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ON THE CONCEPT OF NATURALNESS IN STRING THEORIES

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ABSTRACT

We point out that string theory can provide an explanation for the vanishing of certain parameters in its low-energy limit which cannot be understood through the symmetries of the field-theoretic approximation. This fact may enable us to pinpoint stringy effects in the effective field theoretic limit.

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Any discussion of effective field theories for the description of physical phenomena should aim at an understanding of the magnitude of the parameters under consideration. This is especially true if some of the parameters vanish or have very peculiar relations like, for example, a mass scale tiny compared to another one. In the absence of an explanation of such strange relations the corresponding theory is called unnatural. We usually assume that in a natural theory a free parameter in the action can be very small only if the symmetries of the system are enlarged when this parameter vanishes [1] .

In this paper we want to investigate these questions of naturalness in the framework of string theory and its field-theoretic approximation at low energies. We are interested in the question whether the low-energy approximation allows us to understand its parameters in terms of the symmetries of the effective field theory. If such a situation were the case we could not distinguish between string theory and its field-theoretic limit at low energies. Fortunately, our investigation shows that this is not the case. On the contrary, we find that string theory provides an explanation for the vanishing of certain parameters which cannot be understood within the field-theoretic approximation. This result has important consequences for the interpretation of the low-energy field-theoretic limit. On one hand we need no longer understand all parameters in terms of the low-energy symmetries of the system. String theory could e.g. explain the “gauge hierarchy” problem and this would seem miraculous from the viewpoint of field theory at low energies. On the other hand we have to be careful with our assumptions concerning those terms that are allowed by the symmetries. It could very well be that those terms sometimes urgently needed (e.g. for the generation of quark and lepton masses) are bound to vanish because of the underlying string theory.

Although mechanisms responsible for the absence of terms allowed by low-energy symmetries can in principle be observed in any four-dimensional string theory, we will concentrate on orbifold compactifications. We will use the formalism developed in [2] to study the interactions of the orbifold massless states. For the purpose of determining whether certain couplings vanish or not, it is sufficient to consider a set of selection rules on correlation functions [3-5] . We will refer to these rules as stated in [5] .

It is actually rule 4 which is responsible for the stringy zeros that cannot be understood by the field theoretic symmetries. Rule 4 states that certain correlation functions involving vertex operators with derivatives ∂X^{i*} vanish if $\partial X^i = 0$ is the only solution of the classical

* i refers to the internal (complex) compactified dimensions.

field equation $\bar{\partial}\partial X_{cl}^i = 0$. Vertex operators for the massless states do not contain such derivatives (except for oscillator modes) in the simplest pictures: i.e. $V_{-1/2}$ or V_{-1} for the fermionic or bosonic component respectively. However the picture changing operation introduces explicit factors of ∂X^i .

To compute a tree-level n -point Yukawa-coupling in the superpotential it is appropriate to consider correlation functions of the type

$$\langle V_{-1/2} V_{-1/2} V_{-1} V_0 \dots V_0 \rangle \quad (1)$$

with $n - 3$ vertex operators in the 0-picture, and thus $n - 3$ factors of ∂X . This shows that the consideration of rule 4 is very important for Yukawa couplings that involve more than three external fields.

We now want to search for those cases where $\partial X = 0$ is the only solution of the classical equations of motion. These equations have to be supplemented with the conditions that indicate the transformation of X_{cl} when transported around a collection of twists in the correlator. Typically one finds that $\partial X_{cl}^i = 0$ for all i , when all the twists sit at the same fixed point. Actually if the six-dimensional lattice can be represented as a direct sum of sublattices ∂X_{cl}^i vanishes if all the fields sit at the same fixed point in the i^{th} sublattice. If fields from different fixed points appear, one finds that ∂X_{cl}^i is proportional to the distance between the fixed points. Also, ∂X_{cl}^i vanishes if the correlation does not involve twisted fields. The conclusion to keep in mind is that correlation functions such as (1) involving more than 3 fields will vanish when $\partial X_{cl}^i = 0$ as explained above.

