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VERY LARGE INTERMEDIATE SCALES IN THREE-GENERATION MODELS*

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Abstract

The issue of large intermediate scales is addressed in a class of three-generation models based on the best-known Calabi-Yau compactification of the heterotic string. The renormalisation group equations for this class are studied to determine the scale, $\mu \equiv M_I$, at which the first "intermediate-scale" symmetry breaking occurs. Throughout most of the parameter space specifying the models, M_I is found to be extremely close to the compactification scale, M_c , as is required by phenomenology. This is due to the large number of fields which are massless below M_c . Depending on unknown normalisation factors, different directions of symmetry breaking are possible, including the one thus far preferred on phenomenological grounds.

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Over the past two years, a three-generation superstring model^[1,2,3] based on Calabi–Yau compactification^[4] of the heterotic string^[5] has been evaluated in great detail. This model has provided a first step towards understanding the detailed low-energy phenomenology of the superstring. It employs the Wilson-loop flux breaking mechanism^[6] to reduce the E_6 gauge symmetry to $SU(3)^3$ at the compactification scale, M_c , and then relies on the Higgs mechanism to break the latter down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at an intermediate scale much larger than the electroweak scale, M_W . Actually, this model corresponds to a continuous family of models specified by the complex structure of the compact manifold.

Recently, questions have been raised^[7] as to whether the model is in fact realistic: in particular, whether a large enough intermediate scale can be generated. A large intermediate scale is clearly necessary to give masses to the unobserved fields appearing in the massless spectrum at M_c ,^(F1) and, more importantly, to inhibit proton decay. The latter leads to the constraint on M_I that, at the very least, $M_I > 10^{14} \text{ GeV}$, although it has been argued that 10^{16} GeV is more realistic^[7]. Small enough neutrino masses also seem to rely on a large intermediate scale. In what follows we perform a quantitative examination of intermediate-scale symmetry breaking in the above models through one-loop renormalisation group analysis of the scalar potential.

Before concentrating on this specific model, some general remarks on large intermediate scales^[8] are in order. A first worry is whether the consequences of a very large intermediate scale in the effective four-dimensional theory are to be believed, as string effects may be important. However, until an honest string calculation proves the opposite, one must assume, as is implicit in the following, that radiative corrections calculated in the effective field theory

^(F1) Furthermore, if M_I is not large, the gauge couplings become too small by the time symmetry breaking occurs (since the number of massless fields for scales $\mu > M_I$ is large).

(via the renormalisation group) yield a good approximation to the real physics. Secondly, large intermediate scales appear not only possible (as testified by the specific models discussed below), but necessary in general. As in the case at hand, they can be the final explanation for the absence of proton decay and for the smallness of neutrino masses (which may not be guaranteed solely by the Yukawa couplings in phenomenologically-acceptable models derived from the string ^[9]). Moreover, they will be present in almost any model with a large number of massless fields below M_c (which is the case for many known string vacuum solutions), because, as detailed below, the renormalisation group equations have large coefficients, which trigger symmetry breaking at large intermediate scales as soon as supersymmetry is softly broken. In fact, the cascade of symmetry breaking only stops when the only fields remaining massless are the known chiral multiplets. Otherwise, if many $SU(3)_C \times SU(2)_L \times U(1)_Y$ vector-like fields are left massless (if they are essentially decoupled in the superpotential), the coupling constants of the observed gauge interactions are either not small enough to ensure a perturbative regime from M_c to the electroweak scale, M_W , or else they do not reproduce their observed values at M_W .

Returning to the specific class of models alluded to above, it is distinguished, at present, as having been evaluated in enough detail to allow a quantitative discussion of the scales of symmetry breaking. The renormalisable Yukawa couplings at M_c are known as functions of the complex structure of the Calabi–Yau manifold and the (unknown) field normalisation factors^[2], thus enabling one to evolve the scalar potential from the scale $\mu = M_c$ down to a scale $\mu \equiv M_I$, at which one of the fields obtains a non-zero VEV, $\langle \phi \rangle \sim M_I^{(F2)}$. Evolving *all* the 65 couplings (Yukawa, gauge and supersymmetry-breaking trilinear) and 15 independent masses, we have explored the possible values of M_I for a class of complex structures specified by a single complex variable c , and for different choices of the normalisation factors. This class is appealing be-

^(F2) Note that in what follows M_I is defined as the scale $\mu = M_I$ at which symmetry breaking occurs, not the VEV.

cause the corresponding three-family models possess the maximal set of *exact* discrete symmetries, and there is some evidence^[3] that this choice is favoured.

