Nonperturbative unification and extended supersymmetry

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Abstract

We consider the nonperturbative unification scenario proposed by Maiani, Parisi and Petronzio in $N = 2$ supersymmetric extensions of the Standard Model. Low-energy gauge coupling constants are computed using two-loop renormalization group equations, with a mass spectrum deduced from simple assumptions on the soft terms necessary to break symmetries and to generate chiral fermions. Agreement with the experimental values is obtained when the nonperturbative unification scale is close to the Planck scale.

1. Introduction

Explaining the low-energy values of the coupling constants of strong and electroweak interactions remains a major challenge of unified theories. Actually, two alternatives have been proposed in the framework of gauge theories.

A well-known possibility is that all interactions, unified or not, remain perturbative up to the Planck scale M_P . In this case, the absolute values of the low-energy coupling constants $\alpha_i(M_W)$ cannot be predicted. Only relations among them can be obtained. Moreover, $\alpha_i(M_W)$ turn out to depend critically on $\alpha_i(\Lambda)$, where Λ is a very high energy scale ($\Lambda \gg M_W$), a feature which might seem unpleasant.

An alternative approach was proposed by Maiani, Parisi and Petronzio (MPP) [1]. The MPP scenario is based on the observation [2] that the running coupling constant of an asymptotically divergent interaction at low energy $\alpha(M_W)$ becomes more and more insensitive to its value $\alpha(\Lambda)$ as Λ gets larger and larger compared to M_W . Still, gravity is supposed to cure this ultraviolet divergent behaviour so that Λ is naturally expected to be close enough to M_P . Specifically, the main assumption of MPP is that strong and electroweak interactions are in the nonperturbative regime at a common scale Λ close to M_P . As a result, this scheme allows for an absolute prediction of $\alpha_i(M_W)$ by means of renormalization group techniques. All interactions are then strong and of comparable strengths at $\Lambda = O(M_P)$ and the disparity of the low-energy coupling constants is traced back to the different gauge groups and particle contents characterizing strong and electroweak interactions.

The MPP scenario suffers however from a serious drawback. With the three known fermion generations, $SU(3)_C$ and $SU(2)_L$ are far from asymptotically divergent. This scheme can only work if many additional states are introduced. Clearly, we need a motivation for these new states, and supersymmetry is a plausible possibility. Without supersymmetry, $SU(3)_C$ becomes asymptotically divergent with at least nine generations. $N = 1$ supersymmetry improves the situation, owing to the presence of scalar partners of quarks and leptons and spin 1/2 partners of Higgs scalars and gauge bosons, so that a minimum of five generations is required. Cabibbo and Farrar [3], and subsequently Maiani and Petronzio [4] have shown that the MPP scenario can be successfully implemented in the $N = 1$ supersymmetric extension of

the Standard Model with five generations.

Still, even $N = 1$ supersymmetry seems unsatisfactory in that it fails to provide a motivation for the existence of the full set of new states needed to make the MPP scheme theoretically consistent and phenomenologically viable. Remarkably, the observed three fermion generations are sufficient to make realistic models asymptotically divergent in the context of $N = 2$ supersymmetric gauge theories [5,6], since the existence of the extra new states now follows automatically from the $N = 2$ multiplet structure. As a bonus, $N = 2$ supersymmetry makes the renormalization group calculations more reliable since all contributions to the beta functions beyond one loop vanish in the energy range where $N = 2$ is effective [6].

Observe that one might also turn the question around and ask which kind of physics can naturally be done in the framework of realistic $N = 2$ supersymmetric models. As we said, these models are asymptotically divergent for the three observed quark and lepton generations, and they tend to enter the nonperturbative regime long before they get unified [7]. As a result, perturbative grand unification looks unnatural in the present context, whereas the MPP scenario appears as a natural possibility.

