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TAXONOMY OF HETEROTIC SUPERCONFORMAL FIELD THEORIES

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ABSTRACT

We derive the massless spectrum of all heterotic 2-dimensional superconformal field theories constructed by tensoring models from the discrete series ($c < 3$) using diagonal and exceptional affine invariants (but not including the quotients by discrete groups). A comparison is made between these models and known manifold constructions lending some support to the conjecture that some of these are sigma models with Calabi-Yau target spaces.

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Superstring theories provide the basis for an elegant , probably finite, unification of the fundamental forces including gravity. However only in ten dimensions is superstring propagation in flat Minkowski space free from anomalies. In order to construct viable models in four flat dimensions it is necessary to consider string propagation on a manifold $M \times K$ where M is four-dimensional Minkowski space and K is an internal manifold (6-dimensional for the fermionic string and 22-dimensional for the bosonic string) whose size is of order M_P^{-1} .

There has been considerable work to determine the nature of K and many possibilities [1] have been considered including Calabi-Yau , orbifold and the so-called '4-dimensional string' constructions. From a phenomenological viewpoint the structure of K is crucial for it gives rise to the internal symmetries of the low-energy world and, in principle, determines all the parameters, the masses and coupling constants, of the effective low energy Lagrangian.

Recently a particularly interesting construction has been developed by Gepner [2] in which he forms K implicitly by tensoring products of minimal $N = 2$ superconformal field theories, labelled by the level k of the underlying Z_k parafermionic theory, chosen to cancel the conformal trace anomaly. These are the only known non-trivial $N = 2$ superconformal theories and the technique provides a generalization of the trivial realizations in terms of free fermions which lead to the 4-D constructions and (via their discrete quotients) to the orbifold construction. Remarkably the massless spectrum and discrete symmetries of some of the first such models studied closely match those of known complete intersection Calabi-Yau (CICY) models [3], leading Gepner to the conjecture that all models constructed in such a way correspond to string propagation on Calabi-Yau manifolds (or their orbifold limits) and indeed that such models exhaust the set of Calabi-Yau manifolds.

In this letter we present a first step in cataloguing the models based on this construction, in which we determine the massless spectrum and discrete symmetries of models based on the standard (A) affine invariant, which exists at all levels, and the three exceptional (E) invariants at levels 10, 16 and 28 [4]. Other models of this type are formed by taking their quotient with a discrete group, freely acting or not. In this way one can construct the models based on the only other affine invariants, the D invariants which exist at levels $k = 4j$ and $k = 4j - 2$ ($j = 1, 2, \dots$), as well as other twisted models formed by using the discrete symmetries of the theta function invariants. Thus the class of models considered here gives an upper bound on the discrete symmetries allowed by this type of construction and leads to a preliminary test of the Gepner conjecture by comparing with the set of CICY-manifolds, as well as giving a starting point for the construction of promising phenomenological models.

The minimal $N = 2$ models at level k have primary fields $\Phi_{q,s;\bar{q},\bar{s}}^{l;\bar{l}}$ where l, q, s and $\bar{l}, \bar{q}, \bar{s}$ label the left- and right-moving sectors respectively. The quantum numbers satisfy

$$\begin{aligned}
0 &\leq l \leq k \\
q &:= q(\text{mod } k), \quad |q| \leq l \\
s &:= s(\text{mod } 4) \\
s &= 0, 2(\text{Neveu - Schwarz}(NS)\text{sector}) \\
s &= \pm 1(\text{Ramond}(RA)\text{sector})
\end{aligned} \tag{1}$$

and similarly for the right sector indices. (In constructing the massless states in the product theory it is necessary to include non-primary fields which are conveniently labelled by $s = 2$ (NS sector)[5].) Fields with indices not in this range may be re-expressed in terms of these labels by exploiting the identity

$$\Phi_{q,s;\bar{q},\bar{s}}^{l;\bar{l}} = \Phi_{q+k+2,s+2;\bar{q}+k+2,\bar{s}+2}^{k-l;k-\bar{l}} \tag{2}$$

Notice that it may not be possible to bring the indices of both the right and left sector into the ‘standard’ range (1) simultaneously.

