**THE APPLICABILITY OF PERTURBATIVE QCD
TO EXCLUSIVE PROCESSES**

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ABSTRACT

We re-examine our arguments against the dominance of perturbative QCD over soft, non-perturbative effects in exclusive processes at currently available Q^2 . We continue to find that perturbative contributions are much smaller than soft contributions (which are capable of explaining the data) in the simple test cases of the pion and nucleon electromagnetic form factors. We see no reason why our results should not generalize, and conclude that there is no justification for the continued application of perturbative QCD to exclusive processes.

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1. INTRODUCTION

Since we first sounded a warning [1] against the application of perturbative QCD to exclusive processes [2–5] there have been many developments in both theory and experiment.

The theoretical analysis of such processes has produced results which have radically altered the conventional view [3–5]. Some of the recent developments are:

1) Reanalyses [6–9] of the perturbative calculations revealed an error which reverses, for a given nucleon wave function, the predicted signs of the proton and neutron magnetic form factors. This means, for example, that the asymptotic nucleon distribution amplitudes $\propto x_1 x_2 x_3$ (where x_i is the momentum fraction of quark i) give a value for $Q^4 G_M^n(Q^2)$ which is opposite in sign to the one observed at present. The more general phenomenological ramification of this sign reversal is that ‘non-intuitive’ and very subasymptotic wave functions are required in order to explain the observed nucleon form factors. See Fig. 1 for the analogous pion wave function.

2) Conventional QCD sum-rule methods [10–13] and other methods [14] which avoid the main flaws of the quark-model calculations we presented [1], led to the conclusion, in agreement with us, that both the pion and nucleon electromagnetic form factors were dominated at available Q^2 by non-perturbative contributions.

3) Other sum-rule calculations [8], designed to compute the hadron wave functions directly (or, more precisely, the distribution amplitudes to be defined below), found ‘non-intuitive’ distribution amplitudes of the type required to explain the data in terms of perturbative QCD.

4) Lattice Monte Carlo calculations [15, 16], also designed to ‘measure’ the distribution amplitudes, agreed with the general character of the sum-rule results for the pion distribution amplitude.

5) Model calculations [17, 18], based on an ansatz for boosting rest frame quark-model wave functions to the infinite momentum frame, were able to produce wave functions of the required character. These wave functions are also reported [5] to give small soft contributions. Small soft contributions to the pion form factor F_π have also been found in Ref. [19], and it has been asserted [5] that these calculations supersede ours [1] on the grounds that the former are rotationally invariant.

6) Several empirical studies have explored the requirements on nucleon distribution amplitudes for the perturbative QCD formulae to fit the form-factor data. The authors of Ref. [20] conclude that amplitudes similar to those of Ref. [8] can fit the data with (relatively) small non-perturbative contributions. In Refs. [21] there is a more general fit, allowing ‘non-leading’ contributions corresponding to Pauli (i.e. F_2) form factors. These authors find $F_2^n \gg F_1^n$, in contradiction to the expectations of perturbative QCD.

Theoretical analysis has thus led to the conclusion that exclusive processes are certainly not completely asymptotic in the sense that the hadronic wave functions must, for perturbative QCD to be applicable in any form, be very far from their asymptotic forms. Thus the ‘strong applicability’ of perturbative QCD is ruled out, in that exclusive processes *cannot* be computed in terms of a hadronic distribution amplitude predicted (up to a constant normalization) by the theory. What remains unclear is whether a weaker—but still potentially extremely interesting and powerful—form of applicability remains. In this case the distribution amplitudes for each hadron (i.e. their full functional forms and normalization) must be determined from outside the perturbative approach; but once determined, they could be folded against hard scattering processes (computed in perturbative QCD) to predict high- Q^2 exclusive processes. Matters remain unclear because there is evidence that the data are dominated by soft, non-perturbative processes at high Q^2 , but there is also evidence for distribution amplitudes which would allow perturbative QCD to explain the data.

Fig. 1 : Some pion distribution amplitudes:

- a) the asymptotic amplitude $\propto x(1-x)$;
- b) the amplitude of Ref. [8] $\propto x(1-x)(1-2x)^2$;
- c) a schematic weak-binding amplitude obtained by a free boost of a rest frame harmonic oscillator ground state with $p_T/m = 1/4$;
- d) a schematic 'weak-binding' amplitude obtained by a free boost of a rest frame harmonic oscillator wave function with $p_T/m = 1/2$.

Experiment has also proved to be ambivalent. Recent developments include the following:

1) Experimental tests of the hadron helicity conservation rule predicted by perturbative QCD have been mainly negative. For example, strong spin asymmetries have been seen in pp scattering [22], and observations [23] on $\pi N \rightarrow \rho N$ have shown strong production of ρ mesons via amplitudes forbidden by this rule. Although it is possible that these failures are associated with ‘non-leading effects’ [24], they constitute a *prima facie* case against the use of perturbative QCD in exclusive processes.

2) New heroic measurements of electron–proton elastic scattering [25] have been performed at what may prove to be the highest Q^2 attainable. They show that $Q^4 G_M^p$ has peaked at $Q^2 \approx 10 \text{ GeV}^2$ and is falling slowly. This is, however, consistent with both a slow logarithmic fall-off predicted by perturbative QCD and a slow power-law fall-off predicted by calculations based on the dominance of soft, non-perturbative processes. On the other hand, the data on electron–neutron scattering [26] may, as already mentioned, indicate significant departures from the perturbative predictions [21].

3) New data [27] on $\gamma\gamma \rightarrow \pi^+\pi^-$, K^+K^- and $p\bar{p}$ are not encouraging for perturbative QCD: the meson data disagree with the predictions [28] by a substantial factor; the disagreement with the $p\bar{p}$ predictions [29] is by more than an order of magnitude.*)

Although the trend of these results is perhaps somewhat negative, this must be weighed against the impressive successes of the quark counting rules (which can be given their basis in theory through perturbative QCD [30]; for another point of view, see Refs. [31]), coupled with the possibility that the failures of perturbative QCD may result either from ‘small’ non-leading amplitudes or from Q^2 values being somewhat below those really needed.

Given that the basic issue has not yet been resolved decisively by either theory or experiment, and that there has been a rather radical change in the conventional view of those favouring the use of perturbative QCD in exclusive processes, we have re-examined our arguments against its use. As will be explained in detail below, we find that all our fundamental objections remain fully in force.

2. FIRST OBJECTION: THE UNJUSTIFIED USE OF PERTURBATIVE QCD

In Ref. [1] we showed that simple wave functions suggested either by weak binding or by the asymptotic distribution amplitudes gave values for the pion, and especially the nucleon, form factors which were much smaller than the data. This had been pointed out previously [7]. It is now also clear that such wave functions give the wrong sign for the nucleon magnetic form factors [6–9]. Both these difficulties force one to the conclusion that there is no hope of describing exclusive processes using perturbative QCD unless the nucleon wave functions have rather special properties: they must be very asymmetric in order to reverse the sign of the form factors, and they must have very large intrinsic relative momenta in them to increase the predicted magnitude of the form factors.

In Ref. [1] we considered the possibility of boosting the size of the perturbative contributions to form factors by abandoning ‘intuitive’ wave functions and allowing instead any wave function consistent with general principles. We concluded that it would be very difficult to find a wave function that would serve this purpose. We will repeat our general arguments here and then demonstrate their effect on the use of the various wave functions which have recently been advocated as solutions to the problems of sign and magnitude for the pion and nucleon form factors [5, 8, 17, 20, 21].

As reviewed in Appendix A, form factors can be expressed as convolutions of distribution amplitudes ϕ , which describe the initial and final hadrons, and ‘hard-scattering amplitudes’ T_H

*) Note that earlier data on the unseparated $\pi^+\pi^- + K^+K^-$ channels appeared to agree with the perturbative prediction (see the report in Ref. [5]).

which can be calculated perturbatively for sufficiently large Q^2 . If we consider the lowest-order contributions to T_H for the case of the pion (shown in Fig. 2a), it is intuitively clear that the square of the momentum transfer flowing through the gluons, which is given by $x_2 x_2' Q^2$ for the first twodiagrams and $x_1 x_1' Q^2$ for the second two, must be large in order for perturbation theory to be applicable. More generally, not only Q^2 but also all the invariants (except those proportional to p^2 and p'^2) on which T_H depends must be large, i.e. $x_i x_j' Q^2$ must be large, since otherwise there may be large ‘higher-twist’ contributions of order $M^2/(x_i x_j' Q^2)$ relative to the leading term, and also corrections to the leading term of relative order α_s times logarithms of (large) ratios of invariants. (This condition forces the invariant mass of the virtual $q\bar{q}$ states on the right-hand side of Fig. 2a to be large, and, as explained in Appendix A, T_H contains a projection operator that forces the invariant mass of intermediate $q\bar{q}$ pairs, in diagrams in which additional gluons are exchanged, to be of order Q^2 .)

This discussion shows that high Q^2 does not guarantee the legality of perturbation theory, which fails for $x_i, x_i' \rightarrow 0$. One must therefore take care that the perturbative contribution is not dominated by regions where $x_i x_j' Q^2$ is small compared to the typical hadronic mass scale of 1 GeV^2 . In the following we will show that it is precisely these regions that are responsible for the apparent success of recently proposed wave functions in reproducing the normalization of form-factor data through ‘perturbative’ processes. At the end of this section we explain why similar problems are not encountered in applying perturbative QCD to deep inelastic scattering.

2.1 The pion electromagnetic form factor

In the case of the pion electromagnetic form factor $F_\pi(Q^2)$, the asymptotic normalization of the perturbative contribution is known [2], and is more than a factor of 5 below existing data [32] at $Q^2 \approx 10 \text{ GeV}^2$. Whilst this discrepancy is not as serious as the one we will discuss below for the nucleons, it has been claimed [5] that the perturbative contribution calculated with the wave functions of Ref. [8] is consistent with the normalization and shape of the data for $Q^2 > 1 \text{ GeV}^2$. So we will first address our remarks to this system, in which all the basic issues can be illustrated more simply.

If $\psi_\pi^0(x, \vec{p}_T)$ is the $S = 0, L_z = 0$ component of the (infinite-momentum frame or light-cone) pion wave function in the $q\bar{q}$ sector, and $\phi_\pi(x) \equiv \int d^2 p_T \psi_\pi^0(x, \vec{p}_T)$, where x is the momentum fraction in the infinite-momentum frame and $\int dx d^2 p_T |\psi_\pi^0(x, \vec{p}_T)|^2 = P_{q\bar{q}}^0$, the associated $q\bar{q}$ probability^{*)} ≤ 1 , then asymptotically we have [2–5]

$$F_\pi(Q^2) = \frac{4\alpha_s(Q^2)}{3\pi^2 Q^2} \left| \int_0^1 dx \frac{1}{x} \phi_\pi(x) \right|^2 \quad (1)$$

using the symmetry of the wave function for $x = x_1 \leftrightarrow x_2 = 1 - x$. The derivation of Eq. (1) is reviewed in Appendix A. In Eq. (1) we have absorbed a Q^2 -dependent renormalization factor into ψ and suppressed the fact that the p_T^2 integral should be cut off at a scale of order Q^2 ; the Q^2 dependence due to these factors is very weak, and for practical purposes the integral in Eq. (1) is independent of Q^2 in the regime where perturbation theory is valid. We will therefore assume that it can be ignored. The $1/x$ singularity is generated by the gluon propagator, as noted above. If we use the (rigorously normalized) asymptotic distribution function

^{*)} Note that this convention differs from that of Ref. [3]: their $\psi(x, \vec{p}_T)$ is $(16\pi^3)^{1/2}$ times ours.

Fig. 2a : The hard-scattering amplitude which controls the asymptotic behaviour of the pion form factor. Here $p(p')$ is the four-momentum of the initial (final) pion and $x_1 + x_2 = x'_1 + x'_2 = 1$.

Fig. 2b, c : Hard-scattering diagrams relevant to deep inelastic scattering.

$$\int d^2 p_T \psi_\pi^{\text{asympt}}(x, \vec{p}_T) \equiv \phi_\pi^{\text{asympt}}(x) = \sqrt{3} (2\pi)^{3/2} f_\pi x(1-x) \quad (2)$$

where f_π is the pion decay constant ($f_\pi \approx 130$ MeV), then we recover the classical result [2]

$$Q^2 F_\pi^{\text{asympt}}(Q^2) = 8\pi \alpha_s(Q^2) f_\pi^2. \quad (3)$$

What fraction of this result is reliable in the sense that it is generated by values of x and x' for which perturbation theory can be trusted? The answer to this question depends on the smallest value of the cut-off ($m_{\min}^2$) that we must put on $xx'Q^2$ in order to keep higher-twist contributions and higher-order effects at a reasonably small level. We shall in the first instance simply require $xx'Q^2 > m_{\min}^2$, since it is clear that small x and small x' dominate Eq. (1), so cut-offs on $x(1-x')Q^2$ and $x'(1-x)Q^2$ have a relatively small effect. The invariant $xx'Q^2$ is the square of the momentum transfer carried by the gluons in Fig. 2a or, more generally, the value of the invariant mass in the t -channel. It is therefore probably safe to set $m_{\min}^2 \approx 1 \text{ GeV}^2$ since it is presumably the scale of glueball masses that controls higher-twist effects in the t -channel, but we would consider it not ridiculous to allow a value as small as 0.25 GeV^2 , given that perturbation theory is not totally misleading in deep-inelastic scattering for $Q^2 \sim 1 \text{ GeV}^2$.

Results are given in Table 1 for a variety of Q^2 and $m_{\min}^2$, based on the replacement

$$\left| \int_0^1 dx \frac{1}{x} \phi_\pi(x) \right|^2 \rightarrow \int_0^1 dx' \int_0^1 dx \Theta(xx'Q^2 - m_{\min}^2) \frac{\phi_\pi^*(x') \phi_\pi(x)}{x'x} \quad (4)$$

in Eq. (1). We see from the first set of numbers in the table that, at currently available Q^2 (the highest existing data point is at $Q^2 \approx 10 \text{ GeV}^2$), only a fraction of the asymptotically valid perturbative contribution given by Eq. (1) is legal (even for small $m_{\min}^2$) if the distribution function is of the asymptotic form $\phi_\pi(x) \propto x(1-x)$. The naïve perturbative contribution is more than 90% legal, for $m_{\min}^2 = 1 \text{ GeV}^2$, only for $Q^2 \geq 150 \text{ GeV}^2$! — see Fig. 3a.

Matters are even worse for the more recently advocated [5] ‘asymmetric’ pion distribution function of Chernyak and Zhitnitsky (CZ) in Ref. [8], which is usually parametrized in the form $\phi_\pi(x) \propto x(1-x)(1-2x)^2$, as shown by the second half of Table 1 and by Fig. 3a. This difficulty is to be expected (see Fig. 1) from the fact that this wave function concentrates probability near the end-points. Figure 3b shows the fraction $f_{CZ}(\epsilon)$ remaining as a function of $\epsilon \equiv m_{\min}^2/Q^2$. It also shows the effect of cutting off the end-point regions with an effective gluon mass m_g by replacing $\theta(xx'Q^2 - m_{\min}^2)$ in Eq. (4) by $xx'Q^2/(xx'Q^2 + m_g^2)$. Clearly the effect is qualitatively similar.

