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p-ADIC SUPERSTRINGS

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A B S T R A C T

Different possibilities to construct p-adic superstring amplitudes are discussed. To describe one of these possibilities we formulate a p-adic generalization of the conformal technique. p-adic conformal rules allow us to explicitly write down the amplitudes for massless particles. They do not have the form of the usual kinematical factors multiplied by the p-adic modification of the usual amplitudes. The relation between open and closed p-adic superstring amplitudes is discussed.

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1. - INTRODUCTION

The aim of the present paper is a consideration of various possibilities which can lead to the p-adic superstring theory. The superstring case of p-adic high energy physics is the most interesting one. Namely, a construction of superstrings with non-Archimedean geometry, in addition to the Archimedean one, will provide an understanding of the sub-Planckian physics [1] (see also Ref. [2]). The rôle of superstrings is rather important because they are so far the best candidate for a realistic unified theory [3]. Hence, starting from superstrings one could approach Planck scale physics [4,5].

According to the ideas proposed in Ref. [1] there are two ways to develop a dual p-adic approach for superstrings. The first one, which is the most radical and probably the most promising one, is related to a construction of the algebra of supersymmetric transformations over the field of p-adic numbers [6]. It is connected with a construction of the p-adic-valued amplitude by means of interpolation of the gamma or zeta functions. Following the second way, one can combine the ideas of the dual approach with the idea of constructing a theory on an arbitrary field K . This can be done by means of an amplitude written in the form of a convolution of two characters labelled by elements of the space $\hat{K}$ dual to the space K , e.g.,

$$A(a,b) = \int_K dx \chi_a(x) \chi_b(1-x) \quad (1)$$

In Eq. (1), $a, b \in \hat{K}$ and consequently the object dual to $\hat{K}$, i.e., $\hat{\hat{K}}$, will determine the structure of space-time. Since the integration in Eq. (1) can be associated with an integration over the light-cone-like time, then the most natural case physically is $\hat{\hat{K}} = K$ [7].

In fact, one could give up the requirement $K = \hat{\hat{K}}$ and consider a simpler possibility. In this case the space-time co-ordinates as well as the momenta variables belong to the ordinary (real) numbers and K is an arbitrary locally compact field of numbers. In particular, the case $K = Q_p$, where Q_p is a field of p-adic numbers, may be considered as such a model [8]. Although this model does not exhibit satisfactory physical properties, it sheds new light on the old problems [9-11]. From the point of view of the different possibilities contained in Eq. (1), the case of a Galois field F_q , i.e., $K = F_q$, is also interesting [1,12].

In the present paper, staying within the framework of the simplest model ($a, b \in R^D$, $K = Q_p$), we will investigate the possible consistency of an amplitude presented as a convolution of characters with $D = 10$ supersymmetry.

In the ordinary case it is known that the four-point tree amplitude for the open superstring has the form

$$A_{4p}(k_1, k_2, k_3, k_4) = -\frac{g^2}{2} \frac{\Gamma(-\frac{s}{2}) \Gamma(-\frac{t}{2})}{\Gamma(1-\frac{s}{2}-\frac{t}{2})} K(k_1, k_2, k_3, k_4) \quad (2)$$

where $s = -(k_1+k_2)^2$ and $t = -(k_2+k_3)^2$ are the Mandelstam variables. Since $4P = 4B, 4F$ or $2B2F$, the amplitude (2) accordingly depends on the polarization vectors of the massless vector particles and on the ten-dimensional Majorana-Weyl spinor wave functions. The form of the "kinematic factor" $K(k, \zeta)$ is in agreement with the supersymmetry requirements.

In Section 2 we will analyze an "ad hoc" possibility to generalize formula (2). We also wish to perform the corresponding calculations in the framework of the p-adic conformal technique. Indeed, the most elegant way to obtain the four-point amplitude (2) is to use the conformal approach [13,14]. Hence in Section 3 we will formulate an appropriate p-adic conformal technique. Using it, we will construct the corresponding amplitude for p-adic superstrings in Section 4. The obtained amplitude has the following form

$$A_{4p,p} = -\frac{1}{2} g^2 \left[B_p(1-\frac{s}{2}, -\frac{t}{2}) F_1 + B_p(-\frac{s}{2}, 1-\frac{t}{2}) F_2 \right], \quad (3)$$

where F_1 and F_2 are the corresponding kinematic factors. In Section 5 we will compare the results obtained in Sections 2 and 4. Section 6 is devoted to the construction of the amplitudes for closed p-adic superstrings.

2. - AD HOC GENERALIZATION OF THE TREE SUPERSTRING AMPLITUDE TO THE P-ADIC CASE

Note that the simplest manner to obtain a p-adic analogue of the Veneziano amplitude is to replace the ordinary gamma functions in the Veneziano amplitude

$$A(a, b, c) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)} \quad (4)$$

by their p-adic analogues [15], i.e.,

$$A_p(a, b, c) = \frac{\Gamma_p(a) \Gamma_p(b)}{\Gamma_p(a+b)} \quad (5)$$

In Eqs. (4) and (5), $a = -\alpha(s) = -1 - \frac{1}{2}s$, $b = -\alpha(t)$ and $c = -\alpha(u)$, and they satisfy the mass-shell condition $s + t + u = -8$ or $a + b + c = 1$. Owing to the simple relation

$$\Gamma_p(y) \Gamma_p(1-y) = 1 \quad (6)$$

which is a straightforward consequence of the expression

$$\Gamma_p(y) = \frac{1 - p^{y-1}}{1 - p^{-y}} \quad (7)$$

the p-adic amplitude (5) exhibits total crossing symmetry. $A_p(a,b,c)$ can be presented in the form (1) with $\gamma_a(x) = |x|_p^a$, where $x \in \mathbb{Q}_p$ and $|\cdot|_p$ denotes the p-adic norm. From an adelic point of view, the amplitudes $A_p(a,b,c)$ ($p=2,3,5,\dots$) have been considered [9,10] as partners of the ordinary crossing symmetry amplitude $A_\infty(a,b,c) = A(a,b,c) + A(b,c,a) + A(c,a,b)$, which can also be written in the form (1), where $\gamma_a(x) = |x|^a$ and $x \in \mathbb{R}$.

