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THREE-GENERATION FOUR-DIMENSIONAL $SU(3) \times SU(2) \times U(1)^5$ STRING MODELS

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A B S T R A C T

We generalize the four-dimensional string construction to allow for the removal of the fixed-point degeneracy of the partition function in the twisted sectors. We present two examples, one supersymmetric and the other non-supersymmetric, based on Z_6 discrete symmetry. Both have a rank-8 gauge group $SU(3) \times SU(2) \times U(1)^5$ (no hidden sector), three generations of chiral fermions, Higgs doublets and no Higgs colour triplets.

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1. - INTRODUCTION

There are now various available methods of constructing consistently interacting four-dimensional strings, each one of them having its virtues and being more convenient in some but not all the aspects [1-9]. All in all the number of distinct possibilities is astronomical but most of these are not physically relevant. In the absence of a string theory that distinguishes between all the different possibilities and picks the correct theory as its vacuum state, we must set some guidelines or requirements that help us in our search. An obvious requirement is that the low-energy limit of the candidate string model should give something very near to the standard Weinberg-Salam model. There are currently two lines of thought. In the first one [7] a string model is built (usually obtained by compactifying the ten-dimensional heterotic string on an orbifold) with a gauge group $SU(3) \times SU(2) \times U(1)^n \times$ hidden sector, at the string (or compactification) scale and then take the field theory limit as an effective theory. In the second approach [10], a string model based on a grand unification group such as $SU(5) \times U(1)^4 \times$ hidden sector, is built directly in four dimensions. The field theory limit is then taken and $SU(5)$ is broken into the standard model. In both cases, $N = 1$ supersymmetry is required. In this paper we shall adopt the first strategy, and look directly for a string model with $SU(3) \times SU(2) \times U(1)^5$ gauge group, and no hidden sector. We use the formalism developed in Refs. [8] and [9] for building four-dimensional strings with twisted boundary conditions in the Green-Schwarz formulation. Also, since there are arguments for requiring $N = 1$ supersymmetry such as solving the hierarchy problem in the field theory limit, or against supersymmetry for the difficulty of breaking it spontaneously at the string level [11], we shall cover both possibilities simultaneously. This is easily achieved in our formalism since the only difference between supersymmetric and non-supersymmetric string models is whether or not one of the twists of the space-time fermions is restricted to zero.

Our formalism, although constructed directly in four dimensions, is similar to the strings compactified on orbifolds and shares the problem that in the twisted sectors there is usually a large number of fixed points implying a large number of generations. This problem is resolved in the orbifold compactification by introducing Wilson lines [7] (shift vectors in our case) in the twisted sectors thus removing the degeneracy. We shall slightly generalize the formalism in Refs. [8] and [9] to accommodate this possibility.

The plan of this paper is as follows. In Section 2 we shall present the necessary modifications in the formalism of Refs. [8] and [9] to allow for the removal of the fixed point degeneracy, and write down the resulting conditions from modular invariance. In Section 3 we give two simple models with similar structures, one being supersymmetric and the other not. Because the formalism we are using is very flexible, we shall present both models simultaneously. The spectrum of the

supersymmetric model is a supergravity multiplet, a dilaton, an antisymmetric tensor and a chiral fermion, with $SU(3) \times SU(2) \times U(1)^5$ gauge supermultiplets as well as three chiral supermultiplets $[(\underline{3}, \underline{2}) + 2(\underline{\bar{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{1}) + (\underline{1}, \underline{2} + \underline{2}')] + \text{plus many extra singlets}$. For the non-supersymmetric model, the massless spectrum is amazingly close to the supersymmetric case with a graviton, dilaton, and antisymmetric tensor with $SU(3) \times SU(2) \times U(1)^5$ gauge bosons, three chiral fermionic multiplets in the $[(\underline{3}, \underline{2}) + 2(\underline{\bar{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{1})]$ representations of $SU(3) \times SU(2)$ and three complex bosons in the $[2(\underline{\bar{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{2})]$ representations as well as many singlets. Section 4 is the conclusion.

2. - REMOVING FIXED-POINT DEGENERACY IN FOUR-DIMENSIONAL STRINGS

In Refs. [8] and [9], a consistently interacting four-dimensional string theory has been developed in the Green-Schwarz formulation for both supersymmetric and non-supersymmetric cases. The full detailed analysis is given in Ref. [8] and this is conveniently summarized, with a proper definition of all the quantities to be used in this paper, in Section 3 of Ref. [9]. To avoid repetition, we shall refer frequently to that section and use some of the equations appearing there.

