CERN-TH.5082/88

THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. Altarelli and G.G. Ross^{*)}

CERN - Geneva

ABSTRACT

We show that, due to the anomaly, the gluon contribution to the first moment of the polarized proton structure function, as measured in deep inelastic scattering, is not suppressed by a power of the strong coupling evaluated at a large scale. As a result, the EMC result for the first moment of polarised proton electroproduction is consistent with a large quark spin component.

*) On sabbatical leave from Dept. of Theoretical Physics,
University of Oxford, 1 Keble Road, Oxford, England.

A measurement of the polarized proton structure function in deep inelastic leptonproduction was recently completed by the EMC collaboration at CERN 1). In particular for the integral of the structure function g_1^p at $\langle Q^2 \rangle = 11 \text{ GeV}^2$ the measured asymmetry implies the relation:

$$\int_0^1 dx g_1^p(x, Q^2) \approx 0.114 \pm 0.029 \quad (1)$$

where statistical and systematic errors have been added in quadrature. By combining the EMC results with previous data from SLAC 2) at $\langle Q^2 \rangle = 5 \text{ GeV}^2$ one obtains the improved determination 3)

$$\int_0^1 dx g_1^p(x, Q^2) \approx 0.115 \pm 0.021 \quad (2)$$

if the Q^2 dependence is neglected in the relevant Q^2 range.

In the naive parton model 4)

$$\begin{aligned} \int_0^1 dx g_1^p(x) &= \frac{1}{2} \sum_{i=1}^f e_i^2 \Delta q_i(x) = \frac{1}{2} \left(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) = \\ &= \frac{1}{12} (\Delta u - \Delta d) + \frac{1}{36} (\Delta u + \Delta d - 2\Delta s) + \frac{1}{9} (\Delta u + \Delta d + \Delta s) \end{aligned} \quad (3)$$

Here c quarks are neglected and the following notation was adopted:

$$\Delta q(x) \equiv \int_0^1 dx [q_+(x) + \bar{q}_+(x) - q_-(x) - \bar{q}_-(x)] \approx \langle p, s | \bar{q} \gamma_0 \gamma_5 q | p, s \rangle \quad (4)$$

with $q_{\pm}$ ($\bar{q}_{\pm}$) being the densities of parton quarks (antiquarks) with helicities $\pm 1/2$ in a proton with helicity $+1/2$. $\Delta q(x)$ is connected to the matrix element of the quark axial vector current between polarized protons, as also shown in eq.4. The flavour non singlet components can be obtained from the measured values of the axial-vector matrix elements by the relations $\Delta u - \Delta d = g_A/g_V = F + D$ and $\Delta u + \Delta d - 2\Delta s = 3F - D$ 5). From a recent fit of hyperon decays $F = 0.477 \pm 0.011$ and $D = 0.755 \pm 0.011$ 6). By combining the above results with the EMC experimental value in eq.1 one finds 7):

$$\Delta u \approx 0.74 \pm 0.08 ; \Delta d \approx -0.51 \pm 0.08 ; \Delta s \approx -0.23 \pm 0.08 \quad (5)$$

so that

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s \approx 0.00 \pm 0.24 \quad (6)$$

It has been suggested that the errors have been significantly underestimated 8) allowing for considerably larger values of $\Delta \Sigma$. We do not pursue this possibility, but consider the theoretical implications of a value for $\Delta \Sigma$ in the range given by eq.6.

These results are at first sight surprising because they appear to contradict the intuitive expectation that a substantial fraction of the proton spin is carried by parton quarks and antiquarks ($\Delta \Sigma=1$ if all the proton spin is carried by q and $\bar{q}$) and the guess that Δs should be very small 9). This fact has produced many speculations (including claims of evidence against QCD 10)). In ref.11 it was shown that chiral symmetry and $1/N_c$ expansion imply the vanishing of the SU(3) singlet axial current matrix element. Also the corrections due to finite quark masses and $N_c=3$ were argued to be small on the basis of a skyrmion model of the proton. In this picture most of the proton spin is carried by orbital angular momentum. Explanations based on a rapid Q^2 dependence (the integral in eq.1 is known 12) to change sign at $Q^2=0$ due to the Drell-Hearn sum rule 13)) appear to be discarded by experiment. In fact data at different Q^2 exist 14), down to Q^2 as low as $Q^2=0.5\text{GeV}^2$, and no appreciable Q^2 dependence is visible. Indeed it is found that the asymmetry becomes negative at the N^* resonance 14), however this fluctuation is very local. Actually, as shown in ref.14, even at $Q^2=0.5\text{GeV}^2$ the scaling curve is a good average of the fluctuating asymmetry measured in the resonance region. Note that the position of the N^* resonance is far outside the x range measured by EMC,

