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ON THE TIME DEPENDENCE OF J/Ψ SUPPRESSION
BY THE QUARK-GLUON PLASMA

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ABSTRACT

Time dependence of J/Ψ dissolution by the Quark-Gluon Plasma is formulated within the non-relativistic quantum mechanics. Solutions for simplified situations show that the J/Ψ dissolution time is about $1 - 1.5 fm/c$ and depends only weakly on the Debye screening length. In these simplified situations the J/Ψ is created immediately and starts to dissolve only when the Quark-Gluon Plasma is formed.

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Suppression of J/Ψ in heavy ion collisions has been proposed [1] as a signature of the Quark-Gluon-Plasma (QGP) formation. The effect has been observed by the NA-38 Collaboration at the CERN SPS [2]. Karsch and Petronzio [3] have interpreted the strong p_T dependence of the suppression in $^{16}O - ^{238}U$ in terms of a finite J/Ψ formation time, which permits J/Ψ with large transverse momentum p_T to escape from the region occupied by QGP without being dissolved. The concept of the formation time and its estimate $\tau \approx 1 fm/c$ has been based on semiclassical arguments [3,4]. Within the same semiclassical framework Kapusta [5] obtained a considerably higher estimate of the formation time $\tau \approx 4 fm/c$.

Several authors have pointed out that J/Ψ dissolution by the QGP should be discussed within quantum mechanics, what would permit to clarify the whole concept of the formation time and to obtain reliable quantitative description of the time dependence of J/Ψ dissolution by the QGP. The application of quantum mechanics to this problem is rather straightforward and some simplified situations, not directly connected with the J/Ψ dissolution by the QGP, have already been discussed by Cleymans and Thews [6].

The purpose of the present note is to discuss the time-dependence of the J/Ψ dissolution by the QGP within the framework of non-relativistic quantum mechanics and to present some estimates of how fast is the J/Ψ dissolved. In comparison with Ref. [6] we shall be more directly concerned with J/Ψ in QGP and we shall give results based on numerical integration of the time-dependent radial Schroedinger equation.

To set up a stage we shall first briefly recapitulate the description of J/Ψ within and outside of the QGP.

The production of J/Ψ in hadronic collisions including heavy ion ones is supposed to be a hard and fast process occurring within time interval of about $\hbar/(2m_c c^2)$ in the first stage of the collision. The dimension of the wave packet of created c - and $\bar{c}$ -quarks is about $\hbar/(2m_c c)$. We assume also that colour bleaching possibly accompanying the production of c and $\bar{c}$ in the state of vanishing total colour [7] is fast.

Due to the large c -quark mass the c - $\bar{c}$ system can be approximately described by non-relativistic quantum mechanics [8, 9]. In the absence of QGP the interaction between c and $\bar{c}$ quarks is described by the confining potential [8, 9]

$$V_c(r) = \frac{\alpha_{eff}}{r} + \sigma r \quad (1)$$

where r is the relative coordinate of c and $\bar{c}$, $\alpha_{eff} = (4/3)\alpha_s \approx 0.3 - 0.4$, string tension $\sigma \approx 0.2 GeV^2$ and the mass of the charmed quark is $1.6 - 1.8 GeV/c^2$. Here and in what follows we shall use units $\hbar = c = 1$ in which r is measured in $1/GeV = \hbar c/GeV = 0.2 fm$.

For J/Ψ within the QGP the linear part of the potential is absent and the Coulombic part is screened to

$$V_D(r) = -\frac{\tilde{\alpha}_{eff}}{r} \exp(-r/r_D) \quad (2)$$

where both $\tilde{\alpha}_{eff}(T)$ and the Debye radius $r_D(T)$ are functions of temperature. Non-relativistic Hamiltonians corresponding to potentials $V_c(r)$ and $V_D(r)$ are

$$H_c = -\frac{1}{m_c} \Delta + V_c(r) \quad (3)$$

$$H_D = -\frac{1}{m_c}\Delta + V_D(r) \quad (4)$$

where the standard factor of 2 is missing in the kinetic energy term because the reduced mass of the $c, \bar{c}$ system is $m_c/2$.