The complex bosonic co-ordinates (three right-moving and 11 left-moving) admit in general shifted and twisted boundary conditions. In the untwisted sector the shifts represent quantized momenta that would span a Lorentzian lattice while in the twisted sectors the number of fixed points left invariant by the boundary conditions

$$Z^k(\sigma + \pi) = e^{-2\pi i \theta^k} Z^k(\sigma) + \pi V^k$$

determine the positions of the string centres-of-mass and imply that the string Hilbert space is spanned by the momenta of the winding modes and the centres-of-mass of the twisted modes which are the fixed points. The interactions at different fixed points are all identical in a given sector and since the number of fixed points in most cases is quite large, this degeneracy multiplies any spectrum obtained in the respective sector. This difficulty was resolved in the orbifold compactification by introducing Wilson lines [7] in the twisted sectors in such a way that it makes a one-to-one correspondence between the fixed points and the Wilson lines. This makes the previously identical twisted sectors inequivalent.

We shall now modify slightly the formalism of Refs. [8] and [9] to allow for the possibility of removing the fixed-point degeneracy in the twisted sectors. In Ref. [8], the general partition function was shown to be of the form

$$Z^{\text{total}} = \frac{1}{\tau_2 \eta^2 \bar{\eta}^2} \left(\prod_{m,p} N_m N_p \right)^{-1} \cdot \sum_{a,b,c,d} e^{2\pi i \overline{aW} \cdot \tilde{P}^{ao} \cdot W_o} T_N \left(q^{H^r(AW, P_{ha})} q^{H^l(AW, P_{la})} \cdot e^{2\pi i \left(\sum_m b_m \phi_m + \sum_p (d_p - s_{po}) \psi_p \right)} \right) \quad (2.1)$$

where

$$\phi_m = \sum_n T_{mn}^{ab} a_n - W_m \tilde{P}^{ab} \overline{aW} + \sum_g k_{mg}^{ab} c_g - V_m \tilde{P}^{ab} (\overline{aV} + \overline{cV} - \overline{aV}) + k_{om}^{ab} - W_m \tilde{P}^{ab} N_{\overline{aW}} \quad (2.2)$$

$$\psi_p = \sum_g k_{pg}^{ab} c_g + \sum_m k_{pm}^{ab} a_m - V_p \tilde{P}^{ab} \overline{aV} + \overline{cV} + k_{op}^{ab} - V_p \tilde{P}^{ab} N_{\overline{aW}} \quad (2.3)$$

and H^r and H^l are the Hamiltonians of the left- and right-moving string fields. The quantities appearing in the above equations are defined in Section 3 of Ref. [9].

The untwisted sector is usually defined by taking the $a = 0$ piece of Z^{total} , i.e., when there is no twist in the σ_1 direction. The twisted sectors will then correspond to the $R, \overline{2R}, \overline{3R}, \dots$ twists in the σ_1 directions (i.e., $a=1,2,3,\dots$). However, since one has to perform the trace on the Hamiltonians and this includes the different possible positions of the centres-of-mass, each giving the same contribution, an overall multiplicity factor equal to the number of fixed points is introduced. In a given twisted sector $a = 0$ the boundary conditions are summarized in the vector

$$\overline{\sum_m a_m W_m + \sum_p c_p V_p} \quad (2.4)$$

where $W_m = R_m + V_m$, V_m being the shift vectors accompanying the twists. Thus, for a fixed a_m with every twist vector R_m we only consider a single shift vector V_m . We generalize this by adding different possible shifts which are in one-to-one correspondence with the fixed points. In other words we replace W_m by

$$W_m^{\{c_r\}} = R_m + V_m + \sum_r c_r V_r \quad (2.5)$$

where we associate with every fixed point of co-ordinates $(z_1^o, z_1^o, \dots)$, a sector with a fixed value of $(c_1, c_2, \dots)$. For example in the Z_3 orbifold the centres-of-mass are characterized by (n_1, n_2, n_3) where $n_1, n_2, n_3 = 0, 1, 2$. With this, one can associate the shift vectors $V_m + c_1 V_2 + c_2 V_2 + c_3 V_3$, where $c_1, c_2, c_3 = 0, 1, 2$, or

equivalently (c_1, c_2, c_3) . Thus the only necessary change in Eq. (2.1) is to replace W_m by $W_m^{\{c_r\}}$ everywhere and the matrices T_{mn}^{ab} by $T_{mn}^{ab\{c_r\}}$ defined by

