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THE PATH INTEGRAL
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OF FERMIONIC STRING
PROPAGATOR

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The Path Integral Representation of Fermionic String
Propagator

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Abstract

The representation for the propagator of fermionic Neveu-Schwarz-Ramond string as a path integral over two-dimensional surfaces with fixed boundary curves is proposed. It is shown that in the case of pointlike boundary conditions the result of the computation has a natural interpretation and it can be compared with the result of operator calculation. The ultraviolet and infrared singularities of the propagator are considered.

1. Introduction

Superstring theory is now the most real candidate for the role of the fundamental theory of all interactions. The most developed approach in the string theory is the calculation of scattering amplitudes of states on mass-shell by means of the sum over Riemann surfaces. In fact, computing the scattering amplitudes we consider two-dimensional surfaces with punctures, where the vertex operators, satisfying the on mass-shell conformal invariance condition, are put in. However, one can also consider the objects, defined as path integrals over Riemann surfaces with boundaries, to which the string states (in general off-shell states) are attached. One of the simplest objects of this type is a string propagator, which has been originally computed in Ref.[1] by means of the sum over surfaces[2]. In spite of the difficulties, connected with the definition of non-invariant object, the expression for a propagator can be useful for the study of the singularities in string theories and their causality properties and also for the attempts of constructing the effective field theory of strings.

The calculation of the bosonic string propagator by means of Polyakov path integral was made in refs.[4-9]. It should be pointed out that in Refs.[6,7] the one-loop statistical sum was obtained with the help of the sewing procedure for Green function. This is connected with the problem of the divergences in string theory. The most consistent consideration of the bosonic string propagator from the point of view of the string field theory is presented in Ref.[9]. However all those papers are devoted to the bosonic string. The propagator of the heterotic string in Green-Schwarz formulation was computed in Ref.[10]. The calculation of

heterotic string propagator starting from the action with two-dimensional supersymmetry was carried out in Ref.[11].

The purpose of this paper is to compute the propagator of fermionic string in Neveu-Schwarz-Ramond(NSR) formulation, where the action of string is the action of two-dimensional supergravity [12]. The path integral over supergravity fields is reduced to the finite-dimensional integral (over even and odd modular parameters) like in the case of fermionic point particle (see Refs.[13, 14]). The path integral is taken about some classical string trajectory, the quantum fluctuations give some contribution to the integration measure by means of corresponding determinants. The principal problem is the choice of boundary conditions for quantum fields.

The paper is organized in the following way. In Sect.2 the integration measure for the open Ramond string is calculated. In Sect.3 the result for the pointlike boundary conditions is compared with that of operator computation. Sect.4 is devoted to the open Neveu-Schwarz string. The generalization to the case of closed strings is given in Sect.5. Sect.6 contains the discussion of the obtained results and the conclusion.

2. The Open Ramond String

To calculate the scattering amplitudes in the fermionic string theory one should take the sum over all different boundary conditions i.e. spin structures [15]. This procedure corresponds to the consideration of equal number of bosonic and fermionic states given by GSO projection [16] and realizes the space-time supersymmetry of NSR string. However, in the case of the propagator we fix some boundary conditions for fermions on the bounda-

ries of the surface. Therefore we can speak only of the computation of propagators in different sectors of the fermionic string. Let us start from the calculation of the propagator of open Ramond string, which in the limit $\alpha' \rightarrow 0$ corresponds to the fermionic particle (N=1 one-dimensional supergravity).