Let us now explain why such a behaviour of the correlation functions cannot be understood in terms of the symmetries of the effective field theory of the massless states. Consider for this purpose a simple example: the Z_3 -orbifold with standard embedding. Here we have 27 fixed points with coordinates

$$(n_1 d, n_2 d, n_3 d) \quad (2)$$

where $n_i = 0, \pm 1$ and d is an $SU(3)$ -weight. At each fixed point we have one chiral superfield transforming as a 27 of E_6 in addition to the oscillator modes that transform as a $\bar{3}$ under $SU(3)$. Take now one fixed point, $(0, 0, 0)$ for example, and investigate the selfcouplings of Φ_{000} which is the 27 of E_6 . The gauge symmetries would allow Φ_{000}^{3n} for integer n . Rules 2 and 3 give more restrictions. Φ^{3n} contains $3n - 3$ factors of ∂X , rule 2 implies that they

have to appear in the symmetric combination $[(\partial X^1)(\partial X^2)(\partial X^3)]^{n-1}$ and rule 3 requires $n - 1$ to be a multiple of 3. Thus $n = 3k + 1$ and only terms of the form Φ_{000}^{9k+3} are allowed. This is still not very exciting. Rule 4, however, implies a vanishing correlator unless $k = 0$. How can symmetries explain such a strange behaviour? It is certain that such a fact cannot be explained by a continuous global symmetry. If such a symmetry allows Φ_{000}^3 it also does allow $(\Phi_{000}^3)^n$. The same is true for discrete symmetries. Going to arbitrary high powers in Φ_{000} , any selection rule from a finite order transformation can be fulfilled and this cannot explain the outcome of rule 4.

An alternative could be found in R-symmetries and this case has to be studied very carefully [6]. Let us normalize the R-charge of θ to be 1, and consider Φ_{000} (as a superfield) to have R-charge $\frac{2}{3}$. If we expand $\Phi = \phi + \theta\psi + \theta^2 F$, this implies $R(\phi) = \frac{2}{3}$ and $R(\psi) = -\frac{1}{3}$. As a result

$$\int d^2\theta \Phi_{000}^3 \quad (3)$$

is an allowed contribution to the superpotential and any other power of Φ_{000} is forbidden. This seems to correspond to the behaviour required by rule 4, but we have to be more careful. Consider Φ_{000}^{3n} for some large number n . The R-charge of this term is $2n$ and such a term is forbidden by the R-symmetry as well as rule 4. Now replace some of the fields by those at other fixed points; of course, we have now to satisfy the space group selection rule, but usually this can be achieved quite easily. In our example we might consider

$$\Phi_{000}^{3(n-2)} \Phi_{100} \Phi_{-100} \Phi_{010} \Phi_{0-10} \Phi_{001} \Phi_{00-1} \quad (4)$$

which is allowed by rule 4. Such a term would also be allowed by the R-symmetry if its total R-charge equals 2 to compensate the R-charge -2 of $d^2\theta$. Next consider

$$\Phi_{000}^{3(n-4)} (\Phi_{100} \Phi_{-100} \Phi_{010} \Phi_{0-10} \Phi_{001} \Phi_{00-1})^2 \quad (5)$$

also allowed by rule 4. But now the argument with the R-symmetry breaks down. If (4) is allowed we have only two possibilities: either both (5) and Φ_{000}^{3n} are admissible or both are forbidden, and this is true for continuous as well as discrete symmetries. We thus have to conclude that the strange behaviour required by rule 4 cannot be understood in terms of the symmetries of the field theory of the massless states. Note, however, that such a

behaviour appears quite natural in the underlying string theory, at least in the language of orbifold compactification. It represents a new kind of “pseudo symmetry” under which fields do not have absolute quantum numbers, but only relative ones like here the distance between fixed points. In the extra internal dimensions such a “pseudo symmetry” has some non-local aspect to it, which cannot be understood in terms of the four-dimensional field theory.

Up to now we have only discussed the fields in the twisted sectors. A similar behaviour can be observed for the untwisted charged fields. If we consider these fields exclusively, we again find only trilinear couplings and this could be understood either as an R-symmetry or a classical scale invariance [7]. However, as soon as we consider untwisted-twisted couplings, we will find terms that cannot be understood from symmetry arguments of the field theory.

Having discussed rule 4, we now turn to the other rules to see whether or not they correspond to low-energy symmetries. The first rule states that correlations must be invariant under the action of the orbifold space group. The rotation part of the space group (the point group) is generated by powers of the twist θ with $\theta^3 = 1$ (for Z_3). We then see that point group invariance is equivalent to a discrete symmetry in which a field in a sector twisted by θ^m is multiplied by a factor $e^{2\pi im/3}$ under the symmetry transformation. Invariance under the translation part of the space group requires $\sum_{\alpha} n_i^{\alpha} = 0 \text{ mod } 3$, where the sum runs over all the fields entering in the correlation function and n_i refers the fixed point coordinates as given in (2). Therefore, this also corresponds to discrete symmetries in the field theory. We also notice that since vertex operators are invariant under the combined action of the orbifold point group and the gauge group, every gauge invariant coupling will turn out to be invariant under the point group.

