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ANOMALIES IN WITTEN'S FIELD THEORY OF THE NSR STRING

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ABSTRACT

We show that Witten's action for the NSR superstring on the space of non-smooth vectors generated by SUSY transformations depends on regularization that demonstrates the presence of anomalies in the theory. The necessity of choosing the regularization is connected with the appearance of the product of picture-changing operators at coincident points. Using one of the possible types of regularization, we get the minimal formulation involving truncated fields, and adopting another one, we get the scheme implied in Witten's paper.

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1. - INTRODUCTION

In our previous paper we have demonstrated that the free R-action in Witten's NSR-superstring field theory [1] coincides with the minimal R-action [2-4] and moreover the space-time supersymmetry transformations are the same [5]. This equivalence was proved for the space of regular (smooth) state vectors [5]. The natural conjecture arising from those equivalences is that both free and interacting theories completely coincide (as regards the interaction in the minimal formulation, see Ref. [4]). In this paper, we shall show that this is not the case. The explanation of this will be connected with the fact that Witten's action is not uniquely defined on non-smooth state vectors.

The origin of the ambiguity that arises when extending Witten's action onto singular states is the necessity of choosing the regularization for products of picture-changing operators $X(z)$, $Y(z)$ in coincident points. In this paper, Witten implicitly assumes a regularization admitting everywhere the key property $X(z)Y(z) = 1$. To obtain the minimal action one has to adopt another type of regularization based on the point-splitting in the operator $Y(z)$. So, on the space of singular vectors we shall reveal that the theory depends on the type of regularization chosen, i.e., we shall demonstrate the presence of an anomaly in the theory.

One is forced to deal with singular states (their precise description will be given in what follows) by the explicit form of the SUSY transformations in Witten's theory. In the field theory of the bosonic string, where the set of axioms ensuring the gauge invariance holds only on the space of smooth vectors, the careful separation of the class of singular states also plays an important rôle. These axioms do not hold for a broader class of states, e.g., there are anomalies in the associativity law [6].

The action in Witten's NSR-superstring field theory has the form

$$S = S_0 + S_{int} = \oint A \star Q A + \frac{2}{3} \oint A \star A \star A \quad (1.1)$$

$A = (a, \psi)$, with the quadratic action

$$S_0 = S_{Ns}(a) + S_R(\psi) = -(a, Q_{Ns} a) + (\psi, Y(z) Q_R \psi), \quad (1.2)$$

where $Y(z)$ is the inverse picture-changing operator [1], $z = e^{i\pi/2}$. The action S_0 is uniquely defined for all a and ψ from the space $\mathcal{M}$ of smooth vectors.

The supertransformations are the following:

$$\delta a = W\psi, \quad (1.3a)$$

$$\delta\psi = X(z)W a, \quad (1.3b)$$

with $X(z)$ being the picture-changing operator $X(z) = X_1(z) + X_2(z) + X_3(z) = e^\phi \mathcal{F} + \partial e^\chi + b e^{2\phi} \partial e^{-\chi}$ and W is the covariant fermion-emission vertex [7].

In Ref. [5], it was shown that for every smooth ϕ , S_R can be equivalently rewritten in terms of two-component truncated vectors (we mean the dimension in the space of the superghost's zero modes) $\rho = (\rho_0, \rho_1)$ and the simple change of arguments

$$\Psi = H^{-1} \rho, \quad \Psi = (\Psi_0, \Psi_1), \quad H = \begin{pmatrix} 1 & \chi F \\ 0 & 1 \end{pmatrix}$$

casts S_R into the form of $S_{\min}(\Psi_0, \Psi_1)$ [2,5]. Moreover, the supertransformations for (1.2) and for the action with $S_R = S_{\min}$ coincide while both Ψ_0 and Ψ_1 are supposed to be smooth vectors.

