$b \rightarrow s + \gamma$ and $b \rightarrow s + g$ in $N=1$ supergravity theories

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We compute the $b \rightarrow s + \gamma$ and $b \rightarrow s + g$ decay rates in a spontaneously broken $N = 1$ supergravity theory with radiative electroweak breaking. We find that the *ARGUS* result $BR(B^0 \rightarrow K^{0*} + \gamma) < 4 \cdot 10^{-4}$ implies lower bounds on the masses of the gluinos and the lightest down squark which are in the same region as those provided by the direct (unsuccessful) searches of UA1. An improvement on the existing limit of $BR(b \rightarrow s + \gamma)$ by an order of magnitude can certainly rule out values of squark and gluino masses which are still allowed by the UA1 bounds.

The rates of radiative b decays, $b \rightarrow s + \gamma$ and $b \rightarrow s + g$, are considerably enhanced in the context of supersymmetric (SUSY) extensions of the standard model (SM). The key point is that SUSY exhibits a new source of flavour changing neutral currents (FCNC) in the strong interaction sector in addition to the obvious supersymmetrization of the W mediated one-loop diagrams, which induce FCNC in the SM. Indeed, as it was shown a few years ago [1], gluino vertices can lead to a flavour change. The mixing angles are the same as those which appear in the charged current interactions of SM, but the weak coupling constant is now replaced by the strong one. Moreover, a detailed investigation of the GIM cancellation in the SM and SUSY context reveals that the FCNC semileptonic decays $b \rightarrow sl^+l^-$ and $b \rightarrow s\nu\bar{\nu}$ are irremediably GIM disfavoured in SUSY with respect to the SM. This is not the case for the radiative b decays, which have the typical GIM "power suppression" both in SUSY and SM. These arguments make us expect a substantial SUSY enhancement in the one-loop gluino mediated $b \rightarrow s + \gamma$ and $b \rightarrow s + g$. The correctness of such an expectation has been established by recent quantitative analyses [2].

Previous computations [2] make use of some simplifying assumptions. In particular, since the major source of flavour changes in gluino vertices is present in the $\tilde{g} - q_L - \bar{q}_L$ couplings ($\tilde{g}$ is the gluino, and $\bar{q}_L$ is the scalar partner of the left-handed quark q_L), in first approximation we can avoid the painful diagonalization of the full 6×6 down squark mass matrix restricting ourselves to the 3×3 block with the $\bar{d}_L \bar{d}_L^*$ entries. In this way, one makes use of the so-called super KM basis, where the gluino vertices are made diagonal and the entire amount of flavour change is present in the squark propagators through flavour violating mass insertions. Clearly, then, one has to use an average squark mass. Furthermore, the set of SUSY low energy parameters is entirely determined in terms of the initial conditions at the superlarge scale of SUSY breakdown by solving the ample set of renormalization group equations for the SUSY couplings and masses from that scale down to M_W . In particular, since the amount of flavour change depends on the renormalization of the squark mass matrices, its exact determination in the SUSY parameter space requires the solution of that set of coupled equations. Obviously, also for this problem one can use reasonable approximations, and this was done in the previous analyses of $b \rightarrow s + \gamma$ and $b \rightarrow s + g$.

Although we think that the above-mentioned assumptions are quite justified, and so

we do not expect any drastic difference with the major bulk of results of refs. [2], at the same time the impressive improvement on the present experimental situation concerning radiative b decays and, even more, the near future prospects, have convinced us that there is a need for a more refined analysis. Since the bounds on squark and gluino masses which can be derived from $b \rightarrow s + \gamma$ and $b \rightarrow s + g$ are in the same ball park as the direct ones provided by UA1 at the CERN $p\bar{p}$ Collider, it behoves us to make full use of the predictive power of spontaneously broken $N = 1$ supergravity theories [3] concerning the low energy limit. We shall remove unessential assumptions by considering the full 6×6 down squark mass matrix and by determining the low energy parameters through integration of the renormalization group equations which relate them to the parameters of the original $N = 1$ supergravity model. Even more, we shall require the breaking of $SU(2) \times U(1)$ to originate radiatively from the spontaneous breakdown of the local $N = 1$ SUSY, a feature that we consider now almost theoretically compelling.

The $N = 1$ supergravity Lagrangian after SUSY spontaneous breakdown depends on five SUSY parameters: *a*) m , which sets the scale of the global $N = 1$ SUSY breaking (m^2 is the universal squared mass contribution from SUSY breaking to all the scalars of the theory); *b*) M , the common gaugino mass term at the grand unification scale M_X ; *c*) μ , the dimensional coupling of the superfield bilinear term in the superpotential which mixes the two Higgs doublets H_1 and H_2 ; *d, e*) the dimensionless parameters A and B which enter the trilinear and bilinear terms, respectively, of the SUSY soft breaking sector. The additional crucial parameter for the determination of the low energy features is the mass of the top quark, m_t . The whole set of renormalization group equations whose solution provides the low energy SUSY version of SM depends on the above set of five SUSY parameters plus m_t . The request of a flat Kähler metric further reduces the number of independent SUSY parameters, since $B = A - 1$.

