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**SUPPRESSION OF QED INTERFERENCE CONTRIBUTIONS
TO THE CHARGE ASYMMETRY AT THE Z^0 RESONANCE**

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Abstract: It is shown that, on the top of Z^0 resonance and for loose experimental cut-offs, the contribution from QED initial-final state bremsstrahlung interference to charge asymmetry is suppressed by a factor Γ_Z/M_Z and is below 10^{-3} . In the future experimental tests of the standard model it can be either neglected or easily subtracted. The dependence of this contribution on the centre-of-mass energy and maximum photon energy is given in a compact analytical form.

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The charge asymmetry, called also the forward-backward asymmetry A_{FB} , will be measured in LEP/SLC experiments using high statistics samples of muon pairs produced at the Z^0 peak. Since long it has been regarded as the best source of the experimental information on lepton couplings of the Z^0 and therefore on the electroweak mixing angle θ_W . It appears, however, that in fact the other asymmetry, the spin asymmetry with respect to initial beam polarization A_{LR} , is much more efficient for this purpose due to the unfortunate closeness of $\sin^2 \theta_W$ to $1/4$ resulting in the smallness of the vector-axial coupling constant [1]. Nevertheless, since beam polarization will not be available soon we shall have to be content with A_{FB} as a means of testing the standard electroweak model ⁽¹⁾, one of the main goals of LEP and SLC.

It seems however that, in addition to the smallness of the vector-axial coupling, A_{FB} is also plagued with other problems which may further diminish its efficiency for testing the standard model i.e. its ability to constrain the electroweak mixing angle, the mass of Higgs scalar, the top quark etc. The most serious one is related to the strong dependence of A_{FB} on the centre of mass energy $\sqrt{s}$. Even a small uncertainty on $\sqrt{s} - M_Z$ of the order of $50 MeV$ is not acceptable [3].

In this paper we concentrate on another potential source of uncertainty, related to the interference of QED bremsstrahlung from initial and final state fermions. This contribution is known to be about $+1.5\%$ at PETRA/PEP energies for loose experimental cut-offs and even larger for stronger ones. A contribution to A_{FB} of the same size at the Z^0 would be disastrously large as compared with an experimental precision level $\Delta A_{FB} \sim 0.1\%$ and the value of the asymmetry itself $A_{FB} \sim 3\%$. In such a situation an evaluation of nontrivial two loop, double bremsstrahlung, QED interferences would be necessary ⁽²⁾.

In the following we shall demonstrate an explicit one loop, single bremsstrahlung, QED calculation which indicates clearly that the above interference contribution is rather small at the Z^0 position. This result can be anticipated on general grounds. The Z^0 resonance is relatively narrow, $\Gamma_Z/M_Z \simeq 0.023$, and the initial state bremsstrahlung due to disruption

⁽¹⁾ In fact, the other asymmetry, the final state spin asymmetry, deduced from decay distributions of the τ -lepton is of comparable or slightly better quality in testing the standard model [2].

⁽²⁾ The higher order QED correction will indirectly affect A_{FB} through an effective shift of $\sqrt{s}$ anyway; this we omit from our discussion.

of the electromagnetic field of incoming e^+ and e^- is separated by Z^0 lifetime from the analogous radiation out of the produced muon pair. As a result, we expect the interference among the initial and final state bremsstrahlung to cancel out. It is not trivial, however, to put this statement on a quantitative level, for instance by showing this cancellation in an explicit perturbative calculation. The main obstacle is that one has in this case to integrate without any approximation over all of the real hard photon phase space and to combine very carefully the contributions from real and virtual photon emission. A first example of such a calculation was provided in ref. [4] for the case of the initial state spin asymmetry A_{LR} , i.e., essentially for the total cross section.

Let us now describe briefly how we proceed in order to calculate the contribution from initial-final state QED interference to A_{FB} . We divide, as usual, the entire photon phase space into a hard part where the energy of the photon E^γ is above some infrared cut-off $E_0^\gamma \ll \sqrt{s}$, i.e. $E^\gamma > E_0^\gamma$, and another part $E^\gamma < E_0^\gamma$ called the soft part. In the latter one we also include the Born contribution and contributions from virtual photons, i.e., boxes and vertices (we omit vacuum polarization). In the soft part, the QED interference contribution consists of the interference of the γ - γ and γ - Z (photonic) boxes with the Born amplitude. The amplitude for the γ - Z box is taken from ref. [5] while the hard initial-final state interference differential cross section is taken from ref. [6].

