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STRINGS AND $D = 4$

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Abstract

It is argued that the unique infra-red behaviour of non-Abelian gauge couplings in four dimensions could be responsible, in closed super- (or heterotic) string theory, for dynamical supersymmetry breaking and for compactification to $D = 4$ with string-scale compactified radii.

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1. Why is space-time four dimensional? This intriguing question is one of the outstanding puzzles of Nature.

String theory, if any, looks to be a priori in a worse position than field theory in providing an answer. Indeed, while at least consistent (i.e. renormalizable) field theories can be constructed in $D = 4$, string theories are notorious for requiring extra dimensions.

At a closer look, however, things are not so bad for the string: $D = 4$ quantum field theory is known to have problems when gravity is included, thus consistency in field theory might actually require $D < 4$. As for the extra dimensions needed in string theory, one knows by now how to “compactify” some of them: all we know, after all, is that the *large-scale* structure of space-time is four-dimensional. There is no problem adding extra dimensions as long as they are curled up on scales smaller than the ones tested by experiments.

The question, in string theory, is a different one: since the problem of ultra-violet divergences is supposedly solved in *any* number D of *uncompactified* dimensions, what is the criterion for selecting $D = 4$ and not $D = 10$ or $D = 2$? A related point is that there are by now so many consistent string theories in $D = 4$. What we are lacking is some dynamical principle that could select some (and hopefully just one) of them.

Of course, we shall not give here a complete answer to this fundamental problem: rather we would like to point out that one characteristic feature of $D = 4$, common to field and to string theory, i.e. the singular infra-red behaviour of its non-Abelian gauge interactions, can have some bearing on the issue.

We shall indeed give general arguments, as well as a couple of toy models, supporting the idea that energetic considerations related to these infra-red effects drive a string theory towards $D = 4$ compactification with tiny (i.e. string scale) extra dimensions, at least in comparison with $D > 4$ compactifications. Furthermore, the extra dimensions should compactify so as to yield asymptotically free gauge symmetries. We shall have little to say on why D is not *less* than 4, or on why four-dimensional space-time is asymptotically flat: these questions are clearly tied up with the cosmological constant problem.

In order to put the problem in a physical perspective, it is perhaps useful to recall that, in string theory, all relevant parameters are vacuum expectation values of some fields. Thus the string scale parameter α' can be seen as the normalization of the metric, playing the role of Planck’s constant, the string-loop expansion parameter is related to the expectation value of the dilaton field, the shape and sizes of (internal) spaces are related to various components of the metric, torsion or Wilson line backgrounds, etc.

If one could construct the full effective action of a string theory as a functional of all background fields, one could in principle determine all these parameters, including the fine structure constant, the cosmological constant and the large and small scale structure of space-time itself.

In field theory it is often large distance dynamics which determines the (occasionally non-perturbative) structure of the vacuum, while short distances usually reveal the actual symmetries of the theory. In the classical Coleman-Weinberg example [1], for instance, quantum effects induce logarithms of a scalar field φ (in agreement with the renormalization group) which can be shown to destabilize the tree-level vacuum at $\varphi = 0$. These logarithms are traditionally thought to be of an ultra-violet nature but, in a cut-off theory like string theory, their origin is better seen as infra-red [2].

These logarithms of φ are closely related to the logarithms of Q (the momentum) encountered in field theory loop calculations. We shall thus begin by recalling how running arises in string theory and we shall determine the scale Λ at which the gauge coupling becomes strong as a function of the compactification parameters. We shall then give arguments, based on our present non-perturbative knowledge of non-Abelian gauge theories, suggesting that the largest possible value of Λ should be dynamically selected. This will typically lead us into a ground state corresponding to a four-dimensional model with broken supersymmetry and enhanced gauge symmetry.

2. There has been recently some interest [2,3,4,5] in the question of running in string theories. This is of course an important issue when discussing string unification. Although one can approach the problem with the fully-fledged string loop apparatus, there are some conceptual difficulties due to the on-shell restriction on dual amplitudes. Once these are avoided either by some off-shell continuation [4] or by working in the presence of a background [5], the outcome appears to be the one expected from field theory modulo small string corrections.

This is not surprising if one realizes that running in string theory is better seen as an infra-red phenomenon. Furthermore, the infra-red behaviour of string theory is the one of an effective field theory describing the interactions of its light modes with a momentum cut-off $\mathcal{O}(\Lambda_S = 1/\sqrt{\alpha'})$.