The conformal trace anomaly for the level k theory is given by

$$c_i = \frac{3k_i}{k_i + 2} \quad (3)$$

and the conformal dimension and $U(1)$ charges are given by

$$h = \frac{l(l+2) - \tilde{q}^2}{4(k+2)} + \frac{s^2}{8} \quad (4)$$

$$Q = -\frac{\tilde{q}}{k+2} + \frac{s}{2} \quad (5)$$

where $\tilde{q} = q + s$ in the RA sector, and $\tilde{q} = q$ in the NS sector.

In order to construct an anomaly free theory in $D(= 4, 6, 8)$ dimensions it is necessary to choose a set of r minimal models so that their trace anomaly adds to $\frac{3}{2}(10 - D)$, i.e.

$$c = \sum_{i=1}^r \frac{3k_i}{k_i + 2} = \frac{3}{2}(10 - D). \quad (6)$$

The solution to (6) requires that r be bounded by $(6 - \frac{D}{2}) \leq r \leq c$, where the upper bound is saturated uniquely by c copies of level $k = 1$, i.e. the model 1^c . Moreover the allowed range of k_i is bounded. For example, if we order the levels $k_1 \leq k_2 \leq \dots \leq k_r$, then (6) immediately implies for k_1 the inequality

$$1 - \frac{c}{3r} \leq \frac{2}{k_1 + 2} < r - \frac{c}{3}. \quad (7)$$

Given a choice of k_1 in the range (7) we may use (6) to bound k_2 etc., leading to the allowed sets of r and k_i shown in the tables (see also [6]).

Gepner’s construction consists of taking a tensor product of the r models to describe K and forming a modular invariant solution using these states. In this he

is aided by the fact that under modular transformations the transformation of the partition function for a $N = 2$ theory factorises into a l piece, which is just an affine transformation, a q piece which is a level $(k + 2)$ theta function system, and an s piece which is a level 2 theta function. The complete set of invariants for the theta functions at level m is known and consists of the left-right symmetric solution and quotients of it by any subgroup of the full Z_m symmetry. The set of affine invariants is also known, as discussed above.

Thus it is straightforward to write a modular invariant partition function for the tensor theory. This is then constrained to include only $N = 1$ supersymmetric representations in the D uncompactified dimensions to yield a $N = 1$ space-time supersymmetric theory. It was found possible to do this, in a manner consistent with modular invariance, by identifying the supersymmetry charge with $Q = \exp(i\phi)$, where $J = \partial_z \phi$ is the $U(1)$ current algebra part of the $N = 2$ superconformal algebra, and by requiring that this charge be an odd integer.

The final step in the construction consists of the conversion of the superstring compactification into a heterotic one. The right sector (now describing the bosonic string) uses precisely the same tensor product of minimal $N = 2$ superconformal field theories (this allows for the use of the same modular invariants) contributing the same conformal anomaly as the left sector. The remaining conformal anomaly is made up by $c = \frac{D}{2}$ from the transverse bosons and $c = D + 9$ from the free bosons of an $E_8 \times SO(D + 6)$ Lie group. The observation [2] that allows for immediate use of the modular invariant solutions found for the superstring is that the representation of the modular group for $E_8 \times SO(D + 6)$ theta functions is precisely the same as the representation for $SO(D - 2)$ if one interchanges the vector representation, v , with the singlet, o , and changes the signs of the spinor $(s, \bar{s})$ partition functions. (Here,

for the right sector, the notation denotes the $SO(D + 6)$ representation multiplied by the singlet of E_8 .)

The resultant partition function has the form (without the contribution from the transverse bosons)

$$W = \sum_{\substack{V, \bar{V}, l, \bar{l} \\ V - \bar{V} \in \Lambda \\ V\beta_0, \bar{V}\beta_0 \in \frac{1}{2} + Z \\ V\beta_i, \bar{V}\beta_i \in Z}} \pm A_{l, \bar{l}} Z_V^l Z_{\mu + \bar{V}}^{\bar{l}*} \quad (8)$$

where $A_{l, \bar{l}}$ represents a product of the multiplicities of the $\mathbf{A}_1^{(1)}$ invariants