There is another problem. As mentioned above, the distribution amplitude that controls $F_\pi(Q^2)$ is

$$\phi_\pi(x, Q^2) \equiv \int_{p_T^{\min}(Q^2)}^{p_T^{\max}(Q^2)} d^2 p_T \phi_\pi(x, \vec{p}_T) \quad (5)$$

Table 1

Effects on $F_\pi(Q^2)$ of excluding the contributions of end-point regions where sub-leading (higher-twist) terms are *a priori* likely to be greater than the perturbative result. Entries give the percentage of the full contribution which *remains* after excluding the end-point regions.

Distribution amplitude:	Asymptotic ^{a)} $x(1-x)$			CZ (Ref. [8]) ^{b)}		
$m_{\min}^2$ (GeV ²):	0.25	0.5	1.0	0.25	0.5	1.0
Q^2 (GeV ²):						
1	13	2	0	2	1	0
2	32	13	2	8	2	1
4	52	32	13	16	8	2
8	68	52	32	27	16	8
16	80	68	52	41	27	16

- a) For the asymptotic distribution function, the fraction of the ‘perturbative’ contribution which remains is $(1-\epsilon)(1+5\epsilon) + 2\epsilon(2+\epsilon) \ln \epsilon$, where $\epsilon = m_{\min}^2/Q^2$.
- b) For the Ref. [8] distribution function, the fraction of the ‘perturbative’ contribution which remains is $(1-\epsilon)(1+133\epsilon+808\epsilon^2+456\epsilon^3) + 6\epsilon(6+75\epsilon+128\epsilon^2+24\epsilon^3) \ln \epsilon$.

Fig. 3a : The data for $Q^2 F_\pi(Q^2)$ and a) the naïve asymptotic result, a') the 'legal' component of the asymptotic result with $m_{\min}^2 = 1 \text{ GeV}^2$; b) the naïve result using the wave function of Ref. [8], b') the 'legal' component of b) with $m_{\min}^2 = 1 \text{ GeV}^2$; c2) and c10) are the consistency bounds described in the text with $Q_0^2 = 2 \text{ GeV}^2$ and 10 GeV^2 , respectively.

Fig. 3b : The fraction $f_{CZ}(\epsilon)$ of $Q^2 F_\pi(Q^2)$ calculated with the CZ distribution amplitude [8] remaining as a function of ϵ . The solid curve corresponds to Eq. (4) with $\epsilon = m_{\min}^2/Q^2$; the dashed curve cuts off the gluon propagator with an effective mass m_g by dropping the θ -function in Eq. (4) and replacing $(xx')^{-1}$ by $(xx' + m_g^2/Q^2)^{-1} \equiv (xx' + \epsilon)^{-1}$. Note that using m_g^2 rather than $m_{\min}^2$ as a cut-off which allows dubious ‘perturbative’ contributions in the end-point region, only increases $f_{CZ}(\epsilon)$ appreciably for values of ϵ where it is already small.

Fig. 3c : The fraction $F_{CZ}^P(\epsilon)$ of $Q^4 G_M^P(Q^2)$ calculated with the CZ distribution amplitude [8] remaining as a function of ϵ . The curves are the analogs of those of Fig. 3b as described in the text.

where $p_T^{\max}$ is of order Q . More precisely, as explained in Refs. [3] and [4], $p_T^{\max}$ should be taken to be of order xQ . In the region where perturbation theory is valid, this cut-off contributes to the very mild Q^2 -dependence of the distribution amplitude and the precise value chosen for the cut-off does not change the result to leading order in α_s . At modest Q^2 , however, the cut-off may have a major effect which a) would indicate that it is not really consistent to use perturbation theory and b) allows us to bound the perturbative contribution. In Ref. [1] we showed that if $\psi(x, \vec{p}_T)$ is of the form $x^\eta(1-x)^\eta P(\vec{p}_T)$, this bound lies well below the data. Factorization is, however, a strong assumption, so in Appendix B we generalize our earlier bound and show that

$$Q^2 F_\pi \leq \frac{2\alpha_s(Q_0^2) f^2 Q_0^2}{3\pi} \left[\frac{x_0^2 (x_0 - \ln x_0)}{(1-x_0)^2} - x_0 - \ln(1-x_0) \right], \quad x_0^{-1} = 1 + \left(\frac{Q}{Q_0}\right)^{+1} \text{ if } Q \geq Q_0 \quad (6)$$

Here f is defined by $p_T^{\max} = fxQ$ and Q_0 is a fixed scale at which one has maximized $Q^2 F_\pi$.

The maximum possible value for $Q^2 F_\pi(Q^2)$ at any $Q^2 = Q_0^2$ is thus $4 \ln 2 \alpha_s(Q_0^2) f^2 Q_0^2 / 3\pi$, but if this maximum is obtained at some $Q^2 = Q_0^2$ then $Q^2 F_\pi(Q^2)$ saturates Eq. (6) at *all* values of Q^2 . We show the bound in Fig. 3a with the plausible value $f = 1$ for $Q_0^2 = 2 \text{ GeV}^2$ and 10 GeV^2 . Since the data at $Q^2 = 2 \text{ GeV}^2$ saturate the $Q_0^2 = 2 \text{ GeV}^2$ bound, then (if perturbative QCD applies at 2 GeV^2) the $Q_0^2 = 2 \text{ GeV}^2$ curve bounds the perturbative contribution of all other Q^2 , leading to trouble at $Q^2 = 10 \text{ GeV}^2$. Similarly, the $Q_0^2 = 10 \text{ GeV}^2$ bound leads to trouble at lower Q^2 . This difficulty would not be very severe, since it relies on the single data point at 10 GeV^2 , were it not for the fact that (6) is a strong bound. First, it can only be saturated in the (impossible) case that the pion is not only pure $q\bar{q}$ but also that $L_z = \pm 1$ components of the $q\bar{q}$ infinite-momentum-frame wave function are zero. Second, it requires that the $L_z = 0$ wave function has the highly implausible form (see Appendix B)

$$\Psi(x) \Big|_{x > 1/2} = \frac{1}{\sqrt{4\pi \ln 2} f Q_0} \begin{cases} x^{-1}(1-x)^{-1} & p_T < fxQ_0 \\ (1-x)^{-1} & fxQ_0 \leq p_T \leq f(1-x)Q_0 \\ 0 & p_T > f(1-x)Q_0 \end{cases} \quad (7)$$

In addition, the actual result should only depend logarithmically on f if the use of perturbation theory is legitimate, whereas the bound varies quadratically with f . Our model-independent bound therefore shows that the data cannot be described by perturbative QCD for $Q^2 \leq 10 \text{ GeV}^2$. The underlying problem is that we need small $\langle x \rangle$ and/or large $\langle p_T^2 \rangle$ in order to fit the data and this is incompatible with the condition $p_T \leq xQ$. The same problem occurs for nucleon form factors.

2.2 The nucleon magnetic form factors

Before comparing the predictions of perturbative QCD to the data for nucleon magnetic form factors (a comparison which suffers in an even more lethal form from the problems we have found for the pion), it is convenient to develop a formalism which is more economical than those in the literature. If $\phi_N^{\pm}(x_1 x_2 x_3)$ is the fully antisymmetrized nucleon qqq distribution amplitude for a spin $\pm 1/2$ nucleon (see Appendix C), then (exploiting the symmetry) one finds that asymptotically

$$Q^4 G_M^N(Q^2) = \frac{\alpha_s^2}{9\pi^4} \int dx' \int dx \phi_N^-(x') e_3 t_3 \phi_N^+(x) \quad (8)$$

where e_3 is the charge operator for the third quark, $\int dx \equiv \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \delta(1-x_1-x_2-x_3)$, and

$$t_3 \equiv AS_{1-}S_{2-}S_{3+} + BS_{1-}S_{2+}S_{3-} + CS_{1+}S_{2-}S_{3-} \quad (9)$$

in which

$$A = \frac{-1}{x_1 x'_1 x_2 x'_2} \left(\frac{1}{(1-x'_1)(1-x_2)} + \frac{1}{(1-x'_2)(1-x_1)} \right) \quad (10)$$

$$B = \frac{1}{x_1 x'_1 x_2 x'_2} \left(\frac{1}{(1-x'_1)(1-x_3)} + \frac{1}{(1-x'_3)(1-x_1)} \right) - \frac{2}{x'_1 x_1 (1-x'_3)^2 (1-x_3)^2} - \frac{2}{x'_2 x_2 (1-x'_3)^2 (1-x_3)^2} \quad (11)$$

and

$$C(x'_1 x'_2 x'_3; x_1 x_2 x_3) = B(x'_2 x'_1 x'_3; x_2 x_1 x_3) \quad (12)$$

and where $S_{i\pm} = S_{ix} \pm iS_{iy}$ is the raising (lowering) operator for the spin of quark i . This gives

$$Q^4 G_M^P = \frac{\alpha_s^2}{27\pi^4} \int dx' \int dx \left\{ - (B+C) [S_+(x')S_+(x) + A_{+-}(x')A_{+-}(x)] + (B-C) [S_+(x')A_{+-}(x) + A_{+-}(x')S_+(x)] - 4A [A_{++}(x')A_{++}(x) - \frac{1}{4} \phi_*^P(x') \phi_*^P(x)] \right\} \quad (13)$$

and

$$Q^4 G_M^N = -\frac{1}{2} Q^4 G_M^P - \frac{\alpha_s^2}{54\pi^4} \int dx' \int dx \left\{ + (B+C) [A_-(x')A_-(x) + S_{--}(x')S_{--}(x)] + (B-C) [A_-(x')S_{--}(x) + S_{--}(x')A_-(x)] + 4A [S_{-+}(x')S_{-+}(x) - \frac{1}{4} \phi_*^\lambda(x') \phi_*^\lambda(x)] \right\}, \quad (14)$$

with

$$S_+ \equiv \sqrt{\frac{1}{2}} \phi^S + \frac{1}{2} \phi^\lambda \quad (15)$$

$$A_{+\frac{1}{2}} \equiv \sqrt{\frac{1}{6}} \phi^A + \sqrt{\frac{1}{12}} \phi^P + \sqrt{\frac{1}{3}} \phi_*^P \quad (16)$$

$$A_- \equiv \sqrt{\frac{1}{2}} \phi^A - \frac{1}{2} \phi^P \quad (17)$$

$$S_{-\frac{1}{2}} \equiv \sqrt{\frac{1}{6}} \phi^S - \sqrt{\frac{1}{12}} \phi^\lambda + \sqrt{\frac{1}{3}} \phi_*^\lambda \quad (18)$$

being simple linear combinations of the S_3 -group-irreducible distribution amplitudes of Appendix C.

Having set up this machinery, the argument runs parallel to that for F_π . We make the replacements (B_1 is the first term in Eq. (11), B_2 is the second, and B_3 the third)

$$A \rightarrow \tilde{A} = \theta(x'_1 x_1 - \epsilon) \theta(x'_2 x_2 - \epsilon) A \quad (19)$$

$$B \rightarrow \tilde{B} = \theta(x_1 x'_1 - \epsilon) \theta(x'_2 x_2 - \epsilon) B_1 \\ + \theta(x'_1 x_1 - \epsilon) \theta([1-x'_3][1-x_3] - \epsilon) B_2 \quad (20)$$

$$+ \theta(x'_2 x_2 - \epsilon) \theta([1-x'_3][1-x_3] - \epsilon) B_3 \quad (21)$$

$$C \rightarrow \tilde{C}(x'_1 x'_2 x'_3; x_1 x_2 x_3) = \tilde{B}(x'_2 x'_1 x'_3; x_2 x_1 x_3)$$

in Eqs. (8), (13), and (14); as before, $\epsilon = m_{\min}^2/Q^2$. This imposes a cut-off on the momentum transfer carried by the gluons in the lowest-order contributions to the hard-scattering amplitude T_H which, as in the case of the pion, is certainly necessary in order to control higher-twist and higher-order effects (although perhaps not sufficient as it is somewhat weaker than the general condition $x_i x_j Q^2 > m_{\min}^2$). Table 2 shows that at available Q^2 this cut-off severely depletes the contribution to G_M^n generated by the asymptotic wave function, which was already two orders of magnitude too small to explain the data before the cut-off is imposed. The effect on calculations using the CZ wave function, which it is claimed will solve the 'normalization problem' of perturbative QCD for the nucleon magnetic form factors, is much more serious: their normalization is reduced by two orders of magnitude in the relevant region of Q^2 . The nucleon form factors are thus analogous to F_π but are curtailed even more extremely by imposing the requirement that the perturbative calculation be valid in the region where it is applied. Figure 3c shows the fraction $F_{CZ}(\epsilon)$ of $Q^4 G_M^n(Q^2)$ calculated with the CZ distribution amplitude [8] remaining as a function of ϵ . The solid curve corresponds to Eqs. (19)–(21); the dashed curve cuts off the end-point regions with an effective gluon mass as in Fig. 3b.

There has recently been some discussion [33] regarding the derivation of the CZ wave functions, leading to some modifications of the model. We have checked that these modifications have only a modest effect on the $\epsilon = 0$ result and in no way affect our conclusions. The analysis of Ref. [33] also provided some estimates of the errors that are a result of the modelling of the distribution amplitude from knowledge of its low moments. We find that these errors lead to an uncertainty of about a factor of two in this calculation, essentially because the $\epsilon = 0$ calculation is rather sensitive to high moments which are not constrained by the Ref. [8] and Ref. [33] analyses. This fact makes it perhaps somewhat less surprising that the Ref. [8] model gave such a good result for the normalization of $Q^4 G_M^p$.

Table 2

Effects on $G_M^p(Q^2)$ and $G_M^n(Q^2)$ of excluding the contributions of end-point regions where sub-leading (higher-twist) terms are *a priori* likely to be greater than the perturbative result. Entries give the percentage of the full contribution which *remains* after excluding the end-point regions (negative entries indicate that a sign change has occurred).

Distribution amplitude	Asymptotic ^{a)}		CZ ^{b)}			
Form factor:	G_M^n		G_M^p		G_M^n	
$m_{\min}^2$ (GeV ²):	0.25	1.0	0.25	1.0	0.25	1.0
Q^2 (GeV ²):						
1	0	0	0	0	0	0
5	15	0	1	0	-1	0
10	33	2	1	0	-1	0
25	55	20	1	1	-2	-1

- a) Recall that $G_M^p = 0$ with the asymptotic distribution amplitude $\propto x_1 x_2 x_3$, while the full contribution to G_M^n is two orders of magnitude too small to explain the data..
- b) See Appendix B for the full CZ distribution amplitudes, which generate a full contribution that could have explained the data.