The aim of the present letter is to discuss the MPP scheme in the $N = 2$ supersymmetric version of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, as well as in its left-right symmetric extension $SU(3)_C \times SU(4) \times U(1)$. This last model has a particularly attractive feature: it possesses a natural mirror splitting mechanism, allowing to give heavy enough masses to mirror quarks and leptons [8].

2. Structure of $N = 2$ realistic models

We begin with a short description of the $N = 2$ supersymmetric version of the Standard Model, with gauge group $SU(3)_C \times G_W$. It is convenient not to specify the electroweak group $G_W \supset SU(2)_L \times U(1)_Y$ at this stage. We restrict ourselves to the case of three sequential $N = 2$ families of quarks and leptons.

The particle spectrum is dictated by the $N = 2$ supermultiplet structure [5], which consists of two basic building blocks: the $N = 2$ vector multiplet and the hypermultiplet. The former transforms according to the adjoint representation of

$SU(3)_C \times G_W$ and contains one $N = 1$ vector multiplet $V_a \equiv (v_a^\mu, \lambda_a)$ and one $N = 1$ chiral multiplet $\Phi_a \equiv (\phi_a, \psi_a)$. The latter contains two $N = 1$ chiral multiplets $X \equiv (x, \psi_x)$ and $Y \equiv (y, \psi_y)$, belonging to conjugate representations (R for X and $\bar{R}$ for Y) of the group $SU(3)_C \times G_W$. Since quarks and leptons have only left-handed weak interactions, we are forced to assign all of them to X . Then Y will contain states with $SU(3)_C \times G_W$ conjugate quantum numbers, which will be mirror quarks and leptons with right-handed weak interactions. We see that to each Weyl quark and lepton corresponds one hypermultiplet. This will also be the case for the Higgs fields necessary to break $SU(2)_L \times U(1)_Y$ and to give masses to quarks, leptons and their mirror states. Notice that we need two Higgs hypermultiplets. In terms of $N = 1$ multiplets, they are denoted by H and G , with mirror states H' and G' , respectively. Clearly, the matter fermion representation is $R + \bar{R}$ and then supersymmetric invariant mass terms $X_i Y_i|_{\theta\theta}$ are allowed by gauge invariance. These mass terms would be phenomenologically disastrous, and their coefficients have therefore to be set to zero. This procedure is technically natural, thanks to the $N = 2$ nonrenormalization theorem [9]. In the absence of these invariant mass terms, both ordinary quarks and leptons and their mirror states are chirally protected by $SU(2)_L \times U(1)_Y \subset G_w$. This fact implies that mirror fermions are expected to have masses of order $G_F^{-1/2}$, in the energy range accessible to future experiments.

In order to break G_W spontaneously and supersymmetry explicitly, we consider soft breaking terms [10], which may find their origin in the context of $N = 2$ supergravity. Since soft breaking terms exist which preserve the supersymmetry generated by Q_1 —with respect to which the above $N = 1$ superfields are defined—but not the other generated by Q_2 , we will discuss the general case in which Q_1 is broken at the scale M_1 and Q_2 at M_2 , with, clearly, $M_1 \leq M_2$. The situation $M_1 \neq M_2$ is technically natural, this time because of the $N = 1$ nonrenormalization theorem [11].

Conceptually, however, two options seem more plausible: $M_1 \simeq M_2$ or $M_2 \simeq \Lambda$. In the first case, the theory is effectively $N = 2$ supersymmetric as long as $M_1 \simeq M_2 \ll \Lambda$, so that no higher-loop contributions to the beta functions are present between M_1 and Λ . In contrast, in the second case, $N = 2$ supersymmetry is never realized below Λ . In this regime, only one supersymmetry is effective, but the underlying $N = 2$ structure will affect the choice of supermultiplets (the existence of

mirror states) as well as the structure of the effective superpotential.

