DEPENDENCE OF J/ψ SUPPRESSION IN HEAVY ION COLLISIONS
ON TOTAL TRANSVERSE ENERGY, NUCLEON NUMBERS AND J/ψ TRANSVERSE MOMENTUM

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ABSTRACT

We present a set of semiquantitative predictions for the behaviour of J/ψ suppression in heavy ion collisions as a function of the total transverse energy, nucleon numbers of colliding ions and the J/ψ transverse momentum. These predictions are based on the assumption that all of J/ψ suppression is due to J/ψ and χ disintegrations in collisions with hadrons in a dense hadronic gas formed in the heavy ion collisions. The hydrodynamic evolution of the hadron gas is taken into account. The onset of the quark-gluon plasma formation should manifest itself by clear qualitative deviations from J/ψ suppression calculated in this way, in particular as thresholds or kinks in the E_T -dependence, or as a threshold for J/ψ suppression as a function of the atomic number of the target, expected as Cu or below.

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1. Introduction

J/Ψ suppression observed recently in $^{16}O - ^{238}U$ collisions at 200 GeV per nucleon [1] is an extremely interesting effect. The more so that the effect has been predicted. In their pioneering paper Matsui and Satz [2] argued that J/Ψ suppression is a clear - cut signature of the quark - gluon plasma (QGP) formation in heavy ion collisions. Various aspects of J/Ψ suppression have been studied in further papers [3 - 10] and compared with preliminary data of the NA - 38 collaboration.

At first sight, it might seem that J/Ψ suppression observed by the NA-38 collaboration provides really an evidence for the QGP formation in $^{16}O - ^{238}U$ collisions at 200 GeV per nucleon.

The situation is however not so simple. It has been recently pointed out [11 - 13] that a sizeable J/Ψ suppression is to be expected from disintegration of J/Ψ and χ in collisions with pions and other hadrons in dense hadronic matter undoubtedly formed in heavy ion collisions. It is completely unclear what fraction of J/Ψ suppression observed [1] is due to J/Ψ dissolution by the QGP and what fraction is caused by J/Ψ and χ disintegration in collisions with hadrons. The situation is even more complicated by the fact that nuclear physics effects due to modifications of nucleon structure functions in nuclei can also contribute to the total J/Ψ suppression [14,15,16].

In order to disentangle contributions of various mechanisms one has to study general features of J/Ψ suppression in all of the models and compare their predictions with the data. It seems there are good chances that these models will lead to qualitatively different predictions. If, for instance, J/Ψ is suppressed by QGP, one would expect that J/Ψ suppression would be strongly dependent on the total transverse energy E_T released in the collision. The QGP is formed only when density of matter reaches some critical value. In peripheral collisions corresponding to lower values of E_T this critical density might not be reached, whereas it is reached in central collisions corresponding to higher values of E_T . In J/Ψ suppression caused by the hadron gas one would expect less dependence on E_T since the gas is formed also in peripheral collisions, although not as dense as in the central ones. For the same reason one would expect a stronger dependence of J/Ψ suppression on nuclear numbers of the beam and the target in QGP model than in the hadron gas one.

J/Ψ suppression by nuclear physics effects should also have some typical features. The NA-35 data [17] and their analysis [17, 18] indicate, that in what concerns E_T distributions one can look at the $^{16}O - ^{238}U$ collision as at an incoherent sum of nucleon - ^{238}U collisions. Assuming, although it is rather debatable, that modifications of structure functions of nucleons at the periphery and in the centre of ^{238}U are not much different, one would

not expect much E_T dependence of J/Ψ suppression in this model. We shall not discuss further here nuclear effect - types of models and consider, at least for the time being, QGP and hadron gas models as two possible candidates for the explanation of the effect observed.

The purpose of the present paper is to give semiquantitative predictions for E_T -, A -, and B - dependence of J/Ψ suppression under the assumption that J/Ψ is suppressed only by collisions with hadrons in a dense hadron gas. Strong deviations from qualitative features of these predictions, and agreement with the QGP model, would represent an evidence in favour of the QGP formation in heavy ion collisions.

The paper is organized as follows. In Sect. 2 a qualitative discussion is given of the two interpretations of preliminary NA - 38 data on the J/Ψ suppression. In particular we discuss the implications of the p_T - dependence observed [1] in the $^{16}O - ^{238}U$ collisions. Sect. 3 contains a description of our hadron gas model and predictions for the p_T -, E_T -, A - and B - dependence of J/Ψ suppression in this model. In Sect. 4 we discuss qualitative features of predictions following from our model. Sect. 5 contains comments and concluding remarks.

At the time when we were writing the final version of the manuscript there appeared papers by Blaizot and Ollitrault [19, 20], by Karsch and Petronzio [21] and new presentations of NA-38 data [22, 23]. These papers have clarified the E_T - dependence of J/Ψ suppression from the experimental point of view and brought new predictions for this dependence following from the QGP models. We shall discuss this issue in Sect. 5 and compare the predictions of QGP and hadron gas models with the data.

The reader not interested in technicalities is advised to read Sect. 2, skip Sects. 3 and 4 and read Sect. 5.

2. J/Ψ dissolution or J/Ψ disintegration ?

In this section we discuss in a qualitative way the J/Ψ dissolution by QGP and J/Ψ disintegration by collisions with hadrons.

The energy density of the QGP taken as a gas of free quarks, antiquarks and gluons at $T_c \approx 200 MeV$ is about $2.5 GeV fm^{-3}$. A similar value is obtained also from studies

of the deconfinement phase transition on a lattice. In order to produce QGP in heavy ion collisions the energy density should be somewhat higher than this value. The energy density in $^{16}\text{O} - ^{238}\text{U}$ collisions in the NA - 38 experiment can be estimated in the following way. The transverse energy measured consists of the whole neutral transverse energy plus about 30 % of the transverse energy of charged particles. This corresponds to $0.4 + 0.3 \times 0.6 = 0.58$ of the total transverse energy. An NA - 38 event with 60GeV of measured transverse energy thus corresponds to $E^T \sim 100\text{GeV}$. Since the NA - 38 calorimeter covers the pseudorapidity interval $1.95 < \eta < 4.15$ we get $\Delta E_T / \Delta y \sim 45\text{GeV}$. This is distributed within a volume $\pi R_O^2 c\tau$ where R_O is the ^{16}O radius, and τ is the formation time (of QGP or of the hadron gas). The energy density thus becomes

$$\epsilon \approx \frac{45\text{GeV}}{\pi R_O^2 c\tau} \quad (1)$$

Note that this is the average energy density. The maximal one in the centre is higher by a factor of about 3/2. Taking the standard estimate [24] $\tau = 1\text{fm}/c$ we obtain $\epsilon \approx 1.6\text{GeV fm}^{-3}$. In fact this might be an overestimate. The value $\tau = 1\text{fm}/c$ is supposed to correspond to very high energy collisions when both nuclei are Lorentz contracted to thin pancakes with soft partons occupying a longitudinal distance of about 1fm . At lab. energy of 200GeV per nucleon the c.m.s. energy of a nucleon - nucleon collision is $\sqrt{s} = 20\text{GeV}$. Lorentz contraction factor for each of the nuclei is $\gamma = 10$. The longitudinal dimensions of ^{238}U at energy of $10\text{GeV}/\text{nucleon}$ is about 1.2fm and that of the ^{16}O about 0.6fm . A simple geometrical argument shows that the energy is originally released within a region with longitudinal dimension of about 1fm . Adding to that another 1fm to account for the longitudinal expansion during the formation time we obtain as a more realistic, although somewhat conservative, estimate $\tau = \tau_{eff} = 2\text{fm}/c$. Inserting that into Eq.(1) we have $\epsilon \approx 0.8\text{GeV fm}^{-3}$ what is certainly below the threshold for QGP formation ^{*)}.