near $x=1$. A more physically significant form of Q^2 dependence was advocated in ref.15 where it was stressed that the conservation of the SU(3) singlet axial current, valid for massless quarks, is broken in QCD by the Adler-Bell-Jackiw anomaly (16). In particular while the anomalous dimension of the non singlet axial current vanishes at all orders (17), the singlet axial current anomalous dimension is different from zero at two loops and its value was computed by Kodaira (18). In ref.15 it is proposed that due to this non conservation a rapid variation of $\Delta\Sigma$ takes place in the non perturbative region of Q^2 .

In the present letter we show that the effect of the anomaly in the singlet sector is actually more dramatic and interesting than envisaged in ref.15. The anomaly in fact induces a gluon contribution at all values of Q^2 in the singlet part of the integral of g_1 . This gluon term can in principle be large and can well explain the difference between parton and constituent quarks that the EMC result appears to imply.

We introduce the gluon helicity content in the polarized proton:

$$\Delta g \equiv \int_0^1 dx [g_+(x, Q^2) - g_-(x, Q^2)] \quad (7)$$

with $g_{\pm}$ being the gluon density with helicity ± 1 in a proton with helicity $\pm 1/2$. At order α_s the QCD evolution of $\Delta\Sigma$ and Δg is well known (4):

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} A_{qq}^{(1)} & 2f A_{qg}^{(1)} \\ A_{gq}^{(1)} & A_{gg}^{(1)} \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} 0 & 0 \\ \frac{3}{2}c_F & \frac{11c_A - 4f}{6} \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} \quad (8)$$

where $t = \ln Q^2/\mu^2$, $A_{AB} = \int_0^1 dx \Delta P_{AB}(x)$, $\Delta P_{AB}(x)$ being the lowest order QCD splitting functions for polarized partons given in ref.19. (i.e. $\Delta P_{AB} = P_{A+B+} - P_{A-B+}$), $c_F = 4/3$, $c_A = 3$, $T = f/2$ and f is the number of excited flavours ($f=3$ if c quarks are neglected). The crucial point is that the gg

entry of the evolution matrix in eq.8 is nothing else than $b\alpha_s$, where b is the first coefficient of the QCD β function: $\beta(\alpha)=-b\alpha^2(1+b'\alpha+...)$ with $b=(11c_A-4T)/12\pi$. Then at this order Δg belongs to an eigenvector whose evolution is determined by the factor $[\alpha_s/\alpha_s(t)]^d$ with $d=1$ ($\alpha_s=\alpha_s(0)$). This implies that if a mixing with Δg is present at order $\alpha_s(t)$, induced by the anomaly via the diagrams in fig.1, the gluon component will not vanish at large Q^2 , because $\alpha_s(t)[\Delta g(t)+...]=\alpha_s(t)[\alpha_s/\alpha_s(t)][\Delta g(0)+...]$, where the ellipses indicate the quark completion (at this order) of the relevant eigenvector.

The contribution of Δg to g_1^P at order α_s is obtained from the diagrams in fig.1 by computing the difference of cross-sections

$$\delta\sigma=\sigma(\gamma_L g_+) - \sigma(\gamma_R g_+) \quad (9)$$

where γ_L or γ_R is a left- or right-handed photon of virtual mass $-Q^2$. With a dimensional over-all factor removed, one obtains

$$\delta\sigma \sim \frac{\alpha_s}{2\pi} \frac{T}{f} (x^2 - (1-x)^2) \left[\frac{1}{1-\beta \cos\theta} + \frac{1}{1+\beta \cos\theta} - 1 \right] \quad (10)$$

with θ being the γ -g center of mass angle. Here the normalization is fixed by the correct leading logarithmic splitting function ΔP_{qg} , i.e. the coefficient of $1/(1-\beta \cos\theta)$. Since the first moment of ΔP_{qg} vanishes, the corresponding finite coefficient is well defined, i.e. independent of the regularization procedure. In eq.10 we have adopted the physical quark mass m as a regulator, so that $\beta=p/E=1-2m^2x/[Q^2(1-x)]$.