It is found that a large value of M_I is not only possible but almost inevitable. Furthermore, for all but a limited range of values for c it is very large indeed: $0.1 \leq M_I/M_c \leq 1$. This is true even if the unknown normalisation factors are large such that the Yukawa couplings are much smaller than the gauge couplings at $\mu = M_c$. One may understand this in terms of the large number of fields which are present, and which, through the renormalisation group equations, drive scalar masses negative very rapidly.

The best known^[10] Calabi–Yau space R with three generations can be defined in terms of a manifold $R_0 \subset CP^3 \times CP^3$ with homogeneous coordinates $[x_i; y_i]$ ($i = 0, 1, 2, 3$) obeying^[11]

$$a_{ijk}x_i x_j x_k = 0; \quad b_{ijk}y_i y_j y_k = 0; \quad c_{ij}x_i x_j = 0. \quad (1)$$

The three-generation manifold R is then defined by $R = R_0/G$, where G is a freely-acting Z_3 group defined by the action of its generator g :

$$\begin{aligned} g : [x_0, x_1, x_2, x_3; y_0, y_1, y_2, y_3] \\ \longmapsto [x_0, \alpha^2 x_1, \alpha x_2, \alpha x_3; y_0, \alpha y_1, \alpha^2 y_2, \alpha^2 y_3], \end{aligned} \quad (2)$$

where $\alpha^3 = 1$. Hence (after some linear coordinate redefinitions), R can be specified in terms of *nine* independent complex parameters a_i, b_i, c_k ($i = 1, 2; k = 1, \dots, 5$) specifying its complex structure^[1],

$$\begin{aligned} x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 &= 0, \\ y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 &= 0, \\ x_0 y_0 + c_1 x_1 y_1 + c_2 x_2 y_2 + c_3 x_3 y_3 + c_4 x_2 y_3 + c_5 x_3 y_2 &= 0. \end{aligned} \quad (3)$$

The non-zero Hodge numbers of R are found to be $h^{1,1} = 6$ and $h^{2,1} = 9^{[1,11]}$, corresponding to three generations of chiral supermultiplets transforming as the 27 of E_6 , together with six pairs of $(27 + \overline{27})$ supermultiplets. After flux

breaking, the surviving gauge group is $SU(3)_C \times SU(3)_L \times SU(3)_R$. The massless superfields comprise nine multiplets of leptons $\lambda_i \sim (1, \bar{3}, 3)$, seven multiplets of quarks $Q_i \sim (3, 3, 1)$ and seven multiplets of antiquarks $q_i \sim (\bar{3}, 1, \bar{3})$ all coming from the nine 27s, together with six lepton multiplets $\bar{\lambda}_i$, four quark multiplets $\bar{Q}_i$ and four antiquark multiplets $\bar{q}_i$ coming from the six $\bar{27}$ s. The matter content may be summarised by

$$\begin{aligned}\lambda_i, \bar{\lambda}_i &\in 3 \times 27 + 6 \times (27 + \bar{27}) \\ q_i, \bar{q}_i, Q_i, \bar{Q}_i &\in 3 \times 27 + 4 \times (27 + \bar{27}).\end{aligned}\tag{4}$$

Much of the phenomenology of this model has been evaluated in great detail ^[1,2,3]. The scalar potential possesses “flat” directions along which scalar fields may acquire large VEV’s, $\langle \phi \rangle \sim M_I$, which break the gauge symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. If $\langle \phi \rangle$ is large enough, proton decay is inhibited and the observed low-energy mass spectrum and mixing angles can be reproduced.