$$A_{l, \bar{l}} = \prod_{i=1}^r A_{l_i, \bar{l}_i}^{(k_i)}. \quad (9)$$

The quantity $V (\bar{V})$ is a vector $(a; b; c)$ whose first index ($\frac{D}{2} + 3$ indices), a , labels the $SO(D - 2)$ space-time ($SO(D + 6)$ internal) degrees of freedom. The remaining $2r$ indices b, c of $V (\bar{V})$ are the $q, s (\bar{q}, \bar{s})$ indices of the r minimal $N = 2$ superconformal theories. The condition $V - \bar{V} \in \Lambda$ in (8) restricts the sum of states to include right-left symmetric states and those connected by either the supersymmetry generator $\beta_0 = (\bar{s}; 1; 1)$ or the vectors $\beta_i = (v; 0; 2 \text{ on } i, 0 \text{ elsewhere})$ which span the RA and NS sectors separately for a given l and q . (We use the notation that $o, s, \bar{s}, v$ refer to the weight vectors of the $SO(D - 2)$ space-time (left sector) or $SO(D + 6)$ internal (right sector) degrees of freedom.) Thus Λ is the lattice spanned by these vectors over the integers. The condition $V\beta_0, \bar{V}\beta_0 \in \frac{1}{2} + Z$ in (8) projects onto integer supercharge states, and the condition $V\beta_i, \bar{V}\beta_i \in Z$ projects all the subsectors of the fields in the theory onto either the RA or the NS sector. This is required in order to conserve the total $N = 1$ superconformal gauge condition. Here the product of two vectors n, m is given by $nm = \sum_{i=1}^r \frac{n_i m_i}{2p_i}$, where the i 'th entry labels a level p_i theta function.

Finally, in (8) the appearance of the vector $\mu = (v; 0; 0)$ implements the heterotic construction.

It is now straightforward to determine the massless spectrum of the theory. It consists of the states included in the sum in (8) which, in addition, satisfy the massless condition $h_{TOT} = \frac{1}{2}, 1$ for the left sector and right sector, respectively, where the contribution to h_{TOT} from each of the subtheories is given by (4).

Consider first the left sector component of the chiral supermultiplets of the $N = 1$ theory in $D = 4$ flat dimensions. Using charge conjugation we may always choose these to be left-handed in which case the space-time index a of the left sector is o or s . For the scalar component the contribution to h from space-time vanishes so that $h_{TOT} = h_{INTERNAL}^{LEFT} = \frac{1}{2}$. Using the definition for h and Q from (4) and (5) together with (6) immediately shows that the only possible massless states have $Q_{TOT} = 1$ and $l_i = q_i, s_i = 0$ for their primary field components. (Note that the states $l_i = q_i = 0, s_i = 2$ are null states.) It is now easy to scan through the possible tensor products of such states to determine which give $h_{TOT} = \frac{1}{2}$. Applying the supersymmetry generator β_0 one may always construct the massless spinor partners. (Using (4),(5) and (6) one easily verifies that these indeed have $h_{INTERNAL}^{LEFT} = \frac{3}{8}$ and integer supercharge as required.)

Using this set of left sector (L-) states we now construct the right sector (R-) states by operating on them with the elements of Q and μ . The gauge non-singlet fields transforming as the **27** of E_6 (the choice **27**, not **$\bar{27}$** , is convention) are immediately given for if $V = (o; b; c)$ is a L-state with $h_{TOT}^L = \frac{1}{2}, Q_{TOT}^L = 1$, then $\bar{V} = (v; \bar{b}; \bar{c})$ (where the tilde denotes that they may not be in the standard range) will be a R-state with $h_{TOT}^R = 1, Q_{TOT}^R = 1$. (Note that the gauge contribution to the charge is obtained by replacing v by o [2].) The state $V \otimes \bar{V}$ is a massless

scalar transforming as the **10** of $SO(10)$, and operating by β_0 on the R-state gives first the **16** and then the **1** of $SO(10)$ which together make up the 27-dimensional representation of E_6 .

The states transforming as the **$\bar{27}$** are found by searching for R-states with $h_{TOT}^R = 1, Q_{TOT}^R = -1$, starting with the $\bar{V}$ states and adding elements of Λ which keep the vector gauge component (i.e. $a = v$). This gives the component of the **$\bar{27}$** representation of E_6 transforming as the **$\bar{10}$** , under the $SO(10)$ subgroup, and the remaining **$\bar{16}$** and **1** are found by applying β_0 as before.

