3} \end{aligned} \quad (25)$$

Separating again, as in Eq. (9), the "annihilation" and the "spectator" contributions one has therefore

$$\langle O_{\mu\nu}(m_b) \rangle = M_{\mu\nu}^{(a)} + M_{\mu\nu}^{(s)} \quad (26)$$

with

$$M_{\mu\nu}^{(a)} = B f_B^2(m_b) \frac{1}{N} \left[\frac{1-\beta}{2} M^2 g_{\mu\nu} + \beta 2 P_\mu P_\nu \right] \quad (27)$$

$$M_{\mu\nu}^{(s)} = B f_B^2(m_b) \left[\frac{1+\beta}{2} M^2 g_{\mu\nu} - \beta 2 P_\mu P_\nu \right] \quad (28)$$

In the LLA, $f_B(m_b) \approx f_B(m_W) = f_B$.

If, as in Ref. [12], one chooses a scale $\mu \approx 300$ MeV in order to use there the vacuum insertion approximation ($B_1(\mu) \approx B \approx 1$), then the correction is rather large: $\beta \approx 0.60$. It can be noted, however, that actually one should find a scale μ where $B_2(\mu) \approx B$ and evaluate β at this scale; an overall B factor would multiply f_B^2 in the final expression for Γ_{12} , as it does in the corresponding expression for M_{12} . If this scale μ were larger, as it could possibly be, the correction would be correspondingly smaller. Proceeding as in Ref. [12], namely choosing μ such that $\alpha_s(\mu) = 1$, one obtains

$$(X_{uu}^{(4)} - X_{uc}^{(4)}, X_{cc}^{(4)} - X_{uc}^{(4)}, X_{uc}^{(4)}) = (.112, -.110, .628) \quad (29)$$

As one can see, the effect is large on the individual terms (X_{uc}) but it is rather small on the difference $X_{uu} - X_{uc}$ [that dominates the asymmetry, see Eq. (12)]. This is due to the cancellation of the β dependence in the $O(x)$ terms [see Eqs. (A5) - (A7) in the Appendix].

In the above calculation, we neglected the hybrid logarithmic corrections of the terms $O(\alpha_s/\pi M/E_d)$, given both their uncertainty and their smallness.

As a last point, we consider variations of the parameter $y = (K^{(s)})^2/M^2 \approx m_b^2/M^2$. For $y = 1$ (0.865) corresponding to $m_b = 5.27$ (4.90) GeV the numbers in Eq. (29) would be changed to (0.112, -0.109, 0.679) or (0.112, -0.111, 0.571) respectively; again a 10% variation of the X_{ij} that does not cause a variation of more than 1% in their differences. This is due to the y -independence of the $O(x)$ term, see Eq. (14) and Eqs. (A5)-(A7).

In summary, up to now we have considered several corrections that make it clear that the X_{ij} are separately affected by an uncertainty of about 30%: but in the framework of this calculation, for a fixed value of the charm quark mass m_c , their differences are known with the same accuracy, if not a better one.

4. - THE CP VIOLATING ASYMMETRY a

In order to obtain a numerical estimate for the asymmetry, one could use Eqs. (12) and (3) to obtain

$$a = - \frac{3 \pi M^2}{m_t^2 F\left(\frac{m_t}{m_w}\right) \eta_n} \frac{1}{(1-\rho)^2 + \eta^2} A(X, \rho, \eta) \quad (30)$$

where

$$A(X, \rho, \eta) = \eta (X_{uu} - X_{uc}) - \frac{(1-\rho)\eta}{(1-\rho)^2 + \eta^2} (X_{uu} + X_{cc} - 2X_{uc}) \quad (31)$$

One should further impose the constraints on ρ and η coming from the experimental values of ε , ε'/ε and x_d , and the resulting asymmetry would be essentially free from the uncertainty related to the value of m_b .

However, Eq. (30) would only be useful if we knew the top quark mass: since this is not the case, it is better [11] to eliminate M_t in Eq. (12) in favour of the B decay width, using the experimental value for x_d [Eq. (2)], and further use the spectator model estimate of the width. Following AWBj, one obtains in this way

$$\begin{aligned} a &= - .024 \left(\frac{f_B \sqrt{B}}{140 \text{ MeV}} \right)^2 \left(\frac{5.1 \text{ GeV}}{m_b} \right)^5 A(X, \rho, \eta) \equiv \\ &\equiv - 10^{-3} \left(a_1 \eta + b_1 \frac{(1-\rho)\eta}{(1-\rho)^2 + \eta^2} \right) \end{aligned} \quad (32)$$

The uncertainty due to the experimental errors is about 30%. Note that the use of the spectator model estimate of the width has introduced an m_b dependence in the asymmetry.