In an analogous way the above method can be applied to the superstring amplitude given by Eq. (2) and we can look for its p-adic analogue in the form

$$\mathcal{A}_{4P,p}(a,b,c) = -\frac{g^2}{2} \Phi_p(s,t,u) K(\kappa, \zeta) \quad (8)$$

where the kinematic factor $K(k,\zeta)$ remains unchanged. Taking $\phi_p(s,t,u)$ according to the above procedure we have

$$\Phi_p(s,t,u) = \frac{\Gamma_p(-\frac{s}{2}) \Gamma_p(-\frac{t}{2})}{\Gamma_p(1-\frac{s}{2}-\frac{t}{2})} \quad (9)$$

where $a = -s/2$, $b = -t/2$ and $c = -u/2$ with the mass-shell condition

$$s+t+u = 0 \quad (10)$$

Using the functional equation (6), we obtain the totally crossing-symmetric amplitude

$$\mathcal{A}_{4P,p}(a,b,c) = -\frac{g^2}{2} \Gamma_p(-\frac{s}{2}) \Gamma_p(-\frac{t}{2}) \Gamma_p(-\frac{u}{2}) K(\kappa, \zeta) \quad (11)$$

The amplitude (11) has poles at the real points

$$s=0, \quad t=0, \quad u=0 \quad (12)$$

However, as in the case of an open bosonic string, the expression (11) is not an appropriate candidate for the desired amplitude because it contains poles not only at $s = 0$, $t = 0$ but also a u -pole at $u = 0$.

The structure of Eq. (9) does not allow us to write down the amplitude (11) by means of the convolution of multiplicative characters $\pi_a(x)$, i.e.,

$$A_{4p,p}(a,b,c) \neq -\frac{g^2}{2} K(k,s) \int_{Q_p} \pi_a(x) \pi_b(1-x) dx \quad (13)$$

Furthermore, the lack of a representation of amplitude (11) in the form of the right-hand side of (13) does not permit us to extract the (st) channel amplitude from the crossing symmetric one [16].

In Ref. [17], another approach to the construction of p -adic supersymmetric amplitudes was proposed. There one uses the expression of the totally crossing symmetric ordinary superstring amplitude in terms of the Gel'fand-Graev gamma functions and then generalizes these gamma functions to their p -adic analogues. In this treatment, the corresponding multiplicative characters for gamma functions are of the form $\tilde{\pi}_a(x) = |x|_p^a \text{sign}_\tau x$, where $\text{sign}_\tau x$ is a p -adic generalization of the ordinary sign-function [15]. The amplitude obtained has the form

$$A_p = ig^2 \tilde{\Gamma}_p(-\frac{s}{2}) \tilde{\Gamma}_p(-\frac{t}{2}) \tilde{\Gamma}_p(-\frac{u}{2}) K(k,s) \quad (14)$$

where, for $\tau = \varepsilon$,

$$\tilde{\Gamma}_p(a) \equiv \Gamma_p(\tilde{\pi}_a) = \frac{1+p^{a-1}}{1+p^{-a}} \quad (15)$$

However, the amplitude (14) which is a candidate for the role of a p -adic superstring amplitude contains no poles whatsoever and consequently will not be of particular interest in our further consideration. The amplitude (14) cannot be presented as a convolution of multiplicative characters ($\tilde{\pi}_a(x)$) in the form of the right-hand side of (13) as well. In order to obtain a representation in the form (13) recall that the ordinary amplitude (2) can be obtained from an integral representation in a most elegant way by means of the conformal technique.

3. - p-ADIC CONFORMAL THEORY AND STRING AMPLITUDES

Recall that the usual conformal field in a Euclidean formulation is defined by the propagator

$$\langle x^\mu(z) x^\nu(w) \rangle = -g^{\mu\nu} \ln |z-w|^2, \quad (16)$$

where $g^{\mu\nu}$ denotes the flat metric in D-dimensional space-time and z, w are complex variables. Note that this conformal field is not an analytic one. Sometimes in such cases one uses the symbol $x(z, \bar{z})$. However, this notation is not convenient for a generalization to the p-adic case because, for a given number z , various algebraic extensions lead to several corresponding conjugate numbers. We will denote the standard measure on the complex plane by dz instead of d^2z . The meaning of the symbols will be more obvious from the corresponding contexts. This notation is common in the relevant mathematical literature [15].

The tachyon vertex has the form

$$V(k, z) = : e^{i k_\mu x^\mu(z)} : \quad (17)$$

and the N-point closed string amplitude can be written as follows

$$\mathcal{A}_N(k_1, \dots, k_N) = \int \prod_{i=1}^N dz_i \langle \prod_{i=1}^N V(k_i, z_i) \rangle = \int \prod_{i=1}^N dz_i \prod_{m < n} |z_m - z_n|^{k_m k_n / 2} \quad (18)$$

where dz is the standard measure on the complex plane. The expression (18) is divergent because of the $SL(2, \mathbb{C})$ invariance and after the extraction of the group volume on mass shell $k^2_1 = 2$ can be presented in the Koba-Nielsen form

$$\mathcal{A}_N(k_1, \dots, k_N) = \int dV \prod_{m < n} |z_m - z_n|^{k_m k_n / 2} \quad (19)$$

where

$$dV = \pi^{3-N} \frac{|z_a - z_b|^2 |z_b - z_c|^2 |z_a - z_c|^2}{dz_a dz_b dz_c} \prod_{i=1}^N dz_i \quad (20)$$