The small momentum behaviour of loops certainly depends on the geometry of space-time. In particular, internal geometry determines the spectrum of Kaluza-Klein (KK) excitations (as well as of solitons) which propagate inside string loops, and thus the momentum dependence of the loop itself.

As a simple warm-up example consider the case of a $D = 5$ closed string theory in which one dimension is a circle of radius

$$R = \Lambda_C^{-1} > \sqrt{\alpha'}.$$

For closed or heterotic superstrings the tree-level four-dimensional gauge coupling and Newton's constant are given by:

$$g^2 = \phi / \Lambda_S R, \quad G_N = \alpha' g^2, \quad (1)$$

where ϕ is the string loop expansion parameter related to the vacuum expectation value (vev) of the dilaton field [6]. It is this parameter that controls the size of typical short-distance string-loop effects. We shall thus assume that ϕ is fixed somehow by string-scale dynamics, which cannot be described in field-theoretic terms.

In order to compute the one-loop correction to the effective action we can consider a loop with two external gauge bosons at a scale Q . Such an off-shell extension, compatible with modular and gauge invariance probably does exist [7]. Alternatively, one can put the string in a gauge background and compute the vacuum energy [5]. The result should be equivalent to a field theory loop calculation by the background field method [8]. As far as the singular behaviour near $Q = 0$ is concerned, we can make the field theory approximation:

$$\Pi^{\mu\nu}(Q) = i(Q^\mu Q^\nu - Q^2 g^{\mu\nu})\Pi(Q), \quad (2)$$

where:

$$\begin{aligned} \Pi(Q) &\approx i\beta_0 \sum_{n=0}^{n=\Lambda_S R} \int \frac{d^4 P}{(2\pi)^4} \frac{1}{P^2 + n^2 \Lambda_C^2} \frac{1}{(P+Q)^2 + n^2 \Lambda_C^2} \\ &\approx \frac{\beta_0}{(4\pi)^2} \left[\sum_{n=0}^{n=QR} \log\left(\frac{Q}{\Lambda_S}\right)^2 + \sum_{n=QR+1}^{n=\Lambda_S R} \log\left(\frac{n}{\Lambda_S R}\right)^2 \right] + \dots \end{aligned} \quad (3)$$

The coefficient β_0 includes just the contribution of the massless (zero) modes of KK excitations and explicitly reads:

$$\beta_0 = 2 \sum_K (-1)^{F_K} \left(\lambda_K^2 - \frac{1}{12} \right) C_K, \quad (4)$$

where the label K refers to one tower of KK excitations, F_K denotes the fermion number, λ_K the helicity and C_K the quadratic Casimir in the particle's K representation. We have kept only the KK excitations (and not string excitations), since only these can be light

with respect to Q , and we have inserted the string cut-off by hand assuming that it does not spoil gauge invariance. Massive string excitations only contribute finite corrections to $\Pi(Q)$ as $Q \rightarrow 0$; these we neglect, having assumed that the string loop expansion parameter ϕ is small.

Let us now distinguish two cases:

a) $Q \ll 1/R$. We obtain:

$$\Pi(Q) \approx \frac{\beta_0}{(4\pi)^2} \left[\log(QR)^2 - 2(\Lambda_S R - 1) \right] + \dots \quad (5)$$

where we have approximated the second sum in eq.(3) by an integral. We thus obtain the one-loop-corrected coupling:

$$Q \ll 1/R : \quad g^{-2}(Q) = g^{-2} + \beta_0(4\pi)^{-2} \log(QR)^2 + \mathcal{O}(\phi g^{-2}). \quad (6)$$

We have just rediscovered, in this regime, four-dimensional field-theory running, without any subtraction and/or renormalization procedure!

b) $1/R \ll Q \ll \Lambda_S$. In this region eq.(3) gives:

$$\Pi(Q) \approx -2\beta_0(4\pi)^{-2} R(\Lambda_S - Q) + \mathcal{O}(\phi g^{-2}), \quad (7)$$

corresponding to an effective coupling:

$$1/R \ll Q \ll \Lambda_S : \quad g^{-2}(Q) = g^{-2} + \mathcal{O}(\phi g^{-2}). \quad (8)$$

Thus, in the region $1/R < Q < \Lambda_S$, where the effective field theory is 5-dimensional and non-renormalizable, loops just renormalize (read *change*) the gauge coupling by a finite small amount (because of the assumed smallness of ϕ). They also contribute to some higher order couplings (term proportional to Q in eq.(7)), which are even more negligible well below Λ_S .