It is thus apriori not very probable that QGP can be formed in $^{16}\text{O} - ^{238}\text{U}$ collisions at 200 GeV per nucleon. On the other hand, taking into account a qualitative character of the argument and the fact that we have considered only the average energy density and not the maximal one, the formulation of QGP in these collisions cannot be excluded.

Basic features of the relevant NA - 38 data are easy to summarize:

- i) For $p_T \equiv p_T(J/\Psi) \approx 0$ the survival probability of J/Ψ in events with highest E_T is about 0.5

^{*)} One could argue that c in Eq.(1) is an overestimate of the velocity of the expanding system. The corresponding correction does not help substantially since the velocity of expansion v is related to pseudorapidity η by $\eta = (1/2)\ln[(c+v)/(c-v)]$ and for $\eta \approx 1$ this leads to $v \approx 0.8c$.

- ii) The survival probability $S(p_T)$ depends strongly on the transverse momentum p_T . J/Ψ 's with p_T larger than about $3\text{GeV}/c$ can escape from the collision without being substantially suppressed.

Both "hadron gas" and QGP models of J/Ψ suppression can explain or accommodate both features i) and ii). In both models the probability density for the creation of a $c\bar{c}$ pair in the plane perpendicular to the axis of the collision is proportional to the number of nucleon - nucleon collisions. For a central $^{16}\text{O} - ^{238}\text{U}$ collision this density is proportional to $(1 - r^2/R_O^2)^{1/2}$ where R_O is the oxygen radius and r is the distance from the common centre of both nuclei in the transverse plane.

In QGP models [3, 4, 5, 10] one assumes that the formation time of J/Ψ from a $c\bar{c}$ pair produced essentially in one point is $\tau_{J/\Psi} \approx 0.9\text{fm}/c$. This value of $\tau_{J/\Psi}$ is obtained by dividing the radius of J/Ψ by a mean relative velocity of c and $\bar{c}$ quarks within J/Ψ . In a similar way the formation time of χ is estimated to about $2\text{fm}/c$. J/Ψ and χ are dissolved by the QGP provided that the temperature of QGP is larger than some critical temperature $T_{J/\Psi}$ and T_χ respectively. Regions corresponding to $T > T_{J/\Psi}$ and $T > T_\chi$ are shrinking with the time due to the longitudinal expansion and cooling and with a suitable shrinking prescription one can obtain [3, 5, 10] a 50 % survival probability for J/Ψ with $p_T \approx 0$.

As pointed out by Karsch and Petronzio [5] in a detailed calculation of J/Ψ suppression one has to take into account that about 40 % of J/Ψ 's produced in pn collisions at 200 GeV come from radiative χ - decays [26]. The final "experimental" J/Ψ survival probability is then calculated as

$$S(p_T = 0) = 0.6P_{J/\Psi}(p_T = 0) + 0.4P_\chi(p_T = 0) \quad (2)$$

where $P_{J/\Psi}(p_T = 0)$ is the probability that J/Ψ is formed in the region $T < T_{J/\Psi}$ and therefore it survives. The same notation is used for $P_\chi(p_T = 0)$. The shrinking of regions with $T > T_{J/\Psi}$ and $T > T_\chi$ is essential for the explanation of the strong p_T - dependence of $S(p_T)$.

To illustrate the problem suppose, following the observation by Karsch and Petronzio [4], that a $c\bar{c}$ pair with a certain value of p_T is produced at $r = 0$ (in the centre of the transverse plane). The formation time of J/Ψ is $\tau_{J/\Psi}$, Lorentz dilated by the factor $E_{J/\Psi}/m_{J/\Psi}$. The J/Ψ velocity is $v = p_T/E_{J/\Psi}$ and the distance $c\bar{c}$ pair travels before

*) The opinions on this procedure are not unanimous. For instance Kapusta [25] gives arguments for a larger J/Ψ formation time. We shall discuss the concept of the J/Ψ formation time in more detail elsewhere.

J/Ψ is fully formed is

$$l = \tau_{J/\Psi} \frac{E_{J/\Psi}}{m_{J/\Psi}} \frac{p_T}{E_{J/\Psi}} = \tau_{J/\Psi} \frac{p_T}{m_{J/\Psi}} \quad (3)$$

Taking $p_T \approx 3 \text{ GeV}$ we have $l \approx 1 \text{ fm}$. In a similar way we obtain for χ with $p_T \approx 3 \text{ GeV}/c$ the value $l \approx 2 \text{ fm}$. If J/Ψ with $p_T > 3 \text{ GeV}$ should not be suppressed at all the region with $T > T_{J/\Psi}$ should be limited to $r < 1 \text{ fm}$ at the J/Ψ formation time. In a similar way the region with $T > T_\chi$ should be limited to about $r < 2 \text{ fm}$ when χ with $p_T \approx 3 \text{ GeV}/c$ is formed.

In hadron gas models of J/Ψ suppression [11, 12, 13] one can also describe both basic features i) and ii) of NA - 38 data [1]. This statement might seem to contradict the flat p_T - dependence of J/Ψ in Refs. [11, 12, 13] but, as we shall discuss shortly, this problem can be coped with when one realizes that the concept of formation time has different meanings in QGP and hadron gas models. The survival probability in the hadron gas model is calculated as follows. Suppose, for the moment, that the hadron gas consists only of pions with density ρ_π and consider J/Ψ with transverse momentum $p_T = 0$. Denoting the cross section for

$$\pi + J/\Psi \rightarrow D + \bar{D} + \pi \quad (4)$$

as σ , the time interval over which the pion gas is present as Δt and the probability that pion + J/Ψ c.m.s. energy is above threshold of (4) as $P(s > s_0)$ we have

$$S(p_T) = \langle \exp(-T(p_T)) \rangle \quad (5)$$

with $T(p_T) = \sigma \rho_\pi v P(s > s_0) \Delta t$ where v is the relative velocity of pion and J/Ψ for $s > s_0$. The symbol $\langle \rangle$ in Eq.(5) indicates averaging over positions where J/Ψ has been produced, directions of J/Ψ velocity and momentum distribution of pions in the pion gas. The density of the pion gas can be estimated from the total transverse energy released in the collision. Assuming that each pion contributes 0.5 GeV to E_T , taking E_T from Eq.(1) and assuming further that the density of pions is proportional to the number of nucleon - nucleon collisions in a given point of the transverse plane we obtain

$$\rho_\pi(r) = \frac{3}{2} \sqrt{1 - r^2/R_0^2} \frac{90}{\pi R_0^2 c \tau} \quad (6)$$

For $\tau = 1 \text{ fm}/c$ the average pion density ^{*)} becomes $\langle \rho_\pi \rangle \approx 3 \text{ fm}^{-3}$ and the maximal density at $r = 0$, $\rho_\pi \approx 4.5 \text{ fm}^{-3}$.

^{*)} Note that for pion gas models the difference between densities corresponding to $\tau = 1 \text{ fm}/c$ and $\tau = 2 \text{ fm}/c$ is at present not of a decisive importance since disintegration cross sections are uncertain at least to a factor of two.

The disintegration cross section for the reaction (4) is more difficult to estimate. From the data on photo- and hadro- production of J/Ψ on nuclei [27] it is known that the cross section for $N + J/\Psi \rightarrow N + D + \bar{D}$ is about $1 - 3mb$ above the threshold. Very nice recent analysis [28] leads to somewhat larger value of about $4.5mb$. One can thus expect that the cross section for reaction (4) is $1 - 3mb$ above the threshold. Taking $\sigma \approx 2mb$, $\Delta t \approx 3fm/c$ and calculating $P(s > s_0) \approx 0.2$ we obtain

$$T(p_T \approx 0) \approx 0.2fm^2 \times 3fm^{-3} \times c \times 0.2 \times 3fm/c \approx 0.36$$

$$S(p_T \approx 0) \approx 0.7$$

In a more realistic calculation we have to take into account that about 40 % of J/Ψ production is due to radiative χ decays [5, 26]. The cross section for the disintegration of χ in collisions with pions is larger than in the J/Ψ case, since the radius of χ is larger and the threshold is lower. Taking $\sigma(\pi + \chi) = (\frac{r_\chi}{r_{J/\Psi}})^2 \sigma(\pi + J/\Psi)$ and putting as in Eq.(2)

$$S(p_T) = 0.6exp(-T_{J/\Psi}) + 0.4exp(-T_\chi) \quad (7)$$

we obtain $S(p_T \approx 0) = 0.5$. Note also that this value cannot be, in hadron gas models, calculated accurately since neither the disintegration cross sections nor the composition of hadron gas are well known.