$$T_{mn}^{ab\{c_r\}} = W_m^{\{c_r\}} P^{ab} W_n^{\{c_r\}} \quad (2.6)$$

and sum on all the possible values of $\{c_r\}$ representing the full set of fixed points. Thus only the equations depending on W_m will be now indexed with $\{c_r\}$ with no other changes being necessary. In particular, the modular invariance conditions given in Eqs. (3.24) of Ref. [9] now read

$$W_m^{\{c_r\}} \tilde{P}^{ab} N_{AW\{c_r\}} = \sum_n T_{mn}^{ab\{c_r\}} a_n - W_m^{\{c_r\}} \tilde{P}^{ab} \overline{aW\{c_r\}} + \sum_f k_{mf}^{ab\{c_r\}} c_f - V_m \tilde{P}^{ab} \left(\frac{\sum_n a_n (V_n + \sum_r c_r V_r) + \sum_f c_f V_f}{\sum_n a_n (V_n + \sum_r c_r V_r)} + k_{om}^{ab\{c_r\}} \right) \quad (2.7)$$

$$V_p \tilde{P}^{ab} N_{AW\{c_r\}} = \sum_f k_{pf}^{ab\{c_r\}} c_f + \sum_m k_{pm}^{ab\{c_r\}} a_m + k_{op}^{ab\{c_r\}} - V_p \tilde{P}^{ab} \left(\frac{\sum_n a_n (V_n + \sum_r c_r V_r) + \sum_f c_f V_f}{\sum_n a_n (V_n + \sum_r c_r V_r)} \right) \quad (2.8)$$

In the untwisted sector we set $a = 0$ and we can easily show, by using the fact that the c_r can be independently set to zero, that Eqs. (2.7) and (2.8) reduce to

$$V_{p'} N_{cV} = \sum_f k_{pf} c_f - V_{p'} \overline{cV} + k_{op'} \quad (2.9)$$

where $V_{p'}$ is V_p , V_m or V_r .

The vectors $W_m^{\{c_r\}}$ and V_p satisfy the usual constraints as given in Eq. (3.25) of Ref. [9]. By taking the trace of Eqs. (2.7) and (2.8), it is straightforward to show that

$$\overline{AW\{c_r\}} N_{AW\{c_r\}} = -\frac{1}{2} \overline{AW\{c_r\}} \overline{AW\{c_r\}} + \overline{AW\{c_r\}} W_0 \quad (\text{mod } 1) \quad (2.10)$$

This can also be shown to follow from the level matching condition

$$m^2(\text{left}) = m^2(\text{right}) \quad (2.11)$$

after substituting there expressions from Eqs. (3.26) and (3.27) of Ref. [9].

In the case where we are using a single twist vector R , and in the $a = 1$ twisted sector, Eqs. (2.8) and (2.10) become equivalent to Eqs. (2.7) and (2.8) and thus Eq. (2.7) will not project any massless state out, a fact known for the Z_3 orbifold, but here shown to be generally true. In the $a = 2, 3, \dots$ twisted sectors one has to take Eq. (2.7) into account and indeed it projects some states out.

Before closing this section we mention that we have checked the new modifications to the formalism by reproducing the models of Ref. [7].

3. - Z_6 -MODELS WITH THREE GENERATIONS OF $SU(3) \times SU(2) \times U(1)^5$

In this section we will illustrate the formalism of Section 2 with two models (with and without space-time supersymmetry), based on Z_6 discrete symmetry, with rank-8 $SU(3) \times SU(2) \times U(1)^5$ gauge group (i.e., no hidden sector), three generations of chiral fermions and Higgs doublets. Both models will be presented in parallel and correspond to the twists

$$R = \left(\frac{1}{2} \left(\frac{1}{6} \right)^3, \left(\frac{2}{3} \right)^3 \mid \left(\frac{2}{3} \right)^3, 0^4, \left(\frac{1}{2} \right)^4 \right) \quad (3.1a)$$

and

$$R = \left(0 \left(\frac{2}{3} \right)^3, \left(\frac{2}{3} \right)^3 \mid \left(\frac{2}{3} \right)^3, 0^4, \left(\frac{1}{2} \right)^4 \right) \quad (3.1b)$$

The model defined by (3.1a), that will be called Model a, has no space-time supersymmetry, while the one defined by (3.1b), called Model b, has $N = 1$ space-time supersymmetry. Both twists satisfy the world-sheet supersymmetry and modular invariance conditions (Eqs. (2.12), (2.13) and (3.25) of Ref. [9]) and generate a $Z_6 = Z_3 \times Z_2$ discrete symmetry.