It is natural to consider all vectors with explicit dependence of the point z and/or with an infinite number of components as singular ones. In this sense we say that the supertransformations (1.3b) generate singular states which are the potential source of anomalies. In fact, the supertransformations (1.3) do not generate the true symmetry of the action on singular states as the equality $XY = 1$ fails to hold, due to the possible generation of Schwinger terms on these states [5]. Moreover, the action S_R itself is not well defined on the variation

$$S_R(XW a) = (XW a, Y Q_R XW a) \quad (1.4)$$

due to the singular nature of the product $X(z_1) \cdot X(z_2) \sim (z_1 - z_2)^{-2} \cdot 0(z_1) + \dots$ and, what is more dramatic, the NS action calculated on the second variation $S_{NS}(W X W a)$ seems to be meaningless.

Note that to give a meaning to the interacting term one has also to deal with the same issue of defining the bilinear form

$$f = (X \tilde{\phi}, Y X \phi) \quad (1.5a)$$

because

$$S_{int}(a, 0) = \oint a * a * a = (Xa, YX(a * a)).$$

Both the expression (1.4) and the general bilinear form (1.5) can be defined by means of two different methods. Following the first one it is necessary to calculate the double limit:

$$J^{(I)} = \lim_{z'' \rightarrow z} \left(\lim_{z' \rightarrow z} (X(z'') \tilde{\Phi}, Y(z') X(z) \Phi) \right), \quad (1.5b)$$

i.e., the operator product $Y(z)X(z)$ is calculated neglecting potential singularities appearing when the points z'' and z coincide. For every regular Φ this method gives

$$J^{(I)} = (X(z) \tilde{\Phi}, \Phi). \quad (1.5b')$$

Just this recipe is implied in Ref. [1] and this gives a reason to call it the Witten formulation.

The interacting theory in Ref. [8] is built on the base of this prescription, which results in assigning the following value to the R-action on the fields $\Psi = X\Phi$, $\Phi = Wa$:

$$S_R = (\Phi, XQ\Phi).$$

If the supersymmetry law for these fields is chosen as follows

$$\delta a = WX\Phi, \quad \delta \Phi = Wa,$$

it can be seen that these transformations do not generate a real symmetry, since the value of the action on the first variation $S_{NS}(W X\Phi)$ would be ill defined. This fact is a reflection of the singularity that appears in the value of S_{NS} on the second variation $\delta^2 a = W XWa$, as pointed out above.

The alternative choice is to take into account the factorization property of $Y(z)$ operating in the space with vacuum ghost number $\frac{1}{2}$ *)

$$Y(z) = Y(z) | 0 \rangle \rangle g^{-1}(z) \langle \langle 0 | Y(z). \quad (1.6)$$

*) see Ref. [5] for notations.

The essence of this representation is to treat the operator $Y(z)$ acting from "infinite-dimensional" space into "infinite-dimensional" space as the product of two operators acting from "infinite-dimensional" space to "two-dimensional" space and from "two-dimensional" space to "infinite-dimensional" space respectively. Equation (1.6) is valid in some space of finite vectors.

If one is to follow the second way of defining the form (1.5a), the variables in two Y 's standing to the right and to the left of $g^{-1}(z)$ have to be split. The matrix $g^{-1}(z)$ is also z -dependent and to take this dependence properly into account, we propose to factorize g^{-1} itself

$$Y(z, z') = Y(z|0) h^+(z) (-\delta') h(z') (0|Y(z')). \quad (1.7)$$

As a result we derive the following recipe of the calculation of (1.5a): first the coincident limit for the points in the right-standing product YX should be taken, and after that the same should be done for the left-standing product XY . Adopting this procedure we find that only the first two terms in the t_0 expansion of $\Phi, \tilde{\Phi}$ contribute in the scalar product (1.5a).

