AN ELECTRON-HADRON SEPARATOR FOR DIGITAL SAMPLING CALORIMETERS

(CHARM II Collaboration)

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ABSTRACT

A fast and effective algorithm for electromagnetic and hadronic shower separation has been developed for the digital sampling calorimeter of the CHARM II experiment. It is based on a generalization of the Minimal Spanning Tree concept and can be easily applied to other existing calorimeters. In this particular application, which requires the highest efficiency for retaining electromagnetic showers, one gets, for 99% efficiency, a rejection factor of the order of 100 for hadronic showers.

1. INTRODUCTION

In the following we describe a new powerful e/h separation algorithm, with special emphasis on its applications to digital sampling calorimeters of the CHARM II type for which it has been developed [1]. The simplicity and generality of the basic principles of the algorithm allow its applicability to be extended to other existing calorimeters.

The CHARM II Collaboration has built a very large target calorimeter [2] which is being exposed to the CERN Super Proton Synchrotron (SPS) wide-band neutrino beam (WBB) for the detailed study of the neutral-current processes

$$\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e . \quad (1)$$

The detector acts both as a very massive target and as a calorimeter to measure the energy, the vertex, and the direction of the showers produced by the neutrino-induced reactions. The main task of the experiment is to isolate the signal (1) from a 10^3 times larger background due to muonless neutrino-quark interactions.

The calorimeter, which is of the sampling type [3], is composed of 420 modules. Each module has an active area of $3.7 \times 3.7 \text{ m}^2$ and consists of a 4.8 cm thick glass-target plate (radiation length $X_0 = 10.68 \text{ cm}$; interaction length, 47.02 cm) followed by a plane of 352 plastic streamer tubes. These are of the Mont Blanc type, with 1 cm wire spacing (with direct readout in digital mode); they are equipped with pick-up cathode strips (with analog readout), of 18 mm width and 3 mm pitch, orthogonal to the wires. The wire orientation of two consecutive modules is rotated by 90° , thus allowing two orthogonal views of the event. Figure 1 shows a detail of the calorimeter's basic modules.

The very fine granularity of the detector suggests the development of a new e/h separator which, based on the Minimal Spanning Tree algorithm, is explicitly designed for the characteristic patterns of electromagnetic and hadronic showers exhibited in a digital calorimeter.

The power of separation and the efficiency of such a separator are checked against electron and pion data taken during a calibration run [4]. For this purpose, the low-intensity tertiary test beam X7 at CERN was used. This beam is produced by the interaction of 135 GeV protons on a Be target, and the momenta of secondary particles are selected in the range 3 to 40 GeV/c. The central value of the beam momentum is known with a precision of 1%, and the momentum spread is about 1%. A 20 m long helium-filled threshold Cherenkov counter was placed in front of the calorimeter in order to tag electrons and pions.

2. MINIMAL SPANNING TREE

In each projection, the coordinates of the tubes that are hit, together with the coordinates of the corresponding planes, define a lattice over which the electromagnetic and hadronic showers exhibit the patterns indicated in Fig. 2.

Using the jargon of graph theory, all the tools that are necessary for building what is called a Minimal Spanning Tree (MST) [5] will be defined below.

Given a set of points (*nodes*) as in Fig. 3, one defines: *edge* as a pair of nodes with an assigned *weight* (e.g. their Euclidean distance); *graph* as a set of nodes and edges; *path* as a sequence of edges joining two nodes; *circuit* as a closed path; *connected graph* as a graph which has paths between any pair of nodes; *tree* as a connected subgraph with no circuits; *spanning tree* as a tree which contains all nodes; *minimal spanning tree* as a spanning tree, the weight of which (taken as the sum of weights of all the edges) is minimum among all spanning trees.

The MST concept is widely used in pattern recognition and classification problems (see Refs. [5] to [7] for details and applications). It satisfies a *minimum principle*, so that it is unambiguous in its definition and its uniqueness can be demonstrated. It is also *deterministic* in the sense that it does not depend on random choices in the algorithm or on the order in which the nodes and the edges are selected and examined, but only on the given set of nodes. It is *invariant under similarity transformations*, i.e. under all transformations which preserve the monotony of the metric (monotone invariance) used for its definition (translations, rotations, changes of scale, and even some non-linear distortions). It is also rather insensitive to some amount of noise, randomly spread among the input set of nodes.

Once a metric for the definition of the weights has been chosen, there are several alternative ways of constructing the MST. The procedure which has been used throughout the present analysis is due to a version, elaborated by Ross [8], of Prim's algorithm [9]. It consists in starting with an arbitrary node and adding the edge which has the smallest weight connected to it. This represents the first fragment tree T_1 . The i^{th} fragment T_i results from connecting T_{i-1} to the closest node not belonging to T_{i-1} . The $(N - 1)^{\text{th}}$ fragment tree, N being the number of nodes, is the MST.