The strong p_T dependence of the observed J/Ψ suppression presents a more difficult problem for hadron gas models. In these models the distance l travelled by J/Ψ in the hadron gas is roughly equal to the oxygen radius $l \approx R_O \approx 3fm$. Inserting that into Eq.(3) we can see that in order to escape without interaction, J/Ψ momentum should satisfy the condition

$$p_T \geq l \frac{m_{J/\Psi}}{\tau_{J/\Psi}} \quad (8)$$

For $\tau_{J/\Psi} \approx 1fm/c$ this leads to $p_T \geq 9GeV/c$ in clear contradiction with the data. Including χ with its $\tau_\chi \approx 2fm/c$ improves somewhat the situation but still does not bring $S(p_T)$ to agreement with the data.

The solution to this problem lies in the concept and magnitude of the formation time. For the disintegration of J/Ψ in a collisions with a pion it is not important what is the distance between c and $\bar{c}$, what is important is whether c and $\bar{c}$ have already had enough time to build up their clouds of soft partons. In QGP models it is irrelevant whether c and $\bar{c}$ are current or constituent quarks, but for hadron gas models this is the most relevant aspect. What matters in hadron gas models is thus only the "dressing-up time". Do we have any experimental information about this "dressing-up time"? In principle it should be similar for all hadrons and should manifest itself clearly in cascading in nuclei. Recent

study by Ranft [29] of cascading in the target fragmentation region of $^{16}\text{O} - \text{Pb}$ collisions has shown that the most likely value of the hadron formation time is $2 \pm 1 \text{ fm}/c$. Inserting values of $\tau_{J/\Psi}$ of $2 - 3 \text{ fm}/c$ into Eq.(8) one can see that $S(p_T)$ could reach the value close to 1 already at $p_T \sim 3 - 4.5 \text{ GeV}/c$. The qualitative argument is supported also by results in Fig.1, obtained in the hadron gas model to be described in more detail in Sect. 3. In order to make the figure transparent we have normalized $S(p_T = 0)$ to the same value for all dressing-up times. This requires using different values of the product $\rho\sigma$ for different dressing up times.

The experimental data on $S(p_T)$ fix in this way the value of the dressing up time. In the following we shall use $\tau_{J/\Psi} \approx \tau_\chi \approx 2 \text{ fm}/c$. The product $\rho\sigma$ entering Eq.(5) is also more or less fixed by the overall normalization of the suppression at $p_T \approx 0$ and some other relatively natural assumptions. On this basis we can make predictions of other features of J/Ψ suppression in heavy ion collisions. Disagreement of data with qualitative trends of these predictions would exclude hadron gas models as possible candidates for explanation of J/Ψ suppression in heavy ion collisions lending an indirect support to other candidates.

3. Hadron gas model of J/Ψ suppression in heavy ion collisions

The mean free path of a hadron in a hadron gas with density ρ is $l = 1/\rho\sigma$. Taking $\sigma \approx 20 \text{ mb} \approx 2 \text{ fm}^2$ and $\rho \approx 3 \text{ fm}^{-3}$ we obtain $l \approx 0.15 \text{ fm}$. Since this is much smaller than a typical dimension of the system, $R_0 \approx 3 \text{ fm}$, the hadron gas becomes soon thermalized and expands hydrodynamically. This expansion of the hadron gas has been studied in detail in Refs. [30, 31]. A simplified description of the expansion of the hadron gas consists essentially in a purely longitudinal expansion and in the shrinking due to the rarefaction wave proceeding from outer regions to the centre. The longitudinal expansion is isentropic - the entropy per co-moving volume or per specified rapidity interval remains constant. The entropy is proportional to the density of hadrons and because of that

$$\rho(\vec{r}, t)t = \rho(\vec{r}, t_0)t_0 \quad (9)$$

Since the entropy is also proportional to the third power of the temperature the longitudinal expansion leads to a cooling of the gas according to

$$T^3(\vec{r}, t)t = T^3(\vec{r}, t_0)t_0 \quad (10)$$

In Eqs.(9) and (10) $\vec{r}$ denotes the transverse coordinate and in both cases we have in mind a central collision of two nuclei.

The rarefaction wave proceeds to the centre with a radial velocity equal to the velocity of sound c_s . For an ideal gas $c_s = \sqrt{1/3}$, for a gas consisting of hadron resonances, the velocity of sound is somewhat lower [32], $c_s^2 \approx 0.2$. The rarefaction wave reaches the centre, for the case of $^{16}\text{O} - ^{238}\text{U}$ collision, in time ^{*)} $t_k \approx R_0/c_s \approx 5\text{fm}/c$. The time dependence of the density in $x_{||} = 0$ is then given roughly as

$$\rho(\vec{r}, t) = \rho(\vec{r}, t_0) \frac{t_0}{t} \theta(t_k - t) \theta(R_0 - c_s(t - t_0) - r) \quad (11)$$

Since the temperature is decreasing rather slowly - as a cubic root of time we have simplified the calculations by making the temperature constant. The joint probability density of pions in the ordinary and momentum space has been assumed to factorize

$$\rho(\vec{r}, \vec{p}, t) = \rho(\vec{r}, t) P(\vec{p})$$

with $\rho(\vec{r}, t)$ being given by Eq.(11) and $P(\vec{p})$ parametrised as

$$P(\vec{p}) \sim \exp[-(\vec{p}^2 + m_\pi^2)^{1/2}/T_0]$$

with $T_0 = 200\text{MeV}$. The phenomenologically introduced temperature T_0 corresponds to transverse momentum distributions in multiparticle production and may be thus considered as a sort of average temperature of the expanding hadron gas.

The composition of hadron gas formed in heavy ion collisions is not known. In principle one could include [32] all particles in pseudoscalar and vector meson nonets and of baryon and antibaryon octets and decaplets. But since the disintegration cross - sections are not known this would be an unnecessary complication. We have therefore taken the hadron gas as a mixture of pions, kaons and ρ -mesons. Mean hadron densities in the central $^{16}\text{O} - ^{238}\text{U}$ collision were taken at $t_0 = 1\text{fm}/c$ as ^{**)}

$$\langle \rho_\pi(t_0) \rangle = 2.5\text{fm}^{-3}, \quad \langle \rho_K(t_0) \rangle = \langle \rho_\rho(t_0) \rangle = 1.25\text{fm}^{-3} \quad (12)$$

According to Eq.(1) this mean density of pions corresponds to the following pion density as a function of the transverse coordinate r

$$\rho_\pi(r, t_0) = 2.5\text{fm}^{-3} \cdot \frac{3}{2} \sqrt{1 - r^2/R_0^2} \quad (13)$$

^{*)} In our preceding paper [11] we have used an approximation of a static hadron gas, existing for about $3\text{fm}/c$. This can be viewed as a rough approximation to the expanding hadron gas. The product $\rho_0(t_k - t_0)$ is an approximation to the average density $\int \rho(t) dt$ with $\rho(t) \approx \rho_0 t_0/t$, the integral being taken from t_0 to t_k .

^{**)} Ratios of π , ρ and K as given by Eq.(12) correspond roughly to ratios of directly produced particles in multiparticle production.