The nine parameters a_i, b_i, c_k in Eq. (3) correspond to the $h^{2,1} = 9$ independent deformations of the complex structure of the compact manifold, and should be interpreted as VEV’s of complex gauge-singlet scalar fields ^(F3). These parameters determine the Yukawa coupling of the effective four-dimensional field theory, and for special values give rise to *exact* discrete symmetries. It has been argued^[3] that, after supersymmetry breaking, the theory favours a complex structure which leaves unbroken at least some discrete symmetries. This leads one to consider the class of complex structures leading to the maximal set of discrete symmetries. This class is specified by ^[1]

$$\begin{aligned}a_i &= b_i = c_4 = c_5 = 0, \\ c_1 &= 1, \quad c_2 = c_3 = c,\end{aligned}\tag{5}$$

^(F3) These singlet superfields do not have renormalisable couplings to the gauge non-singlets and are not included in this analysis.

and depends on only one complex parameter, c .

The 27^3 and $\overline{27}^3$ renormalisable Yukawa couplings have been evaluated^[2] in terms of c and the normalisations of the associated fields. For the $\overline{27}$ there is one common normalisation factor, $\overline{N}$, in contrast to the eight independent ones N_i ($i = 1, \dots, 8$) for the 27 's. The Yukawa coupling normalisations, Ω_{ijk} , $\overline{\Omega}$, are related to the field normalisations via

$$\Omega_{ijk} = N_i N_j N_k, \quad \overline{\Omega} = \overline{N}^3, \quad (6)$$

and a large value of Ω or $\overline{\Omega}$ corresponds to a small coupling. In Tables 1 and 2 the Yukawa couplings are listed at the compactification scale in terms of c . (The corresponding normalisation factors are not shown, but are taken into account in the calculation.) The notation for the fields is along the lines of Eq. (4) and is more clearly defined in refs. [1,2].

The one-loop renormalisation group equations for softly-broken supersymmetry are well known^[12]. To write these down for this class of models is straightforward, but special care should be taken to obtain the correct factors. The large number of equations is significantly reduced after taking into account the existing symmetries.

For the fields in the 27 's there are: 25 independent Yukawa couplings, k_i ($i = 1, \dots, 25$); eight independent supersymmetry-breaking scalar masses, m_j ($j = 1, \dots, 8$); and 25 independent supersymmetry-breaking trilinear scalar couplings, A_i ($i = 1, \dots, 25$). Similarly for the fields in the $\overline{27}$ s there are: six Yukawa couplings, $\overline{k}_i$ ($i = 1, \dots, 6$); four independent supersymmetry-breaking scalar masses, $\overline{m}_j^2$ ($j = 1, \dots, 4$); and six trilinear scalar couplings, $\overline{A}_i$ ($i = 1, \dots, 6$). In addition, there are the three $SU(3)$ gauge couplings $g_{C,L,R}$ and gaugino masses $m_{1/2,C,L,R}$. As will be evident below, it is the large number of fields, implying many contributions with large coefficients to the various renormalisation group equations, that drives the large intermediate scales. For example, the corresponding equation for k_1 , the Yukawa coupling for the first term in Table 1,

reads

$$\mu \frac{dk_1}{d\mu} = \frac{k_1}{16\pi^2} (12k_1^2 + 144k_2^2 + 72k_3^2 + 8k_4^2 + 8k_5^2 + 4k_6^2 + 12k_{10}^2 + 12k_{11}^2 + 6k_{12}^2 + 12k_{13}^2 + 6k_{14}^2 - 8g_L^2 - 8g_R^2). \quad (7)$$

Noting that the 27 and $\overline{27}$ sectors are decoupled to this order, they can be treated separately. The initial conditions for the evolution consist of specifying the unified gauge coupling, g_X , and the common supersymmetry breaking parameters, m_0^2 , $m_{1/2}^0$ and A_0 at $\mu = M_c$. Analyses of supersymmetry breaking in superstring models^[13] suggest that m_0^2 and/or $m_{1/2}^0$ are likely to be the dominant sources of supersymmetry breaking, and it shall be henceforth assumed that $A_0 = 0$. (However, non-zero A_i are generated if $m_{1/2}^0 \neq 0$.) Throughout, the gauge coupling is taken to be $g_X = 2$ at M_c . (Although, in this paper, only the relative sizes of the different couplings important, in the final analysis, the correct strength of the strong coupling at low energies should be reproduced.)