We impose now the most stringent requirement: we ask for the radiative breaking of $SU(2) \times U(1)$. In this way a combination of SUSY parameters is fixed in such a way as to reproduce the correct value of M_Z . We choose the basis of the three independent SUSY parameters that we consider more appropriate for our analysis of $b \rightarrow s + \gamma$: the gluino mass $m_{\tilde{g}}$, the mass of the lightest $Q = -1/3$ squark, $m_{\tilde{q}}^{-1}$, and the ratio v_2/v_1 of the

¹ The lightest down squark, $m_{\tilde{q}}$, does not coincide with the partner of the b quark which,

two Higgs vacuum expectation values. The results of our analysis will be presented in the $m_{\tilde{b}} - m_{\tilde{g}}$ plane for fixed values of v_2/v_1 and m_t .

A remark on the choice of the $m_{\tilde{b}} - m_{\tilde{g}}$ plane is in order. As we said, $m_{\tilde{b}}$ denotes the lightest down squark mass eigenstate. What is its relation to the average down squark mass, $\bar{m}$, which was often used in previous analyses? The answer to this question is relevant also if one wants to compare our results to the UA1 bounds on squark and gluino masses. The down squark mass matrix is:

$$M_{d\bar{d}}^2 = \begin{pmatrix} m_{\tilde{d}_L \tilde{d}_L^*}^2 & m_{\tilde{d}_L \tilde{d}_L^c}^2 \\ m_{\tilde{d}_L \tilde{d}_L^c}^{2\dagger} & m_{\tilde{d}_L^c \tilde{d}_L^{c*}}^2 \end{pmatrix} \quad (1a)$$

where

$$m_{\tilde{d}_L \tilde{d}_L^*}^2 = m_d m_d^\dagger + m_Q^2 + c m_u m_u^\dagger, \quad (1b)$$

$$m_{\tilde{d}_L^c \tilde{d}_L^{c*}}^2 = m_d m_d^\dagger + m_D^2, \quad (1c)$$

$$m_{\tilde{d}_L \tilde{d}_L^c}^2 = A_D m m_d + \mu_0 m_d \frac{v_2}{v_1} \quad (1d)$$

The term $c m_u m_u^\dagger$ in (1b) arises when renormalizing $m_{\tilde{d}_L \tilde{d}_L^*}^2$ from some superlarge scale M_X down to M_W . It is the seed of flavour changing effects in $\tilde{g} - d_L - \tilde{d}_L$ vertices. The value of c can be computed in terms of the initial values of the SUSY parameters by solving the aforementioned set of renormalization group equations. In the previous computations of $b \rightarrow s + \gamma$ and $b \rightarrow s + g$ in SUSY c was taken to be at a fixed value, $c = -0.5$. In our approach, c is determined point by point in the $m_{\tilde{b}} - m_{\tilde{g}}$ plane for each value of v_2/v_1 and m_t . The terms m_Q^2 and m_D^2 are also computed by integrating the correspondent renormalization group equations; however, their value at the weak scale is only slightly different from m^2 , and we neglect this difference for the moment ². Finally, A_D and μ_0 are the values that the parameters A and μ take at the weak interaction scale. From eq.

in general, is not even a mass eigenstate.

² All the calculations in the following are performed with the actual value of m_Q^2 and m_D^2 at the Fermi scale.

(1) one sees that the larger m_t is, the smaller $m_{\tilde{b}_L \tilde{b}_L^*}^2 = m_b^2 + m^2 + cm_t^2$ ($c < 0$) becomes. The smallest eigenvalue coincides roughly with $m^2 + cm_t^2$ and, thus, it can be substantially smaller than the average squark squared mass $\tilde{m}^2 \simeq m^2$. For instance, we shall plot our results for $m_t = 100$ GeV, $v_2/v_1 = 2$; in this case, taking for example $m_{\tilde{g}} = 70$ GeV and $m_{\tilde{b}} = 100$ GeV would correspond to $\tilde{m} \simeq 123$ GeV.

The constraints imposed on the SUSY parameter space by the requirement of $SU(2) \times U(1)$ radiative breaking have already been extensively analyzed in the literature (see, in particular, ref. [4] for the case of a large top mass, $m_t = 100$ GeV). In our approach we have to translate these previous results into bounds in the $m_{\tilde{b}} - m_{\tilde{g}}$ plane for different values of v_2/v_1 and m_t . In fig. 1 we provide an example with $m_t = 100$ GeV and $v_2/v_1 = 2$ (the forbidden region shrinks for decreasing values of v_2/v_1). Our aim is now to superimpose the bounds coming from $b \rightarrow s + \gamma$ and $b \rightarrow s + g$ to the allowed region of fig. 1.

