THE QUANTUM COSMOLOGY OF AN ANISOTROPIC UNIVERSE

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ABSTRACT

Surveys of the microwave background indicate that the universe is isotropic to more than one part in 10^5 . Due to the arbitrariness of the initial conditions of the universe at the big bang singularity one cannot predict this; it is usually put in by hand. We therefore construct the Quantum Cosmology of an anisotropic universe according to the 'no boundary' prescription of Hartle and Hawking. Such a model has a well defined behaviour at the classical singularity. We then show it also implies that a large universe, such as ours, is isotropic.

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1. INTRODUCTION

In general relativity the metric plays two roles. First, it is a field defined on the space-time manifold which satisfies the Einstein equations. Secondly, it determines the structure of the manifold itself. Attempts at quantizing gravity, to bring it into line with the other fundamental forces of nature, associate the latter property with the classical, background metric configuration and the former with the excitations upon that background. More often than not, the quantization program is frustrated by uncontrollable divergences stemming from the interactions amongst the excitations.

Cosmology concerns itself with the global properties of space-time. One generally assumes, when extrapolating back from our present state towards the origin of the universe, that the metric can be treated as a classical field for most of its evolution. This has an inescapable consequence. Regardless of the behaviour of the matter fields, we would conclude that the universe had evolved from a state of infinite curvature and density, the big bang singularity. We would be unable to measure any particle properties or field configurations at this point; the laws of physics as we know them break down. This would mean that physics is unable to say anything about cosmogony.

A less daunting view is to believe that nature abhors a singularity. An elegant and ingenious way of circumventing the problem has been proposed by Hartle and Hawking [1] in the context of Quantum Cosmology [2]. One constructs a wave functional whose arguments are the metric (and matter) field configurations on some boundary. This is obtained from a path integral over closed euclidean geometries (and matter fields) which match up to their values on this boundary. This wave functional satisfies the Wheeler-DeWitt equation, and is uniquely specified by the above ‘no boundary’ prescription for the path integral. Solutions for lorentzian geometries are obtained by analytic continuation, although this is a rather contentious issue at present [3]. The wave functional is well defined when evaluated at the classical singularity. Successes so far have been in providing a natural mechanism for the universe to enter a period of inflationary expansion, exit into a matter or radiation dominated phase, and generally give the flat isotropic universe we observe today [4].

In this letter we shall illustrate the prescriptions of the Hartle-Hawking proposal by constructing the wave function for a closed Bianchi I cosmology. The following sections consider the construction of the path integral, its evaluation in the semiclassical approximation, and the analytic continuation to lorentzian space. Before astute readers wonder about how they

perform measurements in a universe which contains themselves, we will say that this is left to the discussion section at the end of the paper. Let us calculate first and ask questions afterwards.

2. CONSTRUCTING THE WAVE FUNCTION

The starting point for quantum cosmology is to choose a restricted form of the metric representing the particular classical universe we wish to study. In our case we are interested in homogeneous Bianchi cosmologies [5] and so we consider the line element,

$$ds^2 = N_E(\tau)^2 d\tau \otimes d\tau + h_{ij}(\tau) e^i \otimes e^j \quad (2.1)$$

which is written in a form amenable to calculations in euclidean space (and hence the subscript on N_E). The space is homogeneous by virtue of the spatial part of the metric, h_{ij} , and the lapse function, N_E , being functions of the ‘time’ parameter only. The one-forms which provide a basis for the three-space are time independent and satisfy,

$$de^i = -\frac{1}{2} C_{jk}^i e^j \wedge e^k$$

where C_{jk}^i are the structure constants of the respective Bianchi group. There are nine possible inequivalent choices for the structure constants and their resulting cosmologies are referred to as Bianchi I–IX.

The euclidean action for gravity we shall use in the path integral is simply,

$$S_E = -\frac{1}{16\pi G_N} \int_{\mathcal{M}} d^4x \sqrt{g} (R - 2\Lambda) - \frac{1}{8\pi G_N} \int_{\partial\mathcal{M}} d^3x \sqrt{h} K \quad (2.2)$$

where the last term is required if the space over which we are integrating, $\mathcal{M}$, has a boundary, $\partial\mathcal{M}$ [6]. We take $\partial\mathcal{M}$ to be a slice at constant τ . K is the trace of the second fundamental form on $\partial\mathcal{M}$, *i.e.* $K = h^{ij} K_{ij}$, and for our choice of metric $K_{ij} = \dot{h}_{ij}/2N_E$, which also only depends upon τ . A convenient parametrization of the spatial metric is,

$$h_{ij}(\tau) = v(\tau)^{2/3} \left[e^{2\beta(\tau)} \right]_{ij}$$

and v scales as the volume of the space. The matrix β is traceless and diagonal. Its two

independent parameters, $\beta_{\pm}$, are defined through,

$$\beta_{11} = \beta_+ + \sqrt{3}\beta_-$$

$$\beta_{22} = \beta_+ - \sqrt{3}\beta_-$$

$$\beta_{33} = -2\beta_+$$

and represent the deviation of the three-space from isotropy.