We now consider the following set of boundary conditions

$$a W_{(c_4, c_5)} + \delta_{a0} C_6 V_6 + C_0 V_0 + C_1 V_1 \quad (3.2)$$

with

$$W_{(c_4, c_5)} = R + V_6 + C_4 V_4 + C_5 V_5 \quad (3.3)$$

where $a, c_6 = 0, 1, \dots, 5$, $c_4, c_5 = 0, 1, 2$ and $c_0, c_1 = 0, 1$.

The shift vectors V_0 and V_1 are those constructing the $E_8 \times E_8$ lattice,

$$\begin{aligned} V_0 &= \left(\left(\frac{1}{2} \right)^6 \mid \left(\frac{1}{2} \right)^{22} \right) \\ V_1 &= \left(0^6 \mid 0^6, \left(\frac{1}{2} \right)^8, 0^8 \right) \end{aligned} \quad (3.4)$$

V_6 is the shift accompanying the twist

$$V_6 = (0^6 / 0^6, \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{5}{6} 0 \frac{1}{3} \frac{1}{6}, \frac{1}{6} \frac{5}{6} \frac{1}{2} 0 0 \frac{1}{6} \frac{1}{6} \frac{1}{6}) \quad (3.5)$$

and V_4, V_5 are the shifts that can be interpreted as Wilson lines [7] in orbifold compactifications

$$V_4 = (0^6 / 0^6, 0 0 0 \frac{1}{3} \frac{2}{3} \frac{2}{3} 0 0, 0 \frac{2}{3} 0 0 0 \frac{1}{3} 0 \frac{2}{3})$$

$$V_5 = (0^6 / 0^6, 0 0 0 0 0 0 0 \frac{2}{3}, \frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{1}{3} 0 0 0 \frac{1}{3}) \quad (3.6)$$

The co-ordinates Z^k ($k=1,2,3$) and Z^I ($I=1,2,3$) have 27 fixed points under the action of R and there is a one-to-three correspondence between the twists $W_{(c_4, c_5)}$ and those 27 fixed points. On top of that, the co-ordinates Z^I ($I=8,9,10,11$) have 16 fixed points. The shift vectors V_4, V_5 and V_6 have been chosen to satisfy the modular invariance conditions (Eq. (3.25) of Ref. [9]).

We will first examine the untwisted sector ($a=0$). A quick glance at m^2 (right, $\overline{AW}$) (Eq. (3.26) of Ref. [9]) shows that only subsectors with all entries of the right-moving shift-vectors equal to $\frac{1}{2}$ can have massless states: they are the subsectors $\overline{V_0 + c_1 V_1 + c_6 V_6}$. The massless states for right movers correspond to the ground state

$$|a\rangle_R + |Z^k\rangle_R + |S_-^{l\alpha}\rangle_R \quad (3.7)$$

where Z^k are three complex scalars corresponding to the six compactified dimensions and the index $\ell = 1,2,3,4$ labels the four-Weyl spinors in the right-moving sector. The vector $|a\rangle_R$ is neutral under the Z symmetry and the scalars $|Z^k\rangle_R$ transform under Z with eigenvalues $(\lambda^2, \lambda^2, \lambda^2)$, where $\lambda = \exp(-i\pi/3)$. In Model a the fermionic states $|S_-^{l\alpha}\rangle_R$ transform under Z with eigenvalues $(\lambda^3, \lambda^5, \lambda^5, \lambda^5)$, and in Model b with eigenvalues $(1, \lambda^2, \lambda^2, \lambda^2)$. For left movers, m^2 (left, $\overline{AW}$) is given in Eq. (3.27) of Ref. [9] and different subsectors have to be analyzed separately. Winding states in the untwisted sector transform under Z_6 with eigenvalue $\exp(-2\pi i V_6 \cdot N_{\overline{AW}})$, and $|>_R \otimes |>_L$ Z_6 non-invariant states will be projected out from the physical spectrum.

In the V_0 subsector, the massless states correspond to different possibilities. They are the same for Models a and b. We will just indicate the occupation number operators and/or winding momenta which are different from zero. i) $\tilde{A}_1^b = 1$, for b equal to 1 or 2, gives the vector state $|b>_L$, neutral under Z_6 . When tensored with $|a>_R$ in (3.7) it gives the graviton, an antisymmetric tensor field (axion) and a real scalar (dilaton). In Model b we have in addition the