The procedure described above can be straightforwardly applied to the derivation of the open string amplitude. In such a case the integration in Eq. (18) goes over the simplex on the real axis,

$$z_1 \geq z_2 \geq \dots \geq z_N \quad (21)$$

and the power of $|z_m - z_n|$ should be multiplied by 2 in accordance with the form of the propagator for an open string

$$\langle x^\mu(z) x^\nu(w) \rangle = -g^{\mu\nu} \left(\ln|z-w|^2 + \ln|z-\bar{w}|^2 \right) \quad (16')$$

A divergence related to the $SL(2, R)$ invariance of the integrand can be removed by dividing it by the volume of this group. One obtains the st-channel amplitude by choosing

$$z_1 = \infty, \quad z_2 = 1, \quad z_N = 0 \quad (22)$$

Denoting the characteristic function of a simplex satisfying conditions (21) and (22) by $\theta_{[0,1,\infty]}(z_1 \dots z_N)$ we can write down

$$\begin{aligned} \mathcal{A}_N^{open}(\kappa_1, \dots, \kappa_N) &= \frac{1}{V} \int (\prod dz_i) \theta_{[0,1,\infty]}(z_3, \dots, z_{N-1}) \left\langle \prod_{i=1}^N V(\kappa_i, z_i) \right\rangle = \\ &= \frac{1}{V} \int (\prod dz_i) \theta_{[0,1,\infty]}(z_3, \dots, z_{N-1}) \prod_{m < n} |z_m - z_n|^{K_m K_n} \end{aligned} \quad (18')$$

The extraction of the $SL(2, R)$ [or $SL(2, C)$] group volume is automatically done by introducing ghosts a and b ($a, \bar{a}$ and $b, \bar{b}$)

$$\langle b(z) c(w) \rangle = \frac{1}{|z-w|} \quad (23)$$

and replacing the vertex (17) by

$$V(z) = c(z) : e^{iK_\mu x^\mu(z)} : \quad (24)$$

at the points $z_1 = \infty$, $z_2 = 1$ and $z_N = 0$. Equation (18') can be rewritten now in the

$$\begin{aligned} \mathcal{A}_N^{open}(\kappa_1, \dots, \kappa_N) &= \int_{1 \geq z_3 \geq \dots \geq z_{N-1} \geq 0} \langle c(z_1) e^{iK_1 x(z_1)} c(z_2) e^{iK_2 x(z_2)} \dots \\ &\dots e^{iK_{N-1} x(z_{N-1})} c(z_N) e^{iK_N x(z_N)} \rangle \prod_{i=3}^{N-1} dz_i \end{aligned} \quad (25)$$

Taking into account the correlation function

$$\langle c(z_1) c(z_{N-1}) c(z_N) \rangle = |z_1 - z_{N-1}| / |z_1 - z_N| / |z_{N-1} - z_N| \quad (26)$$

one obtains the well-known result. Notice that in contrast to the usual notation [7] we introduced moduli in (26) due to the ordering of the z variables.

The p -adic generalization of the above formulae is straightforward. Let K be a quadratic extension of $\mathbb{Q}_p$ or an arbitrary locally compact field with norm $|\cdot|_K$. The corresponding conformal field $x^\mu(z)$, where $z \in K$, is defined by the propagator

$$\langle x^\mu(z) x^\nu(w) \rangle = -g^{\mu\nu} \log |z-w|_K^2 \quad (27)$$

and the vertex as well as the string amplitudes acquire the form (17) and (18), respectively, where dz is now the Haar measure on K . Instead of $SL(2, \mathbb{C})$ here we have $SL(2, K)$ invariance.

For an open p -adic string amplitude we can consider formulae (16') and (17) as a definition of a conformal theory, where $z, w \in \mathbb{Q}_p(\sqrt{\tau})$ and $|\cdot|$ should be replaced by the p -adic norm $|\cdot|_p$. The N -point p -adic amplitude is given by Eq. (18') where $\theta_\tau(z_1 \dots z_N)$ will denote one of the possible p -adic analogues of the characteristic function on the simplex (21), i.e.,

$$A_{N, \tau}^{open}(k_1, \dots, k_N) = \int_{\mathbb{Q}_p^N} \langle \prod_{i=1}^N V(k_i, z_i) \rangle \theta_\tau(z_1, \dots, z_N) \prod_{i=1}^N dz_i \quad (28)$$

This divergent expression has an $SL(2, \mathbb{Q}_p)$ invariance and after extraction of the volume of the group on the mass shell $k_i^2 = 2$ it can be represented in the Koba-Nielsen form

$$A_{N, \tau}^{open}(k_1, \dots, k_N) = \int_{(\mathbb{Q}_p)^{N-3}} \langle c(\infty) c(1) c(0) \prod_{i=1}^N V(k_i, z_i) \rangle \theta_{\tau[0,1,\infty]}(z_1, \dots, z_{N-1}) \prod_{i=1}^{N-1} dz_i \quad (29)$$

where $\theta_{\tau, [0,1,\infty]}(z_1 \dots z_N)$ is now the p -adic analogue of the finite simplex defined by (21) and (22). Various possibilities of constructing p -adic analogues of the characteristic function on the segment on the real axis are considered in Ref. [16]. The formulae obtained above for the N -point open p -adic string amplitudes are in agreement with those in Refs. [17-19].