The finite coefficient is then found to be

$$\alpha_s c_g = \frac{\alpha_s}{2\pi} \frac{T}{f} \int_0^1 dx [x^2 - (1-x)^2] \ln \frac{1-x}{x} = -\frac{\alpha_s}{2\pi} \frac{T}{f} \quad (11)$$

Thus we obtain

$$H_1^S(q^2) \equiv \int_0^1 dx \, g_1^S(x, q^2)_{\text{singlet}} = \frac{\langle e^2 \rangle}{2} \left[\Delta\Sigma - 2 \frac{\alpha_s}{2\pi} \frac{T}{f} \Delta g \right] = \frac{\langle e^2 \rangle}{2} \left[\Delta\Sigma - f \frac{\alpha_s}{2\pi} \Delta g \right] \quad (12)$$

where $\langle e^2 \rangle = \sum_i e_i^2 = 2/9$ (see eq.3) and the factor of 2 in front of $c_g \Delta g$ takes into account that there is a term $-\alpha_s/4\pi$ for each quark and each antiquark of a given flavour.

Since $\alpha_s \Delta g$ is conserved by the leading order QCD evolution it proves convenient to define the new density $\Delta\Gamma$ by

$$\Delta\Gamma \equiv \frac{\alpha_s}{2\pi} \Delta g \quad (13)$$

Now the renormalisation group equation (8) takes a very simple form as the variations of $\Delta\Sigma$ and $\Delta\Gamma$ both vanish at order $\alpha_s(t)$. The $\Gamma\Gamma$ entry vanishes because the growth of Δg is compensated by the variation of $\alpha_s(t)$ as discussed above. The Γq entry becomes $O(\alpha_s^2)$ due to the factor of α_s in $\Delta\Gamma$. This means that eq.12 receives only small radiative corrections at $O(\alpha_s^2)$. For practical purposes we may use eq.12 as is and the reader not interested in theoretical niceties should proceed to the phenomenological discussion at the end of the paper. First, however, we consider the QCD evolution of $\Delta\Sigma$ and $\Delta\Gamma$ at second order in α_s . Now the renormalisation group equations read

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix} = [\alpha_s(t)]^2 \begin{bmatrix} \gamma_{qq}^{(2)} & \gamma_{qg}^{(3)} \\ \gamma_{gq}^{(1)} & \gamma_{gg}^{(2)} \end{bmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix} + O(\alpha_s^3) \quad (14)$$

The label (n) in $\gamma^{(n)}$ denotes the number of loops. There is an obvious reshuffling of powers because of the α_s factor in the relation between $\Delta\Gamma$ and Δg .

Quite in general the Q^2 evolution of the singlet component of the first moment is given by:

$$M_1^S(q^2) = \frac{1}{2} \langle e^2 \rangle C(\alpha_s(t)) E(\alpha_s, \alpha_s(t)) O(\alpha_s) \quad (15)$$

where M_1^S is defined in eq.12, $O(\alpha_s) = \left(\frac{\Delta \Sigma}{\Delta \Gamma} \right)$ and to the relevant order in α_s

$$C(\alpha_s) = (1 + c_\Sigma \alpha_s, c_\Gamma + c'_\Gamma \alpha_s) \quad (16)$$

$$E(\alpha_s, \alpha_s(t)) = 1 + \frac{\alpha_s - \alpha_s(t)}{b} \gamma \quad (17)$$