Finally the chiral superfields transforming as singlets under E_6 are found starting with the left-sector states $\bar{V}$ by operating with Λ -vectors and μ as before, but this time requiring that the gauge component of the state be singlet (i.e. $a = o$). The contribution to h and Q from the non-gauge indices are then $h_{NON-GAUGE}^R = 1, Q_{NON-GAUGE}^R = 0, \pm 2$. The states singlet under E_6 are those states satisfying these conditions which are not partners of the **27** and **$\bar{27}$** representations discussed above.

The results of a computer search implementing these rules are presented in Table 2 and Table 3 for the complete set of models which can be formed by using the standard affine (A-)invariant at all levels and the exceptional (E-)invariants at levels 10, 16 and 28. Table 2 contains the results obtained when compactifying on a 2- or 4-dimensional internal space. Since Gepner[2] did not list the complete spectrum of the A-type models he constructed, we list also these for completeness and ease of comparison.

The discrete symmetries of these models follow immediately from the level structure of the component theories and from the affine invariants used. There is a Z_{k+2} discrete symmetry factor associated with each level k component theory. In addi-

tion, for even k there is a Z_2 discrete symmetry associated with each standard affine invariant, and there are various permutation symmetries associated with identical sub-theories. The full symmetry of the product theory is the quotient of this symmetry group G divided by H , the subgroup of G generated by the element $\{2, 2, \dots, 2\}$ where we denote by $\{s_1, s_2, \dots, s_r\}$ an element of the $\prod_{i=1}^r Z_{k_i+2}$ product group. H corresponds to the fact that acting by the vector $4\beta_0$ does not change the state. Some of these symmetries are R-symmetries, namely those which do not satisfy

$$\sum_{i=1}^r \frac{s_i}{k_i + 2} \in \mathbb{Z}. \quad (10)$$

Table 2 (Table 3) also contain the number (g) of extra gauge bosons that appear in these constructions. As detailed in [2] there are always the $E_8 \times E_7$ ($E_8 \times E_6$) set of gauge bosons that one associates with Calabi-Yau compactifications (assuming the standard embedding of the holonomy in the gauge connection). In addition there are r further gauge bosons associated with the independent $U(1)$'s of each of the minimal models. One combination of these is the superconformal $U(1)$ and completes the adjoint of E_7 (E_6), while the remaining $(r - 1)$ bosons give an enhanced gauge symmetry which is at least $U(1)^{r-1}$. However, it may happen that this symmetry is enlarged and to determine this we start with the vector supermultiplet in the left sector which is given uniquely by $(v; 0; 0), (\bar{s}; 1; 1), (s; -1; -1)$. We then operate on $(v; 0; 0)$ with elements of Λ and μ to search for states $(o; b; c)$ in the right sector which have $h = 1, Q = \pm 1$. In this way we construct the (g) massless $SO(12)$ ($SO(10)$) singlet vector states and can determine whether the gauge group is larger than the minimal $E_8 \times E_7 \times U(1)^{r-1}$ ($E_8 \times E_6 \times U(1)^{r-1}$). Since g is the dimension of G in the adjoint and the rank of G is bounded by $(r - 1) \leq rkG \leq (r_{max} - 1)$, where r_{max} is given in Table 1, we can easily list the possible enhanced gauge groups for every model. In general the answer is not unique, and the structure constants must

be extracted from the operator product expansion of the generators in order to settle the question. A simple example is provided by the K3-model $2 \times 4 \times 10E$ (Table 2), where the gauge group is $E_8 \times E_7 \times G$ and G is either $SU(2)^2 \times U(1)^2$ which is the enhanced gauge group of the K3-realisation $1 \times 2^2 \times 4$ [2], or $G = SU(3)$.