4. - p -ADIC SUPERSTRING AMPLITUDES

Formula (25) is very suitable for a straightforward generalization to the case of superstrings. In the NSR formulation of superstrings we usually encounter the vertex operators for fermion emission

$$V_{-1/2}(\kappa, u_\alpha, z) = u^\alpha(\kappa) e^{-\frac{1}{2}\phi(z)} S_\alpha(z) e^{i\kappa x(z)} \quad (30)$$

$$\begin{aligned} V_{1/2}(\kappa, u_\alpha, z) &= [Q_{BRST}, 2\xi V_{-1/2}] = \\ &= u^\alpha(\kappa) \left\{ e^{\frac{\phi}{2}} (\partial x^\mu + \frac{i}{4}(\kappa\gamma)\psi^\mu) \gamma_{\mu\alpha\beta} S^\beta + e^{\frac{3}{2}\phi} \eta^\beta S_\alpha \right\} e^{i\kappa x(z)} \end{aligned} \quad (31)$$

where the wave function $u_\alpha(\kappa)$ characterizes a fermion. We also encounter the vertex operators for the emission of the massless vector boson

$$V_{-1}(\kappa, \xi, z) = \xi^\mu \psi_\mu e^{-\phi} c e^{i\kappa x(z)} \quad (32)$$

$$V_0(\kappa, \xi, z) = \xi^\mu (\partial x^\mu + i\kappa^\nu \psi_\nu \psi^\mu) e^{i\kappa x(z)} \quad (33)$$

where ξ^μ corresponds to the massless vector boson. The fields introduced in Eqs. (30)-(33) are defined by their correlators

$$\langle \psi^\mu(z) \psi^\nu(w) \rangle = - \frac{g^{\mu\nu}}{|z-w|}, \quad (34)$$

$$\langle \chi(z) \chi(w) \rangle = - \langle \phi(z) \phi(w) \rangle = \ell n |z-w|, \quad (35)$$

$$\langle S_\alpha(z) S_\beta(w) \rangle = - \frac{\delta_\alpha^\beta}{|z-w|^{5/4}} + \frac{(\gamma_\mu)_\alpha^\beta \psi^\mu(z)}{|z-w|^{3/4}} + \frac{(\gamma^{\mu\nu})_\alpha^\beta \psi^\mu(z) \psi^\nu(z)}{|z-w|^{1/4}}, \quad (36)$$

$$\langle S_\alpha(z) \psi^\mu(w) \rangle = \frac{(\gamma^\mu)_\alpha^\beta S_\beta(z)}{|z-w|^{1/2}}. \quad (37)$$

We used again a norm in Eqs. (33)-(36) taking into account that the integrations are over an ordered region. The spin operator S_α is usually constructed in an explicit form starting from the spinor field ϕ_μ [13,20,21]. Construction of an analogous operator for the p-adic case is also desirable.

The tree amplitude for scattering of four fermions can be given by

$$\begin{aligned} \mathcal{A}_{4F}(\kappa_1 u_1; \kappa_2 u_2; \kappa_3 u_3; \kappa_4 u_4) &= \\ &= - \frac{1}{2} g^2 \int d^2 z \theta_{[0,1]}(z) \langle V_{-1/2}(\infty) V_{1/2}(1) V_{-1/2}(z) V_{1/2}(0) \rangle \end{aligned} \quad (38)$$

Performing the corresponding calculations we obtain

$$\begin{aligned}
 & \mathcal{A}_{4F}(k_1 u_1; k_2 u_2; k_3 u_3; k_4 u_4) = \\
 & = u^\alpha u^\beta u^\gamma u^\delta \left(-\frac{1}{2}g^2\right) \int_0^1 d\bar{z} |\bar{z}|^{-1+k_2 k_3} |1-\bar{z}|^{-1+k_3 k_4} \left\{ |1-\bar{z}| \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu - |\bar{z}| \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \right\} = \\
 & = \frac{1}{2} g^2 u^\alpha u^\beta u^\gamma u^\delta \left\{ B\left(1-\frac{k}{2}, -\frac{s}{2}\right) \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu - B\left(-\frac{k}{2}, 1-\frac{s}{2}\right) \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \right\} \quad (39)
 \end{aligned}$$

The transition amplitude from two fermions to two bosons can be written in the form

$$\begin{aligned}
 & \mathcal{A}_{2F2B}(k_1 u_1; k_2 u_2; k_3 \zeta_3; k_4 \zeta_4) = \\
 & = -\frac{1}{2} g^2 \int d\bar{z} \theta_{[0,1]}(\bar{z}) \langle V_{-1}(\infty) V_0(1) V_{-\frac{1}{2}}(\bar{z}) V_{-\frac{1}{2}}(0) \rangle \quad (40)
 \end{aligned}$$

Performing the calculations with the corresponding correlators and imposing the kinematical conditions on the mass shell, one obtains

$$\begin{aligned}
 & \mathcal{A}_{2F2B} = -\frac{1}{2} g^2 \int d\bar{z} \theta_{[0,1]}(\bar{z}) |\bar{z}|^{-1+k_3 k_4} |1-\bar{z}|^{-1+k_2 k_3} \cdot \\
 & \left\{ |\bar{z}| (\zeta_3 k_2) (u_2 \gamma_{\mu\nu} u_1) + \frac{1}{2} \zeta_4^\mu k_3^\nu \zeta_3^\lambda u_2 \gamma^{\nu\lambda} \gamma^\mu u_1 - |1-\bar{z}| (\zeta_3 k_4) (u_2 \gamma_{\mu\nu} u_1 + \right. \\
 & \left. + (\zeta_4 k_3) u_2 \gamma_{\mu\nu} u_1 - (\zeta_4 \zeta_3) u_2 \gamma_{\mu\nu} u_1) \right\} \quad (41)
 \end{aligned}$$