These options are related to the difficulties encountered with spontaneously broken $N = 2$ supergravities. Particles and their mirror partners are connected by the $SO(2)$ global symmetry of the $N = 2$ theory. This symmetry must necessarily be broken if a mirror splitting mechanism is to be obtained. Such a breaking is possible if the super-Higgs mechanism is induced by hypermultiplets. However, in general, the two gravitinos have masses of the same order, and obtaining a hierarchical supersymmetry breaking with a large energy range where effectively $N = 1$ supersymmetry remains unbroken is an extremely difficult task, although not impossible (an example of a breaking with different gravitino masses can be found in ref. [12]). The necessity of a sequential supersymmetry breaking also indicates that spontaneously broken globally $N = 2$ supersymmetric theories cannot realistically be used.

The phenomenology of the $N = 2$ supersymmetric $SU(3)_C \times G_W$ models has been analyzed in detail by Del Aguila et al. [8], with a particular emphasis on the issue of fermion masses. Actually, an interesting feature of realistic $N = 2$ models is the predictivity of the fermion mass spectrum, due to the absence of arbitrary Yukawa couplings. These authors have considered two models, corresponding to $G_W = SU(2)_L \times U(1)_Y$ and $G_W = SU(4) \times U(1)_{B-L}$. We will discuss the MPP scenario within a sort of idealization of these models. More specifically, we assume the same multiplet structure as Del Aguila et al., but, differently from these authors, we allow in principle for all possible soft breaking terms in the lagrangian. Moreover, we do not commit ourselves to any particular mechanism of fermion mass generation. Our attitude is motivated by the desire to make the discussion as less dependent as possible on details of the models.

We need however an assumption on the masses of mirror quarks and leptons. We will assume that all of them have a mass M_W , so that they will participate in the renormalization group equations down to this scale. The effective number of generations will then be six at all energies higher than M_W .

3. The $SU(3) \times SU(2) \times U(1)$ model

This is just the $N = 2$ supersymmetric version of the Standard model, which has some special features. The scalar potential considered in [8] can be unbounded

below when supersymmetry breaking terms are included (this problem can however be avoided by choosing a more general potential). Also, there exists unfortunately no justification for the wide disparity of mirror fermion masses compared with ordinary quark and lepton masses which is needed to make the model viable with respect to experiment.

We need to discuss the mass spectrum generated by the soft terms breaking the supersymmetries. We consider first the soft breaking terms which break Q_2 , but not Q_1 . They are of two kinds. The first class contains $\Phi_a^{(3)}\Phi_a^{(3)}|_{\theta\theta}$, $\Phi_a^{(2)}\Phi_a^{(2)}|_{\theta\theta}$ and $\Phi^{(1)}\Phi^{(1)}|_{\theta\theta}$, corresponding to $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ respectively. According to our previous discussion, we take their mass coefficients to be M_2 . Correspondingly, the mirror gauginos ψ_a as well as the mirror gauge scalars ϕ_a have masses of order M_2 . The second class involves the Higgs supermultiplets. Their quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$ are $(1, 2, -1/2)$ and $(1, 2, 1/2)$ for H and G respectively, H' and G' being in the conjugate representations. The possible Q_2 -violating soft breaking terms are then $HG|_{\theta\theta}$ and $H'G'|_{\theta\theta}$. We also consider here the $N = 2$ invariant terms $HH'|_{\theta\theta}$ and $GG'|_{\theta\theta}$: these terms may or may not be present. We include these soft terms in the lagrangian in order to generate tree-level Higgsino masses without ordinary-mirror fermion mixing. Upon elimination of the auxiliary fields, these terms give rise to contributions to the Higgs masses, on top of those coming from Q_1 -violating terms, to be discussed below. We expect $G_F^{-1/2} < M_1 \leq M_2$, and so, to make the hierarchy problem as mild as possible, we take the coefficients of the terms under consideration at the order of M_1^2 , instead of M_2^2 , as might look more natural at first sight. The Higgsino masses are therefore of order M_1 .