We consider first the radiative $b \rightarrow s + \gamma$ decay. The best experimental result on this process concerns the exclusive channel $B^0 \rightarrow K^{0*} + \gamma$. The *ARGUS* collaboration has given $BR(B^0 \rightarrow K^{0*} + \gamma) < 4 \cdot 10^{-4}$ [5]. The most recent estimates of $\Gamma(B^0 \rightarrow K^{0*} + \gamma)/\Gamma(b \rightarrow b + \gamma)$ yield $(4.5 \div 7) \cdot 10^{-2}$ [6]. Thus, the *ARGUS* limit would imply, at the inclusive level,

$$BR(b \rightarrow s + \gamma) < (6 \div 9) \cdot 10^{-3} \quad (2)$$

The dominant contribution in SUSY to $b \rightarrow s + \gamma$ comes from the one-loop diagrams where gluinos and down squarks are exchanged (fig. 2). The decay amplitude is

$$A(b \rightarrow s + \gamma) = \frac{e\alpha_s}{9\pi} \frac{m_b^2}{m_{\tilde{g}}^2} \bar{s} \frac{\sigma_{\mu\nu} q^\nu \epsilon^\mu}{m_b + m_s} (F_2^V + F_2^A \gamma_5) b \quad (3a)$$

where

$$F_2^{V,A} = \left(\Gamma_{L2i}^\dagger \Gamma_{Li3} \pm \Gamma_{R2i}^\dagger \Gamma_{Ri3} \right) P(x_i) + \frac{m_{\tilde{g}}}{m_b} \left(\Gamma_{R2i}^\dagger \Gamma_{Li3} \pm \Gamma_{R2i}^\dagger \Gamma_{Li3} \right) Q(x_i) \quad (3b)$$

$$P(x) = \frac{1}{12(x-1)^4} (x^3 - 6x^2 + 3x + 2 + 6x \log x) \quad (3c)$$

$$Q(x) = \frac{1}{2(x-1)^3} (x^2 - 1 - 2x \log x) \quad (3d)$$

and $x_i = (\tilde{m}_i/m_{\tilde{g}})^2$ [$\tilde{m}_i^2$ is the i -th eigenvalue of the mass matrix (1)]. Finally, the mixing 6×3 matrices $\Gamma_{L,R}$ in (3b) are defined as in ref. [7].

The amplitude (3) yields the decay rate:

$$\Gamma(b \rightarrow s + \gamma) = \frac{\alpha_s^2 \alpha}{81\pi^2} \frac{m_b^5}{2m_{\tilde{g}}^4} (|F_2^V|^2 + |F_2^A|^2) \quad (4)$$

Imposing the experimental bound (2) we further limit the allowed region in the $m_{\tilde{b}}-m_{\tilde{g}}$ plane, as shown in fig. 3a. In the near future it should be possible to lower the limit (2) to the value of 10^{-3} . If we require $BR(b \rightarrow s + \gamma) < 10^{-3}$ the disallowed region enlarges by the amount shown in fig. 3b.

We turn now to $b \rightarrow s + g$. It has been noticed that this represents an exceptional place for the manifestation of SUSY enhancements. Indeed, the reasons leading to a SUSY enhanced $b \rightarrow s + \gamma$ are now compounded by an additional fact, namely the possibility of attaching the emitted gluon to the internal gluino line. The typical color factor of three which appears at this vertex accounts for one order of magnitude of enhancement more at the level of the decay rate.

However, testing $b \rightarrow s + g$ experimentally is obviously more difficult than looking for $b \rightarrow s + \gamma$. Presently, both *CLEO* and *ARGUS* seem to agree that $B \rightarrow K$ +charmless states should not exceed 20% of the total rate. Thus, we impose:

$$BR(b \rightarrow s + g) < 0.2 \quad (5)$$

The dominant SUSY contribution is readily obtained from the diagram of fig. 2 by replacing the emitted photon with a gluon and attaching it to the gluino instead of the squark line. The leading contribution to the decay rate yields:

$$\Gamma(b \rightarrow s + \gamma) = \frac{9\alpha_s^2}{32\pi^2} \frac{m_b^5}{m_{\tilde{g}}^4} (|F_2^V|^2 + |F_2^A|^2), \quad (6a)$$

where $F_2^{V,A}$ are the same as those of eq. (3), except that the two functions $P(x)$ and $Q(x)$ are now replaced by

$$H(x) = \frac{1}{12(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \log x) \quad (6b)$$

$$K(x) = \frac{1}{2(x-1)^3} (3x^2 - 4x + 1 - 2x^2 \log x) \quad (6c)$$

respectively. The bound (5) yields the forbidden area in the $m_{\tilde{t}} - m_{\tilde{g}}$ plane shown in fig. 4.