We calculate the interference contribution in the soft part according to the following formula

$$\delta A_{FB}^* = \frac{3}{2} \frac{1}{\sigma_{tot}} \int_{-1}^1 \frac{d\sigma_{int}}{d\cos\theta} \cos\theta d\cos\theta \quad (1)$$

where $d\sigma_{int}$ includes contributions from photonic box diagrams and the interference part of the real soft photon bremsstrahlung. The combined result is infrared free. For reasons which will be explained later on we define $A_{FB}^* = \frac{3}{2} \langle \cos\theta \rangle$ instead of, conventionally, $A_{FB} = (\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0))/\sigma_{tot}$. The two asymmetries coincide for any distribution of the type $d\sigma/d\cos\theta = c_1 (1 + \cos^2\theta) + c_2 \cos\theta$, i.e., for a Born distribution. The contribution from real soft photons to A_{FB} can be integrated over photon energy and $\cos\theta$ rather easily. The contribution from boxes includes at worst an integral over $\cos\theta$ of a third order polynomial in $\cos\theta$ times a dilogarithm function. It is still feasible analytically.

We cannot even start to calculate the interference contribution to A_{FB} in the hard bremsstrahlung part of the phase space before we have decided how to define a scattering

angle and A_{FB} in this case. It can be done in many ways since in the presence of an additional photon the muons are not antiparallel anymore. Experimentalists define $\cos \theta$ taking direction of one of the muons, say the μ^- , or they take a direction of a vector difference of μ^- and μ^+ momenta, $\vec{q}(\mu^-) - \vec{q}(\mu^+)$, and then use the conventional definition of $A_{FB} = 2\sigma_{\cos \theta > 0} / \sigma_{tot} - 1$. It appears that it is very difficult⁽³⁾ to integrate the bremsstrahlung differential cross section with the condition $\cos \theta > 0$ where $\cos \theta$ is defined in either of the two above ways.

It happens, however, that we are able to calculate A_{FB} analytically rather easily, over all the hard phase space, if its definition is adjusted appropriately. Let us introduce the following definition

$$A_{FB}^* = \frac{3}{2} \frac{1}{\sigma_{tot}} \int c^* d\sigma, \quad \text{where} \quad c^* = \frac{q^3(\mu^-) - q^3(\mu^+)}{q^0(\mu^-) + q^0(\mu^+)}. \quad (2)$$

where we assume that the third axis is directed along the e^- beam. This is clearly an extension of a previous definition $A_{FB}^* = \frac{3}{2} \langle \cos \theta \rangle$ from the soft part since c^* coincides with $\cos \theta$ in the $E^\gamma \rightarrow 0$ limit⁽⁴⁾. With this new definition we are able to integrate analytically the initial-final state QED interference contribution over the hard part of the phase space defined by $E_0^\gamma < E^\gamma < E_1^\gamma$ where an upper limit on photon energy E^γ may be set arbitrarily high ($E_1^\gamma \leq \sqrt{s}/2$).

The combined soft and hard interference contribution

$$\delta A_{FB}^* = \frac{3}{2} \frac{1}{\sigma(E^\gamma < E_1^\gamma)} \int c^* d\sigma_{int} \quad (3)$$

does not depend on the infrared cut-off E_0^γ anymore and is given by the following compact expression

$$\delta A_{FB}^*(\sqrt{s}, E_1^\gamma) = \frac{3}{2} \frac{\sigma_0}{\sigma(E^\gamma < E_1^\gamma)} \frac{\alpha}{\pi} q\bar{q} \operatorname{Re} \left\{ \left(c_0 + \frac{c_1}{2\xi^*} \right) A_{\gamma\gamma} + \left(\frac{c_1}{2\xi} + \frac{c_2}{|\xi|^2} \right) A_{\gamma Z} \right\}, \quad (4)$$

where

$$A_{\gamma\gamma} = -5 \ln k_1 + \frac{65}{36} + 3k_1 + \ln \left(1 - \frac{k_1}{2} \right) - \frac{2}{3} \pi i, \quad (5)$$

⁽³⁾ The analytical calculation is probably still feasible but the result would be given by a lengthy and complicated expression, an attempt in this direction was made in ref. [7].