Table 3

Process-dependence of asymptopia

Process	Perturbative contribution	Potential relative non-perturbative contribution	$Q_{\text{cross-over}}^2/m^2$
Deep inelastic	$(\alpha_s/\pi)^0$	m^2/Q^2	~ 1
$Q^2 F_\pi(Q^2)$	$(\alpha_s/\pi)^1$	m^2/Q^2	~ 10
$Q^4 G_M^N(Q^2)$	$(\alpha_s/\pi)^2$	m^2/Q^2	~ 100
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$Q^{12} (d\sigma/dQ^2) (\pi N \rightarrow \pi N)$	$(\alpha_s/\pi)^8$	m^2/Q^2	$\sim 10^8$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

2.3 Deep-inelastic scattering

We now explain why the perturbative analysis of deep-inelastic scattering does not suffer from the problem we have found for form factors. It is instructive first to consider a model for possible non-leading ('higher-twist') contributions to T_H , the hard-scattering amplitude that controls the asymptotic behaviour of the pion form factor, by including an effective gluon mass m_g in the amplitude corresponding to Fig. 2 and setting

$$T_H \sim \frac{1}{xx'Q^2 + m_g^2}$$

This is not supposed to be a realistic model but this function illustrates features which T_H probably does exhibit. It is obvious that this function can only be approximated by its asymptotic form $(xx'Q^2)^{-1}$ for $xx'Q^2 \gg m_g^2$ and that any conclusions which depend strongly on using the asymptotic form for $xx'Q^2 \ll m_g^2$ cannot be believed: this is the essence of our argument. Another way of stating this is that T_H is only 'mass finite' (i.e., well-behaved as $m_g \rightarrow 0$) for x and x' non-zero; if T_H is expanded in a power series in m_g^2/Q^2 the coefficients are all 'infrared' divergent for x or $x' \rightarrow 0$.

In deep-inelastic scattering the relevant diagram is Fig. 2b and the structure functions are obtained by convoluting an input quark distribution $q_0(y)$ with a 'hard' amplitude $W = \text{Im } T$ which contains all the Q^2 dependence. W is both 'mass' and 'infrared' finite, i.e. it is well-behaved as $p^2 \rightarrow 0$, $m^2 \rightarrow 0$ (where m is any internal mass) and can be expanded*) in a power series in $1/Q^2$ with coefficients which do not diverge as the other relevant invariant $\hat{s} = (p+q)^2 \rightarrow 0$. This *not* true of T as can be seen by considering the dominant perturbative diagram, Fig. 2c, and including a quark mass m to obtain

$$T \sim \frac{1}{\hat{s} - m^2 + i\epsilon}$$

as a model containing possible non-leading terms, in the same spirit as the model of T_H above. This function can certainly not be expanded in a power series in $m^2/\hat{s}$ for $\hat{s} = Q^2(y/x - 1) \rightarrow 0$ [here and below x is the Bjorken variable $Q^2/(2q \cdot p)$]. However, anticipating the fact that $W \sim \delta(\hat{s} - m^2)$ will be convoluted with a smooth function $q_0(y)$, it is clear that W can safely be expanded in powers of m^2 for large Q^2 , the result being

*) This statement is only true 'up to logarithms' if the structure function is represented *exactly* by Fig. 2b. In this case T can be defined so that W is mass finite, and the leading twist part is calculable perturbatively, but W is not analytic in masses; perturbation theory generates non-analytic terms proportional to $m^2 \ln(Q^2/m^2)/Q^2$ which signal the presence of non-perturbative higher-twist contributions. These contributions can be separated by redefining T and pulling out part of the multiparticle (> 2) intermediate state contributions, which are represented by additional diagrams corresponding to multiparticle operators in the operator product expansion [OPE] (T will still contain non-leading quark mass corrections $\sim m^2/Q^2$ which are calculable perturbatively). Studies of higher-twist effects based on the OPE confirm that $1/Q^2$ is the relevant expansion parameter and so the traditional perturbative analysis of the leading-twist terms is sound (except as $x \rightarrow 1$). A systematic analysis of higher-twist contributions to form factors is not available, i.e. there is no machinery analogous to the OPE which can be used to isolate non-analytic non-leading contributions to T_H , so we can only make model-dependent statements about these contributions.

$$q_0 \otimes W \sim q_0(x) + \frac{m^2 x}{Q^2} q_0'(x) + \frac{1}{2} \left(\frac{m^2 x}{Q^2} \right)^2 q_0''(x) + \dots$$

This example shows why deep-inelastic scattering can be treated perturbatively despite the fact that the leading-order contribution is apparently*) generated by the point $m^2/\bar{s} = 1$.

2.4 Summary

The above discussion shows that there is no justification for the claims that perturbative QCD can explain electromagnetic form factors of the size observed in nature. The exclusion of the end-point regions from the perturbative integrals is a higher-twist effect. The dramatic dependence of the results on such an end-point exclusion demonstrates that the 'perturbative QCD predictions' do not apply at existing energies. Indeed, the behaviour of the results as a function of the infrared parameter $m_{\min}^2$ strongly suggests that the physics of these processes is controlled by non-perturbative effects. In the next section we will present some simple ideas on the possible nature of these effects.

3. SECOND OBJECTION: LARGE NON-PERTURBATIVE CONTRIBUTIONS

The objections raised in Section 2 destroy confidence in the applicability of perturbative QCD at available (or foreseeable) Q^2 for what have traditionally been claimed as two of its showcase successes. This then raises the important question: what is the source of $F_\pi(Q^2)$ and $G_M^N(Q^2)$ at currently available Q^2 ? In answering this question we will uncover a second, less rigorous but essentially independent, reason for doubting the explanation of the data offered by advocates of the applicability of perturbative QCD. However, our main motivation is simply to understand the physics of these form factors.

The perturbative calculations of F_π and G_M^N are based on the use of perturbative QCD to calculate the high relative momentum (i.e. $x_i - x_j$ or $\vec{p}_{Ti} - \vec{p}_{Tj}$ large) tail of the wave function due to hard scattering of the quarks belonging to the low-momentum, non-perturbative 'core' of the wave function. This tail controls the asymptotic behaviour of form factors. An obvious condition for the applicability of perturbative QCD at a given Q^2 is, then, that the non-perturbative 'core' of the wave function is smaller than the perturbative 'tail' in the region of the wave function being probed at a given Q^2 .

One might naively expect this cross-over from non-perturbative to perturbative dominance to occur at $Q^2 \approx 1 \text{ GeV}^2$. There is, however, a problem: the perturbative tail is suppressed by α_s^n , where n is the number of hard-gluon exchanges required to produce the tail perturbatively. One should therefore expect the cross-over points to 'asymptopia' to be determined by the condition $\alpha_s^n = m^2/Q_{\text{cross-over}}^2$ since the non-perturbative contributions are typically suppressed by a power of Q^2 but *not* by any small coupling constant. The potential consequences of this possibility are given schematically in Table 3. The main objective of this section is to demonstrate that Table 3 is a realistic assessment of the cross-over to perturbative asymptopia for F_π and G_M^N . In other words, we will conclude that at both available and foreseeable Q^2 , these form factors are dominated by the

*) Although the leading contribution to W is given numerically by Fig. 2c, this is not actually the dominant diagram. The dominant diagrams are obtained by adding propagator and vertex corrections to Fig. 2c (whose numerical effects cancel to leading order due to a Ward identity). When the top propagator is cut open to form W from T , a jet of quarks and gluons is revealed whose invariant mass is much smaller than Q^2 (to leading order in α_s) but nevertheless grows with Q^2 .

non-perturbative wave functions. In Section 4 we will comment briefly on the extension of these conclusions to more complicated processes as indicated in Table 3.

In studying the non-perturbative contributions to F_π and G_M^N , we will be unable to proceed with the degree of rigour that was possible in Section 2. There are several reasons for this. Perhaps the most fundamental reason is that—despite claims to the contrary [5, 17, 18, 19] and as explained in Appendix A—it is not possible, except in trivial cases, to construct infinite-momentum-frame wave functions with good J^P by kinematics alone. As a result, since we are unable to solve QCD, our conclusions about non-perturbative contributions will be frame-dependent and will depend on which component of the electromagnetic current we consider. We therefore do not hope to compute these contributions with precision, but simply to study whether they are comparable in size to the data. Since we already showed in Section 2 that the calculable perturbative contributions are much smaller than the data, this will allow us to draw plausible conclusions about the underlying physics of F_π and G_M^N . Of course, not all approaches to the non-perturbative contributions to these form factors suffer from the flaws of the approach we will take. In particular, calculations of the non-perturbative contributions using the QCD sum-rule method have produced conclusions [10–13] which are in qualitative agreement with ours; we shall refer to this work again later. We have chosen to look at these issues in a flawed framework mainly because that framework is the legacy of the perturbative calculation of Section 2, but also because it is the one into which knowledge of the wave function based on the quark–parton model is most readily placed.

3.1 The pion electromagnetic form factor revisited

As shown in Appendix D, in the absence of perturbative corrections the non-perturbative $L_z = 0$ $q\bar{q}$ wave function of the pion would lead to a pion form factor given approximately**) by

$$F_\pi^{\text{soft}}(Q^2) \Big|_{\text{trans}} = \int_0^1 dx \int d^2 p_T \hat{\Psi}_\pi^{o*}(x, \vec{p}_T + [1-x]Q\hat{e}_T) \hat{\Psi}_\pi^o(x, \vec{p}_T) \quad (22)$$

for transverse momentum kicks ($\hat{e}_T$ is a transverse unit vector), and for longitudinal kicks by

$$F_\pi^{\text{soft}}(Q^2) \Big|_{\text{long}} = \frac{2\zeta}{1+\zeta} \int_0^{\zeta^{-1}} dx \int d^2 p_T \hat{\Psi}_\pi^{o*}(\zeta x, \vec{p}_T) \hat{\Psi}_\pi^o(x, \vec{p}_T) \quad (23)$$

where

$$\zeta^{-1} = 1 + \frac{Q^2}{2\tilde{m}_2^2} - \sqrt{\frac{Q^4}{4\tilde{m}_2^4} + \frac{Q^2}{\tilde{m}_2^2}}. \quad (24)$$

The effective mass $\tilde{m}_2$ and the ‘non-perturbative core wave function’ $\hat{\psi}$ are discussed below; note that $\zeta \rightarrow 1$ as $Q^2 \rightarrow 0$ and $\zeta \rightarrow Q^2/\tilde{m}_2^2$ as $Q^2 \rightarrow \infty$. The identity of these two formulae (and of the related

*) This point was already made in Ref. [1].

**) Aside from assuming $q\bar{q}$ dominance, these formulae ignore Sudakov form factors. See Appendix D.

formulae for momentum transfers along other axes) obviously requires a special (and not obvious) relationship between the x - and p_T -dependence of ψ_π : this is the problem of rotational invariance to which we have alluded earlier and discuss in Appendix A.^{*)} There is a further problem with the use and interpretation of these formulae. If the full wave function ψ_π of the pion were inserted into these formulae, it would produce the full form factor, including the part of it which is due to the perturbative tail. Thus by $\hat{\psi}_\pi(x, \vec{p}_T)$ we are denoting the ‘core’ wave function generated by non-perturbative physics on which perturbative corrections are to be applied. In modelling it—and through it F_π^{soft} —we must therefore be careful to realize that $\hat{\psi}_\pi(x, \vec{p}_T)$ does not represent the physical pion, but rather the pion with transverse one-gluon exchange removed.

Following Ref. [1], we notice that the negligible Q^2 evolution of the wave functions implies that for practical purposes we can identify the wave functions we need with those of QCD cut off at some relatively small mass scale $\mu \leq 1$ GeV. At such scales it is presumably true that hadrons are dominated by $q\bar{q}$ and qqq configurations which can be approximated by those of the constituent quark model. In modelling $\hat{\psi}_\pi(x, \vec{p}_T)$ (and $\hat{\psi}_N$ below), we will therefore rely on physical insight from the quark model.

We will be guided by two observations:

1) According to the constituent quark model, the ρ - π splitting is due to the hyperfine interaction induced by transverse gluon exchange. In discussing non-perturbative contributions to F_π , we therefore consider a world in which $\bar{m}_\pi = \bar{m}_\rho \approx \frac{3}{4}m_q + \frac{1}{4}m_\pi \equiv \bar{m}_2$, this being the combination of physical masses which (approximately) eliminates hyperfine interaction effects; this is the $\bar{m}_2$ appearing in Eq. (24).

2) The quark model also tells us that a rough representation of the rest frame non-perturbative wave functions corresponding to a cut-off scale $\mu \leq 1$ GeV can be obtained by solving the Schrödinger equation for constituent quarks. The ground-state wave functions of the particular variant^{**) of the model we will use for orientation have $\langle p_T^2 \rangle^{1/2} \approx 0.31$ GeV, comparable to the constituent quark mass $m \approx 0.33$ GeV. If we make an (illegal but nevertheless informative) free boost of such a system, we readily conclude (see, for example, Fig. 1) that the infinite-momentum-frame wave functions $\hat{\psi}_\pi(x, \vec{p}_T)$ corresponding to the constituent quark-model rest-frame wave function will i) be very broadly distributed in x , ii) have a $\langle p_T^2 \rangle^{1/2}$ comparable to that in the rest frame, and iii) have components which include not only the S and L quantum numbers of the rest-frame ground state but also, through Wigner rotation, substantial new components subject to J_z having its rest-frame value. In the case of the pion, $L_z = \pm 1$ wave functions (corresponding to the structure of an a_1 meson) appear. See Appendix A for a rigorous discussion.}

With this picture in mind as a starting point (a picture which certainly seems reasonable [35] based on what we know about deep-inelastic structure functions), let us investigate some models for $\hat{\psi}_\pi(x, \vec{p}_T)$. We will initially ignore the presence of components with quantum numbers differing from those in the rest frame, since we will later show that the simple analysis this allows contains all the essential features of the more complex case. Focusing, then, on the wave function $\hat{\psi}_\pi^0(x, \vec{p}_T)$ with $S = L = 0$ introduced in Section 2, and following Appendix B, we first write without loss of generality

*) If Fock space components other than $q\bar{q}$ are important, special relationships between subspaces are also required in general.

**) For background, see Refs. [34]. The variant to which we refer uses a potential $c - (4\alpha_s/3r) + br$, with $c = -0.84$ GeV, $\alpha_s = 0.5$, $b = 0.18$ GeV², and a quark mass $m = 0.33$ GeV.

$$\hat{\Psi}_{\pi}^{\circ}(x, \vec{p}_T) = \sqrt{P_{q\bar{q}}^{\circ}} X(x) P(x, \vec{p}_T) \quad (25)$$

where $|P(x, \vec{p}_T)|^2$ is an x -dependent normalized p_T distribution. The function $x|X(x)|^2$ can then be associated with the high- x part of the valence-quark momentum distribution function $V_{\pi}(x, Q^2)$ in the pion ‘devolved’ to $Q^2 \simeq 1 \text{ GeV}^2$, where $\psi_{\pi}(x, \vec{p}_T)$ may be associated with the (boosted) wave function of the valence-quark model. Measurements [36] of $V_{\pi}(x, Q^2)$ at $Q^2 \equiv m_{\mu\mu}^2$ in Drell–Yan production of muon pairs, when fitted to the form $x^{\gamma(Q^2)} (1-x)^{\beta(Q^2)}$, give the results $\gamma(25 \text{ GeV}^2) \simeq 0.5$ and $\beta(25 \text{ GeV}^2) \simeq 1.2$. The power $\gamma \simeq 1/2$ may be associated with the expected Regge behaviour as $x \rightarrow 0$; $\beta(Q^2)$ when devolved [37] gives $\beta(1 \text{ GeV}^2) \simeq 1$. We therefore expect that $|X|^2 \sim (1-x)$ for x near one and, using the fact that it must be invariant when x is replaced by $(1-x)$, guess that $|X|^2 \propto x(1-x)$, which we will refer to as the $\sqrt{v}$ (v for valence) model.