Comparing figs. 3 and 4 we see that $b \rightarrow s + g$ poses more severe constraints on $m_{\tilde{g}}$ and $m_{\tilde{t}}$ than $b \rightarrow s + \gamma$. In particular fig. 4 [which assumes $BR(b \rightarrow s + g) < 0.2$] tells us that gluinos of few GeV's are excluded unless $m_{\tilde{t}}$ is at least 100 – 120 GeV and, in any case, gluino masses up to 60 – 70 GeV are ruled out for $m_{\tilde{t}} < 100 - 120$ GeV. On the other hand, we think that the experimental bound (5) on $b \rightarrow s + g$ is on a less firm ground than the limit (2) on $b \rightarrow s + \gamma$.

We believe that the bounds given here on $m_{\tilde{g}}$ and $m_{\tilde{t}}$ are so far the most stringent ones, to be derived from virtual effects of SUSY particles in rare low energy processes. It is obvious to ask how they stand in comparison with the direct bounds provided by UA1 [8]. First of all, notice that the requirement of radiative $SU(2) \times U(1)$ breaking forbids a large part of the region of the $m_{\tilde{t}} - m_{\tilde{g}}$ plane which is ruled out by UA1 (compare our fig. 1 with fig. 4 of ref [8], for instance). As for the remaining part, the area excluded by UA1 includes the area forbidden by the condition $BR(b \rightarrow s + \gamma) < (6 \div 9) \cdot 10^{-3}$. However, if we take $BR(b \rightarrow s + \gamma) < 10^{-3}$ there is already some small part of the $m_{\tilde{t}} - m_{\tilde{g}}$ plane which is ruled out by this b decay, but is otherwise allowed by UA1 limits. The same applies to the case of $b \rightarrow s + g$. In any case, the total area forbidden by UA1 [but still allowed by the radiative $SU(2) \times U(1)$ breaking] remains larger than that disallowed by the rare radiative b decays that we are considering here. However, the point that we want to stress is that the direct and indirect searches for SUSY particles are certainly complementary. The bounds that rare B physics provides reach an interval of values comparable to what UA1 has explored so far and, even more important, the two procedures are based on methods and assumptions which are quite different. In the case of UA1 there might be some problem in dealing with the complicated "jettology", and some assumptions are made concerning the lightest supersymmetric particle and the degeneracy of squark masses. In our case we have a free parameter, v_2/v_1 , that we have to fix in order to give bounds on $m_{\tilde{t}}$ and $m_{\tilde{g}}$. Moreover, we assume $B = A - 1$ and the radiative breaking of $SU(2) \times U(1)$. Finally, we should add that QCD radiative corrections are expected to be small in the SUSY case,

since all the particles in the loop are heavy and so no large $\log$ can appear. However, no estimate has been produced so far.

In conclusion, we have shown that the FCNC radiative b decays, $b \rightarrow s + \gamma$ and $b \rightarrow s + g$, provide interesting lower bounds on the masses of the lightest down squark and the gluinos. They are the best indirect bounds on these SUSY masses, *i.e.* bounds obtained from low energy rare processes. Interestingly enough, the present experimental data already give limits on $m_{\tilde{b}}$ and $m_{\tilde{g}}$ which are not far from those provided by direct searches at the CERN $p\bar{p}$ Collider. Future experimental developments in the study of B physics promise to place rare B decays as one of the most powerful tools in the search for low energy SUSY hints.

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Figure captions

Fig. 1. In the dotted region, spontaneous breaking of the $SU(2) \times U(1)$ symmetry takes place at the correct scale.

Fig. 2. Feynman diagram for the supersymmetric contribution to $b \rightarrow s + \gamma$.

Fig. 3. Region of the $m_{\tilde{b}} - m_{\tilde{g}}$ plane where

a) $BR(b \rightarrow s + \gamma) > 10^{-2}$,

b) $BR(b \rightarrow s + \gamma) > 10^{-3}$,

for $m_t = 100$ GeV and $v_2/v_1 = 2$ (shaded area). The region allowed by $SU(2) \times U(1)$ breaking is also shown (dotted area).

Fig. 4. Region of the $m_{\tilde{b}} - m_{\tilde{g}}$ plane where $BR(b \rightarrow s + g) > 0.2$, for $m_t = 100$ GeV and $v_2/v_1 = 2$ (shaded area). The region allowed by $SU(2) \times U(1)$ breaking is also shown (dotted area).

- Fig. 1 -

- Fig. 2 -

- Fig. 3a -

- Fig. 3b -

- Fig. 4 -