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GLUON CONDENSATE AND POMERON STRUCTURE

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A B S T R A C T

The gluon condensate in the non-perturbative QCD vacuum is characterized by a mass M_c that measures its magnitude, and a correlation length a . The phenomenological approach to the Pomeron introduces the strength β_0 of its coupling to quarks and a mass scale μ_0 that controls its form factor. We relate M_c and a to β_0 and μ_0 by modelling Pomeron exchange as the exchange of a pair of non-perturbative gluons. We find that the value required for M_c is consistent with other recent determinations.

1. - INTRODUCTION

Phenomenological Pomeron exchange gives a good description of all high energy hadronic processes that are governed by the long-range part of the strong interaction. The Pomeron is responsible for most of the total cross-section in hadron-hadron collisions, for small t elastic scattering and for diffractive dissociation. By analyzing data for these, we have established [1] that the Pomeron couples to quarks rather like a $C = +1$ isoscalar photon. The strength β_0 of this coupling is determined from total cross-section data. Pomeron exchange also governs the small x limit of νW_2 , and from this we know [2,3] that the coupling of the Pomeron to quarks is not quite pointlike: it has a form factor that involves a mass scale μ_0 of the order of 1 GeV.

The object of this paper is to demonstrate the relationship between the phenomenological parameters β_0 and μ_0 , and fundamental quantities of non-perturbative QCD, namely the magnitude of the gluon condensate [4], M_c^4 , and its correlation length, a , in the vacuum.

It is generally agreed [5] that, to a good approximation, Pomeron exchange is nothing but the exchange of a pair of gluons. As it concerns the long-range part of the strong interaction, the gluon exchange is surely non-perturbative. Using the gluon condensate in the vacuum as their starting point, Landshoff and Nachtmann (LN) have shown [6] how the exchange of a pair of non-perturbative gluons can reproduce the observed experimental features of Pomeron exchange in hadron-hadron reactions. In addition, they derived a relation for β_0 in terms of M_c and a :

$$\beta_0 = \frac{1}{6} M_c^4 a^5 \quad (1.1)$$

Purely hadronic reactions usually do not involve the form factor of the Pomeron, because the hadronic wave functions constrain the participating quarks to be near their mass shells. In order to investigate the Pomeron's form factor, and to obtain a relation for μ_0 , we consider a reaction in which the Pomeron couples to a quark that is far off-shell. We choose the process $\gamma^*p \rightarrow Vp$, where V is some vector meson that is well described by a non-relativistic wave function. This is the simplest process in which to compare the phenomenological approach to the Pomeron's form factor with that obtained from the two-gluon-exchange picture.

We have found previously [2] that for the particular case $\gamma^*p \rightarrow \rho p$ the phenomenological approach gives a rather good description of data. In this approach, the reaction is described by Fig. 1. Of the two quark lines that couple to the upper end of the Pomeron, one is constrained to be near-shell by the wave function of the meson V . The other is off-shell by an amount that varies with the Q^2 of the

initial virtual photon. We have found [2,3] that the simplest assumption that agrees with experiment is to introduce in the Pomeron's coupling a factor

$$\frac{\mu_0^2}{\mu_0^2 + m^2 - \ell^2} \quad (1.2)$$

for each off-shell quark leg ℓ . Here m is the quark mass, so that the factor is 1 for an on-shell leg. We applied this assumption to the process $\gamma^*p \rightarrow pp$, to the small x limit of νW_2 , and to the charm quark structure function of the proton, and found good agreement with data when μ_0 is chosen just a little greater than 1 GeV.

In the two-gluon-exchange model of Pomeron exchange, Fig. 1 is replaced by Fig. 2. LN showed that, when the γ^* is replaced by a hadron of radius $R \gg a$, the third graph in Fig. 2 is unimportant. However, a virtual photon with large Q^2 effectively has very small radius, so in this case the third graph is important. It tends to cancel the first two graphs, and we find that the combined effect of the three graphs is almost exactly the same as Fig. 1 calculated with the form factor (1.2).

We also find that μ_0 is proportional to $1/a$. The value of the constant of proportionality depends on the behaviour of the non-perturbative correlation function in the vacuum at short distance. If it is gaussian in shape, we obtain

$$\mu_0 = \frac{(4\pi^3)^{1/10}}{a} \quad (1.3)$$

For other reasonable choices of the shape, there is an extra factor on the right-hand side that is less than 1.