states $|S_{-}^1\rangle_R \otimes |b\rangle_L$ giving rise to the gravitino and a Weyl fermion and completing thus the graviton supermultiplet. ii) $\tilde{A}_1^{I^*} = 1$, for some I^* from 1 to 22. For $I^* = 1, \dots, 6$ we can form three complex states, and their complex conjugates, transforming under Z_6 as $(\lambda^4, \lambda^4, \lambda^4)$. Also for $I^* = 15, \dots, 22$ we can form four complex states, and their complex conjugates, transforming as $(\lambda^3, \lambda^3, \lambda^3, \lambda^3)$. Those states will be coupled to the ground state (3.7) making gauge singlets: nine scalars and four fermions in Model a, and nine supersymmetric chiral multiplets in Model b. iii) Finally we get the solution $K^{I^*} = (0 \dots \pm 1 \dots \pm 1 \dots 0)$, $I^* = 1, \dots, 22$. Since we have broken the Cartan generators of the $SO(12)$ corresponding to $I^* = 1, \dots, 6$, we need to project out the rest of generators of $SO(12)$. This can be done by introducing (in the untwisted sector) a set of shift-vectors of a form that appears in the trivial compactification of string theories [12]. They are of the form $(\frac{1}{2}0^5 | \frac{1}{2}0^5, 0^{16}) +$ four permutations of the $\frac{1}{2}$ in the first five right and left entries. Consistency with the modular invariance conditions (2.9) requires that $K^{I^*} = 0$, $I^* = 1, \dots, 6$. The remaining states are the non-Cartan generators of $SO(12)$. After using the modular invariance conditions in (2.9)

$$V_{p'} N_{V_0} = 0 \pmod{1}, \quad p' = 0, 1, 4, 5, 6 \quad (3.8)$$

and combined with $|I^*\rangle$, $I^* = 7, \dots, 14$, they form the adjoint representation of $G = SU(3) \times SU(2) \times U(1)^5$. In Model a, they can be tensored with $|a\rangle_R$ in (3.7) giving rise to gauge bosons in the adjoint representation of G . In Model b, they can be tensored with $|a\rangle_R + |S_{-}^1\rangle_R$ giving rise to vector supermultiplets in the adjoint representation of G .

In the $\overline{V_0+V_1}$ subsector the massless states are $K^{I^*} = a^{I^*}$, with $a^{I^*} \in \{0, 1\}$ and $I^* = 7, \dots, 14$. Those states are projected out by the modular invariance condition $V_5 \cdot N_{\overline{V_0+V_1}} = 1/3$.

In the $\overline{V_0+V_6}$ subsector the massless states are in the following $SU(3) \times SU(2)$ representation: $2 \cdot (\underline{\bar{3}}, \underline{2}) + 12 \cdot (\underline{\bar{3}}, \underline{1}) + 4 \cdot (\underline{3}, \underline{1}) + 14 \cdot (\underline{1}, \underline{2}) + 56 \cdot (\underline{1}, \underline{1})$. The modular invariance conditions, Eq. (2.9), can be written as

$$\begin{aligned} V_0 N_{\overline{V_0+V_6}} &= k_{06} + \frac{1}{2} \\ V_1 N_{\overline{V_0+V_6}} &= k_{16} + \frac{1}{2} \\ V_6 N_{\overline{V_0+V_6}} &= k_{60} + \frac{1}{6} \\ V_4 N_{\overline{V_0+V_6}} &= \frac{1}{3} - k_{64} \\ V_5 N_{\overline{V_0+V_6}} &= \frac{1}{3} - k_{65} \end{aligned} \pmod{1} \quad (3.9)$$

In Model a we will make the following choice of k_{pq} parameters:

$$k_{60} = 0, \quad k_{61} = \frac{1}{2}, \quad k_{64} = k_{65} = \frac{2}{3} \quad (3.10a)$$

In that case only $(\bar{3}, 2) + 3 \cdot (1, 1)$ will survive conditions (3.9). They transform under Z_6 with eigenvalue λ and can be tensored with $|S_{-R}^{\lambda}\rangle$, $\lambda = 2, 3, 4$, making three fermions in the $(\bar{3}, 2) + 3 \cdot (1, 1)$ representation of $SU(3) \times (SU(2))$. The $\overline{V_0 + 5V_6}$ will provide the complex conjugate states with opposite helicities. The three subsector fermions in $(\bar{3}, 2) + (1, 1)$ can be identified with the three generations of left-handed quark doublets^{*} and right-handed leptons

In Model b we will make the choice

$$k_{60} = \frac{1}{2}, \quad k_{61} = \frac{1}{2}, \quad k_{64} = \frac{2}{3}, \quad k_{65} = \frac{1}{3} \quad (3.10b)$$

and also the state $(\bar{3}, 2) + 2 \cdot (1, 1)$, with Z_6 eigenvalue λ^4 , will survive the projection (3.9) and, when combined with right-movers, will make three chiral supermultiplets in the $(\bar{3}, 2) + 3 \cdot (1, 1)$. Again the $\overline{V_0 + 5V_6}$ subsector will provide the complex conjugate states with opposite helicities, and we can identify $3[(\bar{3}, 2) + (1, 1)]$ with the three generations of quark doublets and right-handed lepton supermultiplets.