Although the tables are the main results of our paper and present the first step in the classification of these models, we cannot resist making some preliminary observations here:

- (i) The entries in Table 3, with the exception of those with $h^{11} = h^{21} = 21$, appear to be Calabi-Yau or orbifold candidates. Since the identification of **27** and $\bar{\mathbf{27}}$ is convention, we cannot in general rigorously determine the sign of the Euler number χ . However, assuming that the target space really is a Kähler manifold, there must always be at least one (1,1) form, and using that $\chi = 2(h^{11} - h^{21})$ on Calabi-Yau manifolds, we deduce that χ must be positive when either the **27**s or the $\bar{\mathbf{27}}$ s vanish. There are therefore at least two distinct manifolds with positive Euler numbers 168 and 180 in our list. The first of these may be the smooth Calabi-Yau manifold recently constructed by Schoen[7]. It may also be the $Z_3 \times Z_3$ ($SU(3)^3$) orbifold discussed in ref.[8], in which additional U(1) gauge bosons arise as a result of the Frenkel-Katz mechanism.
- (ii) Comparison of Table 3 with the CICY manifolds shows that h^{21} and h^{11} [9] agree for only 19 models (the starred ones). Two of these, number 60 and 117, have been studied in detail by Gepner[2] who showed that they also possess the same discrete symmetries as P_4 [5] and a particular construction of the three-generation model [10], respectively. We have not yet completed a comparison of the discrete symmetries of the remaining candidates.
- (iii) The 18 models of Table 3 with $h^{21} = h^{11} = 21$ do not appear to be Calabi-

Yau compactifications. These Hodge numbers coincide with those for $K3 \times T$. Unlike Calabi-Yau, h^{10} for $K3 \times T$ is unity leading to the prediction [11] that there should be a vector supermultiplet transforming as a $(\mathbf{27} + \bar{\mathbf{27}})$ of E_6 . Only in this case did we indeed find these states in the Gepner construction. The presence of these states that signals the promotion of the gauge symmetry from E_6 to E_7 .

- (iv) There appears to be some systematics relating models in Table 3. The simplest example is provided by the first two models (see also [12]), where exchanging two level 1 models for one level 4 model leads to the same multiplet structure. We find this pattern repeated in the compactifications to six dimensions in Table 2. There are several other highly non-trivial relations of this type evident in Table 3.
- (v) There are two possible six-dimensional compactifications, corresponding to the torus T^2 and the $K3$ surface. For the tori ($h^{11} = 4$) we always find two vector supermultiplets plus their partners making up the $\mathbf{56}$ of E_7 , as well as two fields transforming in the adjoint of $SO(12)$. The latter must belong to the $\mathbf{133}$ of E_7 , which together with the two $\mathbf{56}$'s and three singlets fill out the adjoint of E_8 , implying that the gauge symmetry is increased to E_8 plus additional E_8 singlet gauge bosons. In the $K3$ case we find the two additional singlet vector superfields with $Q_{INTERNAL}^R = \pm 2$ which, together with the superconformal current in the right sector complete the 133 dimensional adjoint representation of E_7 .
- (vi) The singlet chiral superfields for $K3$ were found in ref.[2] to correspond to the 130 states of the $K3$ surface, together with further singlets, partners of the additional gauge factors, and no more. This pattern persists for $K3$ models

using exceptional affine invariants given in Table 2. For the T^2 surface, however, this pattern does not persist in any obvious way, and neither does it for the four-dimensional examples of Table 3. There may be more complicated Higgs mechanisms available in these cases.

- (vii) Every Euler number in Table 3 is divisible by 12 or 16 with the exception of models 60, 117(iv) and 168.
- (viii) The identification of 2^4 and 3^5 with maximally symmetric 'Fermat surfaces' in complex projective spaces persists in higher dimensions. The d -dimensional Calabi-Yau manifold (defined by vanishing first Chern class) $\mathbf{P}_{d+1}[d+2]$ has $h^{1,1} = 1$, $h^{d-1,1} = \binom{2d+3}{d+2} - (d+2)^2$ and $rkH^1(EndT) = (d+2) \left(\binom{2d+2}{d+1} - \binom{2d+3}{d+2} \right)$ for $d > 2$. We have verified that all the models d^{d+2} with $d \leq 8$ ($c \leq 24$) have one 'antifamily' mode, $h^{d-1,1}$ 'family' modes and $h^{d-1,1} + rkH^1(EndT) + (d+2)$ scalars, as expected. Note also that there are precisely $d+2$ scalars with $Q_{INTERNAL}^R = -2$. In no model except the T^2 's shown in Table 2 have we found any scalar with $Q_{INTERNAL}^R = 2$.