The origin of the representation (1.6) is the existence of the non-trivial kernel of $Y(z)$, i.e., the additional fermionic "gauge" invariance of the action S_R . The operator Y is invertible only on transversal fields, i.e., in fact the equality $YX = 1$ holds only on these fields. In particular, the fields of the form $X_i(z)\phi$, $i = 1, 2$ are longitudinal. Defining the bilinear form (1.5a) with X_i instead of X ($i = 1, 2, 3$) by means of an equation like (1.7) makes it possible to omit all the longitudinal components in the action.

Hence the point of view we are going to advocate in this paper is the following. The transformations (1.3) uniquely define the symmetry of the action (1.2) on a space of smooth vectors ϕ only. To clarify the question of their algebra it is necessary to enlarge sufficiently the space so as to include the space $\mathcal{M}_X$ generated by the first-order variations. The symmetry of the action crucially depends on the way chosen to define its value on this extended space although this symmetry has to coincide with transformations (1.3) on smooth vectors. In other words, in order to describe correctly the algebra of supertransformations, one has first to choose the space on which these transformations act, as well as to extend properly the definition of the action on the space. Let us emphasize that conserving the form of SUSY transformations (1.3) for non-smooth vectors forces us to abandon the usual property of symmetry transformations, that the value of the action is well defined on the second variation.

If the second way of extension of S_R on the fields from $\mathcal{M}_X$ is used then one can restrict oneself to dealing with truncated fields only. This provides a way to overcome the problem of definition of the action on the second variation. Besides, it also seems that no problem arises with the second-order gauge invariance in contrast with the first way of definition (1.5) where we have to treat the singular expression $\delta_{g^2 S_{int}}(a, 0) = (Xa, X(a * a * \lambda))$, and λ is the gauge parameter.

So the purpose of this paper is to demonstrate the existence of two different ways of extending the definition of Witten's action S_R on the space $\mathcal{M}_X$. When one follows the first way which is implied in [1] and which means retaining only one term equal to 1 in the operator product $X(z)Y(z)$, the resulting theory does not coincide with the minimal one. Adopting the second way based on the factorized representation (1.6) for $Y(z)$ makes it possible to describe the whole theory in terms of truncated fields. This also leads to a well-defined value of the action on the second variation and apparently allows us to build a gauge-invariant NSR-string theory without contact terms.

One of the attractive features of the minimal formulation is the simple supersymmetric gauge fixing: $a_1 = 0$ and $\Psi_1 = 0$. In the non-minimal scheme the gauge $a_1 = 0$ and $\phi_n = 0$, $n \geq 1$ does not survive under SUSY transformation (1.3).

The paper is organized as follows. In Section 2 we recall the representation of Witten's action on smooth vectors being obtained earlier [5]. In Section 3 we discuss two ways of defining the action on the space $\mathcal{M}_X$ and in Section 4 we discuss the features of the SUSY algebra in those two schemes.

2. - WITTEN'S R-STRING ACTION ON SMOOTH VECTORS

The operator $Y(z)$ has a non-trivial kernel [5]. The vectors of the form $t^2(z)\psi$ and $c(z)\psi$ with smooth vector ψ belong to the $\text{Ker } Y(z)$. This property gives the possibility of making the action on smooth ψ depend on two-component fields only. We form the two-component field $\psi^{tr} = \rho_0 |0\rangle + \rho_1 |1\rangle$ as the solution of the equation $Y\psi = Y\psi^{tr}|0\rangle$. The formulae which give the expression for ρ_0 and ρ_1 in terms of the components of ψ can be found in [5].

It is natural to introduce the operator of truncation $\mathcal{T}$ acting on smooth vectors ψ as: $\psi = \psi^{tr}$, with the components of ψ^{tr} being ρ_1 and ρ_1 . Due to the equality $(\chi, Y\psi) = (\chi^{tr}, g\psi^{tr})$, where $g(z) = |0\rangle\langle 1| + |1\rangle\langle 0| - 2t(z)|1\rangle\langle 1|$, the factorization

$$Y = \mathcal{F}^+ g \mathcal{F} \quad (2.1)$$

on the smooth vectors takes place [5].