The action in the theory of fermionic string is the action of two-dimensional supergravity [12]:

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu x^M \partial_\nu x^M - \frac{i}{2} \bar{\psi}^M \gamma^\mu \partial_\mu \psi^M - \right. \\ \left. - i \bar{\chi}_\mu \gamma^\nu \gamma^\mu \psi^M \partial_\nu x^M - \frac{1}{4} (\bar{\psi}^M \psi^M) (\bar{\chi}_\mu \gamma^\nu \gamma^\mu \chi_\nu) \right] \quad (1)$$

$$\mu, \nu = 0, 1, \quad M = 1, \dots, D$$

invariant under local reparametrizations and local supersymmetry:

$$\delta x = i \bar{\epsilon} \psi \quad \delta \psi = (\partial_\mu x + i \bar{\psi} \gamma_\mu) \delta^M \epsilon \quad (2)$$

$$\delta e^a_\mu = -2i \bar{\epsilon} \gamma^a \chi_\mu \quad \delta \chi_\mu = -D_\mu \epsilon$$

and also under local Weyl, Lorentz and super Weyl transformations [18]. Ramond sector of the open fermionic string is singled out by the following boundary conditions:

$$\psi_1(0, \tau) = \varepsilon \psi_2(0, \tau) \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \varepsilon = \pm 1 \quad (3)$$

$$\psi_2(1, \tau) = \varepsilon \psi_1(1, \tau)$$

(when $\sigma = 0$ and $\sigma = 1$ the sign in the equalities (3) is the same) and the similar expressions are used for gravitino and the parameter of supersymmetry transformation (2). In the case of Ramond boundary conditions the action (1) in the limit turns to the action of the spin-1/2 particle. To demonstrate this let us choose in (1) the following gauge:

$$ds^2 = e^2(\tau, \sigma) d\tau^2 + d\sigma^2 \quad (4a)$$

$$\delta^\mu \chi_\mu = 0 \quad (4b)$$

Then one can define the one-dimensional fermion as:

$$\psi^M(\tau) = \psi_1^M(\sigma, \tau) = \varepsilon \psi_2^M(\sigma', \tau) \quad , \quad (5)$$

taking into account the fact that in the limit when the width of world-sheet tends to zero, ψ_1 and ψ_2 become independent of the parameter σ . It is necessary to stress that the definition (5) does not contradict the boundary conditions (3); the combination of (5) and (3) does not lead to an inconsistency in the case of Ramond string (but the same is not true in the case of Neveu-Schwarz string). One can write an expression similar to (5) for the gravitino: in combination with (46) it leaves the single Grassmann function dependent only on the parameter τ . Using (4) and (5) and averaging in (1) over σ , one obtains (up to the renormalization on finite constants) the action of the one-dimensional supergravity [17]:

$$\int_0^1 d\tau \frac{1}{2} [e^{-1} \dot{x}^M \dot{x}_M - i \psi^M \dot{\psi}_M - i e^{-1} \dot{\chi}^M \psi_M] \quad (6)$$

The action (6) corresponds to the spin-1/2 particle. It is the open Ramond string that gives the action (6) in the limit.

The propagator of the fermionic string can be defined as the following functional integral:

$$\int D\varepsilon \int D\chi \int D\psi \int D\chi \exp \{ -S \} \quad (7)$$

$\chi_i(\sigma_i)$

where S is the action (1) (in euclidean formulation), and the integration over $\chi^M(\sigma, \tau)$ should be done with fixed boundaries. i.e. the surface in x -space is rigidly fixed on two boundary

curves (contours). In the case of open string X^M variables satisfy the free boundary conditions at the two other boundaries.

After one has singled out the integral over the group of local symmetry of the action (1), the computation of the covariant path integral over supergravity fields is reduced to the effective finite-dimensional integral over supermoduli space [20,21]. The quantization can be carried out so that the integration measure is obviously invariant under all the symmetries except Weyl and super Weyl symmetries, which will be, in general, broken in quantum theory. It is shown in Ref.[2] that this way of quantization leads to the dynamical contribution of Weyl and super Weyl modes in all dimensions except the critical one $D=10$. Below we shall stay in the critical dimension, when the dynamical contribution is absent. It should be pointed out that formula (7) defines in general the non-invariant quantity (for example, if the integrations over the reparametrization of the boundaries are not introduced [1] the propagator is directly dependent on the parametrization of the boundaries). These questions will not be discussed here, they are considered in detail for the bosonic case in Ref. [9], it is sufficient to note that the dependence on the boundary values of Weyl and super Weyl modes can take place.