Analogously, one can obtain the four-boson scattering amplitude. The formulae obtained above, (39) and (41), can easily be generalized to the p-adic case. As in the bosonic string case, we have to replace the standard norm by a p-adic one. In particular, the p-adic four-fermion string amplitude can be written as follows:

$$\begin{aligned}
 & \mathcal{A}_{4F}^{(st)}(k_1 u_1; k_2 u_2; k_3 u_3; k_4 u_4) = -\frac{1}{2} g^2 u_1^\alpha u_2^\beta u_3^\gamma u_4^\delta \cdot \\
 & \cdot \int_{Q_p} d\bar{z} \theta_{\tau[0,1]}(\bar{z}) |\bar{z}|_p^{-1-\frac{k}{2}} |1-\bar{z}|_p^{-1-\frac{s}{2}} \left[|1-\bar{z}|_p \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu - |\bar{z}|_p \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \right]. \quad (42)
 \end{aligned}$$

The explicit form of Eq. (42) depends on the chosen form of the p-adic analogue of the characteristic function $\theta_{[0,1],\tau}(z)$, where τ denotes one of the three quadratic extensions. For instance, when $\tau = \varepsilon$ and

$$\theta_{\varepsilon[0,1]}(\bar{z}) = \frac{1}{2} \left(\text{Sign}_\varepsilon \bar{z} - \text{Sign}_\varepsilon(-1) \text{Sign}_\varepsilon(1-\bar{z}) \right), \quad (43)$$

we can write the following p-adic amplitude

$$A_{4F,p,\varepsilon}^{(st)} = u_1^\alpha u_2^\beta u_3^\gamma u_4^\delta (-\frac{1}{2}g^2) \{ [B_p(\tilde{\pi}_{-\frac{t}{2}}, \tilde{\pi}_{-\frac{s}{2}+1}) - B_p(\pi_{-\frac{t}{2}}, \tilde{\pi}_{-\frac{s}{2}+1})] \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu - \\ - [B_p(\tilde{\pi}_{-\frac{t}{2}+1}, \tilde{\pi}_{-\frac{s}{2}}) - B_p(\pi_{-\frac{t}{2}+1}, \tilde{\pi}_{-\frac{s}{2}})] \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \} \quad (44)$$

where $u_\alpha(i) = u_\alpha(k_i)$ and $B_p(\pi_a, \pi_b)$ are p-adic beta functions. So,

$$A_{4F,p,\varepsilon}^{(st)} = u_1^\alpha u_2^\beta u_3^\gamma u_4^\delta (-\frac{1}{2}g^2) (1-\frac{1}{p}) \frac{1+p^{-\frac{u}{2}-1}}{1+p^{\frac{u}{2}}} \cdot \\ \left\{ \frac{p^{\frac{t}{2}-p^{-\frac{t}{2}-1}} + p^{\frac{s}{2}-1} - p^{-\frac{s}{2}}}{(1-p^{s-2})(1-p^{-t})} \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu + \frac{p^{\frac{s}{2}-p^{-\frac{s}{2}-1}} + p^{\frac{t}{2}-1} - p^{-\frac{t}{2}}}{(1-p^{t-2})(1-p^{-s})} \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu \right\}.$$

This amplitude evidently contains poles at the real points $s = 0$, $t = 0$, $s = 2$ and $t = 2$, as well as poles at the complex points.

Performing the integration in Eq. (42) over the whole field Q_p , one obtains the expression

$$A_{4F,p} = -\frac{1}{2}g^2 [B_p(1-\frac{t}{2}, -\frac{s}{2}) \gamma_{\alpha\beta}^\mu \gamma_{\gamma\delta}^\mu - B_p(-\frac{t}{2}, 1-\frac{s}{2}) \gamma_{\alpha\delta}^\mu \gamma_{\beta\gamma}^\mu] \quad (45)$$

which contains a u-pole as well.

5. - COMPARISON OF AMPLITUDES

The origin of Eq. (45) is rather transparent. In fact, the ordinary amplitude (39) allows for a representation not in the form of a beta function but in the form of a sum of two terms which contain beta functions, i.e.,

$$A_{4p}(s, t, u) = -\frac{1}{2}g^2 [B(1-\frac{t}{2}, -\frac{s}{2}) F_1 + B(-\frac{t}{2}, 1-\frac{s}{2}) F_2] \quad (46)$$

In the ordinary case this equation can be reduced to Eq. (2). Equation (45) and similar expressions for the amplitudes A_{2F2B} and A_{4B} have a form which is different from the expected one given by Eq. (8). The reason for this difference is that the p-adic gamma function does not obey the functional equation $\Gamma(1+x) = x\Gamma(x)$. Hence, Eq. (46) cannot be transformed into the form (8), while the analogous Eq. (39) for an ordinary amplitude can be presented in such a form, which is now equivalent to Eq. (2).

Let us also consider the very important question of consistency of the p-adic amplitudes constructed above with supersymmetry. It is possible to prove that the form (46) of a four-point amplitude is consistent with supersymmetry. In other words, formula (2), with the corresponding choice of factor K [which follows from Eqs. (39) and (41)], is a sufficient condition for supersymmetry but not a necessary one.

6. - p-ADIC CLOSED SUPERSTRINGS

Recall that for the ordinary closed superstrings the four-point amplitude for massless particles has the form

$$A_4^{\text{closed}} = K^2 \int_1^{AA'} \int_2^{BB'} \int_3^{CC'} \int_4^{DD'} K_{ABCD}(\kappa/2) K_{A'B'C'D'}(\kappa/2) C(s, t, u) \quad (47)$$

where K_{ABCD} ($K_{A'B'C'D'}$) is a kinematical factor contained also in the corresponding amplitude for the open superstring. The factor $c(s, t, u)$ is an analogue of $\phi(s, t, u)$ for the open superstring and can be obtained from the relation between the amplitude of the bosonic closed string and the amplitudes of the open bosonic string, i.e.,

$$C(s, t, u) = \sin \frac{\pi t}{8} \frac{\Gamma(-s/8) \Gamma(-t/8)}{\Gamma(1-s/8-t/8)} \cdot \frac{\Gamma(-t/8) \Gamma(-u/8)}{\Gamma(1-t/8-u/8)} \quad (48)$$

This result can also be obtained by means of the conformal technique.