Here $c_\Sigma = -3c_F/4\pi$ was computed in ref.17, $c_\Gamma = -f$ is obtained from eq.12 and γ is the matrix in eq.14. The explicit result is then given by

$$M_1^S(q^2) = \frac{1}{2} \langle e^2 \rangle \left\{ \left[1 + \frac{\alpha_s - \alpha_s(t)}{b} (\gamma_{qq}^{(2)} + c_\Gamma \gamma_{qq}^{(1)} - b c_\Sigma) + \alpha_s c_\Sigma \right] \Delta \Sigma + \right. \\ \left. + \left[1 + \frac{\alpha_s - \alpha_s(t)}{b} (\gamma_{gg}^{(2)} + \gamma_{gg}^{(3)}/c_\Gamma - b c'_\Gamma/c_\Gamma) + \alpha_s c'_\Gamma/c_\Gamma \right] c_\Gamma \Delta \Gamma \right\} \quad (18)$$

It is instructive at this stage to establish contact with the analysis using the short-distance operator expansion (OPE). There are only two possible twist-two operators which may contribute to the first moment of g_1^P . They are

$$j_\alpha^5 = \sum_{i=1}^f \bar{q}_i \gamma_\alpha \gamma_5 q_i \\ k_\alpha = \frac{\alpha_s}{2\pi} \epsilon_{\alpha\nu\lambda\sigma} \text{Tr} \left[A^\nu \left(F^{\lambda\sigma} - \frac{2}{3} A^\lambda A^\sigma \right) \right] \quad (19)$$

with $A^\nu = \sum_B A^\nu B_t B$ and $\text{Tr}_t A_t B = 1/2 \delta^{AB}$. In general k_α is gauge dependent. However the diagonal matrix element which is relevant for the first moment of g_1^P is gauge invariant, being related to the topological charge. This is why it makes sense to use it to fill the gap between the parton language (where the first moment is as well defined as the other ones) and the conventional OPE formalism 18) where the first moment is left without gluons.

The renormalization group equations for the operators j_α^5 and k_α take the form 20)

$$\frac{d}{dt} \begin{pmatrix} j_\alpha^5 \\ k_\alpha \end{pmatrix} = [\alpha_s(t)]^2 \begin{pmatrix} \gamma_{11}^{(2)} & 0 \\ \gamma_{21}^{(1)} & 0 \end{pmatrix} \begin{pmatrix} j_\alpha^5 \\ k_\alpha \end{pmatrix} \quad (20)$$

Here the vanishing of γ_{12} and γ_{22} results because the divergence of k_α is $\alpha_s/2\pi \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})$, i.e. is proportional to the U(1) anomaly which is not renormalized in any order (21). The non zero entries $\gamma_{11}^{(2)}$ and $\gamma_{21}^{(1)}$ are related because j_α and k_α are connected by the equations of motion

$$\partial^\alpha j_\alpha^5 = f \frac{\alpha_s}{2\pi} \text{Tr}[F_{\mu\nu} \tilde{F}^{\mu\nu}] = f \partial^\alpha k_\alpha \quad (21)$$

Since this must be true for any renormalisation scale μ^2 , we have

$$\gamma_{11}^{(2)} = f \gamma_{21}^{(1)} = - \frac{3f c_F}{8\pi^2} \quad (22)$$

in agreement with the explicit two loop calculation due to Kodaira (18).

Using these anomalous dimensions we may diagonalize eq.20 to find the eigenvectors O_i and eigenvalues λ_i . They are:

$$\begin{aligned} O_{1,\alpha} &\equiv j_\alpha^5 & ; \quad \lambda_1 &= \gamma_{11}^{(2)} \alpha_s^2 \\ O_{2,\alpha} &\equiv j_\alpha^5 - f k_\alpha & ; \quad \lambda_2 &= 0 \end{aligned} \quad (23)$$

Then the first moment of the polarized structure function has the form

$$M_1^j(q^2) = \frac{\langle e^2 \rangle}{2} \left[E_1(\mu^2, \alpha_s(t)) a_1(\mu^2) \exp \int_{\alpha}^{\alpha(t)} \frac{\gamma(\alpha')}{\beta(\alpha')} d\alpha' + E_2(\mu^2, \alpha(t)) a_2(\mu^2) \right] \quad (24)$$

where the quantities a_i are related to the operator matrix elements between protons of helicity s by

$$\langle p, s | O_{i,\alpha} | p, s \rangle \equiv a_i(\mu^2) s_\alpha \quad (25)$$