To compute the integration measure in (7) one can use a Wess-Zumino gauge [19]. In this gauge the action has the following form:

$$S = \int d^2\sigma (\partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + \chi \bar{\psi} \bar{\partial} X + \bar{\chi} \psi \partial X + \frac{1}{2} \chi \bar{\psi} \chi \bar{\psi}) \quad (8)$$

$$\bar{\partial} = \frac{1}{2} (\partial_2 + i\partial_1), \quad \partial = \frac{1}{2} (\partial_2 - i\partial_1), \quad d^2\sigma = d\tau d\sigma \sqrt{g}$$

(the sum over $M=1, \dots, D$ is omitted). When passing from (1) to (8) the condition $\delta^\mu \chi_\mu = 0$ is used (the result of the super Weyl symmetry $\delta \chi_\mu = \delta_\mu \lambda$) and the fields are redefined on some number constants, for example $X \rightarrow \sqrt{2} X$. The transformation laws for the supergravity set in the chosen gauge have the following form ($\delta h_\theta^a = e_\theta^a \delta e_\mu^a$):

$$\begin{aligned} \delta h_{\bar{z}}^{\bar{z}} &= \bar{\partial} \bar{\xi} - \frac{1}{2} \bar{\chi} \epsilon + \tilde{\delta} h_{\bar{z}}^{\bar{z}} \\ \delta \bar{\chi} &= -\bar{\xi} \partial \bar{\chi} + \frac{1}{2} \bar{\chi} \partial \bar{\xi} + \bar{\partial} \epsilon + \tilde{\delta} \bar{\chi} \\ \delta \bar{h}_{\bar{z}}^{\bar{z}} &= \partial \bar{\xi} - \frac{1}{2} \chi \epsilon + \tilde{\delta} \bar{h}_{\bar{z}}^{\bar{z}} \\ \delta \chi &= -\bar{\xi} \partial \chi + \frac{1}{2} \chi \partial \bar{\xi} + \partial \epsilon + \tilde{\delta} \chi \end{aligned} \quad (9)$$

where $\xi, \bar{\xi}, \epsilon, \bar{\epsilon}$ are the infinitesimal symmetry parameters of the action (8) (ξ and $\bar{\xi}$ transformations contain reparametrization, Weyl and Lorentz transformations). The variations in (9) mean the motion in the directions orthogonal to the gauge transformations.

The exact analysis of those variations that cannot be globally represented as the gauge transformations can be carried out for surfaces ^{without boundaries} in the case of surfaces with boundaries one should consider the supermoduli space derivative from the correspondent one for double surface. It will be assumed below that the strip which is the world sheet for the open string propagator has as the cylinder the single modular parameter - its length.

The variations of gravitino along the supermoduli direction are characterized by the single Grassmann parameter. The reason is that two components of gravitino left in the Wess-Zumino gauge are connected by the boundary conditions on the ends of the open string. Then the variations of the gravitino can be written as:

$$\tilde{\delta}\bar{\chi} = \bar{\rho}\delta\zeta, \quad \tilde{\delta}\chi = \rho\delta\zeta \quad (10)$$

Using (9) and (10) one can see that the condition of orthogonality with gauge transformations takes place if $\bar{\rho} = \text{const}$ and $\rho = \text{const}$. Then from boundary conditions it follows:

$$\rho = \bar{\rho} \quad (10')$$

Introducing the integration over ghost and antighost variables one has for integration measure:

$$\begin{aligned} & \int_{-\infty}^{\infty} dT \int d\zeta \int db dc D\bar{b} D\bar{c} D\beta D\gamma D\bar{\beta} D\bar{\gamma} DX D\psi D\bar{\psi} \langle \eta, b \rangle \\ & \delta(\langle \rho, \beta \rangle) \exp \left\{ - \int d^2\sigma \left[\partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + \right. \right. \\ & + b \bar{\partial} c + \bar{b} \partial \bar{c} + \beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} + \rho \zeta (\bar{\psi} \bar{\partial} X + \frac{1}{2} \bar{b} \bar{\gamma} + \\ & \left. \left. + \frac{1}{2} \bar{\beta} \bar{\partial} \bar{c} + \psi \partial X + \frac{1}{2} b \gamma + \frac{1}{2} \beta \partial c) \right] \right\} \quad (11) \end{aligned}$$