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TABLE 1.

Space-time Dim.	r	A	A+E	A+D+E
2	$5 < r < 12$	3,683	4,629	38,645
4	$4 < r < 9$	168	228	1,176
6	$3 < r < 6$	17	22	71
8	$2 < r < 3$	3	3	6

Table 1: Number of models with standard (A), exceptional (E) and D-type invariants (r is the number of models that can be tensored). D-type models can presumably be obtained from A-type models, but the E-type models must be included.

TABLE 2:

#:	MODEL	Chi	h11	s(2)	s(0)	s(-2)	g(2)	g(0)	g(-2)	v(1)	v(-1)
D=8: T models:											
1:	1 1 1	0	1	0	3	3	0	3	3	1	1
2:	1 4	0	1	0	5	2	0	5	2	1	1
3:	2 2	0	1	0	3	2	0	3	2	1	1
D=6: T2 models:											
3:	1 1 1 2 2	0	4	2	16	12	2	8	6	2	2
4:	1 1 2-10	0	4	2	18	12	2	9	6	2	2
12:	1-10-10	0	4	2	20	12	2	10	6	2	2
D=6: K3 models:											
1:	1 1 1 1 1 1	24	20	0	80	60	1	0	1	0	0
2:	1 1 1 1 4	24	20	0	84	56	1	1	1	0	0
4:	1 1 2 10	24	20	0	82	54	1	0	1	0	0
5:	1 1 4 4	24	20	0	82	54	1	0	1	0	0
6:	1 2 2 4	24	20	0	90	56	1	5	1	0	0
7:	2 2 2 2	24	20	0	80	56	1	0	1	0	0
8:	1 5 40	24	20	0	86	48	1	0	1	0	0
9:	1 6 22	24	20	0	86	48	1	0	1	0	0
10:	1 7 16	24	20	0	86	48	1	0	1	0	0
10:	1 7-16	24	20	0	86	48	1	0	1	0	0
11:	1 8 13	24	20	0	86	48	1	0	1	0	0
12:	1 10 10	24	20	0	86	48	1	0	1	0	0
12:	1-10 10	24	20	0	88	48	1	1	1	0	0
12:	1 10-10	24	20	0	88	48	1	1	1	0	0
13:	2 3 18	24	20	0	84	50	1	0	1	0	0
14:	2 4 10	24	20	0	84	50	1	0	1	0	0
14:	2 4-10	24	20	0	96	50	1	6	1	0	0
15:	2 6 6	24	20	0	84	50	1	0	1	0	0
16:	3 3 8	24	20	0	84	50	1	0	1	0	0
17:	4 4 4	24	20	0	83	51	1	0	1	0	0

Table 2: All A- and E-type (denoted here by -) models with $c = 3, 6$, corresponding to compactification on 2- and 4-dimensional spaces. The Euler number is χ , s_Q is the number of scalars with $Q_{INTERNAL}^R = Q$, g_Q is the number of extra gauge bosons (not coming from the original $E_8 \times E_8$), and v_Q is the number of vector bosons which transform as vectors under $SO(14)$ ($SO(12)$) gauge transformations. In addition, iff the compact space is a torus T (T^2) there is 1 (2) vector boson transforming in the adjoint.

TABLE 3.