Neither Eq.(13) nor Eq.(1) take into account the r -dependence of the number of nucleons within the U-nucleus. Taking this into account we obtain the expression

$$\rho_{\pi}(r, t_0) = 2.5 fm^{-3} \cdot \frac{3}{2} \frac{1}{2R_O} \frac{1}{2R_U} 2\sqrt{R_O^2 - r^2} 2\sqrt{R_U^2 - r^2} \quad (14)$$

The factors $t_O(r) \equiv 2\sqrt{R_O^2 - r^2}$ and $t_U(r) \equiv 2\sqrt{R_U^2 - r^2}$ represent longitudinal dimensions of both O- and U- nuclei at a given value of r for the central O-U collision. The advantage of Eq.(14) lies in the fact that its generalization to non-central collisions and to collisions of nuclei with other nucleon numbers is straightforward. Assuming that the pion density is proportional to the number of nucleon - nucleon collisions we obtain for a non - central collision of nuclei A and B at impact parameter b

$$\rho_{\pi}(r, t_0) = 2.5 fm^{-3} \cdot \frac{3}{2} \frac{1}{2R_A} \frac{1}{2R_B} 2\sqrt{R_A^2 - r^2} 2\sqrt{R_B^2 - (\vec{r} - \vec{b})^2} \quad (15)$$

The geometry of the A+B collision is shown in Fig. 2. Taking into account the full geometry of the collision introduces into ρ_{π} also the dependence on the longitudinal coordinate. We shall not discuss this point here in more detail.

Densities of ρ and K are given by Eq. (15) with $2.5 fm^{-3}$ replaced by $1.25 fm^{-3}$. The time evolution of the pion density due to the longitudinal expansion is obtained simply by multiplying the r.h.s. of Eq.(15) by (t_0/t) and the shrinking due to the rarefaction wave is included by a multiplicative factor describing the shrinking of radii of both nuclei with the radial velocity equal to c_s . The explicit form of this factor is

$$\theta(R_A - c_s(t - t_0) - r) \theta(R_B - c_s(t - t_0) - |\vec{b} - \vec{r}|)$$

For a noncentral collision the rarefaction wave reaches the centre of the "lens" in Fig. 2 in time $t = \Delta(b)/2c_s$. In fact in our calculations we have stopped the evolution of the hadron gas at a time when the transverse dimension of the "lens", diminished by shrinking, reached the value of about $1 fm$. This usually occurs when $t \approx 4 fm/c$ and the density of the hadron gas is about $1 fm^{-3}$. The mean free path of a hadron $l = 1/\rho\sigma$ is about $0.5 fm$ what is comparable with a transverse dimension of the hadron gas and in this situation the hydrodynamical expansion is naturally finished since the hadrons can easily leave the system. The condition for stopping the hydrodynamical expansion is not unambiguously defined and this brings in some indeterminacy into the results, in particularly at low values of E_T (large impact parameters). This indeterminacy will be mentioned in Sect. 5. Results presented in Sect. 4 correspond to stopping time determined from the condition $\Delta(b) - 2c_s(t - t_0) = 1 fm$.

Values of disintegration cross sections assumed and of corresponding thresholds s_0 are summarized in Table 1. The reaction $\pi + J/\Psi \rightarrow D + \bar{D} + \pi$ has a rather large threshold what has serious implications for the rate of J/Ψ suppression (see the discussion concerning the factor $P(s > s_0)$ following Eq.(6)). When considering J/Ψ suppression in collisions with K's and ρ 's the problem of thresholds practically disappears [12]. The reaction $\pi + \chi \rightarrow D + \bar{D}$ has a threshold at $E_0 = 0.235 \text{ GeV}$ and this corresponds to $P(s > s_0) = 0.94$. Reaction $\rho(775) + J/\Psi(3097) \rightarrow D(1865) + \bar{D}(1865)$ is exothermic and $K(775) + J/\Psi(3097) \rightarrow \bar{D}(1865) + D_s(1970)$ has again very low threshold. As pointed out by Gavin et. al. [12] even for $\pi + J/\Psi$ collisions there exists a disintegration reaction, namely $\pi + J/\Psi \rightarrow \eta_c + \pi$, which is exothermic. The problem is that the relative ratios of $\pi + \chi \rightarrow D + \bar{D}$, $\pi + J/\Psi \rightarrow D + \bar{D} + \pi$ and $\pi + J/\Psi \rightarrow \eta_c + \pi$ are unknown. In this situation we have taken the threshold for $\pi + J/\Psi$ disintegration as corresponding to the final state $D + \bar{D} + \pi$.

The density of J/Ψ produced in a heavy ion collision is assumed to be proportional to the number of nucleon - nucleon collisions in a given position in the transverse plane. In general case it is therefore proportional to the r.h.s. of Eq.(15).

The J/Ψ survival probability is calculated by the formula

$$S(p_T) = \langle \exp(-T_\pi - T_K - T_\rho) \rangle \quad (16a)$$

where T_π , T_K and T_ρ are suppression factors due to the disintegration of J/Ψ by π , K and ρ . The averaging in Eq.(16) goes over the positions in which J/Ψ has been created and over directions of its momentum. In calculating the experimentally observed suppression we have used Eq.(2) and weighted J/Ψ and χ contributions to the total suppression according to the assumption that about 40 % of J/Ψ observed comes from radiative χ decays. The spirit of the calculation corresponds to the simple argument in Eq.(5) but we have been using a bit more complicated and more accurate expression for computing the T. The full expression looks as follows

$$T(\vec{r}_0, \vec{p}_1) = \frac{1}{2E_1} \int_{t_1}^{t_2} dt \int \frac{\rho(\vec{r}, \vec{p}_2, t)}{E_2} [(s - s_a)(s - s_b)]^{1/2} \sigma^*(s) d^3 p_2 \quad (16b)$$

Here $\vec{p}_1$ is the transverse momentum of J/Ψ (or χ), $\vec{r} = \vec{r}_0 + (\vec{p}_1/E_1)t$, $\vec{p}_2$ is the transverse momentum of pion (or kaon or ρ - meson), $\sigma^*(s)$ is the disintegration cross section for collinear meson + J/Ψ collisions and other kinematical quantities take care of the fact that the meson + J/Ψ collision is in general a non-collinear one. Eq.(16b) is derived in detail in the Appendix.

Having described the model we shall now present the results. In Fig. 3 we show the p_T - dependence of the J/Ψ survival probability at about the maximal E_T corresponding

to central collisions with impact parameter $b \approx 0$. The curves correspond to formation time $\tau_{J/\Psi} = \tau_\chi = 2fm/c$. Note that by increasing formation times, one can make curves in Fig. 3 to be somewhat steeper (see Fig.1). With increasing B the curves become more flat. Defining the probability for J/Ψ disintegration $D(p_T) \equiv 1 - S(p_T)$ we find that

$$\frac{D(p_T = 3GeV)}{D(p_T = 0)}|_{O+U} \approx 0.25$$

whereas

$$\frac{D(p_T = 3GeV)}{D(p_T = 0)}|_{Pb+U} \approx 0.87$$

The maximal transverse energy E_{TMAX} corresponds to central collisions with $b = 0$. This quantity is experimentally more or less well defined. Transverse energy distribution in the central rapidity region $2 < y < 4$ as observed by the NA - 35 collaboration [17] consists of a long plateau followed by a steep decrease. The analysis of the NA - 35 data [17, 18] shows that the shape of $d\sigma/dE_T$ can be well described on geometrical grounds taking at a given value of the impact parameter the appropriate convolution of the transverse energy distribution in a $p - U$ collision. The tail at the highest transverse energies is given by the 16-fold convolution. The transverse energy at which the steep downfall begins is denoted as E_{TMAX} . It corresponds to a maximum on the curve for the 16-fold convolution of $p - U$ transverse energy distributions and thus to a central collision in which no fluctuations in the transverse energy occur.