⁽⁴⁾ Monte Carlo exercises show that in the presence of bremsstrahlung A_{FB}^* differs rather little from other, more conventional definitions of A_{FB} , especially at the Z^0 .

$$\begin{aligned}
A_{\gamma Z} = & -5 \ln k_1 + 3\xi k_1 + \frac{31}{9}\xi - 9\xi^2 + 4\xi^3 + \left(5 - \frac{15}{2}\xi + 3\xi^2 + \frac{1}{2}\frac{\xi^2}{\xi-2}\right) \ln\left(\frac{k_1 - \xi}{1 - \xi}\right) \\
& - \left(\frac{5}{2} - \frac{11}{2}\xi + 9\xi^2 - 4\xi^3 - \frac{1}{2}\frac{\xi^2}{\xi-2}\right) \ln(1 - \xi) + \left(\frac{11}{6} - \xi - \frac{\xi}{2}\frac{1}{\xi-2}\right) \xi \ln(-\xi) \\
& - \frac{\xi}{\xi-2} \ln\left(1 - \frac{k_1}{2}\right) + 4\xi(1 - \xi)^3 \left(\text{Li}_2\left(\frac{-\xi}{1 - \xi}\right) - \frac{\pi^2}{6}\right), \tag{6}
\end{aligned}$$

and we define

$$\begin{aligned}
k_1 &= \frac{2E_1^\gamma}{\sqrt{s}}, & \xi &= \frac{s - M_Z^2 + iM_Z\Gamma_Z}{s}, & \sigma_0 &= \alpha\pi^2/3s, \\
c_0 &= (q\tilde{q})^2, & c_1 &= 2q\tilde{q}v\tilde{v}, & c_2 &= (v^2 + a^2)(\tilde{v}^2 + \tilde{a}^2). \tag{7}
\end{aligned}$$

The variables $q, v, a, \tilde{q}, \tilde{v}, \tilde{a}$ are the charge, vector and vector-axial coupling constants of the electron and final state muon respectively. The integrated cross section $\sigma(E^\gamma < E_1^\gamma)$ may be obtained by an analytical or a numerical integration of the distributions $d\sigma_{soft}/d\cos\theta$ and $d\sigma_{hard}/dE^\gamma$ which are given in ref. [6].

Our main conclusion results from numerical examination of the formula (4) and is the following: *The contribution from the QED initial-final state interference to A_{FB} is smaller than 10^{-3} in the energy range $M_Z \leq \sqrt{s} \leq M_Z + 2\text{GeV}$ and for $k_1 \geq 0.2$ i.e. for $E_1^\gamma \geq 10\text{GeV}$. It is equal to $5 \cdot 10^{-4}$ at $\sqrt{s} = M_Z$ and for no cut-offs. In the above range of energies and cut-offs and for limited statistics ($< 10^5$ events) this contribution may be neglected completely. At the higher precision level, for stronger cut-offs or further away from Z^0 position, we classify it as being under control i.e. as calculable and removable.*

From formula (4) it is also seen explicitly that for $\sqrt{s} = M_Z$ and $k_1 \simeq 1$ (on top of the Z^0 and no cut-offs) the interference contribution δA_{FB}^* is proportional to $(\alpha/\pi)(\Gamma_Z/M_Z)$. Numerical evaluation reveals that the additional suppression factor Γ_Z/M_Z disappears gradually when cut-off k_1 is lowered. It disappears completely for $k_1 \simeq \Gamma_Z/M_Z \simeq 0.023$ where δA_{FB}^* is again of order 1%, as in the off-resonance and no cut-off case.

We refer the reader for more details on the calculations and more comments on the results to a forthcoming paper [8]. We conclude this paper with a remark on the comparison with another Monte Carlo calculation. A very good agreement between formula (4) and the Monte Carlo of ref. [9] was obtained outside the Z^0 resonance. Near the Z^0 resonance we compared analytical results with a program based on the $O(\alpha)$ calculation i.e. of the class of ref. [9], but with the exact formula for boxes from refs. [5]. The agreement to

within $2 \cdot 10^{-3}$ was obtained. We have also found that in this type of the Monte Carlo the result depends on the dummy infrared cut-off E_0^γ by $\pm 2 \cdot 10^{-3}$ (up to $\pm 5 \cdot 10^{-3}$ in some cases) i.e. more than the contribution itself. Near the Z_0 , in this type of the Monte Carlo, it may be better to set this contribution to zero by hand then to have it wrong. We would like to stress, however, that in an improved Monte Carlo program [10] the QED interference contribution should be definitely included because there are experimentally accessible regions of the phase space where it is rather large.

Finally, let us note that the smallness ($\delta A_{FB} \leq 10^{-3}$) of the initial-final state interference contribution at the Z^0 resonance was conjectured in ref. [11] without actually stating the range of cut-off's and $\sqrt{s}$.

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