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PROTON-NEUTRON CORRELATIONS AND THE LONGITUDINAL NUCLEAR RESPONSE

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A B S T R A C T

We study the influence of the proton-neutron correlations, in particular of the tensor ones, on the longitudinal nuclear response. They generate a high-energy tail that we evaluate. We show that this tail produces an apparent quenching of the Coulomb sum rule. The amount of quenching found is significant, although insufficient to explain the totality of the missing strength.

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The nuclear response to a charge density fluctuation has been measured in ee' scattering with the separation of the longitudinal cross-section from the transverse one. This response which displays a strong collectivity at small momenta recovers a single particle character at large momenta, $q > 2 \text{ fm}^{-1}$. However the total strength is below the value given by the Coulomb sum rule¹. This has led to the speculation of a swelling effect of the nucleon inside the nucleus².

A more conventional interpretation invokes the effect of 2p2h correlations which depopulate the single particle orbitals. With the introduction of 2p2h excitations the region of response extends to higher energies. It is therefore of interest to explore the high energy tail of the response, which is the aim of the present work. Our approach deals with the deuteron-like correlations, i.e. between the proton and neutron pairs. We first consider the Coulomb sum rule $S(q)$, the integral over the energy of the longitudinal response $R_L(q, \omega)$

$$S(q) = \int d\omega R_L(q, \omega) \quad (1)$$

Here q and ω are the momentum and energy transferred to the nucleus in the laboratory system and $\Theta_L(q)$ is the nuclear charge operator defined as

$$\Theta_L(q) = \frac{1}{2} \sum_{i=1}^A [g_s(q^2) + g_v(q^2) \tau_i^3] e^{i\vec{q} \cdot \vec{r}_i} \quad (2)$$

where $g_s(q^2)$ and $g_v(q^2)$ are the isoscalar and isovector nucleon form factors and r_i are the nucleon coordinates with respect to the centre of mass of the nucleus. In a symmetrical nucleus ($N=Z$) it is possible to split $S(q)$ into an isoscalar and an isovector component:

$$\begin{aligned} S(q) &= S_s(q) + S_v(q) \\ &= \frac{1}{4} g_s^2(q^2) \langle 0 | \sum_{i,j} e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} | 0 \rangle + \frac{1}{4} g_v^2(q^2) \langle 0 | \sum_{i,j} e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} \tau_i^3 \tau_j^3 | 0 \rangle \end{aligned} \quad (3)$$

Studying the expansion in q of $S_S(q)$ and $S_V(q)$ one has, up to order q^2 ,

$$\begin{aligned} S_S(q) &= \frac{1}{4} g_s^2(q^2) [A^2 + \langle 0 | q^2 \sum_{i,j} z_i z_j - A q^2 \sum_i z_i^2 | 0 \rangle] \\ &= \frac{A^2}{4} g_s^2(q^2) [1 - \frac{q^2}{3} \langle r^2 \rangle] \end{aligned} \quad (4)$$

$$S_V(q) = \frac{1}{4} g_v^2(q^2) \langle 0 | \sum_{i,j} [z_i^3 z_j^3 + q^2 z_i z_j z_i^3 z_j^3 - q^2 z_i^2 z_i^3 z_j^3] | 0 \rangle \quad (5)$$

To this order $S_S(q)$ is the development of the elastic response which is

$$S_S(q) = \frac{A^2}{4} |F(q)|^2 \quad (6)$$

where $F(q)$ is the elastic form factor.

On the contrary $S_V(q)$ is purely inelastic. Since the first and third terms in equation (5) vanish for symmetric nuclei, $N=Z$, $S_V(q)$ is proportional to the bremsstrahlung -weighted sum rule measured in photoabsorption experiments:

$$S_V(q) = \frac{g_v^2(q^2)}{4} q^2 \langle 0 | \sum_{i,j} z_i z_j z_i^3 z_j^3 | 0 \rangle = \frac{g_v^2(q^2) q^2}{4 \pi^2 \alpha} \int d\omega \frac{\sigma_{\text{dip}}(\omega)}{\omega} \quad (7)$$

where $\alpha = e^2/\hbar c$ and σ_{dip} is the unretarded electric dipole part of the photoabsorption cross-section. In going from the second to the third step of equation (7) the recoil energy of the nucleus has been neglected. At small momenta the isovector strength is thus completely known from the absorption cross-section of real photons. This last cross-section involves the transverse part of the electromagnetic current while the longitudinal response involves the charge. But in the long wavelength limit there exists a relation between the longitudinal and transverse electric multipole matrix elements, which follows from Siegert's theorem