where

$$\langle \eta, b \rangle = \int (\eta b + \bar{\eta} \bar{b})$$

$$\langle \rho, \beta \rangle = \int (\bar{\rho} \beta + \rho \bar{\beta})$$

Let us integrate in (11) over ζ . Then the expression (11)

becomes the "average" of supercurrent - the sum of supercurrent contributions of ghost and matter fields. One can make the substitution by writing all the fields as the sum of classical and quantum components, where the classical component satisfies the free equations of motion and boundary conditions. When averaging, the contribution of the quantum part vanishes. The zero modes of the antighosts correspond to the supermoduli, so the integration measure over antighosts can be written as:

$$Db D\bar{b} = db_0 D'b D'\bar{b} \quad (12a)$$

$$D\beta D\bar{\beta} = d\beta_0 D'\beta D'\bar{\beta} \quad (12b)$$

The integration over zero modes is trivial because of the presence of δ -functions in (11) and as a result one obtains:

$$\int dT (\det \bar{\partial}_{-1} \det \partial_{-1}) (\det \bar{\partial}_{-1/2} \det \partial_{-1/2})^{-1} \times \int d^2\sigma \left[\bar{\Psi} \bar{\partial} \chi + \psi \partial \chi + \frac{1}{2} \bar{b} \bar{\gamma} + \frac{1}{2} \bar{\beta} \bar{\partial} \bar{c} + \frac{1}{2} b \gamma + \frac{1}{2} \beta \partial c \right]_{\alpha} \times (\det -\square_0)^{-5} (\det \bar{\partial}_{1/2} \det \partial_{1/2})^5 \exp \{ -S_{\alpha} \} \quad (13)$$

where

$$S_{\alpha} = \int d^2\sigma [\partial \chi_{\alpha} \bar{\partial} \chi_{\alpha}] + S^*$$

S^* contains the contribution of both the fermionic and the ghost boundary terms. The dependence of the propagator upon the boundary values of the fermionic and ghost fields will not be discussed in this paper, that is why the contribution of S^* will not be considered (as in the Ref. [28]).

Let us compute the dependence of integration measure in (13) on modular parameter T . At first one can notice that for the determinants in (13) the following relation can be written:

$$\det \bar{\partial}_{-1} \det \partial_{-1} \sim \left(\frac{\det \hat{\Delta}_{-1}}{\det N_{-1} \det N_2} \right)^{1/2} = \left(\frac{\det \hat{\Delta}_{-1}}{T} \right)^{1/2} \quad (14)$$

$$\det \bar{\partial}_{-1/2} \det \partial_{-1/2} \sim \left(\frac{\det \hat{\Delta}_{-1/2}}{\det N_{-1/2} \det N_{3/2}} \right)^{1/2} = (\rho^2 T \det \hat{\Delta}_{-1/2})^{1/2}$$

The sign of proportionality means the difference of right hand side and left hand side of (14) which is the exponent of the super-Liouville action. But taking into account the fact that the whole contribution of super Liouville action to the integration measure vanishes in critical dimension one can change the sign of the proportionality in (14) to the sign of the equality. The determinants of the operators $\hat{\Delta}_{-1}$ and $\hat{\Delta}_{-1/2}$ can be calculated

directly from the eigenvalue equations:

$$\hat{\Delta}_{-1} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \begin{pmatrix} -\square & 0 \\ 0 & -\square \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \lambda^{(\xi)} \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} \quad (15a)$$