The simplest possibility is that $P(x, \vec{p}_T)$ is independent of x . This is the factorized case previously discussed in Ref. [1]. With a factorizing wave function, Eq. (23) becomes independent of $P(x, \vec{p}_T)$ and gives simply

$$F_{\pi}^{\text{soft}}(Q^2) \Big|_{\text{long}} = P_{q\bar{q}}^{\circ} \frac{2\ell}{1+\ell} \int_0^1 dx X(x) X(\ell x) \quad (26)$$

With $X(x) = \sqrt{6} x^{1/2} (1-x)^{1/2}$, and assuming that $P_{q\bar{q}}^{\circ} = 1$, we get the result shown in Fig. 4 as curve (b). Equation (24) for $F_{\pi}^{\text{soft}}(Q^2) \Big|_{\text{trans}}$ does not simplify in general. If, however, we assume the Gaussian form $P(x, \vec{p}_T) = (\pi\beta^2)^{-1/2} \exp(-p_T^2/2\beta^2)$, it becomes

$$F_{\pi}^{\text{soft}}(Q^2) \Big|_{\text{trans}} = P_{q\bar{q}}^{\circ} \int_0^1 dx |X(x)|^2 e^{-x^2 Q^2 / 4\beta^2} \quad (27)$$

This equation is easily shown to give $Q^2 F_{\pi} \rightarrow 12 P_{q\bar{q}}^{\circ} \beta^2$ as $Q^2 \rightarrow \infty$. It is also simply integrated numerically to give curve (f) in Fig. 4.

As well as illustrating the naturally large non-perturbative contributions at available Q^2 , this example shows the flaws in our approach for drawing any but the most qualitative conclusions. There is first of all the expected discrepancy between $F_{\pi}^{\text{soft}} \Big|_{\text{long}}$ and $F_{\pi}^{\text{soft}} \Big|_{\text{trans}}$ which is certainly serious. There is also the fact that each calculation is subject to its own uncertainties. Figure 5 illustrates the sensitivity of the nominal curves (b) and (f) of Fig. 4 to changes in the empirical $\sqrt{v}$ exponent, in $\tilde{m}_2$, and in β^{-2} . Even with such serious flaws, however, these calculations are sufficient to show that there is no *a priori* difficulty with the proposition (forced on us by the failure of perturbative QCD to explain the data) that this form factor arises at currently available Q^2 from soft, non-perturbative physics.

Such calculations also allow us to observe another flaw (beyond those exposed already in Section 2) in the proposition that exotic wave functions can solve the problem of perturbative QCD in

* If $\tilde{m}_2$ were equated to m_{π} , $Q^2 F_{\pi}^{\text{soft}} \Big|_{\text{long}}$ would be much smaller than the data. The use of m_{π} in this context would, however, not only contravene normal quark-model practice, but also lead to a pion charge distribution with a size $\sim (6/m_{\pi}^2)^{1/2} \approx$ the pion Compton wavelength instead of the observed size $\approx (6/m_{\rho}^2)^{1/2} \approx$ the QCD scale.

Fig. 4 : Soft contributions to $Q^2 F_\pi(Q^2)$ due to various wave functions discussed in the text. The legend is, with the abbreviations trans. $\leftrightarrow$ Eq. (22), long. $\leftrightarrow$ Eq. (23), $\sqrt{v} \leftrightarrow \sqrt{6x(1-x)}$, CZ $\leftrightarrow$ Ref. [8] distribution amplitude, fac. $\leftrightarrow$ factorized, non-fac. $\leftrightarrow$ non-factorized:

- a) long., $\sqrt{v}$, non-fac.;
- b) long., $\sqrt{v}$, fac.;
- c) long., CZ, fac.;
- d) long., CZ, non-fac.;
- e) trans., $\sqrt{v}$, non-fac. with $\langle p_T^2 \rangle$ as in (f);
- f) trans., $\sqrt{v}$, fac.;
- g) trans., $\sqrt{v}$, non-fac. with $\langle r^2 \rangle$ as in (f);
- h) trans., $\sqrt{v}$, fac. with exponential p_T distribution having $\langle p_T^2 \rangle$ as in (f);
- i) trans., CZ, fac.;
- j) trans., CZ, non-fac.;
- k) trans., $\sqrt{v}$, fac. with exponential p_T distribution having $\langle r^2 \rangle$ as in (f).

The data are from Refs. [32] and [38]. All curves take $P_{q\bar{q}} = 1$.

Fig. 5 : An illustration of the parameter-sensitivity of the results of Fig. 4 for $Q^2 F_\pi^{\text{soft}}$; the modifications, chosen to reduce soft effects, are:

- a) nominal long., $\sqrt{v}$, fac. [curve (b) of Fig. 4];
- a') curve (a) with $(x_1 x_2)^{1/2} \rightarrow (x_1 x_2)^{3/4}$;
- a'') curve (a) with $\tilde{m}_2 \rightarrow 3/4 \tilde{m}_2$;
- b) nominal trans., $\sqrt{v}$, fac. [curve (f) of Fig. 4];
- b') curve (b) with $(x_1 x_2)^{1/2} \rightarrow (x_1 x_2)^{3/4}$;
- b'') curve (b) with $\beta \rightarrow 3/4 \beta$.

explaining the ‘asymptotic’ normalizations of electromagnetic form factors. Curves (c) and (i) of Fig. 4 show, still within the context of factorizing wave functions, the $Q^2 F_\pi^{\text{soft}}|_{\text{long}}$ and $Q^2 F_\pi^{\text{soft}}|_{\text{trans}}$, respectively, arising from the proposed distribution amplitude of Ref. [8]. [Curve (c) is independent of the known zeroth moment of this amplitude; curve (i) is computed by choosing β so that the moment is reproduced. The result for the former can be obtained analytically and is

$$F_\pi = \frac{1260}{5^4(1+\zeta)} \left[\frac{1}{60} \zeta^3 - \frac{5}{84} \zeta^2 + \frac{8}{105} \zeta - \frac{2}{63} \right] ; \quad (28)$$

the latter is easily obtained by numerical methods.] We see that even if we were to forget about the arguments of the previous section, it might not be consistent to ignore the soft contributions arising from the Ref. [8] wave function. To put the matter more positively, it may be that there is nothing terribly wrong with the CZ wave function, but only something wrong with inserting it into a perturbative QCD calculation!

Whilst we stress again that we do not take the quantitative results of this section seriously, we do heed their general implications. However, before we can expect the reader to follow us in this attitude, we must make good our claim that our conclusions are not critically dependent on the various simplifying assumptions we have made so far.

Let us consider the importance of our initial representation of the p_T distribution as Gaussian. First of all it should be recalled that for factorized wave functions $F_\pi^{\text{soft}}|_{\text{long}}$ is independent of $P(p_T)$; curves (b) and (c) of Fig. 4 from Eq. (26) are therefore unaffected by the use of this representation. If $P(p_T)$ is not Gaussian, the simple form of Eq. (27) is lost and Eq. (22) must in general be fully integrated numerically. The result of doing this for an exponential $P(p_T)$ is shown in curves (h) and (k) of Fig. 4. As might have been expected, these curves are higher than the corresponding Gaussian curve (f), no matter whether an ‘equivalent exponential’ is defined as having the same $\langle p_T^2 \rangle$ [curve (h)] or the same (transverse) charge radius [curve (k)] as the Gaussian. We conclude that our Gaussian representation of the p_T distribution is, if anything, producing misleadingly *small* results.

We can also dispense with the idea that the assumption of factorizing wave functions is misleading us with respect to the large size of the non-perturbative contributions to $Q^2 F_\pi$. The soft transverse calculation of F_π is still simple in the more general case: we have

$$F_\pi^{\text{soft}}(Q^2) \Big|_{\text{trans}} = P_{q\bar{q}}^0 \int_0^1 dx |X(x)|^2 \int d^2 p_T P(x, \vec{p}_T + [1-x] Q \hat{e}_T) P(x, \vec{p}_T) \quad (29)$$

corresponding to an $|X|^2$ -weighted average over the overlaps of the p_T wave functions. With a Gaussian $P(x, \vec{p}_T) = [1/\sqrt{\pi} b(x)] \exp [-p_T^2/2b(x)^2]$, we still obtain the analogue of Eq. (27), namely

$$F_\pi^{\text{soft}}(Q^2) \Big|_{\text{trans}} = P_{q\bar{q}}^0 \int_0^1 dx |X(x)|^2 e^{-x^2 Q^2 / 4 b(x)^2} . \quad (30)$$

As an illustration of the difference which a more general $P(x, \vec{p}_T)$ can make, we consider a model [4] in which

$$b(x)^2 = c^2 x(1-x) \quad (31)$$

It might be expected that $\langle p_T^2 \rangle$ vanishing at the end-points would dramatically suppress the soft (transverse) form factors. Curves (e) and (g) illustrate the actual effect of such an x -dependent p_T distribution on curve (f). In curve (e) we have chosen c so that the non-factorized wave function has the same $\langle p_T^2 \rangle^{1/2} = \beta$ as the factorized wave function in Eq. (27). Curve (g) takes c so that it has the same (transverse) charge radius. Whilst changing the asymptotic behaviour [curve (e) $\rightarrow 2400\beta^4/Q^2$, curve (g) $\rightarrow 12800\beta^4/3Q^2$, and recall that curve (f) $\rightarrow 12\beta^2$], these three wave functions give comparable values for $Q^2 F_\pi^{\text{soft}}|_{\text{trans}}$ in the region of interest. The computation of $Q^2 F_\pi^{\text{soft}}|_{\text{long}}$ is also straightforward, and gives curve (a) of Fig. 4.

Similar effects are seen when we study non-factorizing wave functions corresponding to the CZ distribution amplitude. Once again using as an illustration the ansatz (31), and choosing c to reproduce the known zeroth moment of the distribution amplitude, we find the longitudinal curve (d) arising from the factorized curve (c), and the transverse curve (j) arising from the factorized curve (i). In neither case, once again, are there any dramatic changes in the curves in the Q^2 range of interest.

Finally, let us consider the effects of components other than $\psi_\pi^0(x, \vec{p}_T)$ in the full wavefunction $\psi_\pi(x, \vec{p}_T)$, i.e. those with $L_z = \pm 1$ which may be thought of as arising from Wigner rotation. These 'extra' wave functions do not contribute to the distribution amplitude $\phi_\pi(x) \equiv \int d^2 p_T \psi_\pi(x, \vec{p}_T) = \int d^2 p_T \psi_\pi^0(x, \vec{p}_T)$ and so do not play a role in the asymptotic perturbative formulae, but they can certainly make soft contributions to the form factors. To discuss this situation, we write

$$\hat{\psi}_\pi(x, \vec{p}_T) = \hat{\psi}_\pi^0(x, \vec{p}_T) \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) + \frac{1}{\sqrt{2}} (\hat{\psi}_\pi^{1+}(x, \vec{p}_T)\downarrow\downarrow - \hat{\psi}_\pi^{1-}(x, \vec{p}_T)\uparrow\uparrow) \quad , \quad (32)$$

where angular momenta are quantized along $\hat{z}$, and $\hat{\psi}^{1\pm} = \mp (p_x \pm ip_y) \Psi(x, p_T^2)$. We further decompose

$$\hat{\psi}_\pi^i(x, \vec{p}_T) = \sqrt{P_{q\bar{q}}^i} X^i(x) P^i(x, \vec{p}_T) \quad (33)$$

with normalization conditions as before. The new features to be considered are that it is now $\sum_i x P_{q\bar{q}}^i |X^i(x)|^2$ that we should be associating with the high- x behaviour of the pion structure function, and that Eqs. (22) and (23) are modified by the replacement $\psi_\pi^0(x, \vec{p}_T) \rightarrow \psi_\pi(x, \vec{p}_T)$. Thus, for example,

$$F_\pi^{\text{soft}}(0)|_{\text{trans}} = F_\pi^{\text{soft}}(0)|_{\text{long}} = 1 \approx P_{q\bar{q}}^0 + P_{q\bar{q}}^1 \quad , \quad (34)$$

where $P_{q\bar{q}}^{1\pm} = P_{q\bar{q}}^{1\mp} = \frac{1}{2} P_{q\bar{q}}^1$, and we have assumed that at the constituent quark-model scale higher Fock-space components can be neglected.

Now consider the factorized case $P^i(x, \vec{p}_T) = P^i(\vec{p}_T)$. For the generalizations of Eqs. (26) and (27), we have respectively

$$F_\pi^{\text{soft}}|_{\text{long}} = \frac{2\ell}{1+\ell} \int_0^{1-\ell} dx \sum_i P_{q\bar{q}}^i X^i(x) X^i(\ell x) \quad (35)$$

and

$$F_{\pi}^{\text{soft}} \Big|_{\text{trans}} = \int_0^1 dx \sum_i P_{q\bar{q}}^i |X^i(x)|^2 \Pi_i(x, Q^2) , \quad (36)$$

where

$$\Pi_0(x, Q^2) = e^{-x^2 Q^2 / 4\beta_0^2} \quad (37)$$

$$\Pi_{1\pm}(x, Q^2) = \left(1 - \frac{x^2 Q^2}{4\beta_1^2}\right) e^{-x^2 Q^2 / 4\beta_1^2} \quad (38)$$

with β_1 the analogue of $\beta = \beta_0$ for the transverse amplitudes $P^{1\pm}(\vec{p}_T)$.

Since we continue to make the identification $\sum_i P_{q\bar{q}}^i |X^i(x)|^2 = 6x(1-x)$, it is irrelevant to the factorized formulae Eq. (35) how this probability is partitioned, and all our old factorized longitudinal results are recovered. Equations (37) and (38) produce a less trivial result for the effects of the transition from Eq. (27) to (36). The basic situation can be appreciated by noting that the form of Eq. (38) ensures that $\psi^{1\pm}(x, \vec{p}_T)$ makes a non-leading contribution to $F_{\pi}^{\text{soft}}|_{\text{trans}}$ as $Q^2 \rightarrow \infty$, i.e.

$$Q^2 F_{\pi}^{\text{soft}} \Big|_{\text{trans}} \rightarrow 12\beta^2 P_{q\bar{q}}^0 . \quad (39)$$

Since $P_{q\bar{q}}^1$ is probably not small, this might well reduce $Q^2 F_{\pi}^{\text{soft}}|_{\text{trans}}$ by about a factor of 2 over our earlier estimates, and could be the explanation for a large part of the discrepancies found earlier between $Q^2 F_{\pi}^{\text{soft}}|_{\text{trans}}$ and $Q^2 F_{\pi}^{\text{soft}}|_{\text{long}}$ (see Fig. 4). (This is an additional demonstration of the non-triviality of rotational invariance in $\psi(x, \vec{p}_T)$.) Given the qualitative unimportance of the Gaussian and factorizing assumptions in the $\psi_{\pi} \simeq \psi_{\pi}^0$ case, the reader is probably ready to accept that they are also unimportant here. We have checked that this is indeed the case.