$$\lim_{q \rightarrow 0} \langle f \| T_J^{el}(q) \| i \rangle = \left(\frac{J+1}{J} \right)^{1/2} \frac{E_f - E_i}{q} \langle f \| M_J^{oul} \| i \rangle \quad (8)$$

where E_i and E_f are the eigenvalues of the intrinsic nuclear Hamiltonian corresponding to the initial and final state, respectively. Applied to the dipole case this leads to the following relation between the inelastic response and the photoabsorption cross-section

$$R_L(q, \omega) = \frac{g_v^2(q^2)}{4\pi^2\alpha} \frac{q^2}{(E_f - E_i)^2} \omega \sigma_{dip}(\omega) \quad (9)$$

Assuming $E_i - E_f = \omega$ and integrating both sides of eq. 9 over the energy leads to eq. 7. In the deuteron case, as one cannot ignore the recoil energy, the approximation $E_i - E_f = \omega$ does not apply; we have therefore utilized the exact expression (9) and not its simplified version.

In this work we are interested in the high energy tail of the response, beyond the quasi-elastic region where 2p2h excitations are dominant. Our aim is to apply the relation (9) in order to predict the tail of the longitudinal response, in an energy region which is above the giant dipole resonance and where the photoabsorption cross-section is dominated by a 2p2h excitation process, the quasi-deuteron one. The corresponding cross-section $\sigma_{QD}(\omega)$ has an electric dipole character. We tentatively write in the tail region (neglecting the nucleus centre-of-mass effects)

$$R_L(q, \omega) = \frac{g_v^2(q^2)}{4\pi^2\alpha} \frac{q^2}{\omega} \sigma_{QD}(\omega) \quad (10)$$

The quantity $\sigma_{QD}(\omega)$, which dominates the photo-absorption cross-section between the giant dipole resonance and the pion threshold, has been parametrized with the Levinger formula

$$\sigma_{QD}(\omega) = L \frac{ZN}{A} \sigma_d(\omega) \quad (11)$$

where the Levinger factor L depends only mildly on the particular nucleus with values ranging from 5 to 10.

In order to justify the use of relation (10) we have to discuss its range of validity. Since we are dealing with small subsystems of the nucleus (the deuterons) this range is determined by the deuteron size rather than by the nuclear size. In the following we make a detailed investigation of the applicability of the relation (9) for the deuteron itself. For this purpose we have compared the longitudinal response of the deuteron $R_L^d(q, \omega)$ to the value $\overline{R}_L^d(q, \omega)$ derived from the photoabsorption cross-section on the deuteron according to the relation (9) for two values of the energy and a broad range of momenta. The longitudinal response and the electric photoabsorption cross-section are taken from the non-relativistic calculation of ref 3 which includes one body and Siegert operators and also the parts of the pion exchange currents which go beyond the Siegert limit. For the NN interaction the Paris potential is used. We stress that in the energy domain that we will consider, which is well above the quasi-elastic deuteron peak, the deuteron response is very much influenced by the tensor correlations. Without the d wave component the longitudinal response of the deuteron is reduced by a factor which can be as large as two. It is also important to remark that at these energies σ_{dip} is dominated both in the deuteron and in heavier nuclei (if one believes in the quasi-deuteron model) by the pion exchange processes of fig. 1, where the photon is absorbed through the contact term or by a pion in flight. Thus in the case of nuclei heavier than the deuteron the relation (10) is useful to incorporate in R_L the effects arising from the charge exchange force. In the explicit evaluation of $R_L^d(q, \omega)$ these effects do not enter in the operator (which is a charge density operator and not a current one) as meson exchange corrections. Nevertheless they have to be present in a consistent description of the ground and excited states.

The results of this comparison are shown in Fig. 2 which displays the ratio r between R_L^d and the quantity $\overline{R}_L^d$ evaluated from eq.9 as a function of the momentum q for two values of E_{np} , the relative energy of the outgoing neutron-proton pair in the c.m. system. As expected the ratio r is close to unity for small momenta. However it is not exactly one but somewhat smaller. This is mainly due to the contribution in σ_{dip} of the exchange currents which break the Siegert theorem at finite momenta.