In the $\overline{V_0 + 2V_6}$ subsector the massless states are in the $(\bar{3}, 2) + 3 \cdot (\bar{3}, 1) + 7 \cdot (1, 2) + 22 \cdot (1, 1)$ representation of $SU(3) \times SU(2)$. In Model a, after the choice (3.10a), only $1 \cdot (1, 1)$ state, transforming as λ under Z_6 , will survive the modular invariance conditions (2.9). It will give three gauge singlet fermions. In Model b, after the choice (3.10b), all states will be projected out. As usual, the $\overline{V_0 + 4V_6}$ subsector will provide the complex conjugate states with opposite helicities.

In the $\overline{V_0 + 3V_6}$ subsector, $m^2(\text{left}) > \frac{1}{2}$ and, so, there are no massless states.

In the $\overline{V_0 + V_1 + V_6}$ subsector the massless states are $4 \cdot (\bar{3}, 2) + 4 \cdot (\bar{3}, 1) + 4 \cdot (3, 1) + 4 \cdot (1, 2) + 28 \cdot (1, 1)$. After the application of conditions (3.10a) and (3.10b), for Models a and b, respectively, only $1 \cdot (1, 1)$, transforming as λ^5 and λ^2 , respectively, under Z_6 (and the complex conjugate from the $\overline{V_0 + V_1 + 5V_6}$ subsector) survives the modular invariance conditions if $k_{01} = \frac{1}{2}$ ($k_{01} = 0$) respectively. Otherwise, all massless states are projected out.

^{*}) This assignment merely corresponds to choosing γ_5 with an opposite sign to the conventional one.

In the $\overline{V_0+V_1+2V_6}$ subsector the massless states are $10 \cdot (\underline{3}, \underline{1}) + 10 \cdot (\underline{1}, \underline{2}) + 52 \cdot (\underline{1}, \underline{1})$. In Model b, all states are projected out by the modular invariance conditions (2.9). In Model a, only three (one) $(\underline{1}, \underline{1})$, transforming with eigenvalue λ^3 under Z_6 , will survive (2.9) if $k_{01} = 0$ ($k_{01} = \frac{1}{2}$), giving rise to two (one) gauge singlet fermions. Again the $\overline{V_0+V_1+4V_6}$ will provide the complex conjugate states. In the $\overline{V_0+V_1+3V_6}$ subsector the massless states are $256 \cdot (\underline{1}, \underline{1})$. In Model a, only $1 \cdot (\underline{1}, \underline{1})$, transforming under Z_6 with eigenvalue λ^4 , survives (2.9). They can be tensored with $|Z^K_R\rangle$ giving rise to three gauge singlet complex scalars. In Model b, all states are projected out by the modular invariance conditions (2.9).

In the $a = 1$ twisted sector, and for Models a and b, the vacuum energy for right-movers is zero, and for left-movers it is $-1/6$. There are 36 twisted subsectors $\overline{W_{(c_4, c_5)} + c_0 V_0 + c_1 V_1}$ for $c_4, c_5 = 0, 1, 2$, $c_0, c_1 = 0, 1$. Using the one-to-three correspondence between $W_{(c_4, c_5)}$ and the 27 fixed points of (Z^K, Z^I) , $K, I = 1, 2, 3$, the effective degeneracy of subsectors with fixed c_4 and c_5 is 48. It can be proved that all 36 subsectors have $m^2(\text{left}, \overline{W_{(c_4, c_5)} + c_0 V_0 + c_1 V_1}) > 0$ and so no massless state arises from the $a = 1$ twisted sector. The same result applies to the $a = 5$ twisted sector, since it provides the complex conjugate states.