The results for a_1 and b_1 are reported in the Table. The value of b_1 , although very uncertain, is much smaller than a_1 , and therefore the b_1 term in Eq. (32) can be neglected. In fact, present fits in the standard model to ε , ε'/ε and x_d tend to give [20] $\eta \lesssim 0.4$ and $\rho < 0$.

The asymmetry predicted in the "partonic" calculation is

$$a \simeq - 10^{-3} \left(\frac{f_B \sqrt{B}}{140 \text{ MeV}} \right)^2 \left(\frac{m_c}{1.7 \text{ GeV}} \right)^2 \frac{\eta}{0.37} \quad (33)$$

for the dileptons coming from the decay of a $B_d^0 - \bar{B}_d^0$ pair. The corresponding asymmetry for the $B_s^0 - \bar{B}_s^0$ case is

$$a_s = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{X_{cc} - X_{uc}}{X_{uu} - X_{uc}} a \simeq - \left| \frac{V_{td}}{V_{ts}} \right|^2 a \quad (34)$$

and we know that $|V_{td}|^2 < 0.2|V_{ts}|^2$ in the minimal standard model.

An estimate of the uncertainty is more delicate. In the framework of this calculation, the result could be given as a multiplying factor $1_{-0.9}^{+0.5}$, where I have conservatively allowed for an order of magnitude uncertainty of the $O(\alpha_s/\pi M/E_d)$ correction. Another factor of two (or maybe three) could be a reasonable guess [21] for the uncertainty connected with the use of the partonic approach.

It may be recalled that this approach fell into disrepute due to the failure of the spectator model to account for charm decays. Nowadays, it is clear that subasymptotic effects (interference diagrams, annihilation contributions,...) are very large in that case. These effects are expected to be much smaller for the B mesons, and they have been taken into account, albeit in a rough manner, in this calculation.

It will be therefore very difficult to measure the CP violating asymmetry a if the minimal standard model is correct. There are other more promising ways to find CP violation in B meson decays [21,22].

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APPENDIX

At the scale $m_b \ll m_t \approx m_W$, the effective Hamiltonian for $\Delta B = 1$, $\Delta S = 0$ processes can be written in the form

$$\begin{aligned} H(m_b) = \frac{G}{\sqrt{2}} \left\{ c_1 (\bar{d}_\alpha b_\alpha)_{V-A} \left[\xi_c (\bar{c}_\beta c_\beta)_{V-A} + \xi_u (\bar{u}_\beta u_\beta)_{V-A} + \right. \right. \\ \left. + V_{cb} V_{ud}^* (\bar{c}_\beta u_\beta)_{V-A} + V_{ub} V_{cd}^* (\bar{u}_\beta c_\beta)_{V-A} \right] + \\ + c_2 (\bar{d}_\alpha b_\beta)_{V-A} \left[\xi_c (\bar{c}_\beta c_\alpha)_{V-A} + \xi_u (\bar{u}_\beta u_\alpha)_{V-A} + \right. \\ \left. + V_{cb} V_{ud}^* (\bar{c}_\beta u_\alpha)_{V-A} + V_{ub} V_{cd}^* (\bar{u}_\beta c_\alpha)_{V-A} \right] + \\ + (\xi_c + \xi_u) \left[c_3 (\bar{d}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A} + c_4 (\bar{d}_\alpha b_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A} + \right. \\ \left. + c_5 (\bar{d}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A} + c_6 (\bar{d}_\alpha b_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A} \right] \left. \right\} \quad (A1) \end{aligned}$$

In Eq. (A1), I have used the shorthand notation $\bar{q}q = \bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c + \bar{b}b$ and the definition

$$(\bar{q}_\alpha q'_\beta)_{V \mp A} \equiv \bar{q}_\alpha \gamma_\mu (1 \mp \gamma_5) q'_\beta$$