For each energy we display two curves, one in which we keep in R_L^d only the dipole contribution and one in which all multipoles of R_L^d

are kept. With the dipole contribution only r is almost constant. This demonstrates that the long wavelength approximation for R_L^d works surprisingly well also at higher momentum transfer. But even in the case where all multipoles are kept, r remains close to unity in a large region of momenta. However with increasing momentum the influence of the quasi-elastic peak (which occurs in deuteron at about $q = \sqrt{4m\omega}$, i.e approximately at 600 MeV/c for $E_{np} = 100$ MeV and at 900 MeV/c for $E_{np} = 200$ MeV) shows up in the increase of the ratio r . As many multipoles contribute to the response in the quasi-elastic region our simple evaluation from relation (9) breaks down.

We can conclude from the study of the deuteron case and from figure 1 that it is legitimate, for moderate values of the momentum $q \approx 200$ MeV/c, to apply the relation (10) to predict the tail of the longitudinal response which is of quasi-deuteronic nature. We can link it to an experimental quantity ,the quasi-deuteron part of the photoabsorption cross-section. We show in figure 3 this prediction for the value of the momentum, $q = 200$ MeV/c. Here we have used the experimental values of the photo-absorption cross-section of ref 4, assuming it to be dominated by its dipole component and neglecting retardation effects (the error introduced by this assumption is of the order of 50% in the deuteron but it could be smaller in nuclei).

The interest of a prediction of the high-energy tail mostly lies in the missing strength problem which becomes severe at large momenta. Then the energy in the tail is also large ($\omega > 200$ MeV). The photoabsorption cross-section at these energies is no longer dominated by the electric dipole part and the relation (10) loses its interest. In this situation and using the previous case as a guideline we have performed in the tail region a quasi-deuteronic model on the longitudinal response itself. We have taken the longitudinal response of the deuteron of ref 3 and scaled it through the Levinger formula to obtain the nuclear one

$$R_L(q, \omega) = L \frac{ZN}{A} R_L^d(q, \omega) \quad (12)$$

Notice that a somewhat different model has been proposed ⁶ by two of us (W.L. and G.O.), suited to describe also the peak region where

the one-body process is dominant while the present one aims only at the description of the quasi-deuteron region where exchange effects dominate the cross-section. Also Laget⁷ had used a quasi-deuteron model for the unseparated responses.

In order to test the procedure of eq. 12 and to determine the Levinger parameter L in this regime of momenta and energies we have applied it to the transverse response as well. This last response is known to be strongly influenced by the $2p2h$ excitations which fill in the dip between the quasi-elastic peak and the Δ excitation. We have taken the transverse response of the deuteron from the calculation of ref 3, in which the important meson exchange contributions arising from the graphs of figure 1 as well as those coming from the Δ isobar are incorporated. Scaling then the deuteron response in the region of the dip we obtain for ^{40}Ca at $q=410$ MeV/c the curve shown in figure 4 for the value of the Levinger parameter $L=11$. This value gives a good fit to the experimental data; it is slightly larger than the values currently used in the photoabsorption data.

Coming back to the longitudinal response we show in figures 5a and b the tail region in ^{40}Ca evaluated through the quasi-deuteron model (with $L=11$) at $q=450$ and 550 MeV/c. It is important at this stage to stress that this tail arises from the correlations between the proton-neutron pairs in an isospin singlet state and in particular the tensor ones which are particularly effective for these quantum numbers.

With the quasi-deuteron model we are in a situation to evaluate the total amount of strength which is contained in the high energy tail, beyond the maximum energy explored by the experiment (i.e. above $\omega = 265$ MeV at $q=450$ MeV/c and above $\omega = 310$ MeV at $q=550$ MeV/c). With $L=11$ we find about 10% of the Coulomb sum rule at 450 MeV/c and 11% at 550 MeV/c. These values are obviously proportional to the value of L .

The relevance of these numbers for the missing strength problem will now be discussed. First notice that these percentages represent a significant amount but are insufficient, however, to account for the missing strength. In that respect it is legitimate to enquire whether the strength appearing in the quasi-deuteron tail has really been removed from the low-energy sector, thus producing

an apparent quenching (even if insufficient) of the strength, which is not a priori obvious. To show that this is indeed the case we use the following argument.