In summary, due to the absence of running in higher dimensions, hence between Λ_C and Λ_S , ϕ , even in the presence of loop effects, determines the 4D coupling at Λ_C via eq.(1). Running sets in below Λ_C and the coupling evolves as in usual field theory towards the infra-red. The scale at which the gauge interaction becomes strong is thus given by:

$$\Lambda = \Lambda_C \exp(-8\pi^2 \Lambda_S R / \beta_0 \phi). \quad (9)$$

Before turning to the physical applications of these basically obvious results, we have to extend our considerations to the case of a multi-dimensional compact space. We shall

do this explicitly for the case of a d -dimensional torus of radii $r_1, r_2, \dots, r_d$. Straightforward generalizations of the calculations that led us to eq.(6) show that in this case the scale that determines the running of the effective coupling constant is given by:

$$R^2 = \frac{\prod_{i < j} (r_i^2 + r_j^2) \prod_{i < j < k < l} (r_i^2 r_j^2 r_k^2 + r_i^2 r_j^2 r_l^2 + r_i^2 r_k^2 r_l^2 + r_j^2 r_k^2 r_l^2) \dots}{\prod_{i < j < k} (r_i^2 r_j^2 + r_i^2 r_k^2 + r_j^2 r_k^2) \dots}, \quad (10)$$

$$\text{i.e. } R^2 = r_1^2 \quad (d = 1), \quad R^2 = r_1^2 + r_2^2 \quad (d = 2), \dots$$

which can be interpreted physically as saying that the logarithmic running of the gauge coupling occurs only at scales larger than the largest of r_i , below which at least one extra dimension is “seen”. What is interesting is that it is not the total compact volume $V = \prod_i r_i$, or any other obvious geometrical object, that enters the logarithm.

There is one more case in which it is possible to determine explicitly the upper scale of running. In the symmetric orbifold compactification of the heterotic string [9], all massive KK excitations always contain three chiral supermultiplets for each vector supermultiplet. Hence only the massless states contribute infra-red logarithms. As a result, the orbifold upper scale for running is Λ_S itself and does not depend on the orbifold radius. This amusing property of orbifold compactification is probably related to the presence of isolated fixed points on the orbifold. Curvature is seen with probes of arbitrary resolution, down to the string scale.

We have thus seen that the low energy effective theory is determined uniquely by:

- i. the vev of ϕ ,
- ii. the volume of the compactified space V ,
- iii. the scale $R \approx \max\{r_i\}$,
- iv. the quantity β_0 , which is related to the particular gauge group remaining unbroken after compactification¹ and to the representation content of the massless states; both, in turn, are related to the topological properties of the compactified space.

In terms of these parameters we have:

$$\Lambda = R^{-1} \exp(-8\pi^2 V \Lambda_S^d / \beta_0 \phi). \quad (11)$$

It is of course far beyond our present capabilities to try to determine all these quantities in the cases of more complicated compactifications. We have assumed already ϕ

¹There can, of course, be new gauge symmetries generated via compactification. For large R , these have smaller couplings and smaller Λ 's than the gauge interactions generated in the heterotic way and can be neglected in this discussion.

to be determined by string-scale physics. We shall now argue that, from lower energy physics, there is a preference, energetically, for maximizing Λ .

3. We shall give two arguments for dynamical compactification of string theory to $D = 4$. The first argument is based on a non-supersymmetric description of dynamics below the string scale, the second is an attempt to justify the occurrence of dynamical supersymmetry breaking and compactification in the context of a $N = 1$ supergravity model.

The non-supersymmetric argument can be illustrated within the so-called “spaghetti-vacuum” model [10], but, as we shall argue, its basis rather rests on our knowledge of non-perturbative effects in non-Abelian, asymptotically free gauge theories.