Substituting the resulting metric into the action (2.2) we are led to the following,

$$S_E = \frac{6V_3}{16\pi G_N} \int d\tau \frac{v}{N_E} \left(-\frac{1}{9} \left(\frac{\dot{v}}{v} \right)^2 + \dot{\beta}_+^2 + \dot{\beta}_-^2 - \frac{N_E^2}{6} ({}^3R - 2\Lambda) \right) \quad (2.3)$$

Since all parameters depend only on τ we have been able to extract and perform the volume integral over the three-space, $V_3 = \int e^1 \wedge e^2 \wedge e^3$. 3R is the curvature scalar of the three-space, constructed solely from h_{ij} . Its functional form is dictated once we specify the Bianchi group. The simplest cosmology to study is Bianchi I in which case the structure constants vanish. The closed three-space has the topology of a three-torus. A torus is a flat manifold and hence its curvature, 3R , vanishes. Its volume is finite, being the product of its three circumferences.

The (euclidean) canonical momenta conjugate to v and $\beta_{\pm}$ can be read off from (2.3),

$$\begin{aligned} P_v &= -\frac{\mu}{9N_E} \frac{\dot{v}}{v} \\ P_{\pm} &= \frac{\mu v}{N_E} \dot{\beta}_{\pm} \end{aligned} \quad (2.4)$$

where we have defined $\mu = 3V_3/4\pi G_N$ for notational convenience. The hamiltonian is then,

$$\mathcal{H} = -\frac{9v}{2\mu} P_v^2 + \frac{1}{2\mu v} (P_+^2 + P_-^2) - \frac{\mu v \Lambda}{6} \quad (2.5)$$

Rewriting the action in its canonical form,

$$S_E = \int (P_v dv + P_+ d\beta_+ + P_- d\beta_- - N_E \mathcal{H} d\tau) \quad (2.6)$$

we find that variation with respect to N_E gives the constraint, $\mathcal{H} = 0$, on the extremal path. One has the freedom to choose how to label points on this path, or equivalently label

‘time’, by specifying the form of the lapse function. We shall opt for $N_E(\tau) = 1$. Solving the equations of motion and using this constraint we find the extremal path is parametrized by,

$$\begin{aligned} v(\tau) &= \frac{3\sqrt{P^2}}{\mu H} \sin H\tau \\ \beta_{\pm}(\tau) &= \frac{P_{\pm}}{3\sqrt{P^2}} \ln \left[\tan \frac{H\tau}{2} \right] \end{aligned} \quad (2.7)$$

and for the momenta,

$$\begin{aligned} P_v(\tau) &= -\frac{\mu H}{9} \cot H\tau \\ P_{\pm} &= \text{constant} \end{aligned} \quad (2.8)$$

We have defined $H^2 = 3\Lambda$, $P^2 = P_+^2 + P_-^2$ and chosen $v(0) = 0$. Clearly the four-manifold is closed, and has finite volume since v returns to its initial value of zero after a finite time. We have also chosen the $\beta_{\pm}$ to vanish at the point of maximum expansion. Any other value can be absorbed into a redefinition of V_3 .

So much for the classical solution. The quantum mechanical wave function is given by a path integral,

$$\Psi(v, \beta_{\pm}) = \int e^{-S_E} \mathcal{D}[g_{\mu\nu}] \quad (2.9)$$

where we integrate over all closed Euclidean geometries which start at $\tau = 0$ and match up to the values, $(v, \beta_{\pm})$, on some boundary $\partial\mathcal{M}$, which we suppose occurs at $\tau = \tau_f$. Then the integral (2.6) has upper bounds $v(\tau_f) = v$ and $\beta_{\pm}(\tau_f) = \beta_{\pm}$. However, the functional integral is not a gaussian and even the measure is not well defined. Any evaluation of it must rely on some approximation method.