Let us summarize. In this subsection we have *not* claimed to have made a reliable calculation of the non-perturbative contributions to the pion form factor. Indeed, we have emphasized the difficulty of even making such a calculation rotationally invariant. Even if we were to build in rotational invariance partially by demanding p_T distributions which, for a given x -distribution, produced agreement between the transverse and longitudinal calculations of F_{π} (instead of insisting, as we have done, on an *ab initio* estimate of these p_T distributions), we could still easily produce reasonable models which would give results for $Q^2 F_{\pi}$ varying by factors of 2 over the Q^2 range of interest [compare curves (a), (b), (c), and (d)]. Nevertheless, we believe that the calculations described here are sufficient for our purposes, which were: first, to show that it is *possible* that soft contributions can explain the data for F_{π} [32, 38] so that the conclusions of Section 2 need not be a cause for alarm; and secondly, to show that even without the results of Section 2 it would have been difficult to avoid the conclusion that non-perturbative effects are comparable in size to the data in the region where attempts were being made to test perturbative QCD. Recall that this same conclusion was reached independently within the QCD sum-rule framework in Refs. [10], [11], and [13].

3.2 The nucleon electromagnetic form factors revisited

As in Section 2, the nucleon story is more complicated than the of the pion, but it comes to the analogous conclusion.

The main problem in calculating the nucleon soft form factors is that the full nucleon wave function $\psi_N(x_1, x_2, \vec{p}_{T_e}, \vec{p}_{T_\lambda})$ is a complicated object. (See Appendix C for notation.) Recall that the distribution amplitude Eq. (C.1) is the remnant of the full wave function after it is integrated over transverse momenta; before such an integration, ψ_N contains about 40 independent S_3 -irreducible wave-functions!

With this motivation, which is even stronger than in the case of the pion, let us begin by considering the simplest case in which the nucleon wave function is of the form (refer once again to Appendix C for notation)

$$\psi_N^\pm = \sqrt{\frac{1}{2}} (\phi^p \chi_\pm^p + \phi^\lambda \chi_\pm^\lambda) \psi^S \quad (40)$$

where ψ^S is totally symmetric under quark permutations. Following the pion discussion, we again write

$$\psi^S = X(x_1, x_2) P(x_1, x_2, \vec{p}_{T_e}, \vec{p}_{T_\lambda}) \quad (41)$$

and identify

$$f(x) = x \int_0^{1-x} dy |X(x, y)|^2 \quad (42)$$

with the high- x behaviour of the valence-quark momentum distribution function in the nucleon devolved to $Q^2 \approx 1 \text{ GeV}^2$. (We ignore the difference between u and d valence quarks). This leads us to expect [37] that $f(x) \sim (1-x)^\delta$ as $x \rightarrow 1$, where $\delta(1 \text{ GeV}^2) \approx 3$, and therefore that $|X|^2 \approx x_1 x_2 x_3$; this will be our $\sqrt{v}$ model for the nucleons.

Once again the simplest possibility is that $P(x_1, x_2, \vec{p}_{T_e}, \vec{p}_{T_\lambda})$ is independent of x_1 and x_2 , which is the factorized nucleon case previously discussed in Ref. [1]. In this case, for a longitudinal kick we have, independent of the form of the normalized function P ,

$$G_E^{(p)} \Big|_{\text{long}}^{\text{soft}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \int_0^{\zeta^{-1}} dx_1 \int_0^{\zeta^{-1}-x_1} dx_2 X(x_1, x_2) X(\zeta x_1, \zeta x_2) \quad (43)$$

and

$$G_M^{(p)} \Big|_{\text{long}}^{\text{soft}} = \begin{pmatrix} 1 \\ -2/3 \end{pmatrix} \frac{m \ell}{\tilde{M}_3} \int_0^{\zeta^{-1}} dx_1 \int_0^{\zeta^{-1}-x_1} dx_2 \frac{X(x_1, x_2) X(\zeta x_1, \zeta x_2)}{(1-x_1-x_2)[1-\zeta(x_1+x_2)]} \quad (44)$$

where now ζ is given by Eq. (24) with $\tilde{m}_2 \rightarrow \tilde{M}_3$ and $\tilde{M}_3 \approx \frac{1}{2}(M_N + M_\Delta) \approx 1.09 \text{ GeV}$, whilst, assuming $P(\vec{p}_{T_e}, \vec{p}_{T_\lambda}) \propto \exp(-\vec{p}_{T_e}^2/2\alpha) \exp(-\vec{p}_{T_\lambda}^2/2\alpha)$,

$$G_E^{(p)} \Big|_{\text{trans}}^{\text{soft}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 |X(x_1, x_2)|^2 e^{-Q^2(x_1^2 + x_1 x_2 + x_2^2)/2\alpha^2} \quad (45)$$

and

$$G_M^{(p)} \Big|_{\text{trans}}^{\text{soft}} = \left(\frac{1}{-2/3} \right) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 |X(x_1, x_2)|^2 M(x_1, x_2, Q^2) e^{-Q^2(x_1^2 + x_1 x_2 + x_2^2)/2\alpha^2} \quad (46)$$

with

$$M(x_1, x_2, Q^2) \equiv \sqrt{\frac{E^2}{E_+ E_-}} \frac{1}{2} \left\{ \sqrt{\frac{E_+ + m}{E_+ + m}} (1 + x_1 + x_2) + \sqrt{\frac{E_- + m}{E_- + m}} (1 - x_1 - x_2) \right\} \quad (47)$$

in which $E \equiv (\tilde{M}_3^2 + 1/4 Q^2)^{1/2}$ and $E_{\pm} = [m^2 + 1/4(1 \pm x_1 \pm x_2)^2 Q^2]^{1/2}$. Based on Refs. [34], in the following examples we take $\alpha = 0.32$ GeV as indicated by baryon spectroscopy.

These formulae are derived in Appendix D. They are as straightforward as the corresponding formulae for F_{π} , with the exception of Eq. (46), which has an interesting subtlety. In order to obtain in the (z -moving) infinite-momentum frame the correct Lorentz transform of the Breit-frame magnetic scattering of a nucleon from $-(Q/2)\hat{x}$ to $+(Q/2)\hat{x}$, it is essential that the effects of Wigner rotation of the nucleon spin be taken into account. It is thus a minimum requirement that the infinite-momentum frame wave functions used to compute G_M have Wigner rotated (i.e. $L_z \neq 0$) components. Indeed, from the point of view of the infinite-momentum frame, the analogue of the quark-model statement that $\mu_p \approx 3$ because $m \approx 1/3 M$, is the statement that the spinning quarks in the proton with $\langle x \rangle \approx 1/3$ will Wigner-rotate more rapidly than the proton itself. As a result, in this situation we are forced to model the $L_z \neq 0$ parts of the wave function. We do this in a simple manner suggested by the free-particle boosts, as described in the Appendix, but as a result the $G_M^N|_{\text{trans}}$ are more uncertain than our other form factors.

Figure 6 shows the result of using Eqs. (43) and (45) for $G_E^p(Q^2)$ in various circumstances. Curve (c), which lies remarkably close to the dipole formula curve, gives $Q^4 G_E^p$ arising from a transverse kick to the $\sqrt{v}$, factorized wave function with a mean p_T determined from baryon spectroscopy. Curves (a) and (b) show the effect of varying the parameters in that wave function in such a way as to reduce its importance for G_E . Curves (d), (e), and (f) are the analogous curves for longitudinal kicks. Although the lack of anything but an extrapolation of the data on $G_E(Q^2)$ to high Q^2 weakens the impact of these results, they are nevertheless closely reminiscent of the case of F_{π} : in this case we see that non-perturbative effects naturally give a contribution to $Q^4 G_E^p$ with an intrinsic scale of the order of 1 GeV^4 for Q^2 in the $1\text{--}30 \text{ GeV}^2$ range. Even without data at high Q^2 , these results are very damaging to the suggestion that non-perturbative effects are unimportant in this region. Since $G_E^p(Q^2) = F_1^p - (Q^2/4M_N^2) F_2^p$ ($Q^2 \equiv -q^2 > 0$ in the space-like region) and given that $F_2^p(0) = +2.79$, sensible models, including these, all give even larger values for the form factor F_1^p , which is the form factor actually computed in perturbative QCD.

We make this point because the formulae for $G_E(Q^2)$, being basically momentum-space overlap integrals (or, even more simply, Fourier transforms of charge distributions), are very simple and intuitive. Whilst somewhat less simple in origin (especially $G_M^N|_{\text{trans}}$), the $Q^4 G_M^p$ curves shown in Fig. 7 nevertheless convey the same message: there is no apparent difficulty in explaining the observed sizes of nucleon electromagnetic form factors at currently available Q^2 through non-perturbative effects. Indeed, as was the case with F_{π} , it is difficult to avoid large soft contributions to these form factors. In analogy to the situation for F_{π} , even were we without the arguments of subsection 2.2, the large soft contributions arising from essentially any nucleon wave function with reasonable p_T and x distributions (including those of Ref. [8]) would be sufficient, for us, to destroy the credibility of the

Fig. 6 : Soft contribution to $Q^4 G_E^p$ due to various wave functions discussed in the text. The legend is, with the abbreviations trans. $\leftrightarrow$ Eq. (45), long $\leftrightarrow$ Eq. (43), $\sqrt{v} \leftrightarrow \sqrt{120x_1x_2x_3}$, CZ $\leftrightarrow$ see text, fac. $\leftrightarrow$ factorized:

- a) curve (c) with $\alpha \rightarrow 3/4 \alpha$;
- b) curve (c) with $(x_1x_2x_3)^{1/2} \rightarrow (x_1x_2x_3)^{3/4}$;
- c) trans., $\sqrt{v}$, fac.;
- d) curve (e) with $(x_1x_2x_3)^{1/2} \rightarrow (x_1x_2x_3)^{3/4}$;
- e) long., $\sqrt{v}$, fac.;
- f) curve (e) with $\tilde{M}_3 \rightarrow 3/4 \tilde{M}_3$;
- g) trans., CZ, fac.;
- h) long., CZ, fac.

The dotted curve is the empirical dipole formula times Q^4 ; the data are from Ref. [40].

Fig. 7 : Soft contribution to $Q^4 G_M^p$ due to various wave functions discussed in the text. The legend is, with the abbreviations trans. $\leftrightarrow$ Eq. (46), long $\leftrightarrow$ Eq. (44), $\sqrt{v} \leftrightarrow \sqrt{120x_1x_2x_3}$, CZ $\leftrightarrow$ see text, fac. $\leftrightarrow$ factorized:

- a) curve (c) with $(x_1x_2x_3)^{1/2} \rightarrow (x_1x_2x_3)^{3/4}$;
- b) curve (c) with $\alpha \rightarrow 3/4 \alpha$;
- c) trans., $\sqrt{v}$, fac.;
- d) curve (e) with $(x_1x_2x_3)^{1/2} \rightarrow (x_1x_2x_3)^{3/4}$;
- e) long., $\sqrt{v}$, fac.;
- f) curve (e) with $\tilde{M}_3 \rightarrow 3/4 \tilde{M}_3$;
- g) trans., CZ, fac.;
- h) long., CZ, fac.

claimed perturbative explanation of the nucleon form factors. This point is emphasized in Figs. 6 and 7, which also show the nucleon form factors arising from just the totally symmetric (ϕ^S) component of the Ref. [8] distribution amplitude defined in Appendix C*). (The full amplitude, which is very asymmetric, has even more probability available in the end-point regions for generating soft scattering.) These conclusions are all consistent with those reported in the analogous study of soft contributions performed in Ref. [20]. Finally, we recall once again that this conclusion regarding soft form factors has been reached within the QCD sum-rule framework in Refs. [12] and [13].

3.4 Summary

The calculation of the non-perturbative contributions to the pion and nucleon electromagnetic form factors from infinite-momentum frame or light-cone wave functions is an extremely difficult task. In general, all Fock components of the wave function contribute, as does the full complement of infinite-momentum frame wave functions for each component (including ‘Wigner-rotated’ wave functions with orbital angular momenta differing from those of the rest-frame states). To make matters worse, the construction of good representations of J^P requires, in general, the knowledge of non-trivial *dynamical* relationships between all these wave functions.

In view of these difficulties, we have urged the reader not to take very seriously any of the simple models we have presented—they are all (necessarily, given the complexities of QCD) flawed. Despite this, we have presented the results of this section because we believe they illustrate that non-perturbative wave functions with characteristics corresponding to our *empirical* knowledge of hadron structure (in particular of the mean p_T of quarks and of the breadth of their distribution functions in x) naturally give contributions to $Q^2 F_\pi$ and $Q^4 G_M^N$ of the size required to explain these form factors. In providing this illustration, we believe we have removed what might have been an obstacle in the reader’s path to accepting our main conclusion as embodied in Section 2—that the data cannot be explained by *perturbative* QCD.

4. IMPLICATIONS FOR OTHER PROCESSES

The arguments presented here have dealt explicitly with only F_π and G_M^N as they provide simple test cases—and have been acclaimed as great successes—for the perturbative QCD description of exclusive progress. It is difficult, however, to see why our conclusions should not generalize.

Table 3 has already provided a schematic argument for this generalization. The arguments of Section 2 may be viewed as indicating that if the perturbative contributions exceed the naïve estimates of this table, then it is because they have been illegally boosted by application of the perturbative formulae in the end-point regions. (Indeed, with broad distribution amplitudes of the type one expects, and given that it is difficult to imagine exclusive processes with p_T^2 greater than about 100 GeV^2 being observable in the foreseeable future, end-point exclusions may tend to reduce the calculable perturbative contributions even below one’s naïve estimates.) Whilst these arguments seem to have immediate generalization to any exclusive process, those of Section 3 do not. In Section 3 we relied on empirical properties of strong interaction wave functions, but the general soft contribution will involve not only the non-perturbative wave functions but also non-perturbative scattering (or pair creation/annihilation). Nevertheless, these soft processes are also characterized empirically (not to mention theoretically) by the same intrinsic scale as the p_T in the wave functions. If we thus consider the scattering process as creating an intermediate wave function (out of the initial wave functions) characterized by this same p_T , then scattering amplitudes, like form factors, might be

*) For simplicity we have assumed factorization as in the other models considered with the same p_T dependence.

related to wave function overlaps. (For a more concrete and wide-ranging discussion of soft contributions to high momentum transfer processes in models similar to ours, including some insight on how the quark counting rules might be obtained outside perturbative QCD, see Refs. [31]; for interesting discussions of soft spin effects, see also Refs. [40].) A situation in which one might avoid this conclusion would be a scattering process in which *all* the participating particles were characterized by a scale other than Λ_{QCD} . For example, low-lying heavy-quark states have scales $m_Q\alpha_s$, so that $e^+e^- \rightarrow \psi\chi_0$ might be dominated by perturbative QCD even relatively close to its threshold. The difficulty with such processes is that they have extremely small cross-sections.

Of course the search for situations in which one of the many beautiful predictions of perturbative QCD might survive as a testable prediction deserves careful study. These brief comments are only intended to give the general grounds for our pessimistic appraisal of the prospects of such a search.