Owing to this property of $Y(z)$, Witten's action after the change of variables $\Psi = H\phi$ can be rewritten in a minimal form (Eq. (9) in [5]).

In terms of the matrices H and g the factorized representation for Y [Eq. (1.6)] is rewritten in the form (1.7) where $h = H^{-1}g^{-1}$.

Let us recall that all vectors ϕ^{tr} derived by the procedure described above should have components with explicit dependence on z . This makes it natural to expect that the XY product is not equal to unity on such singular states. Moreover, we have for every such vector ϕ^{tr}

$$\lim_{z'' \rightarrow z} \left(\lim_{z' \rightarrow z} X(z'') Y(z') \phi^{tr}(z) \right) |0\rangle = \psi. \quad (2.2)$$

So, the operator $X(z)Y(z)|0\rangle$ acting on the vector $\phi^{tr}(z)$ gives the original vector ϕ . The possibility to reconstruct the complete multi-component vector ϕ from the two-component one stems from the singular nature of the ϕ^{tr} components.

One can give the transparent but not rigorous derivation of Eq. (2.2) by using Eq. (1.6) (the rigorous proof was done in [5]). Really, according to (2.1), $\phi^{tr}(z)$ is defined as

$$\psi^{tr}(z) = g^{-1}(z) ((0|Y(z)\psi$$

that casts the left-hand side of (2.2) into the form

$$X(z'') Y(z') |0\rangle g^{-1}(z) Y(z) \psi. \quad (2.3)$$

It is obvious that the limit for the product $Y(z')|0\rangle g(z)((0|Y(z)$ when $z' \rightarrow z$ exists and is equal to $Y(z)$. Hence, we obtain on the left-hand side of (2.2) the expression $\lim_{z'' \rightarrow z} X(z'') Y(z) \phi$ where we can replace the XY product by unity as we have supposed the vector ϕ to be regular.

It is instructive to note that taking the limits in the formulae in another way we shall derive a different result. Really

$$\lim_{z'' \rightarrow z} \left(\lim_{z' \rightarrow z} (X(z'') Y(z')) \psi^{tr}(z) |0\rangle \right) = \psi^{tr}(z) |0\rangle. \quad (2.4)$$

Just this point gives the possibility of having two different actions for Witten's R-string theory on singular vectors.

The main conclusion derived from the above consideration is the following. If we restrict the states ϕ to be smooth, the theory can be rewritten equivalently in terms of $\phi^{\text{tr}}(z)$. Every ϕ uniquely defines $\phi^{\text{tr}}(z)$ and can be reconstructed from ϕ^{tr} by means of the XY operator, Eq. (2.2).

The question arises whether for each two-component singular field one can find the corresponding "long" field ϕ . To make the question meaningful, we restrict the set of fields. It is natural to demand that $Y(z)$ should be well defined as the bilinear form on these vectors. This leads to admissible singularities like $c(z)$ and $t^n(z)$. It is easy to show that the XY operator transforms the vectors of this type into "long" vectors, but the inverse operation gives in general a two-component vector not coinciding with the initial one:

$$g^{-1}(z) Y(z) X(z) Y(z) \psi(z) - \psi(z) \neq 0. \quad (2.5)$$

In this sense we can say that the set of two-component vectors with the singularities mentioned above is larger than the set of smooth "long" vectors. Following the analogy with QED, the smooth states can be viewed as the transverse ones and the transition to ϕ^{tr} is the special gauge choice that is possible due to the above-mentioned gauge invariance $\phi \rightarrow \phi + \delta\phi$, $\delta\phi \in \text{Ker} Y$.