The two remaining selection rules have to do with conservation of the right moving $\vec{H}$ momentum and rotation of the derivatives ∂X^i . These two rules combined are equivalent to R-symmetries as we now explain for the Z_3 orbifold. An independent phase rotation of the ∂X^i by $\lambda_i = e^{2\pi i \vec{v} \cdot \vec{e}_i}$, where $\vec{v}$ defines the orbifold twist, must be accompanied by a translation in $\vec{H}$, $\vec{H} \rightarrow \vec{H} + 2\pi \vec{v}$, in order to leave the world-sheet supersymmetry current, T_F , invariant. T_F is given by

$$T_F = \partial x^{\mu} \psi_{\mu} + \partial X^i \bar{\psi}^i + \partial \bar{X}^i \psi^i \quad (6)$$

where $\psi^i \sim e^{i\vec{\alpha}^i \cdot \vec{H}}$ with $\alpha^1 = (0, 0, 1, 0, 0)$, $\alpha^2 = (0, 0, 0, 1, 0)$, etc. However, the space-

time supersymmetry generator, which is essentially the gravitino vertex at zero momentum, is not invariant under this transformation. Therefore, the corresponding four-dimensional symmetries would have to be R-symmetries. In fact, it is easy to see from the explicit form of the vertex operators that fermions and bosons transform in different ways.

It is straightforward to check that the restrictions implied by selection rules 2 and 3 combined are equivalent to those that follow from invariance under three R-symmetries, one for each independent coordinate rotation. The charge assigned to each field is read off from the corresponding emission vertex. Vertex operators involve exponentials $e^{i\beta_i H_i}$ and powers of derivatives $(\partial X^i)^{a_i}, (\bar{\partial} \bar{X}^i)^{b_i}$. The charge under the R_i symmetry is then $\beta_i + a_i - b_i$. This assignment does not depend on the picture because T_F is invariant. The superspace fermionic coordinate has charge $-1/2$ under each R_i -symmetry. Notice that since $\lambda_i^3 = 1$, these are mod 3 symmetries. This concludes the list of symmetries relevant for the discussion of the interactions of those fields that could serve as candidates for charged matter. For the “moduli”, corresponding to the radii and angles of the underlying torus, one observes, for instance, the absence of superpotential terms [2][3]. However, this fact alone can usually be understood as invariance under certain non-compact symmetries [8-10].

In conclusion, we have observed that only the consequences of rule 4 are (so far) the sources of stringy zeros. The remaining selection rules correspond to discrete symmetries and gauge invariance. We have not found continuous global symmetries of compact type in agreement with the conjecture that such symmetries are absent in string theories [11]. Notice that we have not discussed the effect of radiative corrections. Rule 4 is an “on-shell rule”; it uses explicitly the classical equations of motion. We might conceive that terms that are absent at the classical level because of rule 4 could receive non-trivial contributions in perturbation theory, although this has still to be checked (by considering non-supersymmetric string models for instance). In this case one would require supersymmetry for the perturbative stability. This stability then has an explanation in the low-energy theory and is not due to stringy effects.

A discussion of naturalness in the couplings of (2,2)-models and (2,0)-models continuously connected to them was presented in reference [6]. There it was concluded that the couplings in these particular cases could be understood from the symmetries of the low-energy theory. Observe that these statements do not necessarily contradict our result. If we consider (2,2)-orbifold models, we know that such orbifolds can be blown up to smooth

manifolds. Taking into account the blowing-up modes at all the different fixed points and giving them non-vanishing vacuum expectation values would correspond to a mechanism to introduce non-trivial values for the stringy zeros. That is, if a coupling

$$U^n T^m \tag{7}$$

is not allowed because of rule 4,^{*} it is because all the T fields are in fixed points coinciding in at least one complex coordinate. But the couplings

$$U^n T^m \langle Y^i \rangle \tag{8}$$

where Y represents any blowing-up mode in any fixed point, will be allowed. Therefore in (2,2) models, rule 4 is at work only in the orbifold limit. However, (2,0) models (not continuously connected to a (2,2)-model) cannot in general be blown-up and therefore rule 4 is more effective in these cases.