5 SUMMARY AND CONCLUSIONS

We have re-examined the question of whether perturbative QCD is applicable to exclusive processes in the light of recent developments. We find that, as anticipated in Ref. [1], the claims that novel wave functions can produce the observed normalizations of F_π and G_M^p are fatally flawed: only about 10% of the former normalization and 1% of the latter remain after excluding from the perturbative integrals the end-point regions which are controlled by higher twist effects. We have also illustrated in more detail (but with the same lack of rigour!) the remarks made in Ref. [1] that i) this failure of perturbative QCD need not be a cause for alarm since non-perturbative effects can naturally explain form factors of the observed strength, and ii) novel wave functions which (illegally) increase the perturbative integrals tend to give non-perturbative contributions to the form factors which are even larger than those given by smooth ones.

This more complete analysis, along with recent studies [20, 21] of the general character required of wave functions which would allow ‘perturbative’ QCD to fit the form factor data, have left us much more pessimistic about the relevance of perturbative QCD to exclusive processes than even we were before. We now consider it impossible to justify the continued application of this framework to the study of such phenomena.

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APPENDIX A: THE LIGHT-CONE FORMALISM AND ITS CONNECTION WITH THE BETHE-SALPETER EQUATION

In this Appendix we briefly review the QCD treatment of the asymptotic behaviour of form factors and the light-cone (LC) wave function formalism, stressing features that are needed in our discussion. We start with the covariant Bethe-Salpeter (BS) treatment and then go on to establish the connection with the LC, because neither the BS formalism, which is more transparent in our opinion, nor its connection with the LC treatment, has been discussed in detail in the literature.*)

The first step is to isolate the mass singularities which, to leading order in $1/Q^2$, are generated by configurations containing the minimum number of partons all moving parallel to the initial or final hadron in light-cone or other 'physical' gauges. In the case of the pion, which we consider here for definiteness and simplicity, we expose these configurations using the representation [41]

$$(p' + p)_\mu F_\pi = \iint \bar{\chi}(p'k') t_\mu \chi(p, k) \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} \quad (\text{A.1})$$

where t_μ represents the sum of all (truncated) two-particle irreducible diagrams, and $(\bar{\chi})\chi$ is the (conjugate) BS wave function, which satisfies

$$\chi(p, k) = S(p, k) \int V(p, k, k') \chi(p, k') \frac{d^4k'}{(2\pi)^4} \quad (\text{A.2})$$

where V is the sum of all two-particle irreducible (truncated) amplitudes and S represents the two external propagators.

*) For ease of comparison with Ref. [3], we adopt their normalization conventions in this Appendix. The ψ_π appearing here is therefore $(16\pi^3)^{1/2}$ times the ψ_π of the text.

Consider the integration over the loop momentum associated with a two-particle state:

$$\int A(\dots, k) S(p, k) B(\dots, k) \frac{d^4 k}{(2\pi)^4} \equiv A \circ SB$$

For $p^2 = m_q^2 = 0$, the integral diverges as $[(p/2) \pm k]^2 \rightarrow 0$ or equivalently, choosing $\vec{p}$ to define the 3-axis, as $k_- \equiv k_0 - k_3$ and $\vec{k}_T^2$ vanish. We isolate this singularity by setting, for example,

$$\begin{aligned} SB &= \frac{1}{\frac{p}{2} + k - m} \hat{B} \frac{1}{-\frac{p}{2} + k - m} + \text{non-singular} \\ &= \frac{(\frac{p}{2} + k + m)}{2m} SB \frac{(-\frac{p}{2} + k + m)}{2m} + \text{non-singular} \\ &= \sum_{h, \hat{h}} \hat{u}(h)_\alpha \left[\frac{\bar{u}(h) SB v(\hat{h})}{4m^2} \right]_{\alpha\beta} \hat{v}_\beta(\hat{h}) + \text{non-singular}, \end{aligned}$$

where u , $\hat{u}$, v , and $\hat{v}$ are on-mass-shell spinors with the following $+$ and transverse momenta:

$$\begin{aligned} u &= u(x p_+, \vec{k}_T) \quad , \quad v = v([1-x] p_+, -\vec{k}_T) \quad , \\ \hat{u} &= u(x p_+, \vec{0}) \quad , \quad \hat{v} = v([1-x] p_+, \vec{0}) \quad , \end{aligned}$$

and h labels 'infinite-momentum frame' helicities $h^i \equiv W_+^i/p_+^i$ [note $p_+^1 = x p_+$, $p_+^2 = (1-x) p_+$], where W_μ is the Pauli-Lubanski vector [the reason for this choice of spin label will emerge below^{*)}]. We can now define a projection operator P_Λ which isolates the singularity in $A \cdot B$ by

*) The 'particularly useful' spinors introduced by Brodsky and Lepage (Appendix A.2) are eigenstates of h ; their use is mandatory in their formalism and in the equivalent formalism developed here.

$$\begin{aligned}
A \circ P_\Lambda \circ B &\equiv \sum_{h, \hat{h}} \hat{v}(\hat{h}) \hat{A} \hat{u}(h) \frac{\bar{u}(h) B v(h)}{4m^2} \delta\left(\frac{2k_+}{p_+} - 2x + 1\right) \theta(\Lambda^2 - \vec{k}_T^2) \frac{d^4k}{(2\pi)^4} \\
&= p_+ \sum_{h, \hat{h}} \hat{v} \hat{A} \hat{u} \iint d^2k_T \int \frac{dk_-}{2} \frac{\bar{u} B v}{4m^2},
\end{aligned} \tag{A.3}$$

wherein $\hat{A}$ is equal to A with $\vec{k}_T$ and k_+ set equal to zero.

We now rewrite the BS equation (A.2) as

$$\chi = S V \circ \chi = [1 - (1 - P_\Lambda) \circ S K]^{-1} \cdot P \circ \chi \tag{A.4}$$

where, thanks to the projector $(1 - P_\Lambda)$, the factor $[1 - (1 - P_\Lambda) \circ S K]$, which contains all the k_+ and $\vec{k}_T^2$ dependence, is free from mass singularities. The expression (A.1) for the form factor can then be rewritten as

$$\bar{\chi} \circ t_\mu \circ \chi = \overline{P_\Lambda \chi} \circ T_\mu^\Lambda \circ P_\Lambda \chi, \tag{A.5}$$

with

$$T_\mu^\Lambda = \overline{[1 - (1 - P_\Lambda) \circ S K]^{-1} \circ t_\mu \circ [1 - (1 - P_\Lambda) \circ S K]}$$

The projectors in Eq. (A.5) put the external legs of T_μ on mass-shell and sandwich it between on-shell spinors, giving a factor which is gauge invariant and may therefore be written

$$T^\circ(x, x', h_i, h'_i, \Lambda) (p' + p)_\mu. \tag{A.6}$$

Defining

$$\psi_\pi(x, \vec{k}_T, h_i) = p_+ \sqrt{x(1-x)} \int \frac{dk_-}{2\pi} \frac{\bar{u}(h_i) \chi v(h_2)}{4m^2} \tag{A.7}$$

and setting

$$\phi^\circ(x, h_i, \Lambda) = \iint \frac{d^2k_T}{16\pi^3} \psi_\pi \tag{A.8}$$

Eq. (A.5) then gives

$$\begin{aligned}
 F_{\pi} &= \sum_{h_i, h'_i} \int \phi^{*0}(x', h'_i, \Lambda) T^0 \phi(x, h_i, \Lambda) \frac{dx}{\sqrt{x(1-x)}} \frac{dx'}{\sqrt{x'(1-x')}} \\
 &= \sum_{h_i, h'_i} \int \phi^* T \phi \frac{dx}{\sqrt{x(1-x)}} \frac{dx'}{\sqrt{x'(1-x')}} \quad (A.9)
 \end{aligned}$$

where

$$\phi \equiv D^{-1}(\Lambda^2, \mu) \phi^0, \quad T \equiv D^2(\Lambda^2, \mu) T^0$$

D being related to the behaviour of the bare quark propagator S^0 by

$$S^0(q) = \frac{i \not{q} D(q^2, \mu)}{q^2 - m^2(\mu)} + O(m(\mu))$$

where μ is the renormalization scale.

The following remarks concerning the master formula (A.9) and the wave function ψ_{π} and distribution amplitude ϕ are now in order:

1) We can choose different scales $\Lambda_{i,f}$ in describing the initial (i) and final (f) pions and, since T_H , ϕ_i , and ϕ_f are separately renormalization group invariant, we can choose different values of μ to describe them, i.e. $\phi_{i,f} = \phi_{i,f}(\Lambda_{i,f}, \mu_{i,f})$, $T_H = T_H(\Lambda_i, \Lambda_f, \mu, Q)$. If μ , Λ_i , and Λ_f are taken to be of order Q , T_H can be calculated perturbatively, the result being

$$T_H = \frac{64\pi\alpha_s(Q^2)}{3Q^2} \left[\frac{Q_2}{x_1 x'_1} + \frac{Q_1}{x_2 x'_2} \right] + \frac{1}{Q^2} O\left(\alpha_s^2, \frac{m^2}{Q^2}, \frac{\Lambda_{QCD}^2}{Q^2}\right) \quad (A.10)$$

where Q_i are the quark charges, and Λ_{QCD}^2/Q^2 indicates the existence of non-perturbative higher-twist corrections [whose symptom in perturbation theory is the appearance of $m^2 \ln(m^2/Q^2)/Q^2$ terms].

2) Taking the derivative of ϕ with respect to Λ , using dimensional analysis to relate the Λ dependence to the μ dependence of D (which is given by the renormalization group), and then setting $\Lambda^2 = \mu^2$, leads to the evolution equation for $\phi(\Lambda)$ [3].

3) We introduced h_i , x_i , and $\vec{k}_T$ in a frame in which $\vec{p}$ had a definite value directed along the 3-axis. We can, however, make a Lorentz transformation [42]

$$e^{-i\vec{\beta}_T \cdot \vec{E}_T - i\beta_3 K_3}$$

where

$$\vec{E}_T = \left[\frac{1}{2} (k_1 + J_2), \frac{1}{2} (k_2 - J_1) \right]$$

which changes the components k_+ and $\vec{k}_T$ of an arbitrary four-vector to

$$k'_+ = k_+ e^{\beta_3}$$

$$\vec{k}'_T = \vec{k}_T + k_+ \vec{\beta}_T e^{\beta_3}$$

with $\beta_3, \vec{\beta}_T$ chosen to give any desired values of p'_+ and $\vec{p}'_T$ (the cases $\vec{\beta}_T = 0$ and $\beta_3 = 0$ are described as longitudinal and transverse boosts, respectively, in the literature, although the latter is not in fact a pure boost). The values of $x_i \equiv k^i_+/p_+, h_i \equiv W^i_+/p_+$ are invariant under this transformation which also leaves $p_+ dk_-$ in Eq. (A.7) invariant but takes $\vec{k}_T$ to $\vec{k}'_T = \vec{k}_T + \vec{p}'_T$. The function $\psi_\pi(x_i, \vec{k}_T, h_i)$ therefore gives the amplitude to find a $q\bar{q}$ pair with $k^i_+ = x_i p_+, W^i_+ = h_i p_+$, and transverse momenta $x_i \vec{p}_T \pm \vec{k}_T$ independent of p_μ .

Quantizing on the light cone at $\tau = t + z = 0$, only the components $\psi_+ = [\gamma_0(\gamma_0 + \gamma_3) \psi]/2$ of the quark field ψ are dynamically independent. They can be expanded in terms of creation and annihilation operators as

$$\psi_+ = \int_{k_+ > 0} \frac{dk_+ d^2 k_T}{k_+ 16\pi^3} \sum_h \left\{ b(\underline{k}, h) u_+(\underline{k}, h) e^{-i\mathbf{k} \cdot \mathbf{x}} + d^\dagger(\underline{k}, h) v_+(\underline{k}, h) e^{i\mathbf{k} \cdot \mathbf{x}} \right\}_{\tau=0} \quad (\text{A.11})$$

where $\mathbf{k} = (k_+, \vec{k}_T)$ and

$$\{ b(\underline{k}, h), b^\dagger(\underline{k}', h') \} = 16\pi^3 k_+ S_{hh'} S^3(\underline{k} - \underline{k}') , \text{ etc.} \quad (\text{A.12})$$

Introducing basis states

$$|q\bar{q} : \underline{k}_i, h_i\rangle = b^\dagger(\underline{k}_1, h_1) d^\dagger(\underline{k}_2, h_2) |0\rangle , \text{ etc.} , \quad (\text{A.13})$$

we can write a Fock representation for the pion:

$$|\pi : p^+, \vec{p}_T\rangle = \sum_{n, h_i} \int \prod_i \left(\frac{dx_i d^2 k_T^i}{\sqrt{x_i} 16\pi^3} \right) \delta(1 - \sum_i x_i) 16\pi^3 \delta(\sum_i \vec{k}_T^i)$$

$$|n : x_i p^+, x_i \vec{p}_T + \vec{k}_T^i, h_i\rangle \psi_n^\pi(x_i, \vec{k}_T^i, h_i)$$

(A.14)

where, by the above argument, the ψ_n^π 's do not depend on p_μ . If we introduce fields $\tilde{\psi}$, and correspondingly spinors $\tilde{u}$ and $\tilde{v}$, so that $\tilde{\psi}_+ = \psi_+$, and $\tilde{\psi}_-$ is related to ψ_- by the *free* Dirac equation, we can invert Eq. (A.11) and express the creation and annihilation operators in terms of $\tilde{u}$, $\tilde{v}$, etc. It is then straightforward to show that ψ_2^π is given by

$$\psi_2^\pi = \sqrt{x(1-x)} p_+ \int \frac{dq_-}{2\pi} \tilde{u} \tilde{\chi}(q) \tilde{v}, \quad (A.15)$$

where

$$\begin{aligned} \tilde{\chi}_{\alpha\beta}(x_1, x_2) &= \langle 0 | T(\tilde{\psi}_\alpha(x_1) \tilde{\bar{\psi}}_\beta(x_2)) | \pi \rangle \\ &= e^{-ip \cdot (x_1 + x_2)/2} \int e^{iq \cdot (x_1 - x_2)} \tilde{\chi}_{\alpha\beta}(q) \frac{d^4 q}{(2\pi)^4}. \end{aligned} \quad (A.16)$$

Equation (A.15) closely resembles Eq. (A.7), except that the latter involves the full BS wave function constructed from the complete quark field ψ , and includes a factor $P \exp(i\int_{x_1}^{x_2} A \cdot dz)$ which makes it gauge invariant. It is not hard to see, however, that in physical gauges—to which we are committed—these differences are immaterial to leading order in $1/Q^2$. Hence ψ_π in Eq. (A.7) and ψ_2^π are effectively interchangeable.