In the $a = 2$ twisted sector, Model a is supersymmetric and so its spectrum will be. For both Models, a and b, the vacuum energy for right-movers is zero and the ground-state for right-movers is the supersymmetric multiplet $(0, \frac{1}{2})$. The vacuum energy for left-movers is $-2/3$ for both Models and there are 36 twisted subsectors $\overline{2W_{(c_4, c_5)} + c_0 V_0 + c_1 V_1}$ with an effective degeneracy equal to three. Out of those 36 subsectors, nine are massive, 19 contain only $SU(3) \times SU(2)$ singlets and eight contain $SU(3) \times SU(2)$ non-singlets. The 19 subsectors containing only $SU(3) \times SU(2)$ singlets are $\overline{2W_{(c_4, c_5)} + c_0 V_0 + c_1 V_1}$ with $(c_4 c_5 c_0 c_1) = (0000), (0010), (0011), (2000), (2010), (0200), (0210), (0201), (0211), (2200), (2210), (2211), (0100), (0110), (0111), (2101), (2111), (1110)$ and (1101) . We will not analyze here the consistency of those states with the modular invariance equations (2.7), (2.8). Many of them will be projected out and others will survive, depending on our choice of the parameters K_{pq} , controlling thus possible $U(1)$ anomalies. We will now analyze the subsectors containing non-singlet states.

In the $\overline{2W_{1,0} + V_0}$ subsector the massless state is $(\underline{\overline{3}}, \underline{1})$. This state is projected out by the modular invariance condition (2.8) $V_1 \cdot N_{\overline{AW}} = 0$.

In the $\overline{2W_{(1,2)} + V_0}$ subsector the massless state is a $(\underline{\overline{3}}, \underline{1})$. In Model a, this state satisfies conditions (2.7) and (2.8), in particular the condition $W_{(1,2)} \cdot N_{\overline{AW}} = -1/6$. From the $a = 4$ twisted sector we will obtain the complex conjugate state $(\underline{3}, \underline{1})$ with the opposite "helicity" $W_{(1,2)} \cdot N_{\overline{AW}} = 1/6$. Notice that in

the twisted sector $W_{(c_4, c_5)} \cdot N_{AW}$ plays the rôle of the helicity and, for the theory to be CPT invariant, representations and their complex conjugates have to be found with opposite helicities. Those states will describe three up (or down) right-handed quark + squark singlets. In Model b, the state $(\underline{3}, \underline{1})$ is projected out by conditions (2.7) and (2.8).

In the $\overline{2W}_{(2,1)}$ subsector the massless states are $2[(\underline{3}, \underline{1}) + (\underline{1}, \underline{2})]$. In Model a, the state $(\underline{3}, \underline{1}) + (\underline{1}, \underline{2})$ survives the modular invariance conditions, if $k_{(2,1)0} = \frac{1}{2}$, with $W_{(2,1)} \cdot N_{AW} = 1/6$. The complex conjugates, with opposite helicities, will come from the $a = 4$ sector. Those states will describe three down (or up) right-handed quark + squark singlets and three left-handed lepton + slepton doublets. In Model b, the three supersymmetric $(\underline{3}, \underline{1}) + (\underline{1}, \underline{2})$ will survive modular invariance with $W_{(2,1)} \cdot N_{AW} = 1/6$ if $k_{(2,1)0} = 0$, the complex conjugates with opposite helicities coming from the $a = 4$ twisted sector.

In the $\overline{2W}_{(1,0)+V_0+V_1}$ subsector the massless states are $(\underline{3}, \underline{1}) + (\underline{1}, \underline{2})$. In Model a, they will be projected out by (2.8) if $k_{01}^{(1,0)} = \frac{1}{2}$. In Model b they satisfy the modular invariance conditions, if we choose $k_{01}^{(1,0)} = 0$ and $k_{(1,0)1} = \frac{1}{2}$, with $W_{(1,0)} \cdot N_{AW} = 1/6$ and again the $a = 4$ twisted sector providing the complex conjugates with opposite helicities.

In the $\overline{2W}_{(1,1)}$ and $\overline{2W}_{(1,1)+V_0+V_1}$ subsectors the massless states are $4 \cdot (\underline{1}, \underline{2})$ and $1 \cdot (\underline{1}, \underline{2})$, respectively. In both Models, a and b, if $k_{01}^{(1,1)} = 0$, $1 \cdot (\underline{1}, \underline{2})$ and $1 \cdot (\underline{1}, \underline{2})$ can survive the modular invariance conditions (2.7) and (2.8), with the corresponding complex conjugates with opposite helicities coming from $a = 4$. If $k_{01}^{(1,1)} = \frac{1}{2}$, all doublets in both sectors will be projected out from the physical spectrum.