Taking matrix elements between quark and gluon states one obtains:

$$M_1^S(\mu^2) = \frac{\langle e^2 \rangle}{2} \left\{ E_1(\mu^2, \alpha_s(\mu^2)) [\Delta\Sigma - f \Delta\Gamma] + E_2(\mu^2, \alpha_s(\mu^2)) \Delta\Sigma \right\} \quad (26)$$

The coefficients E_i can then be derived by comparison with eq.12 which gives $E_1=1$, $E_2=0$ at lowest order in α_s . Evaluating the exponential factor in eq.24 and including the $o(\alpha_s)$ contribution to E_1 then gives

$$\begin{aligned} M_1^S(Q^2) &= \frac{\langle e^2 \rangle}{2} \left[1 - \frac{\alpha_s(t)}{b} (\gamma_{11}^{(2)} - b c_\Sigma) \right] (\Delta\Sigma - f \Delta\Gamma)_{Q^2 \rightarrow \infty} \\ &\approx \frac{\langle e^2 \rangle}{2} \left[1 - \frac{\alpha_s(t)}{\pi} \frac{33-8f}{33-2f} \right] (\Delta\Sigma - f \Delta\Gamma)_{Q^2 \rightarrow \infty} \end{aligned} \quad (27)$$

Here the Q^2 dependent factor is just that found in ref.18. Note that the entire contribution to eq.27 arises from the operator j_α ; the gluon component is introduced because the anomaly gives a finite gluon matrix element to j_α .

We may now compare the parton model analysis of eq.18 with this result. Since it is only the combination multiplying $\alpha_s(t)$ which has a physical meaning independent of all conventions, we can identify $\gamma_{qq}^{(2)+c\Gamma}\gamma_{gq}^{(1)-bc\Sigma}$ with the corresponding quantity obtained in eq.27. This gives $\gamma_{qq}^{(2)}=0$. In other words the quantity $\gamma_{11}^{(2)}$ of eq.20, computed by Kodaira (18), is what here appears as $c\Gamma\gamma_{gq}^{(1)}$. Note that in fact the leading singularity of the first moment of the diagram in fig.2 is precisely given by the probability of finding a gluon in a quark times the finite cross-section of fig.1, i.e. exactly by $c\Gamma\gamma_{gq}^{(1)}$. Thus it follows that in parton terms $\Delta\Sigma$ is conserved, being identified with the combination $(j_\alpha - f k_\alpha)$ which has no gluon matrix element. This combination is conserved (the anomaly cancels) and hence (see eqs.20 and 22) has vanishing anomalous dimensions. Finally, comparison of the $\Delta\Gamma$ term in eq.18 with that in eq.27 shows that $\Delta\Gamma$ and $\Delta\Sigma$ have the same Q^2 dependence.

We now consider the implications of our final result in eq.27 which includes the effects of the anomaly and the QCD corrections to second

order. As anticipated the Q^2 evolution is expected to play no important role -as it arises from second order effects which should be smaller than the moderate scaling violations observed in non polarized deep inelastic scattering processes. However the crucial result is that the anomalous gluon contribution is not expected to be small as it is not suppressed by a power of $\alpha_s(Q^2)$. This means that the experimental value attributed to $\Delta\Sigma$ in the naive parton model should be interpreted as the quantity $\Delta\Sigma - f\Delta\Gamma$. This separately applies to each flavour: $\Delta u \rightarrow \Delta u - \Delta\Gamma$, $\Delta d \rightarrow \Delta d - \Delta\Gamma$, $\Delta s \rightarrow \Delta s - \Delta\Gamma$. There is no reason in principle why $f\Delta\Gamma$ should be small, so it can well nearly compensate $\Delta\Sigma$. Note that if Δg is positive, i.e. if gluons give a positive contribution to the proton helicity, then the sign is correct for this compensation to occur. The rather large and negative value observed for the strange component $\Delta s - \Delta\Gamma$, eq.5, can be attributed to gluons coupled by the anomaly. If one makes the assumption, following Ellis and Jaffe (9) that Δs is small, which we do not especially recommend, then $\Delta\Gamma = 0.23$ and $\Delta u + \Delta d = 0.69$ (with large errors). It is true that one can argue that one could take polarized deep inelastic scattering to define Δq , so that the the anomalous term could be reabsorbed in a redefinition of Δq , but then one should not be surprised if e.g. the strange component is not small. Also our analysis goes beyond mere definitions for a sizeable gluon component can be tested in other moments (via the x distribution) and also in other processes.