3. - WITTEN'S ACTION ON NON-SMOOTH VECTORS

3.1 The definition of the quadratic action

We are forced to deal with non-smooth vectors by the form of the SUSY transformation. Namely, the states $X_i(z)\Phi$, $i = 1, 2$ belong to $\text{Ker} Y(z)$, i.e., they are longitudinal. We find it natural to have an action which vanishes on the fields from $\text{Ker} Y$. For example, consider the expression

$$(X_1 \Phi, Y Q X_1 \Phi) \quad (3.1)$$

which contains the product of two $X_1(z)$ in coincident points. Following the analogy with QED we have to define S_R so that (3.1) vanishes. A possible choice is

$$(X_1 \Phi, Y Q X_1 \Phi) = \lim_{z'' \rightarrow z} \left(\lim_{z' \rightarrow z} (X_1(z'') \Phi, Y(z') Q X_1(z) \Phi) \right) = 0.$$

The same answer is found if (3.1) is regularized by means of the point-splitting in $Y(z)$ as defined in Eq. (1.7).

The most serious difficulties arise when we proceed to define the value of S_R on non-smooth vectors having non-trivial intersection with $\text{Ker} Y$ but which do not fully lie in $\text{Ker} Y$. The state Φ gives an example of such a vector. If we define the expression

$$(X_1(z'')\Phi, Y(z')QX(z)\Phi)$$

in the same way as in (1.5b), we get

$$(X_1\Phi, YQX\Phi) = (X_1\Phi, Q\Phi). \quad (3.2)$$

This answer looks rather strange: the longitudinal field gives a non-zero contribution to the action. The QED analogy for this situation would be

$$(A_\mu^L, (g^{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}) \square A_\nu^{tr}) \neq 0$$

which, generally speaking, could be possible for the fields A_μ^L, A_μ^{tr} which have the wrong behaviour at infinity.

The other point that we are going to consider in this section is whether it is possible to truncate non-smooth vectors. In particular, we would like to know if it is reasonable to obtain the SUSY transformation law for truncated fields by a straightforward truncation

$$\delta\psi^{tr} = ((0/g^{-1}YX)W)a. \quad (3.3)$$

This answer is incorrect if one naively interprets the right-hand side as

$$\lim_{z'' \rightarrow z} \lim_{z' \rightarrow z} g^{-1}(z'') Y(z') X(z) W a.$$

We shall show that a careful arrangement of the successive limits does make it possible to define the variation for truncated fields. Namely, we assume ψ to be a smooth field and define $\delta\psi^{tr}$ so as to get the same answer for the δS_R as in Witten's scheme. Let $\delta\psi^{tr}$ be

$$\delta\psi^{tr}(z) = g^{-1}(z) ((0/Y(z')X(z''))W)a, \quad (3.4)$$

whereas the limits $z' \rightarrow z, z'' \rightarrow z$ will be taken in the matrix elements. Indeed, δS_R can be written as

$$\begin{aligned}
 & (\delta\psi^{tr}(z), g(z) Q_g \psi^{tr}(z)) + (\psi^{tr}(z), g Q_g \delta\psi^{tr}(z)) = \\
 & = (g^{-1}(z) Y(z') X(z'') W a, g(z) (Q\psi)^{tr}(z)) + (\psi^{tr}(z), g(z) Q_g g^{-1}(z) Y(z') X(z'') W a) = \\
 & = (W a, X(z'') Y(z') (Q\psi)^{tr}(z)) + ((Q\psi)^{tr}(z), Y(z') X(z'') W a),
 \end{aligned}$$

where we used the identity $(Q\psi^{tr}|0))^{tr} = (Q\psi)^{tr}$, hence we get

$$\begin{aligned}
 \delta S_R & = \lim_{z'' \rightarrow z} \lim_{z' \rightarrow z} [(W a, X(z'') Y(z') (Q\psi)^{tr}(z)) + ((Q\psi)^{tr}(z), Y(z') X(z'') W a)] = \\
 & = (W a, Q\psi) + (Q\psi, W a).
 \end{aligned}$$

It is obvious that (3.4) could be replaced by

$$\delta\psi^{tr}(z) = g^{-1}(z) ((0| Y(z) X(z'') W a) \quad (3.5)$$

with the same result.