In the application described here, the metric defined for the weight assignment has been chosen to give a smaller contribution — and therefore a larger distance — to the connections of hits along the longitudinal direction as opposed to those separated laterally on the same plane:

$$W_{ij} = \sqrt{(5 \times \Delta p)^2 + (\Delta w)^2} ,$$

where Δp represents the difference in plane numbers between nodes i and j , and Δw the difference in wire numbers. The choice of the factor of 5 is purely arbitrary, and as a matter of fact, such a number is the only free parameter present in the whole theory. Moreover, because of the monotone invariance of the MST, one can linearize the Euclidean distance in order to speed up calculations and without affecting the construction of the MST:

$$W_{ij} \stackrel{\text{def}}{=} |5 \times \Delta p| + |\Delta w| . \quad (2)$$

The MST of a graph containing N nodes is then characterized by the $(N - 1)$ values, $\lambda_i = W_{ij}$, of the weights of its edges, from which one can compute the total weight $\mathcal{L}$ and the mean edge, or average link length, Λ as

$$\Lambda = \frac{1}{N - 1} \sum_{i=1}^{N-1} \lambda_i = \frac{\mathcal{L}}{N - 1} .$$

3. ELECTRON-HADRON SEPARATION

Once the MST has been built for both projections of a given pattern of hits, one makes use of the different signatures of electromagnetic and hadronic showers to generate the separation.

Since electromagnetic showers are narrower and denser than hadronic showers of the same energy, one expects in general, for equal N , $\Lambda_e < \Lambda_h$ and therefore $\mathcal{L}_e < \mathcal{L}_h$. Figure 4 shows the distributions of Λ and $\mathcal{L}$ for electrons and pions of 10 GeV/c momentum (corresponding to an approximately equal number N of hits). Since these distributions are Gaussian, one can define, in the $(\Lambda, \mathcal{L})$ plane, a 1σ correlation ellipse (Λ and $\mathcal{L}$ being strongly correlated via N) as being the region inside which approximately 64% of all events will fall. By plotting, then, $\mathcal{L}$ and Λ for electrons and pions of all energies (as in Fig. 5), one can attempt a first separation, for example at 99.5 CL, by making a convolution of the upper contours of the 2.6σ correlation ellipses for electrons. At this level a rejection power of about 5–6 can be achieved.

A large improvement in the rejection power is obtained by looking at the distribution of the $(N - 1)$ values λ_i . Because of the definition of the metric used [Eq. (2)], all λ_i assume integer values. All links with $\lambda < 5$ lie along the lateral projection of the shower, whereas those with $\lambda \geq 5$ are mostly directed along the longitudinal projection. Indicating with n_λ ($\lambda = 1, \dots, 6$)^{*)} the distribution of the links, one expects for electromagnetic showers, for a given number N of hits, an n_1 larger than for hadronic showers, whereas the contrary is true, for instance, for n_6 . Figure 6 shows the distributions of n_λ for the electron and pion displayed in Fig. 2.

With the aid of the training sample of electrons taken in the calibration run at different energies, one can construct the *canonical* distribution of links for *pure*^{**)} electromagnetic showers, by parametrizing n_λ as a function of the total number N of nodes:

$$n_\lambda^e(N), \quad \lambda = 1, \dots, 6 . \quad (3)$$

Thus a comparison of the MST observables

$$N \quad \text{and} \quad n_\lambda, \quad \lambda = 1, \dots, 6$$

of an individual event with the corresponding canonical values (3) will establish if the event was an electromagnetic shower. For this purpose, one introduces the variable

$$\chi_e^2 = \sum_{\lambda=2}^6 \left(\frac{n_\lambda - n_\lambda^e(N)}{\sigma_\lambda^e(N)} \right)^2 ,$$

which follows a χ^2 distribution, and the corresponding probability $P_e = P(\chi_e^2)$. Here the $\sigma_\lambda^e(N)$ represent the standard deviations of the distributions (3), and the sum extends over five terms only, instead of six, since the n_λ are not linearly independent but are related by the constraint

$$\sum_{\lambda=1}^6 n_\lambda = N - 1 .$$

The top half of Fig. 7 shows the distribution of P_e for the calibration electrons (all energies). The excess of events ($\approx 1.5\%$) at low values of P_e (< 0.005) is clearly established by visual scanning, as being due to a contamination of pions in the electron sample. The bottom half of Fig. 7 shows the corresponding distribution for the calibration pions (all energies). The total number of events is normalized to the number of electrons in the upper plot; 93% of all pions lie in the region $P_e < 0.005$. The flat background is due to electromagnetic showers. It is consistent with the rate expected from the charge-exchange process $\pi^- + p \rightarrow \pi^0 + n$ (a few per cent in this range of energy) plus a small contribution due to a contamination of electrons in the pion sample.