The “spaghetti-vacuum” argument goes as follows: the low energy effective potential must contain a trace anomaly term representing the coupling constant running we have discussed so far. Typically such a term will read:

$$V_{\text{eff}} = \frac{1}{4\phi} F V \Lambda_S^d + \frac{1}{2\delta} F \frac{\beta_0}{(4\pi)^2} \log(G R^\delta), \quad (12)$$

where $F \equiv F_{\mu\nu} F^{\mu\nu}$ and G is some other gauge and renormalization group invariant object of dimension δ . This is the form of V_{eff} dictated by the appropriate RG invariance. It would also follow from a background field calculation of the type given in ref.[5], with the non-zero background providing the IR cut-off.

If G is identified with F , and minimization of V is performed, one obtains for the vev of F :

$$F = R^{-4} \exp\left(\frac{-32\pi^2}{\beta_0\phi} V \Lambda_S^6 - 1\right) = e^{-1} \Lambda^4, \quad (13)$$

corresponding to the energy density:

$$V_{\text{eff}} = -\frac{\beta_0}{128\pi^2} F = -\frac{\beta_0}{128e\pi^2} \Lambda^4 < 0. \quad (14)$$

It is well known that the original “spaghetti-vacuum” reasoning is not rigorous because one cannot trust the approximation leading to eq.(12) in the region where the minimum occurs. On the other hand, both QCD sum rules [11] and direct lattice calculations [12] in asymptotically free gauge theories have confirmed the validity of these naive conclusions. Non-perturbative effects (infinite genus from the string loop expansion point of view) favour condensate formation since it lowers the vacuum energy density by an amount $\mathcal{O}(\Lambda^4)$, with Λ given in eq.(11).

Energetically, maximization of Λ is favoured since it gives the lowest energy. This favours, for given ϕ : i) R and V as *small* as possible, and ii) β_0 positive and as *large* as possible.

One is clearly pushed towards radii and volumes of order of the string scale. As $R \rightarrow \sqrt{\alpha'}$, the field theoretic description of compactification has to be replaced by its string theoretic analogue. All indications are that instead of having quantities running out to infinity as $R \rightarrow 0$, as eqs.(14) and (11) would indicate, things stabilize because of a symmetry of string theory with respect to the replacement $R \leftrightarrow \alpha'/R$. This symmetry, known as duality [13], corresponds to an interchange of momenta and winding numbers, leaving physics invariant. Furthermore, at the fixed points of the duality transformation (moded by $SL_d(Z)$), the gauge symmetry is usually increased, non-Abelian interactions replace the Abelian ones of the generic case and, of course, β_0 becomes positive and, typically, the larger, the larger the gauge symmetry is.

The above is clearly a rough argument; we believe, however, that it contains a point of general validity: the peculiar infra-red behaviour of $D = 4$ non-Abelian gauge theory could be responsible not only for important phenomena like confinement and chiral symmetry breaking: it could also drive energetically all the extra dimensions to curl up over distances of order of the string scale and in such a way as to maximize the beta-function.

An obvious objection to the above argument is that, if one starts with a superstring theory in 10 dimensions, the effective action should be supersymmetric and the energy non-negative. If dynamical supersymmetry breaking is achieved by some non-perturbative effect [14], it corresponds to the absence of a zero-energy supersymmetric vacuum. But then why should the theory compactify when it can apparently stay supersymmetric in $D = 10$ and with zero vacuum energy? In other words, can $D = 4$ with supersymmetry breaking compete energetically with $D = 10$ without breaking? The answer would appear to be an obvious no, were it not for the loophole that with *local* supersymmetry, positivity of energy is not necessary [15].

Indeed this question has already been asked in the literature [16] and the answer that has emerged is quite interesting from our viewpoint: there is no large-compact-volume, small coupling $D = 4$ theory that can compete with flat $D = 10$, but there could instead be competing solutions with string-scale-size compactified radii, i.e. precisely of the type we argued for in the non-supersymmetric argument. Let us re-examine this conclusion from our viewpoint.

As a first requirement, $N = 4$ supersymmetry has to be broken so that a non-zero

beta-function can emerge driving the theory towards large infra-red couplings. We assume this to happen at the string scale, and the theory to be $N = 1$ supersymmetric below Λ_S . Given this, we have actually constructed a toy model, an effective action based on $N = 1$ supergravity, describing the interaction of the $D = 10$ light string fields with the gluino condensate [17] chiral superfield. Such effective Lagrangians had been considered in the past [17,18,19] as models for supersymmetry breaking.