We can draw the following conclusion from the above consideration. Vectors from the space $\mathcal{M}_X$ can be correctly described by the truncated vectors if X and Y are associated with two different points, say, z'' and z as in Eq. (3.5). When $z'' = z$, the vectors like $X(z'')\Phi$ are the smooth ones. The limit $z'' \rightarrow z$ can be taken at the end of the calculation of δS_R .

3.2 Anomalies in the first variation of the R-action

The product XY is not equal to unity when acting on non-smooth vectors:

$$\lim_{z'' \rightarrow z} (\lim_{z' \rightarrow z} X(z'') (Y(z') \psi(z))) \neq \lim_{z' \rightarrow z} (\lim_{z'' \rightarrow z'} X(z'') Y(z')) \psi(z) = \psi(z). \quad (3.6)$$

The non-associativity of OPE in (3.6) leads to non-uniqueness in the definition of δS_R on non-smooth states ψ , i.e., to an anomaly. In fact,

$$\delta S_R = (W a, Q_R X Y \psi) + (X Y \psi, Q_R W a), \quad (3.7)$$

where the expression $XY\psi$ is not uniquely defined for states having explicit dependence on z , cf. (3.6).

In particular, for $\psi(z) = t^2(z)\theta$, $\theta \in \mathcal{M}$ we get zero if the limits are arranged on the left-hand side of (3.6), whereas adopting the rule as on the

right-hand side of (3.6) gives a non-zero result. Hence, if a vector $\phi(z)$ belongs to the $\text{Ker} Y$ (states of just that kind are generated by $X_{1(2)} W a$) and if the product of operators is defined as on the left-hand side of (3.6), then the transformation law for NS fields has to be modified.

To compensate for the dependence of δS_R of the order of the limits, one can introduce just the same dependence into δS_{NS} by changing the transformation law of the NS fields in the following way

$$\delta a = W X Y \psi \quad (3.8)$$

and

$$-\delta S_{NS} = (a, Q_{NS} W X Y \psi) + (X Y \psi, W Q_{NS} a) \quad (3.9)$$

Note that on smooth states ϕ , (3.8) gives the same answer as in (1.3).

So, to ensure the symmetry of the free NSR action irrelevantly of the choice of a regularization prescription on states ϕ having explicit z dependence, we have to set the NS variation as in (3.8). With the regularization being fixed so that $XY = 1$, we obtain Witten's formulation.

We can go one step further and propose the following transformation law:

$$\delta \psi = X(z') W a, \quad \delta a = W X(z') Y(z) \psi \quad (3.10)$$

with z' and z being two different points. These transformations also define a symmetry of the action (1.2) but they do not generate an algebra at all. The second variation

$$\delta^2 a = W X(z') Y(z) X(z') W a \quad (3.11)$$

is not in any way defined because of the presence of the singular product $(X(z'))^2$. To get a sensible result for $\delta^2 a$ one must make all the z -s coincide. We can proceed to calculate (3.11) at the coinciding points using different regularization schemes. Witten's scheme yields

$$\delta^2 a = \lim_{z'' \rightarrow z} \lim_{z' \rightarrow z} W X(z'') Y(z') X(z) W a = W X(z) W a. \quad (3.12)$$

When we adopt the factorized representation (1.7) for Y it is natural to find $\delta^2 a$ in the form

$$\delta^2 a = \lim_{z'' \rightarrow z} \lim_{z' \rightarrow z} \lim_{z''' \rightarrow z''} W X(z') Y(z) h^+(z) (-\delta') h(z'') Y(z'') X(z''') W a.$$

Using the above-mentioned formulae for OPE we easily get

$$\begin{aligned} \delta^2 a &= W \begin{pmatrix} -c_0 F |0\rangle \langle 0| - |0\rangle \langle 0| F c_0 & |0\rangle \langle 0| \\ |0\rangle \langle 0| & 0 \end{pmatrix} W a = \\ &= W |0\rangle \langle 0| [-c_0 F |0\rangle \langle 0| + |0\rangle \langle 0| F c_0] W a + W [|0\rangle \langle 1| + |1\rangle \langle 0|] |0\rangle \langle 0| W a. \end{aligned} \quad (3.13)$$

The two different results (3.12) and (3.13) for $\delta^2 a$ indicate the appearance of two different algebras of SUSY, as will become clear in the next section.