In the p-adic case, having in mind the origin of the factor $c(s, t, u)$, one could naively expect formulae of the same type as the above ones. However, the main obstacle is the impossibility to present the amplitude (3) in the form (9). Application of the Ansätze (47) and (48) to an amplitude (11) would mean that the p-adic amplitude for the closed superstring can be given by Eq. (47), where $c(s, t, u)$ should be replaced by

$$C_p(s, t, u) = \Gamma_{p, \tau}(-s/2) \Gamma_{p, \tau}(-t/2) \Gamma_{p, \tau}(-u/2) \quad (49)$$

In Eq. (49), $\Gamma_{p, \tau}(a) = (1-q^{a-1})/(1-q^{-a})$ for $\tau = \varepsilon, p, \varepsilon p$ and $q = p^2$.

On the other hand, using the conformal technique, one obtains another result which can be written in the form

$$\begin{aligned}
\mathcal{A}_F^{closed}(s, t, u) = & K^2 \int^{AA'} \int^{BB'} \int^{CC'} \int^{DD'} \left\{ K_{ABCD}^{(1)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(1)} \left(\frac{K}{2} \right) B_F \left(-\frac{s}{8}, 1, -\frac{t}{8} \right) + \right. \\
& + K_{ABCD}^{(2)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(2)} \left(\frac{K}{2} \right) B_F \left(-\frac{s}{8}, -\frac{t}{8} + 1 \right) + K_{ABCD}^{(3)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(3)} \left(\frac{K}{2} \right) B_F \left(-\frac{s}{8}, -\frac{t}{8} \right) + \\
& + \frac{1}{2} \left(K_{ABCD}^{(1)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(2)} \left(\frac{K}{2} \right) + K_{ABCD}^{(2)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(1)} \left(\frac{K}{2} \right) \right) \left[B_F \left(-\frac{s}{8}, -\frac{t}{8} \right) - B_F \left(-\frac{s}{8}, -\frac{t}{8} + 1 \right) - B_F \left(-\frac{s}{8} + 1, -\frac{t}{8} \right) \right] + \\
& + \frac{1}{2} \left(K_{ABCD}^{(2)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(3)} \left(\frac{K}{2} \right) + K_{ABCD}^{(3)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(2)} \left(\frac{K}{2} \right) \right) \left[B_F \left(-\frac{s}{8} + 1, -\frac{t}{8} \right) - B_F \left(-\frac{s}{8}, -\frac{t}{8} + 1 \right) - B_F \left(-\frac{s}{8}, -\frac{t}{8} \right) \right] + \\
& + \frac{1}{2} \left(K_{ABCD}^{(1)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(3)} \left(\frac{K}{2} \right) + K_{ABCD}^{(3)} \left(\frac{K}{2} \right) K_{A'B'C'D'}^{(1)} \left(\frac{K}{2} \right) \right) \left[B_F \left(-\frac{s}{8}, -\frac{t}{8} + 1 \right) - B_F \left(-\frac{s}{8} + 1, -\frac{t}{8} \right) - B_F \left(-\frac{s}{8}, -\frac{t}{8} \right) \right] \Bigg\} \quad (50)
\end{aligned}$$

where F denotes a locally compact field. The factors $K^{(i)}$ ($i=1,2,3$) in Eq. (50) are the factors contained in the kinematical factor $K_{4p}(k, \zeta)$ of an open string

$$K_{4p}(k, \zeta) = \int_1^A \int_2^B \int_3^C \int_4^D \left(s K_{ABCD}^{(1)}(k) + t K_{ABCD}^{(2)}(k) + u K_{ABCD}^{(3)}(k) \right) \quad (51)$$

(see the Appendix for explicit expressions for $K^{(i)}$). Note that the last term $K^{(3)}$ appears only in the case $4P = 4B$.

Here we are not going to discuss the derivation of Eq. (50) in detail. However, notice that such representations (50) and (51) for the field $F = \mathbb{C}$ and the corresponding identities

$$\begin{aligned}
B_C \left(-\frac{s}{8}, -\frac{t}{8} \right) &= \left(\frac{u}{8} \right)^2 C(s, t, u) \\
B_C \left(-\frac{s}{8}, -\frac{t}{8} + 1 \right) &= \left(\frac{t}{8} \right)^2 C(s, t, u) \\
B_C \left(-\frac{s}{8} + 1, -\frac{t}{8} \right) &= \left(\frac{s}{8} \right)^2 C(s, t, u)
\end{aligned} \quad (52)$$

give the well-known amplitudes for the ordinary closed superstrings.

Using the explicit forms of $B_F(a, b)$ and taking into account some properties of the kinematical factors $K^{(i)}$, one can easily see the total crossing symmetry of Eq. (50). In particular, for $F = \mathbb{Q}_p(\sqrt{\epsilon})$ we have