3. THE SEMICLASSICAL APPROXIMATION

In the semiclassical approximation to the path integral the wave function is given by,

$$\Psi(v, \beta_{\pm}) \approx N_0 \sum_i A_i e^{-S_i} \quad (3.1)$$

where N_0 is a normalization factor and S_i are the actions of the *classical* solutions which are closed and which have the given three metric $(v, \beta_{\pm})$ on the surface $\partial\mathcal{M}$. The prefactors, A_i ,

are determinants due to small fluctuations about these classical solutions. However, from (2.7) it is evident that a given value of v corresponds to two different τ 's and so there are two different four-geometries corresponding to a given $(v, \beta_{\pm})$ on $\partial\mathcal{M}$. This ambiguity can be resolved by transforming to the ‘momentum representation’ [1].

The wave function in this representation is given by an expression similar to (3.1), but with an action S^p defined through a legendre transformation on S_E ,

$$S^p = S_E - P_v(\tau_f)v(\tau_f) - P_+(\tau_f)\beta_+(\tau_f) - P_-(\tau_f)\beta_-(\tau_f) \quad (3.2)$$

It is the action appropriate to keeping $(P_v, P_{\pm})$ fixed on $\partial\mathcal{M}$. The momentum space wave function is approximately,

$$\Psi(P_v, P_{\pm}) \approx N_p \sum_i A_i^p e^{-S_i^p} \quad (3.3)$$

This turns out to be particularly convenient in the case of Bianchi I since the momenta $P_{\pm}$ are constants. Once the wave function is known in the momentum representation, we can perform an inverse laplace transformation in P_v to obtain the wave function in terms of v and $P_{\pm}$. Taken all together,

$$\Psi(v, P_{\pm}) \approx \frac{N_p}{2\pi i} \int_{\Gamma} dP_v \exp(-S^p - P_v v) \quad (3.4)$$

where P_v is considered a complex variable and the path of integration, Γ , must be chosen to the *left* of all singularities of the exponent. Clearly if $v < 0$ one can close the contour in the P_v plane such that it contains no singularities. This means that $\Psi(v, P_{\pm})$ vanishes for $v \leq 0$.

We now evaluate S^p . Inserting solutions (2.7) and (2.8) into (2.6), and integrating up to final time τ_f , one finds,

$$S_E = \frac{\sqrt{P^2}}{3} (1 - \cos H\tau_f)$$

Whence, inserting the final values of the momenta and coordinates into (3.2) we arrive at,

$$S^p = \frac{\sqrt{P^2}}{3} \left(1 - f\left(\frac{9P_v}{\mu H}\right) \right) \quad (3.5)$$

where $f(x) = \ln(x + (1 + x^2)^{1/2})$. This is expressed in terms of the momenta only. The leading $\sqrt{P^2}/3$ can be absorbed into the normalization N_p in (3.4). In order to perform the

contour integral we need to define an analytic continuation of $\sqrt{P^2}f(x)$. The cuts on x are shown in fig. 1. The cut from $x = +i$ to $-i$ renders $(1 + x^2)^{1/2} = (x - i)^{1/2}(x + i)^{1/2}$ single valued; we choose the branch that yields a positive number when evaluated on the negative real axis. The cut from $x = 0$ to ∞ can be shown to be the standard cut on the argument of the logarithm.

The integral (3.4) is well suited to evaluation by the method of steepest descent. For this we need to know the stationary points of $S^v = S^p + P_v v$. We then choose our path Γ to pass through these saddle points along the path of steepest descent yet still keeping to the left of the cuts. S^v is stationary with respect to P_v when,

$$v = \frac{3\sqrt{P^2}}{\mu H} \left(1 + \left(\frac{9P_v}{\mu H} \right)^2 \right)^{-1/2} \quad (3.6)$$

or equivalently when,

$$x^2 = \left(\frac{9P_v}{\mu H} \right)^2 = -1 + \frac{9P^2}{\mu^2 H^2} \frac{1}{v^2}$$

As we are ultimately interested in the solution for a lorentzian universe we can rotate the (euclidean) momenta, $P_{\pm}$, back to their values in lorentzian space. The rotation is accomplished by replacing N_E by iN_L in (2.1). From (2.4) this is equivalent to replacing $P_{\pm}$ by $-ip_{\pm}$, where the latter are the momenta that would have resulted if we had considered the classical theory in lorentzian space at the outset. Whence there are two stationary points of the action S^v at,

$$x = \pm i \left(1 + \frac{v^{*2}}{v^2} \right)^{1/2} \quad (3.7)$$

and we have defined $v^* = 3\sqrt{p_+^2 + p_-^2}/\mu H$. These are indicated in fig. 1. They occur at real lorentzian values of the momentum conjugate to v . We are now in a position to evaluate the action at these two points.