In Fig. 4 we present $S(p_T)$ for the same heavy ion collisions at $E_T \doteq 0.5E_{TMAX}$. All the curves are somewhat shifted upwards but their slopes are not substantially changed. The curves in Fig. 4 were obtained in the following way. We have assumed that in a very rough approximation the transverse energy E_T produced in an $A+B$ heavy ion collision at a given impact parameter b is proportional to the number of nucleon - nucleon collisions $N_{AB}(b)$ in this geometry. $N_{AB}(b)$ is calculated in the standard way as (see Fig. 2)

$$N_{AB}(b) = \frac{1}{\sigma_{nn}} \int d^2r [\sigma_{nn} \rho^A t_A(\vec{r})] [\sigma_{nn} \rho^B t_B(\vec{b} - \vec{r})] \quad (17)$$

where σ_{nn} is the inelastic nucleon - nucleon cross section, ρ^A, ρ^B are nucleon densities in nuclei A and B, assumed equal and constant, and $t_A(r) = 2\sqrt{R_A^2 - r^2}$, $t_B(\vec{b} - \vec{r}) = 2\sqrt{R_B^2 - (\vec{b} - \vec{r})^2}$ are the longitudinal dimensions of both nuclei at given values of $\vec{b}$ and $\vec{r}$. In this way the transverse energy $E_T \doteq 0.5E_{TMAX}$ corresponds to such a value of b at which $N_{AB}(b) = 0.5N_{AB}(0)$. Having determined this value of b we have generated the position at which J/Ψ has been created within the "lense" in Fig. 2. The probability density $P(\vec{r})$ for creation of J/Ψ has been assumed to be proportional to the number of

nucleon - nucleon collisions at this value or $\vec{r}$, that means to $\rho^A t_A(\vec{r}) \rho^B t_B(\vec{b} - \vec{r})$. The density of the hadron gas within the lense is also proportional to the number of nucleon - nucleon collisions, in the same way as for the central collisions in Eq. (14). The gas then expands in a longitudinal direction and radii of both nuclei shrink with the velocity of sound to take care of the rarefaction wave proceeding from outside to inside. The hadron gas ceases to exist when - due to the rarefaction wave - the transverse dimension of the region occupied by the hadron gas becomes less than $1 fm$. In the calculation of J/Ψ suppression we have integrated (and averaged) over positions in which J/Ψ has been created and over directions of J/Ψ 's transverse momentum. The J/Ψ suppression is calculated by Eqs. (5, 16a, 16b).

In order to see whether the shrinking due to the rarefaction wave influences the shape of $S(p_T)$ substantially we have calculated $S(p_T)$ once in a way as described above and a second time without shrinking but with longitudinal expansion included and a time of existence of the hadron gas being fixed at the value obtained in the former calculation. Results for $^{16}O - ^{238}U$ and $^{207}Pb - ^{238}U$ are presented in Fig. 5. There is no substantial change in the shapes of $S(p_T)$. There is a change in the magnitude of the suppression but this is also natural since the J/Ψ (or χ) has to cross more of the hadron gas when there is no shrinking. This change in the overall magnitude can be obtained also by a slight change in the density of hadron gas or in the disintegration cross section, so that the data on $S(p_T)$ will hardly bring an information about the details of the hydrodynamical evolution of the hadron gas. The dependence of J/Ψ suppression on the total transverse energy E_T is shown in Figs. 6, 7, 8 for $p_T(J/\Psi) = 0, 1 GeV/c$ and $2 GeV/c$ respectively. We have chosen to plot this dependence not directly as a function of E_T but as a function of the ratio

$$x = \frac{E_T}{E_{TMAX}} \quad (18)$$

Assuming that E_T is proportional to the number of nucleon - nucleon collisions at a given value of the impact parameter b , we have selected for any $A+B$ interaction a set of values x_i and for any x_i found a corresponding value of b_i such that

$$x_i = \frac{N_{AB}(b_i)}{N_{AB}(0)} \doteq \frac{E_{T,i}}{E_{TMAX}} \quad (19)$$

At this value of b_i we have calculated the J/Ψ suppression in the same way as described above. For conversion between b and x see Table 2.

In all of the Figs. 6, 7 and 8 we have used a formation (dressing up) times for both J/Ψ and χ equal to $2 fm/c$ and the velocity of sound in the hadron gas $c_s^2 = 0.2$. The curves are rather flat.

4. An attempt at a qualitative understanding of the results obtained

The p_T - dependence of J/Ψ suppression is essentially given by the formation (dressing up) time of J/Ψ and χ . A good qualitative understanding of this feature is provided by the argument in Eq. (3). In order to understand at least in a very rough and approximate way the dependence of suppression on atomic numbers and on $x = E_T/E_{TMAX}$ we have made a very simple model based essentially on Eq. (5). We consider only J/Ψ with $p_T = 0$. The parameter T in the exponent of Eq. (5) is proportional to the density of the hadron gas and the time $\Delta t(b)$ during which J/Ψ is immersed in the hadron gas. For an $A - B$ collision at a given value of the impact parameter b (and therefore at a given value of x) we simply put

$$S_{AB}(b) = \exp[-T_{AB}(b)] \quad (20)$$

$$T_{AB}(b) = T_{OU}(b=0) \frac{\rho_{AB}^{MAX}(b)}{\rho_{OU}^{MAX}(b=0)} \frac{\Delta t_{AB}(b)}{\Delta t_{OU}(b=0)} \quad (21)$$

Here $T_{OU}(b=0)$ is a parameter determined from the data on J/Ψ suppression in $O - U$ collisions at $p_T \doteq 0$ and $E_T \doteq E_{TMAX}$. For instance, for $S_{OU}(p_T \doteq 0) \doteq 0.5$ we have $T_{OU}(b=0) = \ln 2$. The ratio $\rho_{AB}^{MAX}(b)/\rho_{OU}^{MAX}(b=0)$ gives the ratio of maximal hadron gas densities for AB collision at a given value of b and for the OU collision at $b=0$. It is explicitly given as

$$\frac{\rho_{AB}^{MAX}(b)}{\rho_{OU}^{MAX}(b=0)} = \left(\frac{B}{16}\right)^{1/3} \left(\frac{A}{238}\right)^{1/3} R_{AB}(b) \quad (22)$$

where

$$R_{AB}(b) = \left(1 - \frac{b^2}{R_A^2}\right)^{1/2} \text{ for } 0 \leq b < R_A - R_B$$

$$R_{AB} = \left[1 - \left(\frac{R_A - R_B + b}{2R_A}\right)^2\right]^{1/2} \left[1 - \left(\frac{R_B + b - R_A}{2R_B}\right)^2\right]^{1/2} \quad (23)$$

for $b \geq R_A - R_B$.

The factor $\left(\frac{B}{16}\right)^{1/3} \left(\frac{A}{238}\right)^{1/3}$ takes into account different longitudinal dimensions in $^{16}O - ^{238}U$ and $A+B$ collisions and the factor $R_{AB}(b)$ describes the decrease of the product of longitudinal dimensions with increasing value of b . For $b < R_A - R_B$ the smaller nucleus B is still within the larger one, A and we estimate the lengths in the centre of B . For $b > R_A - R_B$ nucleus B is partly outside of A and we estimate $R_{AB}(b)$ from values of longitudinal lengths of both nuclei in the centre of the "lense" in Fig. 2.

The remaining factor in Eq. (21) gives the ratio of time intervals during which a J/Ψ

with $p_T = 0$ is within hadron gas. It is explicitly given as

$$\frac{\Delta t_{AB}(b)}{\Delta t_{OU}(b=0)} = \frac{t_0 + \frac{\Delta(b)/2}{c_s} - t_f}{t_0 + \frac{R_0}{c_s} - t_f} \quad (24)$$

where t_0 is the hadronization time, $t_F = \tau_{J/\Psi} = \tau_\chi$ is the dressing - up time of J/Ψ and χ , R_0/c_s is the time in which the rarefaction wave reaches the centre of the oxygen nucleus ^{*}). Finally $\Delta(b)/2c_s$ is the time the rarefaction wave needs to reach the centre of the lense in Fig. 2. For $b \leq R_A - R_B$ $\Delta(b) = 2R_B$ and for $R_A - R_B < b < R_A + R_B$ we have $\Delta(b) = R_A + R_B - b$. The present simplified toy model cannot be expected to give accurate results since the approximations involved in arriving at Eq. (20) and (21) are rather severe. Still, in order to see what one can obtain in such a rough approximation we compare in Fig. 9 results of this toy model with more accurate calculations of the preceding section. Toy model results were obtained by Eqs. (20) and (21) and the conversion from impact parameter value b to $x = E_T/E_{TMAX}$ were performed by using Table 2.