Suppose first that QGP is not present at all and the $c, \bar{c}$ system is created in a state described at $t = 0$ by the wave packet $\Psi(r, 0)$. Time evolution of this wave packet is described by the Schroedinger equation with confining Hamiltonian H_c given in Eq.(3). Denoting the eigenstates of H_c as $\Phi_n(r)$ with $\Phi_{J/\Psi}(r)$ corresponding to the J/Ψ bound state, we can write the solution to the Schroedinger equation as follows

$$\begin{aligned} \Psi(r, t) &= a_{J/\Psi} \exp(-iE_{J/\Psi}t) \Phi_{J/\Psi}(r) + \sum_n a_n \exp(-iE_n t) \Phi_n(r) \\ &\equiv \Phi_{J/\Psi}(r, t) + \Phi_{rest}(r, t) \end{aligned} \quad (5)$$

with $a_{J/\Psi}$ being given as

$$a_{J/\Psi} = \int d^3r \Phi_{J/\Psi}^*(r) \Psi(r, 0) \quad (6)$$

and a_n expressed in a similar way. The probability to find J/Ψ in the state $\Psi(r, t)$ is independent of time and equal to $|a_{J/\Psi}|^2$. The concept of the formation time is inapplicable to this simplified situation, the amount of J/Ψ created at $t = 0$ is independent of time.

As a second simplified situation consider the J/Ψ state which is at $t = 0$ immersed into the QGP. The wave function of the $c, \bar{c}$ system at $t = 0$ is then given as

$$\Psi(r, t = 0) = \Phi_{J/\Psi}(r) \quad (7)$$

Its time dependence is prescribed by the time dependent Schroedinger equation with the Hamiltonian H_D of Eq.(4). The probability to find J/Ψ in the state $\Psi(r, t)$ at time t is

$$P(t) = \left| \int d^3r \Phi_{J/\Psi}^*(r) \Psi(r, t) \right|^2 \quad (8)$$

This probability shows how fast is the J/Ψ dissolved by the QGP. We have calculated $P(t)$ in this preliminary study in the following way. The wave function $\Phi_{J/\Psi}(r)$ has been approximated by a Gaussian $(2\pi\sigma^2)^{-1/4} \exp(-r^2/4\sigma^2)$ with σ chosen in such a way that a realistic mean square radius $\langle r^2 \rangle^{1/2} = 0.5 fm$ is obtained. The time dependent Schroedinger equation with potential V_D has been solved numerically by using Galerkin's method. Results are presented in Fig.1 for a few values of parameters of the nonconfining potential V_D . The J/Ψ dissolution proceeds with a typical time of 1–1.5 fm needed to dissolve 0.5 of J/Ψ . The dissolution is faster for smaller values of r_D , as expected; but the dependence on r_D is rather weak. Note also that the curve b in Fig.1 corresponds approximately to a realistic situation with $T \approx 1.5T_c$.

We shall study the dependence of the J/Ψ dissolution rate on $\hat{\alpha}_{eff}$ and r_D in more detail elsewhere.

In Fig.1 we also present the time dependence of $\langle r^2(t) \rangle^{1/2}$ for two sets of parameters. The dimension of the wave packet increases in $\Delta t = 2fm/c$ from 0.5 fm to

about 1.5fm. This result had to be expected on the basis of semiclassical arguments by Karsch and Petronzio [3, 4]. The wave packet is expanding with radial velocity of about $2 < p > / m_c \approx 0.5$, after the J/Ψ has been immersed into the QGP. The quantal picture is, however, somewhat different from that sketched in Refs. [3, 4]. In the quantal description J/Ψ is created immediately, see Eq. (6) and the discussion below it, and dissolves only after being immersed into the QGP. In Refs. [3, 4] the authors refer to a very narrow $c, \bar{c}$ wave packet expanding to the normal J/Ψ size. After having reached this size J/Ψ is immediately dissolved by the QGP.