With our choice of analytic continuation of $f(x)$, we find that,

$$(1 + x^2)^{1/2} = \mp i \frac{v^*}{v}$$

for the two values of (3.7). Upon comparison with (3.6) we must continue the $\sqrt{P^2}$ to the

respective values,

$$\sqrt{P^2} = \mp i \sqrt{p_+^2 + p_-^2}$$

for consistency. It is also straightforward to show that,

$$f(x) = \pm i \frac{\pi}{2} - \coth^{-1} \sqrt{1 + \left(\frac{v}{v^*}\right)^2}$$

at the two points. Whence S^v has the two stationary values,

$$S_{\pm}^v = \pm i \frac{\sqrt{p_+^2 + p_-^2}}{3} \left\{ \sqrt{1 + \left(\frac{v}{v^*}\right)^2} - \coth^{-1} \sqrt{1 + \left(\frac{v}{v^*}\right)^2} \right\} - \frac{\pi}{6} \sqrt{p_+^2 + p_-^2} \quad (3.8)$$

which are complex conjugates of each other.

Inspection of the second derivative of S^v shows that the paths of steepest descent pass through the upper and lower stationary points at angles $\pi/4$ and $3\pi/4$. We can distort our contour to pass through them as depicted in fig. 1. Adding up the two resulting contributions to (3.4) we obtain,

$$\Psi(v, p_{\pm}) \approx \mathcal{A} \cos \left(\frac{p}{3} \left\{ \sqrt{1 + \left(\frac{v}{v^*}\right)^2} - \coth^{-1} \sqrt{1 + \left(\frac{v}{v^*}\right)^2} \right\} + \frac{\pi}{4} \right) \quad (3.9)$$

where $p = \sqrt{p_+^2 + p_-^2}$. $\mathcal{A}$ is a prefactor to be determined in the next section. The reason that the contributions added, and hence gave the cosine solution, is that the determinants from the fluctuations, which have been absorbed into $\mathcal{A}$, are even functions of $p_{\pm}$. The wave function is real, as it should be from CPT invariance [1].

4. THE WHEELER-DEWITT EQUATION

If we had kept the N_E explicit in (2.9) and differentiated $\Psi(v, \beta_{\pm}, N_E)$ with respect to N_E , we would have found that Ψ satisfied a functional differential equation: the Wheeler-DeWitt equation. It is an operator version of the classical constraint $\mathcal{H} = 0$. Expressed in terms of the lorentzian momenta,

$$\mathcal{H} = \frac{9v}{2\mu} p_v^2 - \frac{1}{2\mu v} (p_+^2 + p_-^2) - \frac{\mu v \Lambda}{6} \quad (4.1)$$

Side-stepping questions of factor ordering, a problem that is still not fully resolved, we can make $\mathcal{H}$ an operator by formally substituting,

$$p_v^2 \longrightarrow -\frac{\partial^2}{\partial v^2} \quad \text{and} \quad p_{\pm}^2 \longrightarrow -\frac{\partial^2}{\partial \beta_{\pm}^2}$$

Actually, since $\hat{p}_{\pm}$ commute with $\hat{\mathcal{H}}$, the eigenvalues $p_{\pm}$ are constants and thus label orthogonal eigenstates. These are the wave functions of (3.9). The Wheeler-DeWitt equation for these functions reads,

$$\left(-\frac{9v}{2\mu} \partial_v^2 - \frac{1}{2\mu v} (p_+^2 + p_-^2) - \frac{\mu v \Lambda}{6} \right) \Psi(v, p_{\pm}) = 0 \quad (4.2)$$

This can be solved in terms of Bessel functions. However, an approximate solution is found by fixing the leading behaviour of the prefactor. Inserting (3.9) into (4.2) and solving the differential equation we get,

$$\mathcal{A} = \mathcal{N}_p v^{1/2} \left(1 + \left(\frac{v}{v^*} \right)^2 \right)^{-1/4} \quad (4.3)$$

where $\mathcal{N}_p$ is a normalization factor which only depends on $p_{\pm}$. Here we have explicit verification that Ψ vanishes as $v \rightarrow 0$, a result alluded to above. The wave function is well defined at the classical singularity.

5. DISCUSSION

Quantum cosmology, with the ‘no boundary’ condition, is a tentative prescription for applying the principles of quantum mechanics to the universe as a whole. One might think, at first, that such a proposal is slightly absurd. For example, the usual concepts of observer, system, and measurement, which work for laboratory experiments, seem to be meaningless if there is only one grand system containing everything. Moreover, performing a sequence of measurements would be rather complicated [7] as there is no explicit dependence on time in quantum cosmology. Clearly we have to depart from the conventional Copenhagen interpretation if the theory is to make any sense at all.