If $(m_{1/2}^0)^2 \ll m_0^2$ then the effects of $m_{1/2}$ and A_i can be neglected, and intermediate-scale symmetry breaking will be signalled by a scalar mass-squared going negative. For illustration, Fig. 1 shows the value of M_I as a function of the normalisation factors for the 27 and $\overline{27}$ sectors, assuming a common normalisation factor, $\Omega_{ijk} \equiv \Omega$, for all 27^3 couplings. One can see that very large intermediate scales are indeed generated with $\overline{\lambda}_1$ obtaining a non-zero VEV in the $\overline{27}$ sector (independent of c), and a combination of $(p_1\lambda_1 + p_2\lambda_2 + p_3\lambda_3 + p_4\lambda_4)$ in the 27 sector for $c = 1, 10$ (and λ_{5+} for $c = 0.1$). Taking $\Omega, \overline{\Omega} = 1$ means that the Yukawa couplings will be of the same order of magnitude as the gauge couplings (for $c = 0(1)$) at M_c . Even for larger values of $\Omega, \overline{\Omega}$, corresponding to smaller Yukawa couplings at M_c , a very large intermediate scale can be obtained with the colour gauge coupling $g_C(\mu = M_I)$ a reasonable magnitude. It is evident that for $\Omega \approx \overline{\Omega} < 1$, it is the 27 sector, and not the $\overline{27}$ sector, which in general is responsible for the generation of M_I .

In Fig.2, M_I/M_c is plotted in the 27 sector as a function of real positive c for a common 27^3 normalisation, $\Omega_{ijk} \equiv \Omega$. It is clear that even for very large

values of Ω , M_I is very close to M_c for all but a limited range of values for c . The cusps in the figure at $c = 0.87$ correspond to the boundary between the regions in which $\langle \sum_{i=1}^4 \lambda_i \rangle$ becomes non-zero ($c > 0.87$) and $\langle \lambda_{5+} \rangle$ becomes non-zero ($c < 0.87$). Very similar behaviour was also found along other directions in the complex c plane, with an intermediate scale which is again typically very large.

In cases in which $(m_{1/2}^0)^2 > m_0^2$, significant non-zero trilinear scalar couplings can be generated. In this case the criterion for a minimum is not as simple as $m^2 < 0$. However, since $A_0 = 0$ and M_I is in general very close to M_c , the A_i will not have enough time to grow very large before symmetry breaking occurs, provided $m_{1/2}^0$ is not much larger than m_0 . Consequently, the A -terms should not significantly alter the potential in the neighbourhood of the minimum. For $m_{1/2}^0 \leq 10m_0$ we have checked that, to a reasonable approximation, M_I is still be given by the scale at which m^2 becomes negative. The dashed curve in Fig. 3 shows the results for $m_{1/2}^0 = 10m_0$ in the $\overline{27}$ sector. It is qualitatively similar to the case of $m_{1/2}^0 \ll m_0$. For the 27 sector, the curves changed very little for $m_{1/2}^0 = 10 m_0$.

Of course, if $m_\phi^2 < 0$ at a scale $\mu = M_I$ then a large VEV will develop only if the field, ϕ , corresponds to an F-flat and D-flat direction of the potential. Such flat directions necessarily involve combinations of fields from the 27 and $\overline{27}$, and a minimum really only develops when $\sum m^2 < 0$, where the sum is over all these fields. The fields obtaining a negative mass-squared in the above analysis actually correspond to ‘components’ of flat directions. In fact, the discrete symmetries guarantee that the following directions are flat^(F4)

$$(\bar{\lambda}_1 \pm \bar{\lambda}_2); \text{ and } (\lambda_1 \pm \lambda_3), (\lambda_2 \pm \lambda_4), \lambda_{5\pm}, \lambda_7, \text{ or } \lambda_{8\pm}.$$

It has been checked that, along all these directions, a negative mass-squared

^(F4) There exist other leptonic flat directions, but for these it does not appear to be possible to obtain $m^2 < 0$ —at least not at this first stage of symmetry breaking.

can indeed be obtained for various choices of c and N_i . For example, if $N_{\lambda_{5+}} > N_j (j \neq \lambda_{5+})$ then it is not λ_{5+} , but any combination of λ_8 and λ_9 —in particular, $\lambda_{8\pm}$ — that gets a negative mass-squared for small $c^{(F5)}$. One can therefore expect very large VEVs along the above flat directions.