Eliminating a between (1.1) and (1.3), we have

$$M_c = \left\{ \frac{3\beta_0 \mu_0^5}{\pi^{3/2}} \right\}^{1/4} \quad (1.4)$$

In the next section we review the LN model. In Section 3 we evaluate M_c , and show that other recent estimates [7-11] are in a range of values consistent with ours, and investigate the connection between the correlation length in the vacuum and the mass scale μ_0 . Section 4 contains a brief discussion and summary of our results.

2. - THE LN MODEL

The starting point of the LN model [6] is that the vacuum consists of a gluon condensate, such that

$$\langle 0 | g^2 G_{\mu\nu}(0) G^{\mu\nu}(0) | 0 \rangle = M_c^4 \quad (2.1)$$

is non-zero [4]. For different arguments of the two field-strength tensors G this becomes

$$\langle 0 | g^2 T G_{\mu\nu}(x) G^{\mu\nu}(0) | 0 \rangle = M_c^4 f(x^2/a^2) \quad (2.2)$$

with $f(0) = 1$. Since f is a dimensionless function, this relation introduces some fixed length a , which may be thought of as a correlation length for the gluon field in the vacuum.

In the correct, non-Abelian gauge theory, the left-hand side of (2.2) is gauge-dependent. This is one of several severe technical problems that one encounters in trying to incorporate colour in the LN model. We do not attempt to solve this important problem here, but continue to make the same approximation as LN and treat the gluons as Abelian. However, we do give the quarks colour.

If one calculates (2.1) in perturbation theory, the answer is divergent, and by definition this divergent part is subtracted off [4]. A similar subtraction must be made in (2.2); as we shall discuss in Section 4, at the practical level there is some ambiguity about how to do this. We work with a gluon propagator that has the perturbative contribution subtracted off at short distance, but is unsubtracted at long distance. This means that in momentum space it goes to zero more rapidly than $1/k^2$ at large k^2 . At $k^2 = 0$ nothing is subtracted off and the propagator is assumed to be finite. This reflects the property that, being confined, gluons do not propagate through long distances, and LN showed that it is a necessary requirement in order to obtain the quark counting rule for total cross-sections.

We work in a Feynman-like gauge, so that the subtracted propagator is $g^{\mu\nu} D(k^2)$. Write

$$g^2 D(k^2) = -\frac{1}{6} M_c^4 a^6 F(a^2 k^2) \quad (2.3)$$

where F is a dimensionless function [with $F(0)$ assumed to be finite]. Then the relation (2.1) is

$$\frac{1}{16\pi^2} \int_0^\infty dz z^2 F(-z) = 1 \quad (2.4)$$

which requires $F(-z)$ to go to zero faster than z^{-3} as $z \rightarrow \infty$. In order to fix the definition of a , LN also imposed

$$\frac{1}{8\pi} \int_0^\infty dz [F(-z)]^2 = 1 \quad (2.5)$$

LN showed that the model reproduces correctly the low p_T phenomenology of hadron-hadron collisions. The strength β_0 of the coupling of the Pomeron to light on-shell quarks is given by [6]

$$\beta^2(t) = \frac{\beta_0^2 a^2}{8\pi^2} \int d^2 k_T F(-a^2(\underline{k}_T - \frac{1}{2}\underline{q}_T)^2) F(-a^2(\underline{k}_T + \frac{1}{2}\underline{q}_T)^2) \quad (2.6)$$

where $q_T^2 = -t$ and $\beta_0 = M_c^4 a^5 / 6$ as is quoted in (1.1). Because of (2.5), at $t = 0$ we have $\beta^2 = \beta_0^2$.

The shape of the function F is not known, so for purposes of illustration we shall use a form that makes the integral (2.6) simple to perform:

$$F(-z) = A e^{-\lambda^2 z} \quad (2.7)$$

Then A and λ are determined from (2.4) and (2.5); in particular

$$\lambda = (4\pi^3)^{-1/10} \quad (2.8)$$

From (2.6) and (2.7),

$$\beta^2(t) = \beta_0^2 e^{\frac{1}{2} a^2 \lambda^2 t} \quad (2.9)$$

In the simple LN model the quark-quark scattering amplitude is just $\gamma \cdot \gamma \beta^2(t)$. However, experiment requires [1] Pomeron exchange to behave as $\gamma \cdot \gamma \beta_0^2 (s/s_0)^{\alpha(t)-1}$, with

$$\begin{aligned} \alpha(t) - 1 &= \varepsilon + \alpha' t \\ \varepsilon &= 0.086 \quad \alpha' = 0.25 \text{ GeV}^{-2} \end{aligned} \quad (2.10)$$

This Regge behaviour arises from t -channel iterations of some ladder-like structure. The constant s_0 is not well-determined, either theoretically or from data [1]. However, we may absorb in $\log s_0$ the quantity $\frac{1}{2} a^2 \lambda^2 / \alpha'$, so that the exponential in (2.9) no longer appears explicitly.