Various comments are now appropriate:

1) It is only in LC quantization that the Fock representation exists. In equal time quantization all states are dominated by vacuum fluctuations, of infinite norm, which contribute to both disconnected and connected amplitudes, and normalizable state vectors cannot be constructed (this is the content of Haag's theorem). In contrast, vacuum fluctuations can be ignored in light-cone quantization as they contribute only to disconnected diagrams, i.e. we can write $|\pi\rangle_{LC} = |\hat{\pi}\rangle_{LC} \otimes |vac\rangle$ and consider a Fock representation for the normalizable state $|\hat{\pi}\rangle_{LC}$ since $|vac\rangle$ does not contribute to connected matrix elements^{*)}. This is only a formal advantage, however, since we can

*) This is a result of the fact that the light-cone Hamiltonian H_{LC} conserves k_+ . It is sometimes incorrectly stated (e.g. in Ref. [4]) that, because of this, all vacuum diagrams vanish and the bare vacuum is an eigenstate of H_{LC} (if it were true, this would prohibit spontaneous symmetry breaking!). In fact disconnected diagrams are sufficiently singular that the point $k_+ = 0$ contributes and cannot be ignored, as it can in all other diagrams (see Ref. [43]). The derivation [44] of light-cone Feynman rules from the conventional rules by letting $\vec{p} \rightarrow \infty$ also appears formally to lead to the conclusion that vacuum diagrams vanish, but this derivation involves interchanging the order of taking the limit with performing integrations, which is illegitimate for vacuum diagrams, whose Lorentz-invariant contributions cannot be removed by changing frame.

construct components of connected amplitudes which play the role of wave functions without invoking light-cone quantization, as we did in Eq. (A.7), and we can talk of the $q\bar{q}$ state dominating the pion wave function—meaning that it dominates connected matrix elements—although in fact the conventional equal t-wave function is overwhelmed by vacuum fluctuations.

2) By working in the frame in which $q_+ = 0$, $\vec{q}_T^2 = Q^2$, considering the '+' component of the electromagnetic current, and using the fact that $\bar{u}(k'_+, h') \gamma_+ u(k_+, h) = 4(k_+ k'_+)^{1/2} \delta_{h'h}$, it is easy to find exact expressions for form factors such as

$$F_\pi(Q^2) = \sum_{n,h} \sum_q \eta_q Q_q \int \prod_i \frac{dx_i d^2 k_T^i}{16\pi^3} \delta(1 - \sum x_i) \delta^2(\sum \vec{k}_T^i) \cdot \psi_n^{\pi^*}(x_i, \vec{k}_T^i) \psi_n^\pi(x_i, \vec{k}_T^i) \quad (A.17)$$

where $\eta_q = +1$ for quarks and -1 for antiquarks, and $\vec{k}_T^i = \vec{k}_T^i - x_i \vec{q}_T + \vec{q}_T$ for the struck quark q and $\vec{k}_T^i = \vec{k}_T^i - x_i \vec{q}_T$ for the others. Similar formulae may be derived in other frames, but they involve additional kinematical factors, as in Eq. (23).

3) It follows from Eq. (A.11) to (A.14) that the LC wave functions satisfy the normalization condition

$$1 = \sum_{n,h} \int \prod_i \frac{dx_i d^2 k_T^i}{16\pi^3} \delta(1 - \sum x_i) 16\pi^3 \delta^2(\vec{k}_T^i) |\psi_n^\pi|^2 \quad (A.18)$$

This is closely related to Eq. (A.17) for $Q^2 \rightarrow 0$ and, like Eq. (A.17), has only a formal meaning unless the theory is provided with an ultraviolet cut-off. Equation (A.12) implies that the quark fields are normalized so that the ψ_n^π 's are divergent, cut-off-dependent quantities. (Note, however, that the cut-off dependence of D cancels that in T^0 and ϕ^0 so that T and ϕ in Eq. (A.9) remain finite when the cut-off is removed.)

4) We imagine working with a real cut-off Λ which would not change the physics at scales much less than Λ , up to corrections of order $1/\Lambda$. The solution of QCD with Λ of a (few) GeV would resemble the constituent quark model, and Eq. (A.18) would be dominated by ψ_2^π , which would presumably generate a reasonable approximation to the cut-off-independent distribution amplitude $\phi(Q^2)$ for $Q^2 \ll \Lambda^2$, which can then be evolved to higher Q^2 .

5) The unrenormalized wave function is constrained by the relation

$$\langle 0 | J_\mu^w | \pi(p) \rangle = f_\pi p_\mu, \quad (A.19)$$

where the isospin raising/lowering current J_μ^w is constructed from unrenormalized fields. Equations (A.14) and (A.19) give

$$f_\pi = 2\sqrt{2} N_c \iiint \frac{dx d^2 k_T}{16\pi^3} \psi_2^\pi(x, \vec{k}_T) \quad (A.20)$$

where N_c is the number of colours. We can write the right-hand side of Eq. (A.20) in terms of the renormalized amplitude ϕ if we calculate with a cut-off Λ , since then $D(\Lambda^2, \Lambda) = 1 + O[\alpha_s(\Lambda)]$, which tends to one as $\Lambda \rightarrow \infty$, so that

$$\int \frac{dx d^2 k_T}{16\pi^3} \psi_2^\pi = \lim_{\Lambda \rightarrow \infty} \int dx \phi(x, \Lambda) . \quad (\text{A.21})$$

This constrains ϕ and leads to the classic prediction (3) for the asymptotic behaviour of F_π . We have spelt out the derivation of the ϕ - f_π relation because we feel that the treatment of renormalization is not always clear in discussions in the literature.

6) Whereas it is possible to write quite general covariant BS wave functions and fixed t -wave functions which describe states of definite J^{PC} , the same is not true for the LC wave functions. The reason is that, in contrast to the basis states in fixed- t quantization which are eigenstates of $\hat{P}$ and have known properties under rotations, the basis states $|xp_+, \vec{k}_T\rangle$ are not eigenstates of P_3 . We know how they transform under rotations about the 3-axis but not under general rotations (exact eigenstates of $\hat{P}_-$ would turn out to have definite J^{PC} but we cannot construct an ansatz with definite J without solving the eigenvalue equation for $\vec{J}^2$, which is as hard as solving the light-cone Schrödinger equation). Setting $\vec{p}_\pi^T = 0$, the pion must have $J_z = 0$, which restricts ψ_2^π to four terms with the structure $(k_1 - ik_2) \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow$, and $(k_1 + ik_2) \downarrow\downarrow$, each multiplied by a function of x and $\vec{k}_T^2$, in an obvious notation. By considering the combined effect of a rotation through π about an axis in the xy plane perpendicular to $\vec{k}_T$ and a reflection in the origin, we obtain

$$\psi_2^\pi \left(h_1 = \frac{1}{2}, h_2 = -\frac{1}{2} \right) = -\psi_2^\pi \left(-\frac{1}{2}, \frac{1}{2} \right) = \psi^\pi(x, \vec{k}_T^2) \quad (\text{A.22})$$

$$\psi_2^\pi \left(\frac{1}{2}, \frac{1}{2} \right) = (k_x - ik_y) \Theta(x, \vec{k}_T^2) \quad (\text{A.23})$$

$$\psi_2^\pi \left(-\frac{1}{2}, -\frac{1}{2} \right) = (k_x + ik_y) \Theta(x, \vec{k}_T^2) . \quad (\text{A.24})$$

APPENDIX B: DERIVING A BOUND ON THE PERTURBATIVE CONTRIBUTION TO F_π

Consistency of the perturbative analysis for $F_\pi(Q^2)$ requires that we use for the distribution amplitude at Q^2 ,

$$\phi_\pi(x, Q^2) \equiv \int_{f_{xQ}} d^2 p_\tau \Psi_\pi^0(x, \vec{p}_\tau) \quad (\text{B.1})$$

This allows us to bound

$$Q^2 F_\pi(Q^2) = \frac{4\alpha_s(Q^2)}{3\pi^2} \left| \int_0^1 dx \frac{1}{x} \phi_\pi(x, Q^2) \right|^2. \quad (\text{B.2})$$

To derive the bound, first note that we can always write

$$\Psi_\pi^0(x, \vec{p}_\tau) = \sqrt{P_{q\bar{q}}^0} X(x) P(x, \vec{p}_\tau) \quad (\text{B.3})$$

with $P_{q\bar{q}}^0$ the $q\bar{q}$ probability in the S-wave piece of the pion (see Appendix A), P a normalized transverse-momentum amplitude dependent on x ,

$$1 = \int d^2 p_\tau |P(x, \vec{p}_\tau)|^2 \quad (\text{B.4})$$

and $X(x)$ also normalized:

$$1 = \int dx |X(x)|^2. \quad (\text{B.5})$$

Then

$$\int_0^1 dx \frac{1}{x} \phi_\pi(x, Q^2) \leq \int_0^1 dx \frac{1}{x} X(x) \int_{f_{xQ}} d^2 p_\tau P(x, \vec{p}_\tau) \quad (\text{B.6})$$

since $P_{q\bar{q}}^0 \leq 1$. Using the fact that X and P must be unchanged under $x \leftrightarrow 1 - x$, we can write

$$I = \int_0^{1/2} dx X(x) \left[\left(\frac{1}{x} + \frac{1}{1-x} \right) \int_0^{f_{xQ}} d^2 p_\tau P(x, \vec{p}_\tau) + \frac{1}{1-x} \int_{f_{xQ}}^{f_{(1-x)Q}} d^2 p_\tau P(x, \vec{p}_\tau) \right]. \quad (\text{B.7})$$

We will now minimize I at a given $Q = Q_0$. The Schwartz inequality gives

$$\int_a^b d^2 p_T P(x, p_T) \leq \left[\pi \left(\int_a^b d^2 p_T |P|^2 \right) \left(\int_a^b d^2 p_T^2 \right) \right]^{1/2} \quad (\text{B.8})$$

so that setting

$$A(x) \equiv \int_0^{fxQ} d^2 p_T^2 |P|^2 \quad (\text{B.9})$$

$$B(x) \equiv \int_{fxQ}^{f(1-x)Q} d^2 p_T^2 |P|^2 \quad (\text{B.10})$$

we have

$$I \leq \sqrt{\pi} f Q_0 \int_0^{1/2} dx \frac{1}{1-x} X(x) \left[\sqrt{A(x)} + \sqrt{B(x)(1-2x)} \right] \quad (\text{B.11})$$

It is easy to maximize the square bracket in (B.11) subject to the condition $A + B \leq 1$, the result being

$$A = \frac{1}{2(1-x)} \quad (\text{B.12})$$

$$B = \frac{1-2x}{2(1-x)} \quad (\text{B.13})$$

Eq. (B.11) combined with the Schwartz inequality then gives

$$I \leq \sqrt{2\pi} f Q_0 \int_0^{1/2} dx \frac{1}{\sqrt{1-x}} X(x) \quad (\text{B.14})$$

$$\leq \sqrt{2\pi} f Q_0 \left[\int_0^{1/2} dx |X|^2 \cdot \int_0^{1/2} dx \frac{1}{1-x} \right]^{1/2} = \sqrt{\pi \ln 2} f Q_0 \quad (\text{B.15})$$

Eq. (B.15) is saturated by setting

$$\psi = \frac{1}{\sqrt{4\pi \ln 2} f Q_0} \begin{cases} x^{-1}(1-x)^{-1} & p_T < f x Q_0 \\ (1-x)^{-1} & f x Q_0 \leq p_T \leq f(1-x) Q_0 \\ 0 & p_T > f(1-x) Q_0 \end{cases} \quad (\text{B.16})$$

for $x \leq 1/2$ and using $\psi(x) = \psi(1-x)$ to determine ψ for $x > 1/2$.

From (B.15) we obtain

$$Q_0^2 F_\pi(Q_0^2) \leq \frac{4 \ln 2 \alpha_s(Q_0^2) f^2 Q_0^2}{3\pi} \quad (\text{B.17})$$

as the maximum at any given value of Q_0^2 . If the data saturate this bound at some point $Q = Q_0$, then—if the use of perturbation theory is valid—the wave function must be given by (B.16) which can then be used to calculate

$$Q^2 F_\pi(Q^2) = \begin{cases} Q_0^2 F_\pi(Q_0^2) \left[\frac{Q_0^2(x_0 - \ln x_0) - Q^2(x_0 + \ln(1-x_0))}{Q^2 2 \ln 2} \right] & Q > Q_0 \\ Q_0^2 F_\pi(Q_0^2) \left[\frac{Q^2(\hat{x}_0 - \ln \hat{x}_0) - Q_0^2(\hat{x}_0 + \ln(1-\hat{x}_0))}{Q_0^2 2 \ln 2} \right] & Q < Q_0 \end{cases} \quad (\text{B.18})$$

at other values of Q^2 . In (B.18), $x_0 \equiv (1 + Q/Q_0)^{-1}$ and $\hat{x}_0 = (1 + Q_0/Q)^{-1}$. The ramifications of these conclusions are discussed in the text.

APPENDIX C: THE NUCLEON DISTRIBUTION AMPLITUDES IN AN S_3 SYMMETRIC BASIS

It is convenient to discuss the nucleon infinite-momentum frame/light-cone distribution amplitudes in terms of a set of functions constructed of representations of the permutation group on three objects (S_3). The most general such distribution amplitude with the required isospins and J_z values $\pm$ are^{*)} (suppressing the overall antisymmetric colour wave function)

$$\begin{aligned} \phi_N^\pm(x_1, x_2, x_3) = & \sqrt{\frac{1}{2}} (\phi_N^p \chi_\pm^p + \phi_N^\lambda \chi_\pm^\lambda) \phi^s(x_1, x_2, x_3) \\ & \pm \sqrt{\frac{1}{2}} (\phi_N^p \phi_\star^p(x_1, x_2, x_3) + \phi_N^\lambda \phi_\star^\lambda(x_1, x_2, x_3)) \chi_{\frac{3}{2}^\pm}^s \\ & + \sqrt{\frac{1}{2}} (\phi_N^p \chi_\pm^\lambda - \phi_N^\lambda \chi_\pm^p) \phi^A(x_1, x_2, x_3) \\ & + \frac{1}{2} (\chi_\pm^p \phi_N^p \phi^\lambda(x_1, x_2, x_3) + \chi_\pm^\lambda \phi_N^\lambda \phi^p(x_1, x_2, x_3) + \chi_\pm^p \phi_N^\lambda \phi^p(x_1, x_2, x_3) - \chi_\pm^\lambda \phi_N^p \phi^\lambda(x_1, x_2, x_3)) \end{aligned} \quad (C.1)$$

where S, A, and (ρ, λ) refer to functions which are symmetric, antisymmetric, and of mixed symmetry under S_3 with the convention (P_{ij} interchanges indices i and j)

$$P_{12} M^p = -M^p \qquad P_{12} M^\lambda = +M^\lambda \quad (C.2)$$

$$P_{13} M^p = \frac{1}{2} M^p - \sqrt{\frac{3}{4}} M^\lambda \qquad P_{13} M^\lambda = -\frac{1}{2} M^\lambda - \sqrt{\frac{3}{4}} M^p \quad (C.3)$$

(Note that the full wave function has many more components since a given J_z can be obtained from $M_L \neq 0$ as well as $M_L = 0$, but in forming the distribution amplitude [$\vec{p}_\rho = \sqrt{1/2}(\vec{p}_1 - \vec{p}_2)$, $\vec{p}_\lambda = \sqrt{1/6}(\vec{p}_1 + \vec{p}_2 - 2\vec{p}_3)$])

$$\phi_N^\pm(x) \propto \iint d^2 p_T^p d^2 p_T^\lambda \psi_N^\pm(x, \vec{p}_T^p, \vec{p}_T^\lambda) ,$$

$m_L \neq 0$ components of the wave function are annihilated.) For explicit forms for the flavour and spin wave functions, see, for example, Ref. [34].