Once this first stage of symmetry breaking is realised, many fields get masses of order M_I , but many others remain massless. Then the further evolution of the renormalisation group equations will lead to a second stage of symmetry breaking and a second intermediate scale; however, the details will depend on the specific direction of the first. Nevertheless, even after these two stages of symmetry breaking, some exotic fields remain massless. They will acquire large masses (in order that the theory remain perturbative and give the correct low-energy gauge couplings) and, in doing so, define yet another large mass scale^(F6). For the case of $\langle \lambda_{8\pm} \rangle \neq 0$, a detailed analysis down to the electroweak scale will be presented in a forthcoming publication^[14].

To conclude, let us reiterate the important result that very large intermediate scales seem to be characteristic of this class of superstring models. The discrete symmetries lead to flat directions of the potential along which symmetry breaking can occur at a large intermediate scale. It is both the flatness along these directions and the great number of fields which ensure that M_I is large.

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^(F5) Not all the parameter space for the normalisation factors gives breaking along lepton directions.

^(F6) In the particular case in question, the masses and scale of decoupling are expected to be mainly set by the $27^2 \overline{27}^2$ couplings.

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TABLE CAPTIONS

Table 1. 27^3 Yukawa couplings at the compactification scale, M_c . Note that, for clarity, the normalisation factors $1/\Omega_{ijk} = 1/N_i N_j N_k$ (where i, j, k refer to the relevant fields) have been omitted. These global factors satisfy: $N_{\lambda_1} = N_{\lambda_2} = N_{\lambda_3} = N_{\lambda_4}$; $N_{\lambda_8} = N_{\lambda_9}$; $N_{q_1} = N_{q_2}$; $N_{q_4} = N_{q_5} = N_{q_6} = N_{q_7}$; and $(q \rightarrow Q)$.

Table 2. $\overline{27}^3$ Yukawa couplings at the compactification scale, M_c . Note that the common normalisation factor $1/\overline{\Omega} = 1/\overline{N}^3$ has been omitted.

FIGURE CAPTIONS

- Fig. 1. M_I in units of the compactification scale. For the 27 sector, the solid, dashed and dot-dashed curves represent $c = 1, 0.1$ and 10 respectively, and the normalisation Ω is assumed to be common to all Yukawa couplings. The dotted curve represents M_I in the $\overline{27}$ sector as a function of $\overline{\Omega}$.
- Fig. 2. M_I as a function of $\text{Re } c$ in the 27 sector for $\Omega = 1$ (solid line), 10 (dashed) and 100 (dot-dashed).
- Fig. 3. M_I as a function of the Yukawa normalisation $\overline{\Omega}$ in the 27 sector for $m_{1/2}^0 = 0$ (solid line) and $10m_0$ (dashed).