Some, but not all, of these difficulties can be resolved if one adopts the ‘relative state’ formulation [8] of quantum mechanics. It is particularly well-suited to problems of this sort. A readable account of this has been written by DeWitt [9]. The wave function, $\Psi(v, p_{\pm})$, is the probability amplitude for the universe to have a volume v , given that it is in an eigenstate labelled by $p_{\pm}$. What the wave function does is to set up correlations amongst these variables; there will be certain combinations of v and $p_{\pm}$ for which the probability is large. Thus the expectation of some function of $p_{\pm}$ should be dependent on v . We shall illustrate this by extracting an expression for the anisotropy of the universe from our model.

How much energy is locked up in the anisotropy of the universe, and therefore not contributing to its expansion, is related to the shear. The shear, σ , is given by,

$$\sigma^2 = 3(\dot{\beta}_+^2 + \dot{\beta}_-^2)$$

or equivalently in terms of the momenta, $p_{\pm}$,

$$\sigma^2 = \frac{3}{\mu^2 v^2} (p_+^2 + p_-^2)$$

Let the probability density of momentum eigenstates be $\rho(p_{\pm})$. The expectation of the shear is then,

$$\langle \sigma^2(v) \rangle = \frac{3}{\mu^2 v^2} \frac{\int dp_+ dp_- |\Psi(v, p_{\pm})|^2 \rho(p_{\pm}) (p_+^2 + p_-^2)}{\int dp_+ dp_- |\Psi(v, p_{\pm})|^2 \rho(p_{\pm})} \quad (5.1)$$

We can estimate this in the classical limit which corresponds to either sufficiently large volumes, or sufficiently small volumes. Let us first consider large volumes. Suppose that ρ

falls off rapidly for $\sqrt{p_+^2 + p_-^2}$ greater than some value p_0 . For volumes $v \gg 3p_0/\mu H$ the wave function (3.9) has the asymptotic form,

$$\Psi(v, p_{\pm}) \sim \mathcal{N}_p \sqrt{v^*} \cos\left(\frac{v}{\mu H} + \frac{\pi}{4}\right)$$

and we conclude from (5.1) that,

$$\langle \hat{\sigma}^2(v) \rangle \sim \frac{3b_1}{\mu^2 v^2}$$

where,

$$b_1 = \frac{\int_0^{p_0} dp_+ dp_- \mathcal{N}_p^2 v^* \rho(p_{\pm}) (p_+^2 + p_-^2)}{\int_0^{p_0} dp_+ dp_- \mathcal{N}_p^2 v^* \rho(p_{\pm})}$$

is just a number. Thus the shear falls off as $1/v^2$ for a sufficiently large universe, in keeping with what one would expect from classical theory.^{*}

In the limit of small volumes, a closer analysis of (5.1) reveals that the only contribution to the integrals comes from the region of ‘large’ values of $\sqrt{p_+^2 + p_-^2}$, where the square of the cosine can be averaged to $1/2$. We then find that the asymptotic behaviour of (5.1) is,

$$\langle \hat{\sigma}^2(v) \rangle \sim \frac{3b_2}{\mu^2 v^2}$$

where,

$$b_2 = \frac{\int_0^{\infty} dp_+ dp_- \mathcal{N}_p^2 \rho(p_{\pm}) (p_+^2 + p_-^2)}{\int_0^{\infty} dp_+ dp_- \mathcal{N}_p^2 \rho(p_{\pm})}$$

Summarizing, we can say that a small universe is very likely to be *anisotropic*, whilst a large universe is very likely to be *isotropic*.

We mention in closing that quantum cosmology has taken another turn in recent months. The introduction of topology changing configurations, such as wormholes, has some remarkable consequences [11]. One of these is that the coupling constants in our universe are not fundamental numbers, but rather the eigenvalues of certain operators arising out of worm-hole dynamics. Furthermore, it has been argued that the cosmological constant vanishes in

^{*} Previous investigations of anisotropy have been in the context of Bianchi IX groups [10]. Notice, however, that our result disagrees with the one derived by Amsterdamski [10] who finds that for large v the shear decays faster than an exponential. This is peculiar since in the classical regime the Bianchi I and Bianchi IX cosmologies have the same asymptotic behaviour for large v .

theories of this type [12]. Aside from these latest developments, it appears that the simplest quantum cosmology, such as our model, yields reasonable physical results. The wave function of everything is a theory of the universe.

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FIGURE CAPTION

1. The stationary values of the action in the complex plane are shown by crosses. The contour, Γ , has been distorted to pass through these, along the path of steepest descent. Also shown are the cuts required for single-valuedness of $f(x)$.

Figure I