#	MODEL	Chi	h21	h11	s
1:	1 1 1 1 1 1 1	168	0	84	252
2:	1 1 1 1 1 1 4	168	0	84	252
3:	1 1 1 1 1 2 2	0	21	21	185
4:	* 1 1 1 1 2 10	-120	62	21	250
4:	1 1 1 1 2-10	0	73	21	186
5:	* 1 1 1 1 4 4	-144	21	11	250
5:	1 1 1 1 2 2 4	-48	35	21	231
6:	1 1 1 1 2 2 2	0	21	21	182
7:	1 1 1 1 5 40	-48	47	16	321
8:	1 1 1 1 1 6 22	-72	52	8	285
9:	1 1 1 1 1 7 16	-120	62	2	231
10:	* 1 1 1 1 7-16	-120	41	17	249
11:	1 1 1 1 1 8 13	-48	79	7	321
12:	1 1 1 1 1 10 10	-144	62	2	250
12:	* 1 1 1 1 10 10	-120	21	21	187
13:	1 1 1 1 1-10-10	0	21	21	180
14:	* 1 1 1 1 2 3 18	0	62	2	249
14:	1 1 1 1 2 4 10	-120	35	11	231
15:	1 1 1 1 2 6 6	-48	21	21	180
16:	1 1 1 1 3 3 8	0	0	84	285
17:	1 1 1 1 4 4 4	168	46	10	183
18:	1 1 1 1 2 2 2 10	-72	21	21	252
18:	* 1 1 1 1 2 2 4 4	0	51	3	261
20:	1 1 2 2 2 2 4	-96	61	1	285
21:	2 2 2 2 2 2	-120	0	90	536
22:	1 1 2 2 1 1 154	180	71	71	440
23:	1 1 2 12 82	0	76	40	426
24:	1 1 2 13 58	-72	86	26	426
25:	1 1 2 14 46	-120	86	26	440
26:	1 1 2 16 34	-168	100	16	234
26:	1 1 2-16 34	-72	46	10	330
27:	1 1 2 18 28	-48	55	31	181
27:	1 1 2 18-28	0	21	21	316
28:	1 1 2 22 22	0	41	41	536
29:	1 1 2 22 22	-240	131	11	414
30:	1 1 3 6 118	0	55	55	334
31:	1 1 3 7 43	-96	67	19	328
32:	1 1 3 8 28	-96	67	19	181
32:	1 1 3 8-28	0	21	21	256
33:	1 1 3 10 18	0	31	31	181
33:	* 1 1 3-10 18	0	21	21	402
34:	1 1 4 5 40	-192	103	7	336
35:	1 1 4 6 22	-96	65	17	308
36:	1 1 4 6 22	-120	70	10	312
37:	* 1 1 4 7 16	-144	79	7	

37:	1	1	4	7-16	-120	62	2	231
38:	1	1	4	8 13	-72	48	12	248
39:	1	1	4	10 10	-192	101	5	372
39:*	1	1	4	-10 10	-120	62	2	249
39:	1	1	4	-10-10	-48	35	11	231
40:	1	1	5	5 19	-96	65	17	314
41:	1	1	6	6 10	-48	43	19	256
41:	1	1	6	6-10	0	21	21	181
42:	1	1	7	7 7	-216	112	4	400
43:	1	2	2	5 40	0	35	35	331
44:	1	2	2	6 22	-120	68	8	298
45:	1	2	2	7 16	-48	43	19	291
45:	1	2	2	7-16	0	21	21	179
46:	1	2	2	8 13	0	27	27	251
47:	1	2	2	10 10	-168	89	5	344
47:	1	2	2	-10 10	-72	46	10	235
47:	1	2	2	-10-10	0	21	21	184
48:	1	2	3	3 58	-48	47	23	314
49:	1	2	3	4 18	-48	39	15	268
50:	1	2	4	4 10	-144	74	2	284
50:*	1	2	4	4-10	-96	51	3	276
51:	1	2	4	6 6	-96	55	7	294
52:*	1	3	3	3 13	-144	75	3	252
53:	1	3	3	4 8	-48	39	15	344
54:	1	4	4	4 4	-204	103	1	280
55:	2	2	4	3 18	-120	65	5	266
56:*	2	2	4	4 10	-132	69	3	261
56:	2	2	4	-10	-120	61	1	298
57:*	2	2	2	6 6	-168	86	2	251
58:	2	2	3	3 8	-48	39	15	287
59:	2	2	4	4 4	-108	60	6	330
60:*	3	3	3	3 3	-200	101	1	1779
61:	1	5	41	1804	0	251	251	1401
62:	1	5	42	922	-240	257	137	1285
63:	1	5	43	628	-336	263	95	1023
64:	1	5	44	481	0	143	143	1195
65:	1	5	46	334	-480	287	47	1195
66:	1	5	47	292	-480	287	47	791
67:	1	5	49	236	0	107	107	817
68:	1	5	52	187	-240	173	53	1285
69:	1	5	54	166	-624	335	23	701
70:	1	5	58	138	-144	131	59	1401
71:	1	5	61	124	-720	377	17	907
72:	1	5	68	103	-384	221	29	611
73:	1	5	76	89	0	83	83	1779
74:	1	5	82	82	-960	491	11	867
76:	1	6	24	310	-216	174	66	823
77:	1	6	25	214	-264	180	48	807
78:	1	6	26	166	-312	190	34	815
79:	1	6	28	118	-360	204	24	414
79:	1	6	-28	118	0	55	55	863
80:	1	6	30	94	-408	222	18	521
81:	1	6	31	86	-48	81	57	