Most of our subsequent work will concern $t = 0$, where the energy dependence of the amplitude is just $(s/s_0)^\epsilon$. The experimental value of ϵ is small (we have shown recently [12] that it is mostly attributable to light-quark insertions in the Pomeron). That is, at least at $t = 0$ the ladder-like insertions have only a small effect on the energy dependence. We shall assume that they similarly have small effect on the coupling strength β_0 , so that we may compare its experimental value with the calculations of the LN model.

3. - MAGNITUDE OF THE GLUON CONDENSATE AND FIELD CORRELATION LENGTH

We consider the process $\gamma^*p \rightarrow Vp$, which is the simplest reaction in which to study the non-pointlike structure of the Pomeron. We treat the Pomeron as the exchange of two non-perturbative gluons, and evaluate the sum of the three graphs of Fig. 2, where the lower fermion line is one of the valence quarks of the proton. The objective is to compare the result with that of the phenomenological calculation described by Fig. 1.

We make the comparison for the simplest case, where the vector meson V is described by a non-relativistic wave function, so that in the integration round the upper loops of the graphs of Fig. 2, the dominant contribution occurs when each of the two quarks that form the vector meson takes half its four-momentum. The vertex can be normalized [13] in terms of the decay constant f_V that describes the decay $V \rightarrow e^+e^-$.

We quote results from the square root of the appropriate trace of the square of the amplitude. Figure 1 gives [2]

$$\frac{2\sqrt{6} m e_q f_V \beta_0^2}{\frac{1}{2}Q^2 + 2m^2 - \frac{1}{2}t} (\epsilon \cdot \epsilon_V) s \left(\frac{s}{s_0} \right)^{\alpha(t)-1} \frac{\mu_0^2}{\mu_0^2 + (\frac{1}{2}Q^2 + 2m^2 - \frac{1}{2}t)} \quad (3.1)$$

Here m is the quark mass and ϵ, ϵ_V are the polarization vectors of the virtual photon and the meson. For the sum of the three graphs of Fig. 2, there is no Regge factor $(s/s_0)^{\alpha(t)-1}$ and the last factor is replaced by

$$\frac{a^2}{8\pi^2} \int d^2k_T \frac{2k_T^2 + \frac{1}{2}t}{2k_T^2 + \frac{1}{2}t + (\frac{1}{2}Q^2 + 2m^2 - \frac{1}{2}t)} \times F(-a^2(k_T - \frac{1}{2}q_T)^2) F(-a^2(k_T + \frac{1}{2}q_T)^2) \quad (3.2)$$

The similarity between the two results is striking. In (3.1) the last factor is just the phenomenological form factor (1.2), while (3.2) results from a cancellation among the three graphs of Fig. 2

If we specialize to $t = 0$ and large Q^2 , we obtain, by comparing (3.1) and (3.2),

$$\mu_0^2 = \frac{1}{4\pi a^2} \int dz z \left[F(-z) \right]^2 \quad (3.3)$$

With the particular choice (2.7) for F , we find the result quoted in (1.3):

$$\mu_0 = \frac{1}{\lambda a} = \frac{(4\pi^3)^{1/10}}{a} \quad (3.4)$$

which, with (1.1), leads to the relation (1.4) for M_c in terms of the experimentally measurable quantities β_0 and μ_0 .

In order to test the sensitivity of (1.4) to the choice of F , we have calculated it also for

$$F(-z) = \frac{A}{(1 + \lambda^2 z/n)^n} \quad (3.5)$$

The values of A and λ are again determined from (2.4) and (2.5), with now

$$\lambda = \left\{ \frac{4\pi^3 [(n-1)(n-2)(n-3)]^2}{n^5 (n-\frac{1}{2})} \right\}^{-1/10} \quad (3.6)$$

while (3.3) gives

$$\mu_0 = \frac{1}{\lambda a} \sqrt{\frac{n}{n-1}} \quad (3.7)$$

Hence (1.4) becomes

$$M_c = \left[\frac{3\beta_0 \mu_0^5}{\pi^{3/2}} \right]^{1/4} \left[\frac{(n-\frac{1}{2}) (n-\frac{3}{2})^3}{(n-2)^2 (n-3)^2} \right]^{1/8} \quad (3.8)$$