Differences between results of both calculations are not shocking but one would still like to know the qualitative reason for very flat $S(x)$ shapes for $x \geq 0.3$ in more accurate calculations in Sect. 3.

5. Comparison with the preliminary NA-38 data, comments and conclusions

Although the NA - 38 data [22, 23, 33] in particular those on the E_T - dependence of J/Ψ suppression are still preliminary and the role played by the nuclear effects is still not completely clarified it is difficult to resist the temptation to compare our results with the data and speculate about what the data are telling.

It seems to us that one can look at the data and their implications at least from three different points of view:

A) assuming that the hadron gas contribution is the dominant one

^{*}) In fact one should rather replace $\Delta(b)$ in Eq. (24) by $\Delta(b) - 1 fm$ and R_0 by $R_0 - 0.5 fm$ to simulate our conditions for stopping the expansion. The replacement has only a small effect on results.

B) assuming that the main role is played by the QGP

C) avoiding any conclusions before the contribution of nuclear effects is understood.

We shall now discuss these three possible attitudes in more detail and point out possible weak points and/or expected complications.

A. Hadron Gas Model interpretation of the data

Hadron gas models of J/Ψ suppression in heavy ion collisions contain three basic parameters

- i) the product $\rho\sigma P(s > s_0)$ (see Eq. (5)) at the hadronization time, or its generalizations and refinements, corresponding to the presence of more sorts of hadrons in the hadron gas.
- ii) the radial velocity of the rarefaction wave which determines the effective life - time of the hadron gas. In a simplified version of Eq. (5) this parameter appears in the form of Δt .
- iii) J/Ψ and χ dressing - up times $\tau_{J/\Psi} \approx \tau_\chi$.

All three parameters can be fixed by the $^{16}O - ^{238}U$ data under the assumption that the whole suppression observed is due to disintegration of J/Ψ and χ in collisions with hadrons in the hadron gas.

The product $\rho\sigma P(s > s_0)\Delta t$ with $\Delta t \sim R_0/c_s$ is essentially determined by $S(p_T \approx 0)$, the dressing up time by the $S(p_T)$ dependence and the radial velocity of the rarefaction wave enters in substantial way $S(x)$ dependence of J/Ψ suppression via the time of existence of hadron gas for different values of the impact parameter b .

On the other hand there exists independent information, determining - with some inaccuracies - these parameters. The density of the hadron gas can be estimated from the total transverse energy E_T as in Eqs. (1) and (6). A weak point here is, of course, that some value for the formation time τ of the hadron gas has to be assumed.

The hydrodynamics of the expanding hadron gas stands on its own reasonably firm basis [30, 31] and the magnitude of the dressing up time of J/Ψ and χ should be similar to hadron formation times estimated from intranuclear cascading [29]. In the hadron gas model the data on $S(p_T \approx 0)$ and on the shape of $S(p_T)$ fix parameters to such an extent that E_T - dependence of J/Ψ suppression becomes a crucial test of the model.

An open question in analysing these data is whether the lowest E_T bin is already suppressed or not. We take here the attitude that **no-suppression should correspond to p-U data**. The relative magnitude of the suppression in the lowest E_T bin can be obtained from p-U data on J/Ψ over the continuum ratio presented by Baglin [23].

In Fig. 10 we compare our results with the preliminary NA - 38 data [23] on the E_T - dependence of J/Ψ suppression in $^{16}O - ^{238}U$ collisions. It should be stressed that this comparison is based on the assumption that there is suppression already in the lowest E_T bin. As can be seen from Fig. 10 the data, interpreted in this way, do not contradict hadron gas model calculations. Results similar to ours have been obtained also by Blaizot and Ollitrault [20].

With this interpretation of data the QGP formation could manifest itself as a kink [34] in the E_T - dependence of the J/Ψ suppression. A speculative possibility of such a kink is shown in Fig. 11. The data do not indicate a kink, but present error bars leave ample space for such a structure.

It is sometimes claimed that the hadron gas model is in a disagreement with data on the p_T - dependence of J/Ψ suppression. The contradiction appears only in models with a short [11, 12, 13] or vanishing [20] J/Ψ formation time. As explained in Sect. 2 there are no reasons to believe that J/Ψ dressing up time is smaller than $1 fm/c$ or vanishing. Dressing up times of $2-3 fm/c$ make the agreement between the hadron gas model and the data quite acceptable. The situation is illustrated in Fig. 12 where $\tau_{J/\Psi} = \tau_\chi = 2 fm/c$ is used. Increasing $\tau_{J/\Psi}$ and τ_χ to $3 fm/c$ would make the agreement still better.

It is true that analyses [27, 28] leading to estimates of $\sigma(J/\Psi + N)$ assumed vanishing formation time of J/Ψ [20]. We believe that these estimates should be re-analysed ^{*)}. Such a re-analysis would certainly increase the estimates of $\sigma(J/\Psi + N)$ and as a consequence also the estimates of $\sigma(\pi + J/\Psi)$, etc. Such an increase would permit to describe the observed pattern of J/Ψ suppression with lower values of the hadron gas density.

The model of J/Ψ suppression by the hadron gas presented here could be improved in more respects. The relation between $x = E_T/E_{TMAX}$ is based on a very simplified assumption of proportionality between the number of nucleon - nucleon collisions and E_T . The available models of ion - ion collisions [36] permit to make the x - b conversion in a more accurate and realistic way. One should also include explicitly the time - dependence of temperature into the calculations. We do not expect, however, that these improvements would change qualitative features of the results obtained.

In heavy ion collisions there is certainly some absorption by the hadron gas present and this can hardly be ruled out by the experiment. But a "radical" hadron gas model assuming that this mechanism gives a dominant contribution to J/Ψ suppression can be ruled out by the experiment at least on two counts:

- i) by showing that the J/Ψ "dressing-up" time is significantly shorter than $2 fm/c$. This

^{*)} This point has been recently made also by Brodsky and Mueller [35]

would lead to clear disagreement with $S(p_T)$.

- ii) by demonstrating that the absorption by the hadron gas model is not responsible for the difference between J/Ψ suppression in p-U and O-U collisions in the lowest E_T bin. This would lead to disagreement between the model and the E_T - dependence of J/Ψ suppression.

B. Quark Gluon Plasma Model

In this model one assumes [20, 21] that the ratio (Ψ/N_c in a given E_T - bin) over (Ψ/N_c in the lowest E_T - bin) corresponds to the J/Ψ suppression by QGP. The assumption is justified if nuclear effect simulating J/Ψ suppression enter in a factorizable, energy independent way, and if there is no significant contribution from the J/Ψ absorption by the hadron gas. Comparison with the data is shown in Fig. 13. The freedom left in the model and the error bars certainly permit to make the agreement acceptable.

The hadron gas model is in a somewhat worse agreement with the data interpreted in this way. The reason for disagreement is that the hadron gas model gives some suppression already in the lowest E_T - bin and this interpretation of data says by definition that there is no suppression.