$$\hat{\Delta}_{-1/2} \begin{pmatrix} \epsilon \\ \bar{\epsilon} \end{pmatrix} = \begin{pmatrix} -\square & 0 \\ 0 & -\square \end{pmatrix} \begin{pmatrix} \epsilon \\ \bar{\epsilon} \end{pmatrix} = \lambda^{(\epsilon)} \begin{pmatrix} \epsilon \\ \bar{\epsilon} \end{pmatrix} \quad (15b)$$

$$\square = \left(\frac{\partial}{\partial \sigma^1} \right)^2 + \left(\frac{\partial}{\partial \sigma^2} \right)^2, \quad 0 < \sigma = \sigma^1 < 1, \quad 0 < \tau = \sigma^2 < T$$

To find the eigenvalues λ in (15) one should choose some boundary conditions. The usual conditions of fixed boundary are used for ξ^1 and ξ^2 parameters:

$$\sigma = 0, 1 \quad \xi^1 = 0, \quad \partial_1 \xi^2 = 0 \quad (16)$$

$$\tau = 0, T \quad \xi^2 = 0, \quad \partial_2 \xi^1 = 0$$

The choice of boundary conditions for the Grassmann parameters is the principal question. To find the eigenfunctions of (15b) let us at first remember that for ϵ and $\bar{\epsilon}$ we have:

$$\epsilon(0, \tau) = -\varepsilon \bar{\epsilon}(0, \tau), \quad \epsilon(1, \tau) = -\varepsilon \bar{\epsilon}(1, \tau) \quad (17)$$

$$\varepsilon = \pm 1$$

Therefore one can introduce the mixed components:

$$\epsilon_+ = \frac{1}{\sqrt{2}} (\epsilon + \bar{\epsilon}), \quad \epsilon_- = \frac{1}{\sqrt{2}} (\epsilon - \bar{\epsilon}) \quad (18)$$

which are also the eigenfunctions of the Laplace operator but they satisfy the natural conditions when $\sigma = 0, 1$:

$$\epsilon_- = 0, \quad \frac{\partial}{\partial \sigma} \epsilon_+ = 0 \quad (19)$$

or vice versa. From (19) it follows that all eigenfunctions ϵ_- are dependent on σ , and ϵ_+ can be either dependent^{on} or independent of σ zero modes $\epsilon_+^{(0)}$. Let us choose the following boundary conditions:

$$\epsilon_+^{(0)}(\tau=0) = 0 \quad , \quad \epsilon_+^{(0)}(\tau=T) = 0 \quad (20a)$$

$$\epsilon_+(\tau+T, \sigma) = -\epsilon_+(\tau, \sigma) \quad (20b)$$

$$\epsilon_-(\tau+T, \sigma) = -\epsilon_-(\tau, \sigma)$$

i.e. the "zero modes" satisfy Dirichlet boundary conditions but the other eigenfunctions are considered on the sewed strip (on the cylinder) and satisfy the antiperiodic boundary conditions. These conditions correspond to "asymptotic" supersymmetry, they give the correct result in the limit of the fermionic particle. They also give the correct distribution function for the states in the string spectrum. For the determinants in (14) one has:

$$\det \hat{\Delta}_{-1} = \left[\prod_{n=1}^{\infty} \prod_{m=1}^{\infty} \left(\pi^2 n^2 + \frac{\pi^2 m^2}{T^2} \right) \right]^2 \prod_{n=1}^{\infty} \pi^2 n^2 \prod_{m=1}^{\infty} \frac{\pi^2 m^2}{T^2} \quad (21a)$$

$$\det \hat{\Delta}_{-1/2} = \left[\prod_{n=1}^{\infty} \prod_{m=0}^{\infty} \left(\pi^2 n^2 + \frac{\pi^2 (2m+1)^2}{T^2} \right) \right]^2 \prod_{m=1}^{\infty} \frac{\pi^2 m^2}{T^2} \quad (21b)$$