Let us now discuss the implications of these stringy zeros. As a first observation we find them to be responsible for the vast continuous vacuum degeneracy of four-dimensional string models as observed in reference [5]. In fact, many of the flat directions in the framework of degenerate orbifolds have their origin in rule 4. Apart from the R- symmetries discussed above, rule 4 is actually the only mechanism that can give rise to such vacuum degeneracies. In the attempt to construct realistic $SU(3) \times SU(2) \times U(1)$ models, rule 4 seems to play a crucial role [12]. As an example one could propose the $SU(3) \times SU(2) \times U(1)^n$ models of reference [13] as a starting point for model building. Here one has then to ask the question, whether one can give non-trivial vev's to fields along flat directions that break the group to $SU(3) \times SU(2) \times U(1)_Y$. Taking only into account the symmetries of the field theory of the massless states, one would conclude that such a mechanism is not possible for some of the models [14]. A full calculation including rule 4 shows however, that such an argumentation is not necessarily true [12].

The main importance of such a miraculous absence of terms in the superpotential should reveal itself in an application to the gauge hierarchy problem, i.e. the question about the reason why the breakdown scale of the weak interactions is so small compared with the

^{*} Here U and T refer to arbitrary untwisted and twisted fields respectively.

Planck scale or a grand unification scale. Even if one considers this question in the framework of supersymmetric models, one has to worry about a Higgs mass term in the superpotential. Let us consider the renormalizable terms allowed by gauge symmetries and an R-symmetry (to forbid proton-decay by dimension 4 operators)

$$W = \mu H \bar{H} + g_L L L \bar{E} + g_D Q H \bar{D} + g_U Q \bar{H} \bar{U} \quad (9)$$

We do not understand why μ is small compared to the Planck mass. One might therefore consider putting $\mu = 0$ and this leads to an additional global U(1) symmetry only broken by an anomaly. This possibility is phenomenologically unacceptable because of the presence of an axion. This situation is known as the μ -problem [15]. Some time ago it was conjectured that the anomalous U(1) symmetry is only a symmetry of the renormalizable terms in the superpotential. One could consider a situation in which the symmetry is broken by non-renormalizable terms, and the possibility of a simultaneous solution of the μ problem and the strong CP-problem has been pointed out [15]. More recently a similar attempt included non-symmetric terms in the Kahler potential [16]. In both cases one had carefully constructed symmetries to render such a solution “natural”. As a result, such solutions appeared to be quite complicated and suggested a more general framework in which such a mechanism could be understood.

String theory provides such a mechanism. It is actually here that the consequences of rule 4 play a major role. In the investigation of the 3-family models of reference [13] where the additional U(1) symmetries are broken by the vev’s of massless fields, one in general observes the phenomenon that even in the presence of these vev’s some Higgs-doublets remain massless, although a mass term would be allowed by the symmetries of the low-energy field theory. An explicit discussion of these models can be found in reference [12].

We thus can have $\mu = 0$ not accompanied by additional embarrassing symmetries. In such a theory one would then expect radiative corrections to induce a non-vanishing value for μ and give a mass to the candidate axion. With unbroken supersymmetry, of course, we would have a perturbative stability of $\mu = 0$, and as a result we would expect μ to be proportional to the gravitino mass. One should add a word of caution here. Our observation does not really imply that the μ problem is solved in string theory, it only indicates there is the possibility of $\mu = 0$ without a symmetry in a wide class of string models. Given a certain model, one has to sit down and compute all relevant quantities to see what value of

μ is induced after supersymmetry breakdown and whether this value is phenomenologically acceptable.

In fact these “stringy zeros” of certain couplings render the discussion of the low-energy limit more difficult than previously thought. We can no longer rely on the fact that all terms allowed by the symmetries are actually present. We have to convince ourselves through a string calculation whether such terms exist or not. This point is particularly important for the discussion of the mass matrix of quarks and leptons. Here it still has to be checked whether all the terms allowed by discrete and gauge symmetries are really available for the generation of quark and lepton masses as well as the mixing angles [17] [18] .

All in all, however, we think that the observation of the “stringy zeros” due to selection rule 4 could be very helpful. It teaches us to rethink the notion of naturalness in the framework of string theory. Ultimately it might give us a tool to read off from a low-energy effective Lagrangian whether string theory rules are acting that could not be understood in the framework of the low-energy theory. At the moment we cannot yet pinpoint a special parameter that could serve as a test, because the breakdown of supersymmetry and its effect on the “stringy zeros” has to be better understood. One should also try to obtain a better understanding of rule 4, which at the moment has its only transparent interpretation in the geometric language of orbifold compactification.

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