Consider next the soft breaking terms which violate both Q_1 and Q_2 . Our assumptions tell that their mass coefficients should be M_1 . Among these terms are Majorana masses for the gauginos, which are therefore of order M_1 . Clearly, squarks, sleptons, mirror squarks and mirror sleptons are also as heavy as M_1 . Finally come the Higgs mass terms. The soft terms giving masses to the Higgs scalars should be of order M_1 , so that all Higgs and Higgsino masses would seem to be of order M_1 . On the other hand, the breaking of $SU(2)_L \times U(1)_Y$ at $G_F^{-1/2}$ is induced by the Higgs potential resulting from these terms as well as from the supersymmetric mass terms which we assumed of order M_1 . A natural situation would be to find $M_1 \sim M_W$. But

a hierarchical situation $M_W \ll M_1$ could arise with particular (fine-tuned) values of the mass coefficients such that some Higgs scalars have a vacuum expectation value of order $G_F^{-1/2}$ even though all soft terms are of order M_1 . In this case, in the unbroken phase, all Higgs fields will have masses of order M_1 , but they will be widely split by the symmetry breaking. Another possibility is to have a vanishing supersymmetric mass term for some Higgs doublets, in which case these states and their Higgsino partners would have masses of order M_W , and the weak breaking scale would be, for instance, radiatively induced. We then allow for some Higgses and Higgsinos with masses of order M_W only in the renormalization group equations.

The evolution of the gauge coupling constants at two-loop is dictated by the renormalization group (RG) equations

$$\mu \frac{\partial}{\partial \mu} g_i(\mu) = -\frac{1}{16\pi^2} b_i g_i^3(\mu) - \frac{1}{(16\pi^2)^2} \sum_j b_{ij} g_i^3(\mu) g_j^2(\mu) \quad (1)$$

($i = 1, 2, 3$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively). Notice the absence of Yukawa coupling contributions, simply because they are forbidden by $N = 2$ supersymmetry invariance. They are generated by radiative corrections once all supersymmetries are broken. The coefficients of these equations are easily obtained from the results of refs. [13]. We will first give them for the three different energy intervals, $M_W \leq \mu < M_1$, $M_1 \leq \mu < M_2$ and $M_2 \leq \mu < M_P$, which correspond to different spectra of light states.

Below M_1 , according to our assumptions on the soft breaking terms, all supersymmetric partners have become massive. The coefficients of the RG equations are then given by

$$\begin{aligned} b_3 &= 11 - \frac{4}{3}n, \\ b_2 &= \frac{22}{3} - \frac{4}{3}n - n_H, \\ b_1 &= -\frac{20}{9}n - n_H \end{aligned} \quad (2a)$$

for the one-loop coefficients and

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 136/3 & 0 \\ 0 & 0 & 102 \end{pmatrix} - \begin{pmatrix} 95/27 & 1 & 44/9 \\ 1/3 & 49/3 & 4 \\ 11/18 & 3/2 & 76/3 \end{pmatrix} n - \begin{pmatrix} 3/2 & 9/2 & 0 \\ 3/2 & 25/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H, \quad (2b)$$

where n is the number of fermion generations (ordinary or mirror), which will be six in the following and n_H is the number of light Higgs and Higgsino multiplets $(1, 2, 1/2) + (1, 2, -1/2)$. The weak hypercharge $U(1)_Y$ is normalized according to the usual definition of the electric charge, $Q = T_{3L} + Y$, so that on each generation, $\text{Tr } Y^2 = \frac{5}{3} \text{Tr } T_{3L}^2$.

Between M_1 and M_2 , we have $N = 1$ supersymmetry with n generations (ordinary or mirror) and n'_H Higgs supermultiplets $(1, 2, 1/2) + (1, 2, -1/2)$. The coefficients read (see also [4])

$$\begin{aligned} b_3 &= 9 - 2n, \\ b_2 &= 6 - 2n - n'_H, \\ b_1 &= -\frac{10}{3}n - n'_H, \end{aligned} \tag{3a}$$

and

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{pmatrix} - \begin{pmatrix} 190/27 & 2 & 88/9 \\ 2/3 & 14 & 8 \\ 11/9 & 3 & 68/3 \end{pmatrix} n - \begin{pmatrix} 1 & 3 & 0 \\ 1 & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix} n'_H. \tag{3b}$$