4. - SPACE-TIME SUPERSYMMETRY

The supersymmetry of the action (1.2) with S_R being taken in minimal form is well known [2,4]

$$\delta a = [W - W c_0 F |0\rangle \langle 1|] \Psi |0\rangle, \quad (4.1)$$

$$\delta \Psi = (-\delta')^{-1} \langle 0| [W - |1\rangle \langle 0| F c_0 W] a. \quad (4.2)$$

It is obvious that these transformation rules neither bring any singularity nor increase the zero-mode dimension of Ψ . Moreover, they transform any finite state into some finite one.

To compare the transformations (4.1) and (4.2) with (1.3) it is necessary to compare the smoothness constraints for Ψ and ϕ .

Usually, Ψ_0 and Ψ_1 have been assumed to be smooth fields, so the corresponding ρ -field possesses the only singularity in $\check{c}(z)$. The corresponding "long" field ϕ will be

$$\begin{aligned} \Psi &= X(z) Y(z) |0\rangle [(\Psi_0 + \check{c}(z) F \Psi_1) |0\rangle + \Psi_1 |1\rangle] = \\ &= [(\Psi_0 - c_0 F \Psi_1) |0\rangle + \Psi_1 |1\rangle] |0\rangle. \end{aligned} \quad (4.3)$$

We see that ϕ is recast into the form of the "consistently truncated" field. The latter has been proposed in Refs. [3,2] as the building block for R-string field theory.

Inverting the problem we can ask which smooth field ϕ will be transformed into a smooth Ψ , i.e., when the expression

$$H^{-1}(z) g^{-1}(z) ((0|Y(z)\Psi \quad (4.4)$$

will be free from any singularities. It is obvious that such a ϕ has only two components in the t_0 expansion. That gives

$$\begin{aligned} & H^{-1} g^{-1} ((0|Y(z)\Psi = \\ & = H^{-1} ((0|C(z) [\psi_{0,0}|0\rangle + \psi_{1,0}|1\rangle + c_0 \psi_{0,1}|0\rangle + c_0 \psi_{1,1}|1\rangle] |0\rangle) = \\ & = (\psi_{0,0} - \check{c} \psi_{0,1} - \check{c} F \psi_{1,0}) |0\rangle + (\psi_{1,0} - \check{c} \psi_{1,1}) |1\rangle. \end{aligned}$$

To cancel the $\check{c}(z)$ dependence one has to define $\psi_{0,1} = -F\psi_{1,0}$ and $\psi_{1,1} = 0$, which demonstrates that ϕ has to be a "consistently truncated" field again.

The fact that Witten's action for $\Psi = H^{-1}g^{-1}((0|Y\phi$ takes the form of the minimal one leads to the natural conjecture that supertransformations (4.1) and (4.2) are nothing but (1.3) for a special class of fields.

Indeed, by using the reconstruction operator XY we can write down

$$\delta a = WXY\psi^{tr}|0\rangle) = WXYH\Psi|0\rangle).$$

For smooth Ψ this gives

$$\delta a = W[\psi_0|0\rangle + \psi_1|1\rangle - c_0 F \psi_1|0\rangle] |0\rangle),$$

which totally coincides with (4.1).