Before accepting completely the QGP model calculations one would like to understand in more detail at least two issues:

- i) the magnitude of the J/Ψ and χ formation times is so far based on a rather qualitative semi - classical argument. The calculation of formation time should be performed in a quantum - mechanical framework [37, 38]. Such a calculation would at least clarify the present disagreement between standart values and larger estimates by Kapusta [25].
- ii) The original mechanism of J/Ψ suppression [2] by Debye screening is based on a tacit assumption that the mean free path of J/Ψ in a dense and hot matter changes abruptly at the phase transition from hadron gas to QGP or at some specific value of temperature. In view of the fact that dense hadron gas can lead to sizeable absorption of J/Ψ this assumption should be examined critically from a microscopic point of view. (for some steps in this direction see Ref. 39). If it would turn out that the mean free path would not vary abruptly at some critical temperature, the calculations should include also a contribution from absorption by matter below the critical point.

A typical feature of predictions of the QGP model is the presence of thresholds or kinks in the E_T - dependence and of a threshold in the dependence of J/Ψ suppression on the mass of the target. This is connected with low energy density of the matter created in the heavy ion collision for low values of E_T and low values of A - the atomic number of

the target. Absence of such thresholds and nonvanishing J/Ψ suppression at low values of E_T and A are inconsistent with QGP. In calculations by Blaizot and Ollitrault [20] no J/Ψ suppression by QGP is obtained already for the Cu - target. The hadron gas model prediction is compared with the data in Fig. 14. It is uncertain whether this threshold is present already at Cu or somewhat lower, but a systematic study of J/Ψ suppression on targets lighter than copper can decide the question of the dominant mechanism of J/Ψ suppression.

C. Nuclear effects

Nuclear effects are of two different types. The former is due to J/Ψ absorption by nucleons present in the original nuclei. The effect has been studied by Gerschel and Hüfner and by Capella et. al. [28] under the assumption that J/Ψ and χ dressing up times are negligible. The latter one is due to modifications of parton structure functions of nucleons by their binding in nuclei. The experimental [15, 16] and theoretical [14] studies indicate that these effects might be large. In the first approximation these effects should be E_T - independent as being given by the hard processes in the first stage of a collision. But one would feel safer if this E_T - independence would be verified in detail by experiment. The data by the NA - 38 collaboration [23] do show that $\Psi/\text{continuum}$ ratio in p-U interactions is practically independent of E_T . The highest E_T - bin for p-U interactions contains all events with $E_T(p-U) > 3.4\text{GeV}$ what corresponds roughly to $16 \times 3.4\text{GeV} = 54.4\text{GeV}$ of E_T in O-U interactions. More detailed information about $E_T(p-U) > 3.4\text{GeV}$ is wellcome and it would be fine if this E_T - independence in p-U collisions were checked also at various p_T 's.

Note also that suppression of other particles [40] may bring useful information on the presence of QGP or hadron gas in heavy ion collisions.

To conclude: both hadron gas and QGP model of J/Ψ suppression are able to reproduce roughly the available data. The crucial features of experimental data are thresholds in E_T - and A - dependence of J/Ψ suppression. Such thresholds would speak clearly in favour of the QGP model. In order to decide this question experimentally, more data with smaller errors on E_T - dependence of J/Ψ suppression are required as well as more detailed studies on targets with atomic numbers of Cu and below.

More detailed understanding of nuclear effects is needed before definitive conclusions can be made.

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Appendix

We shall derive here Eq.(16) which gives the survival probability of J/Ψ and after that, with some approximations, we shall obtain the simplified form of Eq. (5). Quantities related to J/Ψ (or χ) will have index 1, those related to hadrons, index 2.

Suppose a beam of J/Ψ 's with density ρ_1 and velocity $\vec{v}_1$ moves through a cylinder of cross section S and length dl containing hadron gas with density ρ_2 moving with velocity $\vec{v}_2$. The number of disintegrated J/Ψ 's within time interval dt is

$$dn = \rho_1 \rho_2 |\vec{v}_1 - \vec{v}_2| \sigma dt S dl$$

where σ is the disintegration cross - section. The probability dP of disintegrating a single J/Ψ is obtained by dividing dn by the number of incident J/Ψ 's:

$$dP = \frac{\rho_1 \rho_2 |\vec{v}_1 - \vec{v}_2| \sigma dt S dl}{v_1 \rho_1 S dt} = \frac{|\vec{v}_1 - \vec{v}_2| \rho_2 \sigma dt S dl}{v_1} \quad (A.1)$$

Collisions of J/Ψ with particles in the hadron gas are in general non - collinear. It is therefore useful to express σ in terms of the collinear cross section σ^* . Making use of Lorentz invariance of the number of disintegrations in an invariant volume $dV dt = dV^* dt^*$ we obtain

$$E_1 E_2 |\vec{v}_1 - \vec{v}_2| \sigma = E_1^* E_2^* |\vec{v}_1^* - \vec{v}_2^*| \sigma^* \quad (A.2)$$

where the asterisk denotes c.m.s. quantities. The r.h.s. is equal to $(1/2)(s - s_a)^{1/2}(s - s_b)^{1/2}\sigma^*$ where $s_a = (m_1 + m_2)^{1/2}$ and $s_b = (m_1 - m_2)^{1/2}$. In this way we get

$$|\vec{v}_1 - \vec{v}_2| \sigma = \frac{(s - s_a)^{1/2}(s - s_b)^{1/2}}{2E_1 E_2} \sigma^* \quad (A.3)$$

Making use of $v_1 E_1 = p_1$ we find

$$d\tilde{P} = \frac{1}{p_1} \frac{\rho_2}{E_2} (s - s_a)^{1/2} (s - s_b)^{1/2} \sigma^* dl \quad (A.4)$$

Particles in the hadron gas are in general described by a joint probability density $\rho_2 = \rho_2(\vec{r}, \vec{p}_2, t)$ and Eq. (4) is modified to

$$d\tilde{P} = \frac{dl}{p_1} \int \frac{\rho_2(\vec{r}, \vec{p}_2, t)}{E_2} (s - s_a)^{1/2} (s - s_b)^{1/2} \sigma^*(s) d^3 \vec{p}_2$$

Let $P(l)$ denote the probability that J/Ψ is disintegrated when travelling the path of length l in the hadron gas. Then

$$dP = [1 - P(l)] d\tilde{P}$$

and consequently

$$\int_0^{P(l)} \frac{dP}{1-P} = -\ln |1 - P(l)| = \int_0^l d\tilde{P}$$

After simple rearrangements we obtain

$$P(l) = 1 - \exp(-T) \quad (A.5)$$

where

$$T = \frac{1}{2p_1} \int_0^l dl \int \frac{\rho_2(\vec{r}, \vec{p}_2, t)}{E_2} (s - s_a)^{1/2} (s - s_b)^{1/2} \sigma^*(s) d^3\vec{p}_2 \quad (A.6a)$$

The survival probability $S \equiv 1 - P$ and we get the final expression

$$S = \exp(-T) \quad (A.6b)$$

In actual calculations the survival probability $S(p_T)$ or $S(x)$ is obtained from (A.6b) by integrating over positions in which J/Ψ has been created and over directions of J/Ψ transverse momentum.