*) where

$$n_6 \stackrel{\text{def}}{=} \sum_{\lambda \geq 6} n_\lambda ,$$

since for $\lambda > 6$, in the case of electromagnetic showers, the number of edges is statistically very limited.

**) A small amount of contamination in the training sample cannot affect the distribution n_λ because of the different values of N for electrons and pions of the same energy.

4. 'ELECTROMAGNETICITY'

The procedure used to establish the canonical electromagnetic link distributions [formula (3)] can be applied to the training sample of pions taken in the calibration run in order to infer the canonical distribution of links for hadronic showers:

$$n_{\lambda}^{\pi}(N), \quad \lambda = 1, \dots, 6 .$$

This, combined linearly with formula (3), allows the possibility of treating hadronic showers with variable amounts of electromagnetic component by defining a function F:

$$F(\alpha) = \sum_{\lambda=2}^6 \left(\frac{n_{\lambda} - [\alpha n_{\lambda}^e(N) + (1 - \alpha)n_{\lambda}^{\pi}(N)]}{\sigma_{\lambda}^e(N)} \right)^2 , \quad (4)$$

α being a free parameter.

The value of α which minimizes Eq. (4), $\alpha_{\min}$, can be interpreted as the electromagnetic content of the shower (*electromagnetism*). Clearly, all electrons will exhibit values of $\alpha_{\min}$ around 1, whereas, $\alpha_{\min}$ around 0 will be expected for pions. In Fig. 8 the value of $\alpha_{\min}$ is plotted for the electrons and pions of the calibration data.

5. RESULTS

The fact that the probability distribution P_e for electrons is uniform, independent of their energy, not only ensures the correctness of the theory, but also gives a direct indication of the efficiency for retaining electromagnetic showers for any given cut on P_e . On the other hand, Fig. 7 indicates that the pion contamination extends up to approximately $P_e = 0.10$. The remaining events show a flat distribution in P_e which is compatible with that expected from charge-exchange processes (a few per cent) which produce electromagnetic cascades, plus a small contribution due to bad tagging in the Cherenkov counter. Therefore, for a 90% electron efficiency, a cut at $P_{\text{cut}} = 0.10$ will leave a hadronic shower contamination of a fraction of a per cent.

An attempt to rescue the rejected electrons can be made with the help of $\alpha_{\min}$. In fact, in the $P_e < 0.10$ sample, by rejecting only the events with $\alpha_{\min} < 0.7$ the electron efficiency is increased to 99% at the expense of a pion contamination of 1%.

6. CONCLUSIONS

A straightforward application of the notions of graph theory has allowed the introduction of a new, powerful separator for hadronic and electromagnetic showers. For the needs of CHARM II, which favours small *significance* (inefficiency for selecting electromagnetic showers) over large *power* (rejection factor for hadronic showers), a pion contamination of 1% for 99% electron efficiency can be reached.

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Figure captions

Fig. 1: Details of the calorimeter's basic modules of the CHARM II detector.

Fig. 2: Typical hit pattern: a) 10 GeV pion and b) 10 GeV electron. Only one projection is shown.

Fig. 3: Graph as a set of nodes to illustrate the concepts needed for the definition of the Minimal Spanning Tree.

Fig. 4: 10 GeV electrons and pions: a) mean MST edge Λ ; b) total weight $\mathcal{L}$ of the MST.

Fig. 5: Mean values of Λ and $\mathcal{L}$ for electromagnetic and hadronic showers of different energies. The error bars indicate one standard deviation of their distributions. The solid line represents the convolution of the upper contours of the 2.6σ correlation ellipses for electrons. The shower energy in GeV is given next to the data points.

Fig. 6: Distribution of links of the electromagnetic and hadronic showers of Fig. 2.

Fig. 7: Probability distribution $P_e = P(\chi_e^2)$. Top: test beam electrons (all energies). A small contamination of pions in the electron sample is visible in the first bin for $P_e < 0.005$. Bottom: test beam pions (all energies). The flat background is mainly due to the charge-exchange process ($\pi^- + p \rightarrow \pi^0 + n$).

Fig. 8: Electromagneticity, $\alpha_{\min}$, for calibration data (both electrons and pions).

THE LAST 5 MODULES OF ONE RACK

CALORIMETER MODULES

Fig. 1

$\Delta x = 10$ cm (distance between planes)
 $\Delta y = 1$ cm (distance between wires)

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

e^- and π^- normalized to same number of events

Fig. 7

Fig. 8