When the two-component variation $\delta\Psi$ is compared with the multicomponent $\delta\phi$ the main difficulty comes from the explicit singularities of $\delta\phi = X(z)W\alpha$. We have just discussed the same question in Section 3, but now we have some additional information about the smoothness. This allows us to clarify whether it is possible to use a truncated state description for X in the bilinear form $(XW\alpha, YQ\psi)$. Since we suppose the Ψ field to be smooth, both fields ϕ and $Q\phi$ will be "consistently truncated" fields with smooth components. Using Eq. (1.6), we can write down

$$(X\phi, YQ\psi) = (X\phi g^{-1}|0\rangle) ((0|YQ\psi)$$

which can be identically rewritten in the form

$$((- \delta')^{-1} \langle 0 | H^+ Y X \Phi, (- \delta') \langle 0 | H^{-1} g^{-1} Y Q \Psi).$$

The right-hand side state in the scalar product is regular, hence $\langle 0 | Y X$ acts as the reconstruction operator only on elements of the matrix H^+ : $\langle 0 | F_c Y X = - \langle 0 | F_c$ and we get

$$\delta \Psi = (- \delta')^{-1} \langle 0 | [1 - F_c] | 1 \rangle \langle 0 | W a$$

which gives the transformation law (4.2).

Performing the supertransformations (4.1) and (4.2) consecutively we get

$$\begin{aligned} \delta^2 a = W \{ & (|0\rangle \langle 1| + |1\rangle \langle 0|) |0\rangle \langle 0| - \\ & - [c_0 F |0\rangle \langle 0| + |0\rangle \langle 0| c_0 F] |0\rangle \langle 0| \} W a \end{aligned} \quad (4.5)$$

This expression completely corresponds to Eq. (3.13) which explicitly demonstrates that the description of the theory in terms of smooth two-component fields originates from the special regularization procedure in Witten's action, based on the point-splitting in the inverse picture-changing operator $Y(z)$.

5. - DISCUSSION

In this paper, some specific features of Witten's NSR-superstring field theory related to the presence of picture-changing operators $X(z)$, $Y(z)$ in the theory are presented. As we have noted before [5], the inverse picture-changing operator $Y(z)$ serves as the metrics in the quadratic R-string action and the existence of the non-trivial kernel for $Y(z)$ leads to the additional "fermionic gauge invariance". In other words, the true variables for the free R-string appear not to be the vectors of the Fock space F , but the elements of the coset space $F/\text{Ker} Y(z)$. We have seen that fixing the "gauge" in two different ways we obtain two self-consistent form of the free R-string field theory, namely, the first gauge corresponds to the restriction of the theory to smooth vectors and the second one to two-component vectors [5]. Note that this "gauge invariance" is not specific for Witten's theory but appears also when $Y(z)$ is replaced by Yamron's picture-changing operator Y [9,5].

In this paper, we have seen that in close analogy with usual SUSY gauge theories, Witten's space-time SUSY transformation spoils the gauge fixing. So, to

make the theory supersymmetric it is necessary to define it not only on a special section, say smooth vectors, but on the whole space F , especially on the states of the form $X(z)\phi$. This immediately forces one to consider the singular products of picture-changing operators at coincident points. The non-associativity of OPE makes it possible to regularize the products in different manners. In this paper we have discussed two regularization schemes, one corresponding to Witten's theory and the other to the minimal formulation. These schemes produce two different algebras of SUSY transformations.

In conclusion, we would like to make a few remarks about the interacting case. Firstly, pursuing the analogy with QED we have to make sure that the full action does not depend on longitudinal fields. While this question is rather important for quantization, the known explicit form of the R-R-NS vertex does not give a definite answer. Secondly, there are problems with the g^2 -order gauge invariance of Witten's action [10] which make Wendt introduce the contact counter terms à la light-cone string field theory. (The contact terms appear also from other points of view [11,12].) The lack of this invariance jeopardizes the ϕ^3 programme [13]. The origin of this difficulty is the appearance of two X operators at coincident points. Note that the formula for Y , like the factorized representation (1.7), permits us to cancel two X operators instead of one cancelled in Witten's prescription, which gives us a chance to get the g^2 -order gauge invariance in the minimal formulation [4]. The study of these problems is in progress.

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