Because we are requiring $F(-z)$ to be finite at $z = 0$, and because its normalization is fixed by (2.4) and (2.5), the main effect of changing n is to change the behaviour at large $z = a^2 k^2$. That is, the value of n reflects the short-distance correlation in the gluon condensate. For the integral (2.4) to be convergent $n > 3$. We assume that n must be an integer, so that $\infty > n \geq 4$ and the factor in square brackets in (3.8) lies between 1 and 1.4. When $n = \infty$, the form (3.5) for F is just the exponential (2.7).

The experimental values of β_0 and μ_0 are [1,3]

$$\beta_0 = 2.0 \text{ GeV}^{-1} \quad \mu_0 = 1.1 \text{ GeV} \quad (3.9)$$

so that M_c varies between 1150 and 1600 MeV. This is somewhat larger than the value [4,14,15] quoted by LN, but it is very much in line with more recent evaluations. Marrow and Shaw [7] have argued forcibly that the earlier values are much too small, and by fitting accurate charmonium data they find $1.14 < M_c < 1.29$ GeV. On the basis of a non-relativistic potential model, Bell and Bertlmann [8] conjectured that the moment procedures used in the early evaluations underestimate the gluon condensate M_c^4 by a factor of about 3. Similar features are apparent [9] in two-dimensional QCD. More recently, Bertlmann et al. [10] have analyzed the ρ -meson channel using FESR's and estimate that the gluon condensate is a factor 2 to 5 larger than the original "standard" value. Finally, Dominguez and Sola [11] have estimated the two-gluon condensate using recent data on the semi-leptonic τ decays $\tau \rightarrow \nu_\tau + n\pi$. They conclude that these data only constrain the condensate magnitude to lie in the range from the early "standard" value to three times this.

With $\infty \leq n \leq 4$, the values of λ in (3.6) lie between 0.6 and 1.0. Hence, from (3.4) with $\mu_0 = 1.1$ GeV we obtain values of the correlation length a between 0.3 and 0.2 fm. Attempts in the literature to determine the correlation length are much less numerous [16,17] than evaluations of the magnitude of the gluon condensate, and there is no general agreement as to its precise definition. To obtain an independent check on our result, we use the relationship established by Dosch [17] for the string tension:

$$\sigma = \frac{\pi^2}{18} \frac{M_c^4}{4\pi^2} \int_{-\infty}^{\infty} d\tau \int_0^{\infty} dr f\left(\frac{-\tau^2 - r^2}{a^2}\right) \quad (3.10)$$

Here f is the correlation function of the gluon condensate in the vacuum, defined in (2.2). It is normalized to 1 at $\tau = r = 0$ and, up to a constant factor, is [6] the Fourier transform of $k^2 F(a^2 k^2)$. With the simple choice (2.7) for F ,

$$f\left(\frac{-\tau^2 - r^2}{a^2}\right) = \left\{1 - \frac{\tau^2 + r^2}{8\lambda^2 a^2}\right\} \exp\left\{-\frac{\tau^2 + r^2}{4\lambda^2 a^2}\right\} \quad (3.11)$$

and hence

$$\sigma = \frac{\pi}{72} \lambda^2 a^2 M_c^4 = \frac{1}{72} \left(\frac{\pi}{2}\right)^{2/5} a^2 M_c^4 \quad (3.12)$$

If we take the canonical value $\sigma = (2\pi)^{-1} \text{ GeV}^2$, then for $M_c = 1.15$ GeV we obtain $a = 0.46$ fm. Compostrini et al. [16] obtain an appreciably smaller value for the correlation length, but their definition differs from ours and we are unable to relate it to a . We can conclude only that our range of values for a , 0.2 to 0.3 fm, is sensible. Note that a small value for a is important for the consistency of the LN model: it is needed [6] to explain why the Pomeron couples only to a single

quark at a time in diffractive processes involving hadrons composed of light quarks, so giving the additive quark rule for total cross-sections.

Consider finally (3.2) for large Q^2 , and $t = 0$. Then only the exponential choice for F allows a straightforward analytic evaluation of the integral, with the result that μ_0^2 acquires an explicit t dependence and must be replaced by

$$\mu^2 = \mu_0^2 + \frac{1}{2} t \quad (3.13)$$

That is, μ decreases slowly as $-t$ increases.