Finally, for $M_2 \leq \mu < M_P$, we have $N = 2$ supersymmetry and the beta function only receives contributions at one loop, of the form

$$\begin{aligned} b_3 &= 6 - 4n', \\ b_2 &= 4 - 4n' - n'_H, \\ b_1 &= -\frac{20}{3}n' - n'_H, \end{aligned} \tag{4}$$

where n' is now the number of $N = 2$ generations, including mirror states ($n' = 3$ in the following discussion).

We integrate these equations numerically, starting from Λ and running the coupling constants to lower energies. Since we certainly want to obtain $\alpha_{em}(M_W) = 128^{-1}$, this information will be an input in the computation. Moreover, according to the MPP philosophy, the low-energy values of the gauge coupling constants should be insensitive to their values at Λ . To be definite, we discuss throughout the numerical results obtained with $\alpha_i(\Lambda) \equiv g_i^2(\Lambda)/4\pi = 1$. We have verified the weakness of the sensitivity of the numbers in a change of this initial value. We then only keep the solutions corresponding to acceptable values of $\sin^2 \theta_W(M_W)$ and $\alpha_3(M_W)$ [14],

$$\begin{aligned} \sin^2 \theta_W(M_W) &= .229 \pm .007, \\ \alpha_3(M_W) &= .122 \pm .016. \end{aligned} \tag{5}$$

We present in table 1 the results obtained in the case $M_1 = M_2$ without Higgs or Higgsino left massless below the supersymmetry breaking scale. We always start with $n_H = 2$ at the cut-off Λ , as required in $N = 2$ theories. In the ranges

$$\begin{aligned}\Lambda &\sim (.8 \text{ to } 2.5) \cdot 10^{17} \text{ GeV}, \\ M_1 = M_2 &\sim (.7 \text{ to } 3) \cdot 10^{10} \text{ GeV},\end{aligned}\tag{6}$$

the values of $\sin^2\theta_W(M_W)$ and $\alpha_3(M_W)$ are in the acceptable intervals (5). As it should, Λ is close to the Planck mass, but the supersymmetry breaking scale turns out to be very large, eight orders of magnitude higher than the weak interaction scale. Thus, using supersymmetry breaking to induce weak breaking represents a serious hierarchy problem. The effect of the second order corrections on the values of the gauge couplings in this case is small: less than .5 % for the weak mixing angle, and about 5 % for the strong coupling. This is related to the absence of higher order terms above M_1 , where $N = 2$ supersymmetry is effective and where the coupling constants become large. These results are then certainly reliable in perturbation theory. The supersymmetry breaking scale on the other hand is much more sensitive to higher order corrections: one has to take the exponential of the solutions to the RG equations. The two loop contributions increase this scale by about a factor two. Keeping some Higgses and Higgsinos below $M_1 = M_2$ results in a large increase of the strong coupling constant, leading in general to unacceptable values.

Table 1 also contains the results obtained in the case $M_2 = \Lambda$. Roughly, with respect to the previous case, the effect is to lower Λ by a factor 20 and M_1 by about two orders of magnitude. Acceptable values are then obtained when

$$\begin{aligned}M_2 = \Lambda &\sim (.5 \text{ to } 1) \cdot 10^{16} \text{ GeV}, \\ M_1 &\sim (.24 \text{ to } 2.2) \cdot 10^8 \text{ GeV}.\end{aligned}\tag{7}$$

These results are much more sensitive to two loop corrections which are also present in the energy range where the couplings become large. They change $\sin^2\theta_W(M_W)$ and $\alpha_3(M_W)$ by about 2 % and 25 % respectively. It is clear that intermediate cases with $M_1 < M_2 < \Lambda$ will also lead to solutions intermediate between (6) and (7).