We shall now derive from general expression Eq. (A.6a) the simplified version used in Eq. (5). Since we are interested in slow J/Ψ 's we write $dl = v_1 dt$, $v_1 = p_1/E_1$ and integrate over the time interval (t_1, t_2) during which J/Ψ is within the hadron gas. Denoting as $\vec{r}_0$ the place in which J/Ψ has been created we have

$$T(\vec{r}_0, \vec{p}_1) = \frac{1}{E_1} \int_{t_1}^{t_2} dt \int \frac{\rho_2(\vec{r}, \vec{p}_2, t)}{2E_2} (s - s_a)^{1/2} (s - s_b)^{1/2} \sigma^*(s) d^3\vec{p}_2 \quad (A.7)$$

Here, $\vec{r} = \vec{r}_0 + (\vec{p}_1/E_1)t$ is the J/Ψ position in time t , t_1 is the larger one of the J/Ψ formation time t_f and of the formation time t_i of the hadron gas, t_2 is the smaller of the time t_e when hadron gas ends its existence and the time t_{out} when J/Ψ leaves the region with hadron gas. Eq. (A.7) forms the basis of our calculations. For $\vec{p}_1 = 0$ we have $(s - s_a)(s - s_b) = 2m_1 p_2$ where $p_2 = |\vec{p}_2|$ and the expression in (A.7) simplifies to

$$T(\vec{r}_0, 0) = \int_{t_1}^{t_2} dt \int \rho_2(\vec{r}_0, \vec{p}_2, t) v_2 \sigma^*(s) d^3\vec{p}_2 \quad (A.8)$$

We assume that ρ_2 factorizes : $\rho_2(\vec{r}_0, \vec{p}_2, t) = \rho(\vec{r}_0) w(\vec{p}_2)$, where $w(\vec{p}_2)$ is the probability density in momentum space normalized to one. With isotropic $w(\vec{p}_2)$ we can write $w(\vec{p}_2) = 4\pi p_2^2 w(p_2)$, with $w(p_2)$ normalized as $\int w(p_2) dp_2 = 1$. We can now define the probability density in E_2 as $w(E_2) = (E_2/p_2) w(p_2)$. Assuming further that the density of the hadron gas is constant and the gas exists within the time interval $t_1 < t < t_e$ we obtain from Eq. (A.8)

$$T(\vec{r}_0, 0) = \rho(\vec{r}_0) (t_e - t_1) \int \frac{1}{4\pi E_2 p_2} w(E_2) v_2 \sigma^*(s) d^3\vec{p}_2$$

After simple manipulations

$$T(\vec{r}_0, 0) = \rho(\vec{r}_0)(t_e - t_1) \int_{m_2}^{\infty} w(E_2) v_2 \sigma^*(s) dE_2$$

Assuming that $\sigma^*(s)$ vanishes below the threshold at E_0 and is constant above it we get

$$T(\vec{r}_0, 0) = \rho(\vec{r}_0)(t_e - t_1) \sigma \int_{E_0}^{\infty} w(E_2) v_2 dE_2 \quad (A.9)$$

Defining $P(E_2 > E_0) = \int_{E_0}^{\infty} w(E_2) dE_2$ and $\langle v_2 \rangle = \int_{E_0}^{\infty} w(E_2) v_2 dE_2 / P(E_2 > E_0)$ we find the final result

$$T(\vec{r}_0, 0) = \rho(\vec{r}_0)(t_e - t_1) \sigma \langle v_2 \rangle P(E_2 > E_0) \quad (A.10)$$

used for estimating J/Ψ suppression in Eq. (5) and serving as a motivation for a toy model mentioned in Sect. 4.

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Table 1. Cross sections and disintegration thresholds

Collision	Disintegration cross section [mb]	Threshold s_0 [GeV ²]
$\pi + J/\Psi$	2.	15.054
$\pi + \chi$	4.67	13.988
$K + J/\Psi$	1.	15.840
$K + \chi$	2.4	16.128
$\rho + J/\Psi$	2.	15.031
$\rho + \chi$	4.67	18.404

The disintegration cross sections for $\pi + J/\Psi$ and $\rho + J/\Psi$ were put equal to each other, the K has been assumed to have a smaller cross section by a factor of two and the cross sections on χ and J/Ψ are related by the ratio $r_\chi^2/r_{J/\Psi}^2$ with $r_{J/\Psi} = 0.45 fm$ and $r_\chi = 0.70 fm$.

Densities at hadronization time are given in Eq. (12).

Table 2. Conversion from impact parameter b to $x = E_T/E_{TMAX}$ for selected ion - ion collisions.

x	$^{16}O - ^{238}U$ b [fm]	$^{32}S - ^{238}U$ b [fm]	$^{40}Ca - ^{238}U$ b [fm]	$^{207}Pb - ^{238}U$ b [fm]
1	0.0	0.0	0.0	0.0
0.9	3.06	2.93	2.89	2.04
0.8	4.18	3.974	3.88	3.14
0.7	4.89	4.67	4.6	4.10
0.6	5.45	5.28	5.23	5.01
0.5	5.97	5.87	5.84	5.93
0.4	6.48	6.47	6.47	6.86
0.3	7.02	7.11	7.15	7.88
0.2	7.62	7.83	7.91	9.02
0.1	8.39	8.74	8.88	10.51

The conversion is performed on the basis of Eqs. (17), (18) and (19) and under the assumption that nuclei are spheres with radii $R_A = 1.2A^{1/3}$ and constant density. Radii of nuclei in this Table are $R_U = 7.44 fm$, $R_{Ca} = 4.10 fm$, $R_S = 3.81$, $R_O = 3.01 fm$.

Figure Captions

- Fig.1 J/Ψ survival probability as a function of $p_T(J/\Psi)$ for a set of values of the “dressing up time” $\tau_{J/\Psi} = \tau_\chi$ calculated by the model of Sect. 3
- Fig.2 Geometry of A+B collision (as viewed from the beam axis): b - impact parameter, $\bar{r}$ - transverse coordinate calculated from the centre of the A nucleus
- Fig.3 J/Ψ survival probability at $E_T \doteq E_{TMAX}$ for $B-^{238}U$ collisions. The curves plotted correspond to $^{16}O-^{238}U$, $^{32}S-^{238}U$, $^{40}Ca-^{238}U$ and $^{207}Pb-^{238}U$ collisions.
- Fig.4 J/Ψ survival probability at $E_T = 0.5E_{TMAX}$ for the same heavy ion collisions as in Fig. 3
- Fig.5 $S(p_T)$ for shrinking hadron gas (full lines) and for the non-shrinking evolution (dashed lines) for $^{16}O-^{238}U$ and $^{207}Pb-^{238}U$ central collisions.
- Fig.6 Dependence of J/Ψ suppression on $x = E_T/E_{TMAX}$ for $p_T(J/\Psi) = 0$ for O-U, S-U, Ca-U and Pb-U collisions. Formation times for both J/Ψ and χ are $2fm/c$, $c_s^2 = 0.2$. Other parameters specified in Table 1 and Eq. (12)
- Fig.7 The same as in Fig. 6 for $p_T(J/\Psi) = 1GeV/c$
- Fig.8 The same as in Fig. 6 for $p_T(J/\Psi) = 2GeV/c$
- Fig.9 Comparison of results on the x - dependence of J/Ψ suppression for $p_T(J/\Psi) = 0$ obtained from the toy model of Sect. 4 (dashed lines) and from more complete calculations of Sect. 3 (full lines).
- Fig.10 Comparison of NA - 38 preliminary data obtained from Ref. 23 under the assumption of no J/Ψ suppression in pU collisions. Solid curve - our results with $\tau_{J/\Psi} = \tau_\chi = 2fm/c$, other parameters specified in Table 1 and Eq. (12) and $c_s^2 = 0.2$. Dashed curve - our calculations with all cross - sections in Table 1 multiplied by 1.36 to obtain the same J/Ψ suppression as the data at $x = 1$. Other parameters remain unchanged. E_{TMAX} fixed at 100 GeV. The J/Ψ suppression in the lowest bin obtained from p-U data on $(J/\Psi)/\text{continuum ratio}$ [23].
- Fig.11 An illustration of a speculation on possible manifestations of QGP by kink in the E_T - dependence of J/Ψ suppression.
- Fig.12 Comparison with data [23] on p_T - dependence of J/Ψ suppression with our calculations a,b) Dressing up times of J/Ψ and χ equal to $3fm/c$, c,d) Dressing up times equal to $2fm/c$. Cross sections and initial densities for c) specified in Table 1 and Eq. (12)

For other cases the products of cross sections and initial densities are somewhat higher to take into account increased suppression at $p_T = 0$ or longer dressing up time.

- Fig.13 Comparison of data with the QGP model, under the assumptions described in the text. Dash - dot calculations by Karsch and Petronzio [21], dot - by Blaizot and Ollitrault
- Fig.14 Comparison of our calculations with preliminary data [23] on the E_T dependence of the ratio of J/Ψ to continuum for O-Cu collisions. E_{TMAX} fixed at 70 GeV.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14