4. - DISCUSSION AND SUMMARY

It is clear that a lot of work still remains to be done. Most notably, the LN model is an Abelian model and we cannot assess what effect the inclusion of colour (assuming it to be possible) will have on our results. Also the LN model is too simple to display the correct Regge features of Pomeron exchange. We have previously discussed [12] the importance of quark loop insertions in the two-gluon exchange, but gluon loops must also be considered. The work that has been done on this so far (for example, Ref. [18]) has dealt only with perturbative gluon exchange, which is surely not correct. The LN model cannot investigate this while it remains Abelian.

It is surely correct that the gluon exchange should be predominantly non-perturbative, as Pomeron exchange is a long-range effect. Indeed, Richards [9] has demonstrated that if the Pomeron were to be generated from perturbative gluons its coupling would not even factorize, and so would not agree with experiment. Our calculations in this paper have been in terms of the subtracted propagator $D(k^2)$, which has the perturbative contribution removed from it. It is this subtracted propagator that is normalized in terms of the gluon condensate M_c^4 [Eqs. (2.1), (2.3) and (2.4)]. Nevertheless, the expressions (2.6) and (3.3) for β^2 and μ_0^2 should have added to them contributions from the large- k^2 perturbative tail. In order to avoid conflicting with experiment such a contribution, at least in the case of β^2 , should not be large.

We may check that this is plausible; a precise calculation is not possible because of the ambiguity in the procedure for the subtraction of the perturbative propagator from D . The perturbative contributions are (at $t = 0$)

$$\begin{aligned}
 (\beta^2)_{\text{pert.}} &= \frac{1}{8\pi} \int dk_T^2 g^4(k_T^2)/k_T^4 \\
 (\beta^2 \mu_0^2)_{\text{pert.}} &= \frac{1}{4\pi} \int dk_T^2 g^4(k_T^2)/k_T^2 \quad (4.1)
 \end{aligned}$$

where $g(k_T^2)$ is the usual perturbative running coupling constant. In order that these integrals may make sense, some lower limit κ^2 must be introduced (and it will be recalled that, in any case, we said in Section 2 that we make the subtraction from D only at large k^2).

The choice of κ is somewhat arbitrary, as it depends both on the form taken for the non-perturbative propagator and the value of Λ (as the two propagators must be matched in some sense) but it is certainly of the order of a few GeV. Choosing $\kappa = 4$ GeV and $\Lambda = 100$ MeV, then the perturbative contribution to β^2 is about $\frac{1}{2}\%$. In writing the second integral in (4.1) we have assumed that Q^2 is so large that the $\frac{1}{2}Q^2$ in the denominator in the integral (3.2) dominates over $2k_T^2$. Because of the very slow convergence of the integral, this actually requires Q^2 to be very, very large, so that in practice we should replace the second relation in (4.1) with

$$\frac{(\beta^2 \mu_0^2)_{\text{pert.}}}{\frac{1}{2} Q^2} = \frac{1}{4\pi} \int dk_T^2 \frac{g^4(k_T^2)}{k_T^2 (2k_T^2 + \frac{1}{2} Q^2)} \quad (4.2)$$

We deduce that at $Q^2 = 20$ GeV² the perturbative correction to μ^2 is some 3%, while at $Q^2 = 1000$ GeV² it has risen to more than 20%. Of course, these results can only be indicative since they depend on the choice of κ^2 , but they do imply that our results are not too sensitive to this particular uncertainty.

Despite these outstanding questions, we find the success of our calculations somewhat remarkable. We have demonstrated that the LN [6] model is consistent with what we know about the coupling of the Pomeron to off-shell quarks, and we have justified our previous [2,3] ansatz for the form of the Pomeron form factor. Using the phenomenological values of the strength β_0 of the Pomeron's coupling to quarks and the mass scale μ_0 that controls the Pomeron form factor, we find values for the magnitudes of the two input quantities to the LN model. These are the magnitude of the gluon condensate, which we find to be in good agreement with other recent determinations, and the correlation length in the vacuum, which has a sensible value.

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FIGURE CAPTIONS

- Fig. 1 Deep inelastic production of a vector meson by Pomeron exchange. The quark leg indicated with an arrow is far off-shell.
- Fig. 2 Two-gluon exchange in deep inelastic production of a vector meson. The dotted lines with the black spots denote the non-perturbative gluon propagator.

- Fig. 1 -

- Fig. 2 -