Our effective Lagrangian differs in two respects from those considered earlier: i) We assume no important role to be played by the torsion field (no $H_{\mu\nu\rho}$ condensation), and ii) We incorporate the information contained in the running formula, eq.(6), through the supersymmetric version of eq.(12), i.e. through a superpotential:

$$f = \frac{1}{4}SW + \frac{\beta_0}{96\pi^2}W \log \left[W(ST)^{\frac{3}{8}} \right], \quad (15)$$

where S and T are the chiral superfields of ref.[6] and $W \equiv W_\alpha W^\alpha$, with W_α the usual gauge supermultiplet. Note that the scale of the log, which was taken to contain a parameter μ in refs.[18,19], is now itself a chiral superfield, a combination of the superfields S and T . When the system is coupled to gravity in the usual $N = 1$ supergravity way, the flat direction in T is removed precisely because of the appearance of T under the log.

The resulting scalar effective potential, which is no longer positive definite, can be studied and minima can be searched for with the following results:

- a) If the scale Λ is much smaller than Λ_S , there is no state that can compete with the “vacuum at infinity” corresponding to flat $D = 10$ space-time. The other candidate vacuum, also at zero energy, occurs at an unphysical value for S ($S = -\beta_0/6$ [19], whereas S is, by definition, positive).
- b) If we allow Λ to approach $1/\sqrt{G_N}$, a state of negative energy can possibly exist with acceptable values of S and T .

It is difficult, of course, to argue on the basis of a field theory effective action in this region of Λ . All we want to suggest is that, on the basis of the example, dynamical supersymmetry breaking at the string scale *can* be dynamically favoured.

4. Let us summarize and discuss a bit further our results. We have looked at string theory as a theory of gauge and gravitational interactions with a fixed, physical cut-off $\sqrt{\alpha'}$. This string scale parameter, which is the basic Planck constant of string theory [2], can be also thought of as the lattice spacing of a lattice regularization of an effective field theory. Let us now compare, in this effective lattice theory, $D > 4$ and $D = 4$. At $D > 4$ both the gauge and the gravitational interactions are non-renormalizable (in the usual

sense). They represent irrelevant operators, leading to a trivial theory if the cut-off is removed, or to a weakly-coupled perturbative theory at distances much larger than $\sqrt{\alpha'}$. By contrast, if $D = 4$, non-Abelian gauge interactions suffer non-trivial renormalization effects and correspond to relevant operators. Physics at scales much larger than $\sqrt{\alpha'}$ can be extremely rich with formation of condensates, confinement, spontaneous breaking of gauge and global symmetries etc. If all these processes take place (as seems to be the case in explicit lattice calculations), that must mean that the energy is lowered by their occurrence.

We thus conclude that, in general, compactification from $D > 4$ (say $D = 10$) to $D = 4$ pays off energetically if AF gauge interactions are originated (or kept unbroken) in the process. Since all the above-mentioned non-perturbative large distance effects know only the scale Λ at which the couplings get large, the effects get larger and larger with increasing Λ . Thus a theory with a given Λ is unstable in the direction of increasing Λ . In string theory, Λ is a function of various fields occurring in the theory and the above effects would drive these fields towards the region that gives maximization of Λ . This by itself seems to imply dynamical supersymmetry breaking and a string-scale compactification that preserves non-Abelian gauge symmetries with large positive beta-functions.

The remaining point is understanding what stops D from shrinking further down, say to $D = 2$. This problem is not easy to discuss, since in $D < 4$ IR phenomena are even more singular than in $D = 4$. It could indeed be that they are too singular! As an example, in the Schwinger model, the singular IR nature of vacuum polarization induces a mass for the photon, and this makes the theory somewhat trivial again. Possibly, along these lines, one could justify that $D = 4$ is the best compromise with just the “right amount” of IR singularities.

Another approach to this question would be to tie it up with the cosmological constant problem. We have been working all the time with a flat $D = 4$ space (plus extra curled up dimensions), but why should the $D = 4$ space not curl up as well? The cosmological constant, as any other constant of Nature, is a dynamical quantity in string theory, depending e.g. upon the dilaton vev. One might thus hope that some dynamics, one related to gravitational interactions this time, will select vanishing cosmological constant, perhaps along the lines advocated by Hawking [20].

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