To conclude this section, we see that acceptable values for the low-energy parameters with Λ close to the Planck mass can be obtained with the MPP scenario applied to the $N = 2$ version of the standard model, at the expense of a rather severe hierarchy problem between supersymmetry breaking scale and $G_F^{-1/2}$.

4. The $SU(3)_C \times SU(4) \times U(1)_{B-L}$ model

This model [8] is a simple left-right symmetric extension of the Standard Model, where $SU(2)_L \times SU(2)_R$ is enlarged to $SU(4)$. Thus, parity violation in weak interactions arises from the breakdown of $SU(2)_L$ and $SU(2)_R$ at different scales. One family of ordinary and mirror quarks and leptons transforms under $SU(3)_C \times SU(4) \times U(1)_{B-L}$ according to

$$(1, 4, -1/2) + (3, 4, 1/6) + (1, \bar{4}, 1/2) + (\bar{3}, \bar{4}, -1/6) \quad (8)$$

which, as expected, is a real representation. Four Higgs hypermultiplets are introduced:

$$(1, 4, -1/2) + (1, \bar{4}, 1/2) + (1, 4, 1/2) + (1, \bar{4}, -1/2). \quad (9)$$

We assume that $SU(4) \times U(1)_{B-L}$ is spontaneously broken directly into $SU(2)_L \times U(1)_Y$ at the same scale M_1 where the last supersymmetry, generated by Q_1 , is also broken. This gauge symmetry breaking is controlled by the Higgs hypermultiplets, with the help of Q_1 and Q_2 violating soft breaking terms with mass coefficients M_1 . Therefore we take all Higgs and Higgsino masses of order M_1 , except for the Higgs and their Higgsino partners necessary to break $SU(2)_L \times U(1)_Y$ at M_W .

Remarkably enough, this model contains a natural mirror splitting mechanism [8]: all mirror fermions have tree-level generation-independent masses $O(G_F^{-1/2})$, whereas ordinary fermions get their masses via radiative corrections. This circumstance is due to the fact that the scalar potential has a global minimum for which the vacuum expectation value of the $SU(4)$ mirror gauge scalar $\langle \phi_a^{(4)} \rangle$ is off-diagonal and breaks $SU(2)_L \times U(1)_Y$. Such a situation arises in turn from the presence of a Q_1 and Q_2 violating mass term for $\phi_a^{(4)}$ which has then a mass coefficient of order M_1 . Since the supersymmetry generated by Q_1 is broken at M_1 , the $SU(4)$ mirror gauginos must also have masses at this scale. As in the previous model, all squarks, sleptons, mirror squarks and mirror sleptons have Q_1 and Q_2 violating mass terms, so that their masses are of order M_1 . The same is true for the gauginos of $SU(3)_C \times SU(4)$, since their Majorana masses break both Q_1 and Q_2 . Finally, our assumptions imply that the mirror gauginos and the mirror gauge scalars of $SU(3)_C \times U(1)_{B-L}$ have masses of order M_2 since they arise from the Q_1 conserving soft breaking term $\Phi_a^{(3)} \Phi_a^{(3)}|_{\theta\theta}$ and $\Phi^{(1)} \Phi^{(1)}|_{\theta\theta}$.

Notice that the $SU(4)$ gauge group contains interactions connecting fermions to their mirror states. The corresponding $N = 2$ vector multiplets will have masses M_1 . However, the K_L - K_S mass difference gives rather strong lower limits on the masses of these gauge scalars [15], indicating that M_1 should not be lower than $10^{5-6} GeV$.