$$\begin{aligned}
A_{P, \tau}^{closed}(s, t, u) = & K^2 \int_1^{AA'} \int_2^{BB'} \int_3^{CC'} \int_4^{DD'} \left[(1,1) \frac{1-q^a}{1-q^{-a-1}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^{c-1}}{1-q^{-c}} + \right. \\
& + (2,2) \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^b}{1-q^{-b-1}} \frac{1-q^{c-1}}{1-q^{-c}} + (3,3) \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^c}{1-q^{-c-1}} + \\
& + (1,2) \left[\frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^c}{1-q^{-c-1}} - \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^b}{1-q^{-b-1}} \frac{1-q^{c-1}}{1-q^{-c}} - \frac{1-q^a}{1-q^{-a-1}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^{c-1}}{1-q^{-c}} \right] + \\
& + (2,3) \left[\frac{1-q^a}{1-q^{-a-1}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^{c-1}}{1-q^{-c}} - \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^b}{1-q^{-b-1}} \frac{1-q^{c-1}}{1-q^{-c}} - \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^c}{1-q^{-c-1}} \right] + \\
& + (3,1) \left[\frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^b}{1-q^{-b-1}} \frac{1-q^{c-1}}{1-q^{-c}} - \frac{1-q^a}{1-q^{-a-1}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^{c-1}}{1-q^{-c}} - \frac{1-q^{a-1}}{1-q^{-a}} \frac{1-q^{b-1}}{1-q^{-b}} \frac{1-q^c}{1-q^{-c-1}} \right].
\end{aligned} \tag{53}$$

where (i,j) denotes the product of the kinematical factors $K^{(i)}$ and $K^{(j)}$. In Eq. (53), $a = -s/8$, $b = -t/8$, $c = -u/8$ and $a+b+c = 0$. Amplitude (53) has real poles at the points $s = 0$, $t = 0$, $u = 0$ and $s = 8$, $t = 8$, $u = 8$, and poles at some complex points too. Notice that the terms in Eq. (53) with factors (i,j) , for $i \neq j$, do not lead to poles at $a = 0$ (consequently at $b = 0$ and $c = 0$) since the corresponding residues at these points are equal to zero.

7. - CONCLUSION

In this paper some of the possible generalizations of the usual supersymmetric amplitude to the p-adic case are proposed. Note that it is just the context of supersymmetry and superanalysis where the p-adic numbers appear for the first time in mathematical physics. We have obtained the p-adic superstring amplitude by using the p-adic conformal technique. Maybe it will be possible to generalize the Neveu-West group theory approach [29] for the p-adic case as well. It would also be interesting to consider p-adic superstring amplitudes connected with higher-order algebraic extensions. These amplitudes will be the generalization of the Gervais amplitude [23] and one can relate with them the supermembrane theory [24].

The p-adic approach is supposed to deal with sub-Planckian physics, so it is natural to compare the p-adic supersymmetric amplitude with the high energy scattering amplitudes recently obtained by Amati, Ciafaloni and Veneziano [4] and by Gross and Mende [5]. Strictly speaking [1], the results of Amati-Ciafaloni-Veneziano and Gross-Mende cannot be applicable beyond the Planck scale because they have been obtained starting from the usual string in the usual space-time and at

the Planck scale the space-time structure must be changed. However, one can hope that the asymptotic behaviours of the usual string amplitudes are not only extrapolated by the p-adic string amplitude, but they coincide totally. In this case, one can say that the p-adic amplitude corresponds to some special measure on the moduli space and the correspondence principle will be guaranteed on the S-matrix level.

In the formal limit $\alpha' \rightarrow \infty$ one does not obtain the particle spectrum, i.e., in this limit one has no particles, and no scale parameter at all. It is interesting to note that the p-adic string amplitude obtained by p-adic interpolation of the Veneziano amplitude has no poles as well [1]. So one can believe that the following scenario takes place. At the sub-Planckian distance scale, there appears a p-adic string, after that at the Planck scale one goes to a massless theory of an infinite number of particles [25], and then to the usual string theory.

In this paper we mostly discuss the p-adic string. However, we have in mind a more general approach dealing with arbitrary number fields. The starting point of this approach is the general principle according to which the fundamental physical laws should be invariant under a change of the number field [1]. This principle has some analogy with the Einstein principle of general relativity invariance. The principle of independence from the number field puts strong restrictions on the possible forms of the theory. One of the examples of these restrictions is the possibility to write down the scattering amplitude for the gravitons corresponding to an arbitrary field K in superstring theory and the impossibility to do this in the bosonic case.

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APPENDIX

The kinematical factors $K_{4p}^{(i)}$ have the following form:

$$K_{4B}^{(1)} = -\frac{1}{8} \left[t(\xi_1 \xi_3)(\xi_2 \xi_4) + u(\xi_2 \xi_3)(\xi_1 \xi_4) \right] + \frac{1}{2} \left[(\xi_1 \kappa_4)(\xi_3 \kappa_2)(\xi_2 \xi_4) + \right. \\ \left. + (\xi_2 \kappa_3)(\xi_4 \kappa_1)(\xi_1 \xi_3) + (\xi_1 \kappa_3)(\xi_4 \kappa_2)(\xi_2 \xi_3) + (\xi_2 \kappa_4)(\xi_3 \kappa_1)(\xi_1 \xi_4) \right],$$

$$K_{4B}^{(2)} = -\frac{1}{8} \left[s(\xi_1 \xi_3)(\xi_2 \xi_4) + u(\xi_1 \xi_2)(\xi_3 \xi_4) \right] + \frac{1}{2} \left[(\xi_2 \kappa_1)(\xi_4 \kappa_3)(\xi_3 \xi_1) + \right. \\ \left. + (\xi_3 \kappa_4)(\xi_1 \kappa_2)(\xi_2 \xi_4) + (\xi_2 \kappa_4)(\xi_1 \kappa_3)(\xi_3 \xi_4) + (\xi_3 \kappa_1)(\xi_4 \kappa_2)(\xi_2 \xi_1) \right],$$