The coefficients c_i are calculated in the LLA in the standard way from the matrix of the anomalous dimensions of the relevant operators [23]. In particular, one has

$$c_{\pm(2)} = (c_+ \mp c_-) / 2$$

with c_+ , c_- defined in Eq. (17). Here and in the following equations, I give numerical values for $\Lambda_4^{LL} = 100$ MeV, $m_b = 5.1$ GeV and $m_t \approx m_W$. One has

$$(c_1, \dots, c_6) = (-.211, 1.088, .009, -.022, .007, -.026)$$

Let us define

$$f(x, y; z, t) = xz + yt + (xt + yz)/N$$

$$g(x, y; z, t) = N^2 xz + yt + N(xt + yz) \quad (A2)$$

Then one has [Eq. (16)]

$$\eta^{(s)} = f(c_1, c_2; c_3, c_4) = 1.075$$

$$\eta^{(a)} = g(c_1, c_2; c_5, c_6) = 0.208$$

We define for further use

$$B_c = f(c_3, c_4; c_5, c_6) + f(c_5, c_6; c_3, c_4) = .001$$

$$F_c = g(c_3, c_4; c_5, c_6) + g(c_5, c_6; c_3, c_4) \simeq 0$$

$$B_2 = 2 f(c_3, c_4; c_5, c_6) = 0.001$$

$$F_2 = 2 g(c_3, c_4; c_5, c_6) \simeq 0$$

$$X_L^{(s)} = f(c_1, c_2; c_3, c_4) = -0.021$$

$$X_L^{(a)} = g(c_1, c_2; c_3, c_4) = 0.003$$

$$X_R^{(s)} = f(c_1, c_2; c_5, c_6) = -0.026$$

$$X_R^{(a)} = g(c_1, c_2; c_5, c_6) = -0.003$$

(A3)

and

$$B_u = 3 B_c$$

$$F_u = 3 F_c$$

$$A_u = \eta^{(s)} + 2 X_L^{(s)} + B_u$$

$$E_u = \eta^{(a)} + 2 X_L^{(a)} + F_u$$

$$A_c = \eta^{(s)} + 2 X_L^{(s)} + B_c$$

$$E_c = \eta^{(a)} + 2 X_L^{(a)} + F_c$$

$$D_u = X_L^{(s)} + B_u$$

$$G_u = X_L^{(a)} + F_u$$

$$D_c = X_L^{(s)} + B_c$$

$$G_c = X_L^{(a)} + F_c$$

$$A_2 = 2 X_R^{(s)} + B_2$$

$$E_2 = 2 X_R^{(a)} + F_2$$

$$D_2 = X_R^{(s)} + B_2$$

$$G_2 = X_R^{(a)} + F_2 \quad (A4)$$

One can see that in the limit of no mixing with Penguin operators all these constants vanish, except $A_u = A_c = \eta^{(s)}$ and $E_u = E_c = \eta^{(a)}$.

In the following formulae, the masses of all the light quarks (u,d,s) have been neglected. With all the corrections discussed in the text, the final equations

are ($N = 3$, $x = m_c^2/M^2$, $y = m_b^2/M^2$)