-120	62	2	231	5
-72	48	12	248	4
-192	101	5	372	4
-120	62	2	249	5
-48	35	11	231	7
-96	65	17	314	4
-48	43	19	256	4
0	21	21	181	8
-216	112	4	400	4
0	35	35	331	5
-120	68	8	298	4
-48	43	19	291	5
0	27	21	179	7
0	89	27	251	5
-168	46	5	344	4
-72	21	10	235	5
0	47	21	184	9
-48	39	23	314	4
-48	74	15	268	6
-144	51	2	284	4
-96	55	3	252	6
-96	75	7	276	6
-144	39	3	294	4
-48	103	15	252	6
-204	65	1	344	4
-120	69	5	280	4
-132	61	3	266	4
-120	86	1	261	7
-168	39	2	298	4
-48	60	15	251	5
-108	101	6	287	5
-200	251	1	330	4
0	257	251	1779	3
-240	263	137	1401	3
-336	143	95	1285	3
0	287	143	1023	3
-480	287	47	1195	3
-480	287	47	1195	3
0	107	107	791	3
-240	173	53	817	3
-624	335	23	1285	3
-144	131	59	701	3
-720	377	17	1401	3
-384	221	29	907	3
0	83	83	611	3
-960	491	11	1779	3
-216	174	66	867	3
-264	180	48	823	3
-312	190	34	807	3
-360	204	24	815	4
0	55	55	414	4
-408	222	18	863	3
-48	81	57	521	3

153:	3	4	8-28	-48	39	15	252	6
154:	3	4	10 18	-72	49	13	271	3
154:	3	4	-10 18	-48	39	15	268	6
155:	3	4	13 13	-192	103	7	402	4
156:	3	5	5 68	-96	71	23	387	3
157:	3	6	6 18	-112	63	7	295	3
158:	3	8	8 8	-288	145	1	495	3
159:	4	4	5 40	-156	86	8	371	3
160:	4	4	6 22	-168	90	6	353	3
161:	4	4	7 16	-192	99	3	361	3
161:*	4	4	7-16	-96	53	5	229	3
162:	4	4	8 13	-108	61	7	271	3
163:	4	4	10 10	-252	128	2	443	3
163:	4	4	-10 10	-144	74	2	284	4
163:*	4	4	-10-10	-96	51	3	252	6
164:	4	5	5 19	-96	65	17	314	4
165:	4	6	6 10	-120	66	6	281	3
165:	4	6	6-10	-96	55	7	276	6
166:	4	7	7 7	-216	112	4	400	4
167:	5	5	5 12	-240	122	2	415	3
168:	6	6	6 6	-296	149	1	503	3

Table 3: The 228 A- and E-type models with $c = 9$, corresponding to compactification

on 6-dimensional spaces. The total number of E_6 singlet scalars is s , and g is g_0 plus the $r - 1$ bosons from the conformal algebras. There are additional states iff the compact manifold is $T \times K3$ ($\chi = 0$, $h^{10} = h^{20} = 1$, $h^{11} = h^{21} = 21$). They have precisely one vector in the adjoint of $SO(10)$ and $v_1 = v_{-1} = 1$. These 18 models, nos. 3, 4, 7, 12, 13, 15, 16, 18, 27, 32, 33, 41, 45, 47, 98, 107, 142 and 145, also all have $g_2 = 1$ while g_{-2} varies: 3, 3, 4, 3, 4, 4, 4, 4, 4, 4, 3, 4, 4, 4, 3 and 4, respectively.