The coefficients of the RG equations in the range $M_1 \leq \mu < M_2$ are given by

$$\begin{aligned} b_4 &= 8 - 4n - 2n_H, \\ b_3 &= 9 - 4n, \\ b_1 &= -\frac{8}{3}n - 4n_H, \end{aligned} \tag{10a}$$

and $(i, j = 1, 3, 4)$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 56/27 & 32/9 & 0 \\ 4/9 & 136/3 & 0 \\ 4/3 & 16 & 0 \end{pmatrix} n - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} n_H. \tag{10b}$$

n is the number of generations (8) and n_H the number of combinations of Higgs multiplets (9). Notice that the two loop coefficients b_{i4} always vanish: between M_1 and M_2 the $SU(4)$ representation content is still the full $N = 2$ spectrum, and there is then no second order contribution from $SU(4)$ loops. In the energy interval $M_2 \leq \mu < M_P$, the $N = 2$ coefficients are

$$\begin{aligned} b_4 &= 8 - 4n - 2n_H, \\ b_3 &= 6 - 4n, \\ b_1 &= -\frac{8}{3}n - 4n_H. \end{aligned} \tag{11}$$

and for $\mu < M_1$ the RG equations are given again by (2).

The numerical results for this model are reported in table 2. The MPP scenario works only if all Higgs and Higgsino doublets have masses of order M_W , corresponding to $n_H = 2$ in eqs. (2). Removing some Higgs states at M_1 always leads to too small values for $\alpha_3(M_W)$. This is also the case when the option $M_1 = M_2$ is considered. In fact, the MPP scenario works only in the extreme case $M_2 = \Lambda$, where $N = 2$ supersymmetry is never realized in the perturbative regime, below Λ . In this case, acceptable values (see eq. 5) are obtained for

$$\begin{aligned} M_2 = \Lambda &\sim (1.7 \text{ to } 2.5) \cdot 10^{17} GeV, \\ M_1 &\sim (.7 \text{ to } 5) \cdot 10^9 GeV. \end{aligned} \tag{12}$$

Again, the supersymmetry breaking scale M_1 is much higher than the weak scale, but the necessary Higgs fields are now present at $G_F^{-1/2}$. Notice also that the scale M_1 corresponds to the breaking of left-right symmetry. Such a high value for M_1 is safely beyond the experimental limits on right-handed weak currents (see for instance [16]), and also beyond the more specific limits of ref [15].

The two loop corrections essentially affect the value of the strong coupling. They decrease its low-energy value by about 30 % and are therefore responsible for the too small values of $\alpha_3(M_W)$. The reliability of the two-loop computations is then certainly questionable.

The MPP scenario applied to the $SU(3)_C \times SU(4) \times U(1)_{B-L}$ model seems to lead to the conclusion that $N = 2$ supersymmetry should be already broken at the cut-off scale Λ , which turns out to be less than two orders of magnitude smaller than the Planck scale. It is then reasonable to identify M_2 with the scale of the super-Higgs mechanism in the supergravity theory.

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Table captions:

Table 1:

Numerical results for $SU(3)_C \times SU(2)_L \times U(1)_Y$. All Higgs and Higgsino states have masses M_1 .

Table 2:

Numerical results for $SU(3)_C \times SU(4) \times U(1)_{B-L}$. All Higgs and Higgsino doublets have masses M_W .

Table 1:

$\Lambda (GeV)$	$\sin^2\theta_W(M_W)$	$\alpha_3(M_W)$	$M_1 (GeV)$
$M_1 = M_2:$			
10^{17}	.233	.111	$7.49 \cdot 10^9$
$2 \cdot 10^{17}$	.228	.127	$2.75 \cdot 10^{10}$
$M_2 = \Lambda:$			
$5 \cdot 10^{15}$	.236	.113	$2.45 \cdot 10^7$
$8 \cdot 10^{15}$	.233	.134	$1.38 \cdot 10^8$
10^{16}	.231	.139	$2.16 \cdot 10^8$

Table 2:

$\Lambda (GeV)$	$\sin^2\theta_W(M_W)$	$\alpha_3(M_W)$	$M_1 (GeV)$
$M_2 = \Lambda:$			
$1.6 \cdot 10^{17}$	.226	.103	$6.16 \cdot 10^8$
$2 \cdot 10^{17}$	.224	.113	$1.88 \cdot 10^9$
$2.5 \cdot 10^{17}$	.222	.125	$5 \cdot 10^9$