$$K_{4B}^{(3)} = -\frac{1}{8} \left[s(\xi_2 \xi_3)(\xi_1 \xi_4) + t(\xi_1 \xi_2)(\xi_3 \xi_4) \right] + \frac{1}{2} \left[(\xi_2 \kappa_2)(\xi_4 \kappa_3)(\xi_3 \xi_1) + \right. \\ \left. + (\xi_3 \kappa_4)(\xi_2 \kappa_1)(\xi_1 \xi_4) + (\xi_1 \kappa_4)(\xi_2 \kappa_3)(\xi_3 \xi_4) + (\xi_3 \kappa_2)(\xi_4 \kappa_1)(\xi_1 \xi_2) \right],$$

$$K_{2B2F}^{(1)}(u_1 \xi_2 \xi_3 u_4) = \bar{u}_1(\gamma \xi_3) u_4 (\kappa_3 \xi_2) - \bar{u}_1(\gamma \xi_2) u_4 (\kappa_2 \xi_3) - \bar{u}_1(\gamma \kappa_3) u_4 (\xi_2 \xi_3),$$

$$K_{2B2F}^{(2)}(u_1 \xi_2 \xi_3 u_4) = -\frac{1}{2} \bar{u}_1(\gamma \xi_2)(\gamma(\kappa_3 + \kappa_4))(\gamma \xi_3) u_4,$$

$$K_{2B2F}^{(1)}(u_1 \xi_2 u_3 \xi_4) = \frac{1}{2} \bar{u}_1(\gamma \xi_4)(\gamma(\kappa_3 + \kappa_2))(\gamma \xi_2) u_3,$$

$$K_{2B2F}^{(2)}(u_1 \xi_2 u_3 \xi_4) = \frac{1}{2} \bar{u}_1(\gamma \xi_2)(\gamma(\kappa_3 + \kappa_4))(\gamma \xi_4) u_3,$$

$$K_{4F}^{(1)}(u_1 u_2 u_3 u_4) = -\frac{1}{2} \bar{u}_2 \gamma^\mu u_3 \bar{u}_4 \gamma_\mu u_1$$

$$K_{4F}^{(2)}(u_1 u_2 u_3 u_4) = -\frac{1}{2} \bar{u}_1 \gamma^\mu u_2 \bar{u}_4 \gamma_\mu u_3$$

REFERENCES

- [1] I.V. Volovich - Teor.Mat.Fiz. 71 (1987) 337; Class.Quant.Grav. 4 (1987) L83;
I.V. Volovich - "Number Theory as the Ultimate Physical Theory", CERN
Preprint TH. 4781 (1987).
- [2] I.Ya. Aref'eva and I.V. Volovich - Strings, Gravity and p-adic Space-time,
Seminar on Quantum Gravity, Moscow, May 1987 (to be published, World
Scientific, Singapore, 1988);
I.Ya. Aref'eva - in "Conformal Invariance and String Theory", Poina, Brasov,
September 1987 (Academic Press, 1988).
- [3] M.B. Green, J.H. Schwarz and E. Witten - Superstring Theory (Cambridge
University Press, Cambridge, 1987).
- [4] D. Amati, M. Ciafaloni and G. Veneziano - CERN Preprints TH. 4782 and 4886
(1987).
- [5] D.J. Gross and P.F. Mende - Preprint PUPT-1067 (1987).
- [6] V.S. Vladimirov and I.V. Volovich - Theor.Math.Phys. 59 (1984) 3-27.
- [7] I.V. Volovich - "Harmonic Analysis and p-adic Strings", Preprint (Oct. 1987),
(to be published in Lett.Math.Phys.)
- [8] P.G.O. Freund and M. Olson - Phys.Lett. B199 (1987) 186.
- [9] P.G.O. Freund and E. Witten - Phys.Lett. B199 (1987) 191.
- [10] B. Grossman - Rockefeller University Preprint DOE/EP/40325-7 (1987).
- [11] I.Ya. Aref'eva, B.G. Dragovic and I.V. Volovich - "On the Adelic String
Amplitudes", Preprint IF-13/88 (1988) (to be published in Phys.Lett.B).
- [12] B. Grossman - Phys.Lett. B197 (1987) 101.
- [13] D. Friedan, E. Martinec and S. Shenker - Nucl.Phys. B271 (1986) 93;
J. Cohn, D. Friedan, Z. Qiu and S. Shenker - Nucl.Phys. B278 (1987) 577.
- [14] E. Cremmer and J.-L. Gervais - Nucl.Phys. B90 (1975) 410;
A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov - Nucl.Phys. B241 (1983)
333.
- [15] I.M. Gel'fand, M.I. Graev and I.I. Piatetskii - "Shapiro, Representation
Theory and Automorphic Functions" (Nauka, Moscow, 1966).
- [16] I.Ya. Aref'eva, B. Dragovic and I.V. Volovich - "Open and Closed p-adic
Strings and Quadratical Extensions of Number Fields", Preprint IF-14/88; CERN
Preprint TH. 5076 (1988).
- [17] L. Brekke, P.G.O. Freund, M. Olson and E. Witten - Nucl.Phys. B302 (1988)
365.
- [18] P.H. Frampton and Y. Okada - Phys.Rev.Lett. 60 (1988); Phys.Rev. D37 (1988)
3077.
- [19] E. Marinari and G. Parisi - Phys.Lett. B203 (1988) 52.
- [20] A. Le Clair, M.E. Peskin and C.R. Preitschopf - "String Field Theory on the
Conformal Plane, I, II, III", pp. 4306, 4307 and 4464.
- [21] E.F. Corrigan and D. Olive - Nuovo Cimento 11A (1972) 749;
E.F. Corrigan and P. Goddard - Nuovo Cimento 18A (1973) 339.

- [22] A. Neveu and P. West - CERN Preprint TH. 4903 (1987).
- [23] J.-L. Gervais - Phys.Lett. B201 (1988) 306.
- [24] M.J. Duff - Class.Quant.Grav. 5 (1988) 189.
- [25] D.J. Gross - Preprint PUPT-1085 (1988).