TABLE 1

i	TERM IN 27^3	k_i
1	$\lambda_1 \lambda_4 \lambda_9 + \lambda_2 \lambda_3 \lambda_8$	$(2c^3 + 1)/c$
2	$\lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \lambda_4^3$	$c(2c^3 + 1)/6(c^3 + 2)$
3	$\lambda_8^3 + \lambda_9^3$	$1/6c^2$
4	$\lambda_{5+}(\lambda_1 \lambda_3 + \lambda_2 \lambda_4)$	$2\sqrt{2}(2c^3 + 1)/c(c^3 + 2)$
5	$\lambda_7(\lambda_1 \lambda_3 - \lambda_2 \lambda_4)$	$-2(2c^3 + 1)(c^3 + 1)/c(c^3 + 2)$
6	$\lambda_{5+} \lambda_8 \lambda_9$	$\sqrt{2}/3c^2$
7	$\lambda_{5-}^2 - \lambda_{5+}$	$4\sqrt{2}/3$
8	λ_{5+}^3	$8\sqrt{2}(1 - c^3)/3c^2(c^3 + 2)$
9	$\lambda_{5+} \lambda_7^2$	$-4\sqrt{2}(2c^3 + 1)/3c^2(c^3 + 2)$
10	$\lambda_1 q_2 Q_7 + \lambda_2 q_2 Q_6 + \lambda_3 q_5 Q_1 + \lambda_4 q_4 Q_1 + (q \leftrightarrow Q)$	$(2c^3 + 1)/c$
11	$\lambda_1 q_3 Q_4 + \lambda_2 q_3 Q_5 + \lambda_3 q_6 Q_3 + \lambda_4 q_7 Q_3 + (q \leftrightarrow Q)$	$(2c^3 + 1)/2(c^3 + 2)$
12	$\lambda_1 q_6 Q_6 + \lambda_2 q_7 Q_7 + \lambda_3 q_4 Q_4 + \lambda_4 q_5 Q_5$	$c(2c^3 + 1)/2(c^3 + 2)$
13	$\lambda_8 q_1 Q_6 + \lambda_8 q_5 Q_2 + \lambda_9 q_4 Q_2 + \lambda_9 q_1 Q_7 + (q \leftrightarrow Q)$	$\frac{1}{2}$
14	$\lambda_8 q_4 Q_7 + \lambda_9 q_5 Q_6 + (q \leftrightarrow Q)$	$-1/6c$
15	$\lambda_{5+}(q_1 Q_2 + q_2 Q_1)$	$-2\sqrt{2}$
16	$\lambda_{5-}(q_1 Q_2 - q_2 Q_1)$	$2(c^3 + 1)/c$
17	$\lambda_{5+} q_3 Q_3$	$-\sqrt{2}(2c^3 + 1)/3(c^3 + 2)$
18	$\lambda_{5+}(q_4 Q_6 + q_5 Q_7 + (q \leftrightarrow Q))$	$\sqrt{2}(c^3 - 1)/3c(c^3 + 2)$
19	$\lambda_{5-}(q_4 Q_6 + q_5 Q_7 - (q \leftrightarrow Q))$	$-\frac{1}{3}$
20	$\lambda_7(q_4 Q_6 - q_5 Q_7 + (q \leftrightarrow Q))$	$(2c^3 + 1)/3c(c^3 + 2)$
21	$q_1^3 + q_2^3 + Q_1^3 + Q_2^3$	$c/6$
22	$q_3^3 + Q_3^3$	$c(2c^3 + 1)/6(c^3 + 2)$
23	$q_4^3 + q_5^3 + q_6^3 + q_7^3 + (q \rightarrow Q)$	$c/2(c^3 + 2)$
24	$q_3 q_4 q_6 + q_3 q_5 q_7 + (q \rightarrow Q)$	$-(2c^3 + 1)/6(c^3 + 2)$
25	$q_1 q_2 q_3 + Q_1 Q_2 Q_3$	$(2c^3 + 1)/c$

TABLE 2

i	TERM IN $\overline{27}^3$	$\overline{k}_i$
1	$\bar{\lambda}_1^3$	2
2	$\bar{\lambda}_1 \bar{\lambda}_2^2$	-1
3	$\bar{\lambda}_1(\bar{\lambda}_3 \bar{\lambda}_5 + \bar{\lambda}_4 \bar{\lambda}_6)$	$\frac{3}{2}$
4	$\bar{\lambda}_2(\bar{\lambda}_3 \bar{\lambda}_5 - \bar{\lambda}_4 \bar{\lambda}_6)$	$-\left(\frac{3}{2}\right)^{3/2}$
5	$\bar{\lambda}_1(\bar{Q}_1 \bar{q}_2 + \bar{Q}_3 \bar{q}_4 + (\bar{Q} \leftrightarrow \bar{q}))$	1
6	$\bar{\lambda}_2(\bar{Q}_1 \bar{q}_2 - \bar{Q}_3 \bar{q}_4 + (\bar{Q} \leftrightarrow \bar{q}))$	$-\left(\frac{3}{2}\right)^{1/2}$

FIG.1

FIG.2

FIG.3