It is to be noted that at large values of $\langle r^2(t) \rangle^{1/2}$ the approximation in which the QGP is treated as a continuous medium might break down. The dissolution of the J/Ψ is then caused by the recombination of c and $\bar{c}$ with neighbouring non-charmed quarks and the dissolution proceeds probably faster than calculated in the continuum model of the QGP.

In a general situation the $c, \bar{c}$ system is immersed in the QGP for $t_1 \leq t \leq t_2$ where t_1 is the QGP formation time and t_2 is the time when the QGP goes via the phase transition into the hadron gas or into the mixed phase. The temperature of the QGP also depends on time. In this situation one should solve the time-dependent Schroedinger equation with $\Psi(r, t=0) = \Psi_0(r)$ being a very narrow wave packet with a width of the order of $\hbar/m_c c$ describing the $c, \bar{c}$ system created at $t=0$. The Hamiltonian is time dependent

$$\begin{aligned} H(r, t) &= H_c(r) \quad \text{for } t \leq t_1 \\ H(r, t) &= H_D(r, T(r, t)) \quad \text{for } t_1 \leq t \leq t_2 \\ H(r, t) &= H_c(r) \quad \text{for } t_2 \leq t \end{aligned} \tag{9}$$

The ratio $r(t_2, t_1)$ of the probability to find J/Ψ undissolved at t_2 to the probability with which J/Ψ has been created at $t=0$ (and lived up to t_1) gives the survival probability of J/Ψ

$$r(t_2, t_1) = \frac{|\int \Phi_{J/\Psi}^*(r) \Psi(r, t_2) d^3r|^2}{|\int \Phi_{J/\Psi}^*(r) \Psi(r, t_1) d^3r|^2} \tag{10}$$

This scheme can provide a useful approximation only if the time scale of changes of $H_D(r, T(t))$ is larger than a typical time for the J/Ψ system.

The wave function $\Psi(r, t)$ in Eq. (10) consists of two parts $\Phi_{J/\Psi}(r, t)$ and $\Phi_{rest}(r, t)$ as specified in Eq. (5). Neglecting $\Phi_{rest}(r, t)$ we have $r(t_2, t_1) = P(t_2 - t_1)$ with $P(t)$ given by Eq. (8). We shall study the dependence of $r(t_2, t_1)$ on t_2 and t_1 in more detail elsewhere.

One can naturally ask where appears the concept of the J/Ψ formation time within the quantal description of J/Ψ dissolution by the QGP. We conjecture that it will reappear in the $r(t_1, t_2)$ dependence on t_1 for small values of t_1 . Since the original wave packet $\Psi(r, 0)$ is rather narrow it will need some time to grow up to dimensions large enough to be able to feel the difference between the confining potential $V_c(r)$ and the deconfining one $V_D(r)$. This speculation has, of course, to be confirmed by a numerical analysis.

To conclude: The J/Ψ dissolution by the QGP is not immediate but it takes about 1 – 1.5 fm to dissolve 0.5 of J/Ψ 's created. We believe that this effect should be taken into account in models analyzing J/Ψ dissolution by the QGP assumed to be produced in heavy

ion collisions. For larger values of $\langle r^2(t) \rangle^{1/2}$ the present description should be replaced by a microscopic one [10]. This will make J/Ψ to dissolve faster at large t than given by our model.

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Figure Caption

Figure:1 Survival probability $P(t)$ (left hand vertical scale) for J/Ψ in the QGP. The potential for the $c\bar{c}$ pair within the QGP is given by Eq.(2) with $\bar{\alpha}_{eff} = 0.18$ and a) $r_D = 5.0 fm$, b) $r_D = 0.2 fm$, c) $r_D = 0 fm$ (free particle wave packet spreading). For the cases a) and c) we present also the dependence of $\langle r^2(t) \rangle^{1/2}$ vs. t to show how fast the corresponding wave packets are spreading (right hand vertical scale). The curves $\bar{a}$ and $\bar{c}$ correspond to cases a) and c).