The photon being coupled to the proton charge, the longitudinal response probes the proton-proton dynamical correlations. It can be the self-correlation, i.e. of a proton with itself, or the pair correlation. On the other hand the sum rule, integral over the energy of the response, probes this correlation at equal times. With this in mind the effect of the quasi-deuteron mechanism can be inferred from that of the deuteron. Here the longitudinal response measures the self-correlation of its unique proton, which reduces at equal times to a δ function in space. In momentum space, the Fourier transform of this function which gives the sum rule $S(q)$ is therefore unity, independent of the pn correlations i.e. on the deuteron wave function. On the contrary the detailed distribution of strength, i.e. the non-equal-time self-correlation of the proton, does depend on the wave function.

In nuclei the pp pair correlations enter in addition to the self correlation which is again a δ function in space. The Coulomb sum rule can be modified only through a modification of the p-p pair correlation function. But we stress that we are using a quasi-deuteron model in which the different pn pairs act incoherently. The quasi-deuteron mechanism therefore does not generate correlations between pp pairs which remain unaffected by the type of pn correlations which lead to the quasi-deuteron absorption. We can therefore conclude that the total strength remains the same and that the strength which appears in the quasi-deuteron tail has been removed from the low energy sector. The percentages given previously indeed have a connection to the missing strength problem as they represent the fraction of strength in the tail beyond the experimental limit of detection. At low momentum, in the hypothetical case where all the quasi-deuteron tail would escape detection, a situation not encountered in reality, the loss of strength interpreted in terms of a nucleon swelling would be equivalent to a radius increase

$$\Delta \langle r_p^2 \rangle = \frac{3}{2\pi^2 d} \int d\omega \frac{\sigma_{\text{QD}}(\omega)/A}{\omega} \quad (13)$$

an expression given by Ericson and Rosa-Clot ⁸ for the nucleon swelling of pionic origin.

In summary we have studied the high energy tail of the longitudinal nuclear response, in an energy region well above the quasi-elastic peak where the 2p-2h excitations are dominant. We focused our attention on processes where the virtual photon absorption excites a proton-neutron pair. At moderate momenta, $q < 200$ MeV/c, we have linked this tail to the quasi-deuteron component of the photoabsorption cross-section. At larger momenta we have performed directly a quasi-deuteron model on the longitudinal response itself. We have scaled the longitudinal response of the deuteron with a Levinger formula to obtain the nuclear one. The Levinger parameter has been determined by applying the same procedure to the tail region of the transverse response. Its value is compatible with those of photoabsorption.

From the evaluation of the quasi-deuteron tail it is possible to estimate the amount of strength contained in this tail. We have shown that this strength has to be removed from the quasi-elastic region, thus producing an apparent quenching of the Coulomb sum rule. The amount of quenching, of the order of 10%, is significant but insufficient to account for the totality of the missing strength.

In the framework of the influence of 2p-2h excitations there exist another possibility to reduce the strength, which has recently been suggested⁹. It is the influence of the Δ isobar, as the 2p 2h correlations with a Δ should be treated on the same footing as the ones with nucleons only. The present treatment offers a frame for the extension to the incorporation of these excitations.

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Figure Captions

Fig.1 Pion exchange graphs dominating the photo-absorption process

Fig.2 Ratio r between the longitudinal response of deuteron and the one calculated through equation (9) as a function of the momentum for the fixed values of the energy $E_{np} = 100$ MeV (fig 2a) and $E_{np} = 200$ MeV (fig.2b). The continuous line represents the case where only the dipole component is retained in R_L while in the dotted curve all multipoles are kept. The dashed parts refer to the time-like region.

Fig.3 The quasi-deuteron tail predicted from the relation (9). In this equation we have used for σ_{QD} the data of ref 4. The experimental points are from ref 5.

Fig.4 The transverse response of ^{40}Ca at $q=410$ MeV/c. The curve represents the tail calculated with a quasi-deuteronic expression similar to (12) with $L=11$. The experimental points are from ref 5.

Fig.5 The longitudinal tail of ^{40}Ca from equation (12) with a value $L=11$ for a momentum $q=450$ MeV/c (fig 5a) and $q=550$ MeV/c (fig 5b). The experimental points are from ref 5.

- Fig. 1 -

- Fig. 2a -

- Fig. 2b

- Fig. 3 -

- Fig. 4 -

- Fig. 5a

- Fig. 5b -