To conclude we have found it is possible to have a consistent picture of the proton with a large component of the spin being carried by valence quarks balanced by a sizeable gluon contribution arising from the anomaly.

While this work was in preparation we received the paper in ref.22. We agree with these authors on the existence of a large mixing between $\Delta\Sigma$ and $\Delta\Gamma$, but we differ on a number of important points. First we have directly

computed the crucial coefficient c_g in eq.11 and obtain a different result than quoted in ref.22. Then we reach completely different conclusions on the QCD evolution. In particular for us $\Delta\Sigma$ is conserved while in ref.22 the conserved quantity is $\Delta\Sigma+2f\Delta T$. Finally in ref.22 this conserved quantity is identified with twice the total proton helicity, i.e. the authors of ref.22 set $\Delta\Sigma+2f\Delta T=1$. We consider this statement not justified in any case, because in parton terms the gluon contribution to the proton helicity is by definition Δg (see eq.7). Also the total helicity carried by the partons is not conserved because of the orbital angular momentum.

We gratefully acknowledge useful discussions with C.Becchi and J.Ellis.

References

- (1) J.Ashman et al.(EMC Collaboration),Phys.Lett.B206,364(1988)
- (2) G.Baum et al.,Phys.Rev.Lett.51,1135(1983)
- (3) G.Baum et al.,Bielefeld-Yale preprint(1988)
- (4) G.Altarelli,Phys.Rep.81,1(1982)
- (5) J.D.Bjorken,Phys.Rev.148,1467(1966);D1,1376(1971)
- (6) M.Bourquin et al.,Z.Phys.c21,27(1983)
- (7) J.Ellis,M.Karliner,SLAC-PUB-4592(1988)
- (8) F.E.Close,R.G.Roberts,Phys.Rev.Lett.60,1471(1988)
- (9) J.Ellis,R.L.Jaffe,Phys.Rev.D9,1444(1974);E D10,1669(1974)
- (10) G.Preparata,J.Soffer, Milan preprint(1988); A.Giannelli et al. Phys.Lett.150B,214(1985)
- (11) S.J.Brodsky,J.Ellis,M.Karliner,Phys.Lett.b206,309(1988)

- (12) B.L.Ioffe, V.A.Khoze, L.N.Lipatov, "Hard Processes", Vol.1, North-Holland, 1984; V.M.Belyaev, B.L.Ioffe, Ya.L.Kogan, Phys.Lett.151B, 290(1985)
- (13) S.D.Drell, A.C.Hearn, Phys.Rev.Lett.16, 908(1966); S.B.Gerasimov, Yad.Fiz.2, 598(1966)
- (14) G.Baum et al., Phys.Rev.Lett.45, 2000(1980)
- (15) R.L.Jaffe, Phys.Lett.B193, 101(1987)
- (16) S.L.Adler, Phys.Rev.177, 2426(1969); J.S.Bell, R.Jackiw, Nuovo Cim. A51, 47(1969)
- (17) J.Kodaira et al., Phys.Rev.177, 2426(1979); Nucl.Phys.B159, 99(1979)
- (18) J.Kodaira, Nucl.Phys.B165, 129(1979); M.A.Ahmed, G.G.Ross, Nucl Phys.B111, 441; K.Sasaki, Prog.Theor.Phys.54, 1816(1975)
- (19) G.Altarelli, G.Parisi, Nucl.Phys.B126, 1977(298)
- (20) D.B.Kaplan, A.Manohar, Harvard preprint HUTP-88/A024(1988)
- (21) W.A.Bardeen, Nucl.Phys.B75, 246(1974)
- (22) A.V.Efremov, O.V.Teryaev, Dubna preprint E2-88-287(1988)

Fig. 1: Diagrams contributing to a finite mixing of order α_s between g_1^P and the polarized gluon parton density.

Fig. 2: The typical diagram contributing to the leading logarithms in the Q^2 dependence of g_1^P at next-to-leading order.