In the $\overline{2W}_{(1,2)+V_0+V_1}$ and $\overline{2W}_{(1,0)+V_0+V_1}$ subsectors the massless states are $2 \cdot (\underline{1}, \underline{2})$ and $3 \cdot (\underline{1}, \underline{2})$, respectively. Out of those states, for Models a and b, $1 \cdot (\underline{1}, \underline{2})$ and $3 \cdot (\underline{1}, \underline{2})$, respectively, can survive the conditions (2.7) and (2.8), or can be projected out, depending on the values of $k_{01}^{(c_4, c_5)}$ and $k_{(c_4, c_5)1}$ in every subsector. They will contribute to the U(1) anomalies, as well as the singlets, and we will postpone the detailed analysis of it to a forthcoming paper.

Finally in the $a = 3$ twisted sector, $\overline{3W}_{(c_4, c_5)+c_0V_0+c_1V_1} = \overline{3W+c_0V_0+c_1V_1}$, where $3W = 3R+3V_6$, for all values of (c_4, c_5) . In Model a the vacuum energy for right-movers is $-\frac{1}{2}$ and there is a tachyonic state at $m_R^2 = -\frac{1}{2}$ and 16 massless states (four fermions $|S_{\frac{1}{2}}^j\rangle_R$ and 12 scalars at $K^j = \pm 1$, $j = 1, \dots, 6$) for subsectors $\overline{3W+V_0}$ and $\overline{3W+V_0+V_1}$. The vacuum energy for left-movers is $-\frac{1}{2}$ and so $m^2(\text{left}, \overline{3W+V_0}) \geq 1$ and

$m^2(\text{left}, \overline{3W+V_0+V_1}) > \frac{1}{2}$. Therefore all tachyonic and massless states are projected out from the physical spectrum. For Model b, the vacuum energy for left-movers is again $-\frac{1}{2}$ and the massless states are equally projected out by the mass level matching condition.

Before closing this section we will summarize the massless spectrum obtained from the twisted and untwisted sectors for matter fields. For Model a we get

$$3 \left[(\underline{3}, \underline{2}) + (\underline{1}, \underline{1}) \right]_F + 3 \left[2(\underline{\bar{3}}, \underline{1}) + (\underline{1}, \underline{2}) \right]_{F+B} + 3(1+n) \cdot (\underline{1}, \underline{2})_{F+B} + \text{singlets} \quad (3.11a)$$

and for Model b we obtain chiral supermultiplets in the $SU(3) \times SU(2)$ representation

$$3 \left[(\underline{3}, \underline{2}) + 2(\underline{\bar{3}}, \underline{1}) + (\underline{1}, \underline{2}) + (\underline{1}, \underline{1}) \right] + 3(2+n) (\underline{1}, \underline{2}) + \text{singlets} \quad (3.11b)$$

where $0 \leq n \leq 5$. So in Model a (Model b) we allow for a minimum of 3 (6) and a maximum of 18 (21) Higgs doublets.

4. - CONCLUSIONS

In this paper we have studied the modular invariance of the total partition function of four-dimensional strings in the general case where the degeneracy of the partition function, corresponding to different centres-of-mass string co-ordinates, is lifted by a one-to-one correspondence between fixed points and shift vectors (Wilson lines for orbifold compactifications). The removal of the degeneracy has the effect of breaking the gauge group (but not affecting the rank) and decreasing the number of generations. We have presented two phenomenologically interesting models (with and without space-time symmetry) based on Z_6 -discrete symmetry with $SU(3) \times SU(2) \times U(1)^5$ gauge group, three generations of chiral fermions and Higgs doublets. In the hidden sector, E_8 is broken to $U(1)^8$ by the shift-vectors and the rank-8 is destroyed by a Z_2 twist in the corresponding four complex co-ordinates.

In both models all Higgs colour triplets are projected out by the modular invariance condition, preventing thus baryon number violating dimension-5 and dimension-6 operators. Of course, the Yukawa couplings $\tilde{d}_{LL}^c q_L$ and $\tilde{d}_{LL}^c u_L^c$ could mediate fast proton decay if these are not forbidden by string considerations. A detailed analysis of Yukawa couplings, in both models, as well as the $U(1)^5$ hyper-

charges and the contribution of singlets to possible $U(1)$ anomalies, is now in progress.

The last issue we want to discuss is related to the cosmological constant in non-supersymmetric models. It has been argued [13] that if the number of bosonic (n_B) and fermionic (n_F) light degrees of freedom are equal then the cosmological constant, though non-zero, could be exponentially suppressed. In the non-supersymmetric model we have presented, this number is small ($n_F - n_B = 10$) as compared to the number of massless fermions, or bosons ($\sim 10^2 - 10^3$). The reason is that the only sector with non-supersymmetric massless spectrum is the untwisted sector. It would be interesting and probably feasible to perform a (computerized) search of non-supersymmetric models with $n_B = n_F$.

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