The authors of Ref. [8] introduce, instead of the four irreducible distribution amplitudes $\phi^S, \phi^A, (\phi^p, \phi^\lambda)$, and $(\phi_\star^p, \phi_\star^\lambda)$, the amplitudes V and A in terms of which one has the state vector representation

^{*)} See, for example, Ref. [46].

$$\begin{aligned}
|p\uparrow\rangle \propto \int dx \left\{ \frac{1}{2} [V(x) - A(x)] |u\uparrow(x_1)u\downarrow(x_2)d\uparrow(x_3)\rangle \right. \\
+ \frac{1}{2} [V(x) + A(x)] |u\downarrow(x_1)u\uparrow(x_2)d\uparrow(x_3)\rangle \\
\left. - T(x) |u\uparrow(x_1)u\uparrow(x_2)d\downarrow(x_3)\rangle \right\} , \quad (C.4)
\end{aligned}$$

where $V(x_1x_2x_3) = V(x_2x_1x_3)$, $A(x_1x_2x_3) = -A(x_2x_1x_3)$, and where

$$T(x_1x_2x_3) = \frac{1}{2} [V(x_1x_3x_2) + V(x_2x_3x_1) - A(x_1x_3x_2) - A(x_2x_3x_1)] . \quad (C.5)$$

One can easily determine the relationship between these two representations to be that

$$\phi^S = -\sqrt{\frac{1}{6}} (1 + P_{13} + P_{23}) V \quad (C.6)$$

$$\phi^A = \sqrt{\frac{1}{18}} (1 - P_{13} - P_{23}) A \quad (C.7)$$

$$\left\{ \begin{aligned} \phi^P &= -\frac{2}{3} (1 + \frac{1}{2} P_{13} + \frac{1}{2} P_{23}) A \end{aligned} \right. \quad (C.8)$$

$$\left\{ \begin{aligned} \phi^\lambda &= \sqrt{\frac{1}{3}} (P_{13} - P_{23}) A \end{aligned} \right. \quad (C.9)$$

$$\left\{ \begin{aligned} \phi_*^P &= \frac{1}{2} (P_{23} - P_{13}) V + \frac{1}{3} (1 + \frac{1}{2} P_{13} + \frac{1}{2} P_{23}) A \end{aligned} \right. \quad (C.10)$$

$$\left\{ \begin{aligned} \phi_*^\lambda &= \sqrt{\frac{1}{3}} (1 - \frac{1}{2} P_{13} - \frac{1}{2} P_{23}) V - \sqrt{\frac{1}{12}} (P_{13} - P_{23}) A \end{aligned} \right. \quad (C.11)$$

The advantage of forming these irreducible wave functions is that the resulting formulae are simpler than those expressed in terms of V and A.

In terms of the model of Ref. [8] for V and A, we have

$$\phi^S = -\sqrt{\frac{1}{6}} F_N x_1 x_2 x_3 [31.52 (x_1^2 + x_2^2 + x_3^2) - 8.82] \quad (C.12)$$

$$\phi^A = 0 \quad (C.13)$$

$$\left\{ \begin{aligned} \phi^P &= + F_N x_1 x_2 x_3 [6.72 (x_1^2 - x_2^2)] \end{aligned} \right. \quad (C.14)$$

$$\left\{ \begin{aligned} \phi^\lambda &= + \sqrt{\frac{1}{3}} F_N x_1 x_2 x_3 [6.72 (x_1^2 + x_2^2 - 2x_3^2)] \end{aligned} \right. \quad (C.15)$$

$$\left\{ \begin{array}{l} \phi_*^p = -F_N x_1 x_2 x_3 [2.09(x_1^2 - x_2^2) - 0.84(x_1 - x_2)] \\ \phi_*^\lambda = -\sqrt{\frac{1}{3}} F_N x_1 x_2 x_3 [2.09(x_1^2 + x_2^2 - 2x_3^2) - 0.84(x_1 x_2 - 2x_3)] \end{array} \right. \quad (C.16)$$

$$\left\{ \begin{array}{l} \phi_*^p = -F_N x_1 x_2 x_3 [2.09(x_1^2 - x_2^2) - 0.84(x_1 - x_2)] \\ \phi_*^\lambda = -\sqrt{\frac{1}{3}} F_N x_1 x_2 x_3 [2.09(x_1^2 + x_2^2 - 2x_3^2) - 0.84(x_1 x_2 - 2x_3)] \end{array} \right. \quad (C.17)$$

where F_N is an overall constant that is analogous to f_π and related to the f_N of Ref. [8]. That $\phi^A = 0$ is a peculiarity of the model of Ref. [8] arising from its restriction to quadratic functions.

APPENDIX D: SOFT FORM FACTORS

Here we sketch the derivation of the formulae we use for calculating soft contributions to the form factors.

We begin in the Breit frames where we have simply

$$F_{\pi}(Q^2) = \frac{(2\pi)^3}{2E_{Q/2}} \langle \pi(+\frac{\vec{Q}}{2}) | j^0 | \pi(-\frac{\vec{Q}}{2}) \rangle \quad (D.1)$$

$$G_E^N(Q^2) = (2\pi)^3 \langle N(+\frac{\vec{Q}}{2}, +) | j^0 | N(-\frac{\vec{Q}}{2}, +) \rangle \quad (D.2)$$

$$G_M^N(Q^2) = \frac{2M_N(2\pi)^3}{Q} \langle N(+\frac{Q}{2}\hat{x}, -) | j^3 | N(-\frac{Q}{2}\hat{x}, +) \rangle \quad (D.3)$$

$$= -\frac{2M_N(2\pi)^3}{Q} \langle N(+\frac{Q}{2}\hat{z}, -) | \frac{1}{2}(j^1 - ij^2) | N(-\frac{Q}{2}\hat{z}, +) \rangle \quad (D.4)$$

Here all spins are quantized along the unit vector $\hat{z}$, and $E_{Q/2} \equiv (m^2 + 1/4 Q^2)^{1/2}$ for the relevant mass m . If we boost an original $-(Q/2)\hat{z} \rightarrow +(Q/2)\hat{z}$ scattering along $\hat{z}$, we get the longitudinal formulae

$$F_{\pi}|_{\text{long}} = \frac{(2\pi)^3}{P' + P} \langle \pi(P'\hat{z}) | j^0 | \pi(P\hat{z}) \rangle \quad (D.5)$$

$$G_E^N|_{\text{long}} = (2\pi)^3 \sqrt{\frac{M_N^2}{P'P}} \langle N(P'\hat{z}, +) | j^0 | N(P\hat{z}, +) \rangle \quad (D.6)$$

$$G_M^N|_{\text{long}} = -\frac{2M_N(2\pi)^3}{Q} \langle N(P'\hat{z}, -) | \frac{1}{2}(j^1 - ij^2) | N(P\hat{z}, +) \rangle \quad (D.7)$$

where $P'/P = (E_{Q/2} + Q/2)/(E_{Q/2} - Q/2) = \zeta$ as defined by Eq. (23) of the text. These formulae are, of course, just the Lorentz transforms of (D.1), (D.2), and (D.4), with the special feature that a spinor quantized along its direction of motion has an invariant spin under a further boost along that direction. If we boost an original $-(Q/2)\hat{x} \rightarrow +(Q/2)\hat{x}$ scattering along $\hat{z}$, we get the transverse formulae

$$F_{\pi}|_{\text{trans}} = \frac{(2\pi)^3}{2P} \langle \pi(P\hat{z} + \frac{Q}{2}\hat{x}) | j^0 | \pi(P\hat{z} - \frac{Q}{2}\hat{x}) \rangle \quad (D.8)$$

$$G_E^N|_{\text{trans}} = \frac{M_N}{P} (2\pi)^3 \langle LN(+\frac{Q}{2}\hat{x}, +) | j^0 | LN(-\frac{Q}{2}\hat{x}, +) \rangle \quad (D.9)$$

$$G_M^N|_{\text{trans}} = \frac{2M_N E_{Q/2}}{Q P} (2\pi)^3 \langle LN(+\frac{Q}{2}\hat{x}, -) | j^3 | LN(-\frac{Q}{2}\hat{x}, +) \rangle \quad (D.10)$$

where LN is the boosted nucleon as $P \rightarrow \infty$:

$$|LN(\pm\frac{Q}{2}\hat{x}, \mp)\rangle = \sqrt{\frac{E_{Q/2} + M_N}{2E_{Q/2}}} \left\{ |N(P\hat{z}, \mp)\rangle + \frac{Q/2}{E_{Q/2} + M_N} |N(P\hat{z}, \pm)\rangle \right\} \quad (D.11)$$

which contains a Wigner-rotated component with spin opposite to the original spin.

Consider first Eq. (D.8). In the approximation of ignoring all but the $q\bar{q}$ sector of Fock space, we can write (with h the total $q\bar{q}$ spin along $P\hat{z}$)

$$|\pi(\vec{P})\rangle = \sum_h \sqrt{2} m \int \frac{dx d^2p_T}{\sqrt{x(1-x)}} \psi^h(x, \vec{p}_T) |q(x, \vec{p}_T, s) \bar{q}(1-x, -\vec{p}_T, \bar{s})\rangle, \quad (D.12)$$

where we have suppressed the spin, flavour, and colour indices on ψ , and where with $\langle q(p's')|q(ps)\rangle = (E/m) \delta_{s's} \delta^3(\vec{p}' - \vec{p})$ and $\langle \pi(\vec{P}') | \pi(\vec{P}) \rangle = 2E \delta^3(\vec{P}' - \vec{P})$ the ψ^i are normalized so that $\sum_i \int dx d^2p_T |\psi^i|^2 = 1$ with an assumed $q\bar{q}$ dominance. Since ψ^h is invariant under a change of direction of $\vec{P}$, the two states $|\pi[P\hat{z} \pm (Q/2)\hat{x}]\rangle$ required in Eq. (D.8) differ only by having total quark [antiquark] transverse momenta $\pm x(Q\hat{x}/2) + \vec{p}_T [\pm(1-x)(Q\hat{x}/2) - \vec{p}_T]$; the rotation of the spin quantization axis can be ignored as $P \rightarrow \infty$. One therefore easily obtains the result

$$F_\pi(Q^2)|_{\text{trans}} = \sum_h \int dx d^2p_T \psi^{h*}(x, \vec{p}_T + [1-x]Q\hat{x}) \psi^h(x, \vec{p}_T) \quad (D.13)$$

which leads to Eqs. (22) and (36) in the text. It should be clear to the reader how to generalize this formula to an arbitrary Fock decomposition of the pion: the result is the exact formula (A.17) derived in Ref. [45].

Before proceeding to further derivations, let us pause to consider the implications of approximating the exact result (A.17) by (D.13). In the spirit of the constituent quark model, it is natural to represent the pion form factor at low to intermediate Q^2 as

$$F_\pi(Q^2) = \sum_h \int dx d^2p_T \psi^{h*}(x, \vec{p}_T + [1-x]Q\hat{x}) \psi^h(x, \vec{p}_T) F_q(Q^2) \quad (D.14)$$

where $F_q(Q^2)$ is a constituent quark form factor. This is an approximation to Eq. (A.17) in which:

- the (unrenormalized) $q\bar{q}$ wave function is replaced by the (renormalized) ‘non-perturbative’ constituent quark-model wave function, calculated in a theory with an ultraviolet cut-off of the order of a (few) GeV and therefore lacking the perturbative power-law tail at large p_T^2 ;
- higher Fock-state contributions are neglected, except those that contribute to the quark form factor $F_q(Q^2)$. For $x \rightarrow 1$, which tends to dominate Eq. (22) since the constituent quark-model wave function falls off very rapidly with $\vec{p}_T^2$, the struck quark is close to mass shell before and after it absorbs the virtual photon; for large

Q^2 , this quark has a tendency to radiate gluons copiously; consequently the probability of scattering without radiation is suppressed by a form factor. This 'Sudakov' form factor F_q depends on the invariant masses p_1^2 and p_2^2 of the initial and final struck quarks and might behave approximately as [47]

$$F_q(Q^2) \sim \exp \left\{ - \frac{2\alpha_s}{3\pi} \ln \frac{Q^2}{p_1^2} \ln \frac{Q^2}{p_2^2} \right\}.$$

We do not include this factor in our calculations, but we should not forget its presence, which will tend to decrease the predictions based on our soft scattering formulae (by a factor of perhaps 2 to 10 for $Q^2 \approx 10 \text{ GeV}^2$)^{*}.

The derivation of $F_{\pi|_{\text{long}}}$ from Eq. (D.5) proceeds along similar lines (although it is somewhat more straightforward since $\vec{P}'$ and $\vec{P}$ are parallel) leading to Eqs. (23) and (35) in the text. Similarly, the longitudinal nucleon form factors of Eqs. (43) and (44) are easy to derive. Moreover, the resulting formulae remain sensible when truncated to the $L_z = 0$ (i.e. $h = \text{pion or nucleon helicity}$) wave function only, as in Eqs. (22), (23), (43), and (44). For example, the truncated formulae contain the weak binding limit in which other components of the wave function are unimportant.

As mentioned in the text, a complication arises for the transverse nucleon formulae, and in particular for $G_{M|_{\text{trans}}}^N$. The complication is apparent from Eq. (D.11): the boost of a transverse Breit-frame scattering of the spinning proton produces Wigner rotation. One might therefore anticipate that in these cases inconsistencies could occur with the $L_z = 0$ truncations. Indeed, since the spin of the nucleons is carried by the quarks, a minimum requirement if we are even to recover the weak binding limit is that the quarks undergo the Wigner rotation analogous to Eq. (D.11). Of course the true Fock-space wave function has automatically the required properties, but as emphasized in Appendix A the desired relationships between the 'Wigner rotated' wave functions and the $L_z = 0$ wave function are dynamical in nature. Since the only case where these relationships are known is for free particles, as an alternative to modelling the Wigner-rotated wave functions independently we have chosen them in a simple way that guarantees that we will recover the weak binding limit: we take them to be the wave functions that arise from Wigner rotation of the quarks in the parent nucleons with momenta $\mp(Q/2)\hat{x}$ in the approximation that quark i has momentum $\mp x_i(Q/2)\hat{x}$ (i.e. we neglect intrinsic transverse momentum). This leads to Eq. (46). Since G_E^N has the correct weak binding limit at low Q^2 without these complications, for simplicity we ignore them in Eq. (45) (although we have checked that they would not make much difference there).

It will be clear to the reader that the soft-scattering formulae in the text are useful only as rough guides to the non-perturbative contributions to the form factors. As discussed in the text, $L_z \neq 0$ wave functions could easily change the calculations for the pion by a factor of 2. Similar sensitivities can be expected for the other form factors, excepting G_M^N which depends directly on such wave functions and is correspondingly more uncertain.

^{*}) Note that there is no form factor in the perturbative contribution that dominates at asymptotic Q^2 , because the colour-singlet $q\bar{q}$ pairs that enter/leave the hard-scattering amplitude T_H are close together in coordinate space. The probability that they radiate soft gluons is therefore small—except for $x_i \rightarrow 0, 1$, where higher-order contributions of order $\alpha_s[\ln x_i/\ln(1-x_i)]$ are encountered which must sum up to give a form factor suppression in this region; this is another and independent reason why perturbative calculations in which the $x_i \approx 0, 1$ regions dominate should not be trusted.

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