$$\begin{aligned}
 X_{uu} = & y \left[A_u \frac{1+\beta}{2} + B_2 \ 2 \frac{x}{y} \sqrt{1-4 \frac{x}{y}} + \right. & (A5) \\
 & + B_c \sqrt{1-4 \frac{x}{y}} \left(\frac{1+\beta}{2} - 2 \frac{x}{y} \frac{1-\beta}{2} \right) + \frac{\sigma}{48} \eta \left. \right] + \\
 & + \frac{1}{3} \left[E_u \frac{1-\beta}{2} + F_2 \ 2x \sqrt{1-4x} + \right. \\
 & + F_c \sqrt{1-4x} \left(\frac{1-\beta}{2} - 2x \frac{1+\beta}{2} \right) - \frac{2}{27} \sigma \eta \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 X_{cc} = & y \left[A_c \sqrt{1-4 \frac{x}{y}} \left(\frac{1+\beta}{2} - 2 \frac{x}{y} \frac{1-\beta}{2} \right) + \right. & (A6) \\
 & + A_2 \ 2 \frac{x}{y} \sqrt{1-4 \frac{x}{y}} + B_u \frac{1+\beta}{2} + \\
 & + \frac{\sigma}{48} \eta \left(\sqrt{1-4 \frac{x}{y}} \left(1+2 \frac{x}{y} \right) - 12 \frac{x^2}{y^2} \log \frac{1-2 \frac{x}{y} + \sqrt{1-4 \frac{x}{y}}}{2 \frac{x}{y}} \right) \left. \right] + \\
 & + \frac{1}{3} \left[E_c \sqrt{1-4x} \left(\frac{1-\beta}{2} - 2x \frac{1+\beta}{2} \right) + \right. \\
 & + E_2 \ 2x \sqrt{1-4x} + F_u \frac{1-\beta}{2} - \\
 & - \frac{2}{27} \sigma \eta \left(\sqrt{1-4x} (1-7x+6x^2) + 12 x^3 \log \frac{1-2x + \sqrt{1-4x}}{2x} \right) + \\
 & + \frac{\sigma}{3} \mathcal{L} \times \left(3 \sqrt{1-4x} (1-2x) - 2 (1-4x+6x^2) \log \frac{1-2x + \sqrt{1-4x}}{2x} \right) \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 X_{uc} = & \gamma \left[D_u \frac{1+\beta}{2} + D_2 2 \frac{x}{y} \sqrt{1-4 \frac{x}{y}} + \right. & (A7) \\
 & + D_c \sqrt{1-4 \frac{x}{y}} \left(\frac{1+\beta}{2} - 2 \frac{x}{y} \frac{1-\beta}{2} \right) + \\
 & + \eta^{(s)} \left(1 - \frac{x}{y} \right)^2 \left(\frac{1+\beta}{2} + \frac{x}{y} \beta \right) + \\
 & \left. + \frac{\sigma}{48} \pi \left(1 + 3 \frac{x^2}{y^2} - 4 \frac{x^3}{y^3} + 6 \frac{x^2}{y^2} \log \frac{x}{y} \right) \right] + \\
 & + \frac{1}{3} \left[G_u \frac{1-\beta}{2} + G_2 2x \sqrt{1-4x} + \right. \\
 & + G_c \sqrt{1-4x} \left(\frac{1-\beta}{2} - 2x \frac{1+\beta}{2} \right) + \\
 & + \eta^{(a)} (1-x)^2 \left(\frac{1-\beta}{2} - x\beta \right) - \\
 & - \frac{\sigma}{27} \mathcal{J} \left(2 - 9x + 18x^2 - 11x^3 + 6x^3 \log x \right) + \\
 & \left. + \frac{\sigma}{6} \mathcal{L} x (3 - 4x + x^2 + 2 \log x) \right]
 \end{aligned}$$

The coefficients $A_u, \dots, G_2$, in Eqs. (A5)-(A7), have been defined in Eqs. (A3) and (A4) and β in Eq. (25). I have also used the definitions

$$\sigma \equiv \frac{\alpha_s(m_c)}{\pi} \frac{M}{E_d} \simeq 1.35 \quad (\text{A8})$$

and [9]

$$\begin{aligned} \mathcal{J} &= c_2^2 \simeq 1.18 \\ \mathcal{L} &= c_2 (c_2 + 3c_1) \simeq 0.50 \\ \mathcal{N} &= \frac{16}{9} c_1 (3c_1 + 2c_2) \simeq -0.57 \end{aligned} \quad (\text{A9})$$

From Eqs. (A5)-(A7), one obtains the final results, $X_{ij}^{(4)}$ in Eq. (29), if one uses

$$\beta = \left(\frac{1}{\alpha(m_c)} \right)^{-\frac{8}{27}} \left(\frac{\alpha(m_c)}{\alpha(m_b)} \right)^{-\frac{8}{25}}$$

From the same formulae, with $\beta = 1$, one obtains the $X_{ij}^{(3)}$, Eq. (21). Putting furthermore $\sigma = 0$ one obtains $X_{ij}^{(2)}$, Eq. (20). Finally, neglecting the contribution of the Penguin operators, one reproduces Eq. (15).

	a_1			b_1		
m_b (GeV)	5.27	5.1	4.9	5.27	5.1	4.9
$N = \infty$	2.3	2.8	3.4	0.17	0.24	0.35
(2)	2.6	3.1	3.8	0.17	0.24	0.36
(3)	2.4	2.9	3.5	~ 0	0.04	0.12
(4)	2.3	2.7	3.3	-0.07	-0.05	-0.02

Table - Values of a_1 , b_1 , see Eq. (32), for $m_c = 1.7$ GeV and $f \sqrt{B_B} = 140$ MeV. The rows are labelled in correspondence to calculations described in the text immediately before the equations mentioned: $N = \infty$ [Eq. (19)], (2) [Eq. (20)], (3) [Eq. (21)], (4) [Eq. (29)].

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