The last product in the right hand side of (21b) corresponds to the non trivial contribution of the "zero modes". The determinants (21) are calculated regularizing the infinite products and introducing corresponding counterterms into the integration measure. Then for (14) one obtains:

$$\begin{aligned} \det \bar{\partial}_{-1} \det \partial_{-1} &\sim \prod_{n,m=1}^{\infty} \left(\pi^2 n^2 + \frac{\pi^2 m^2}{T^2} \right) \sim \\ &\sim e^{-\frac{\pi T}{12}} \prod_{n=1}^{\infty} (1 - e^{-2\pi n T}) \equiv \eta(T) \end{aligned} \quad (22a)$$

$$\det \bar{\partial}_{-1/2} \det \partial_{-1/2} \sim \rho T \prod_{n=1}^{\infty} \prod_{m=0}^{\infty} \left(\pi^2 n^2 + \frac{\pi^2 (2m+1)^2}{T^2} \right)^2 \sim \quad (22b)$$

$$\sim \rho T e^{-\frac{\pi T}{12}} \prod_{n=1}^{\infty} (1 + e^{-\pi n T})^2 \equiv \rho T \Omega_R(T)$$

To calculate the determinants of the operators, acting on the fermionic matter fields the antiperiodic boundary conditions for the quantum fields should be used as in the case of the fermionic particle. Then:

$$\det \bar{\partial}_{1/2} \det \partial_{1/2} \sim (\det \hat{\Delta}_{1/2})^{1/2} \sim$$

$$\sim \prod_{n=1}^{\infty} \prod_{m=0}^{\infty} \left(\pi^2 n^2 + \frac{\pi^2 (2m+1)^2}{T^2} \right)^2 \sim \Omega_R(T) \quad (23)$$

Finally, the determinant of Laplace operator, acting on the scalar two-dimensional fields has been previously calculated many times:

$$\det -\square_0 \sim T \eta(T) \quad (24)$$

where $\eta(T)$ is defined in (22a). Substituting (22), (23) and (24) to (13) one gets:

$$\int_0^T \frac{dT}{T^5} \prod_{n=1}^{\infty} \left(\frac{1+e^{-\pi n T}}{1-e^{-\pi n T}} \right)^4 \frac{1}{T} \int d^2 \sigma \left[\bar{\psi}_\alpha \bar{\partial} \chi_\alpha + \psi_\alpha \partial \chi_\alpha \right] \cdot \quad (25)$$

$$\times \exp \{ -S_\alpha \}$$

In the expression (25) and everywhere below the contribution of "classical" ghost fields ^{has been} omitted.

3. The Pointlike Boundary Conditions

Let us consider formula (25) in the case of the pointlike boundary conditions.

For the

fixed boundary values of the $X^M(\sigma, \tau)$ variables when $\tau = 0, T$ the following expansion can be written:

$$X^{i,t}(\sigma) = X_0^{i,t} + \sum_{n=1}^{\infty} X_n^{i,t} \cos \pi n \sigma \quad (26)$$

The pointlike boundary conditions mean that

$$X_n^{i,t} = 0, \quad n = 1, 2, \dots \quad (27)$$

Then the classical action in (25), which in general has the form:

$$S_{cl} = S(X_{cl}) + S^*(\psi_{cl}) = \frac{(X_0^+ - X_0^i)^2}{2T} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{\pi n}{\sinh \pi n T} \left\{ [(X_n^+)^2 + (X_n^i)^2] \cosh \pi n T - 2X_n^+ X_n^+ \right\} + S^*(\psi_{cl}) \quad (28)$$

simplifies up to:

$$S(X_{cl}) \Big|_{X_n^{i,t}=0, n>0} = \frac{(X_0^+ - X_0^i)^2}{2T} \equiv \frac{(\Delta X)^2}{2T} \quad (29)$$

In the expressions (28) and (29) X^M are normalized as in (1) but not as in (8). The classical contributions of the matter fields to the supercurrent in (25) is:

$$\frac{1}{T} \int d^2 \sigma [\bar{\psi}_{cl} \bar{\partial} X_{cl} + \psi_{cl} \partial X_{cl}] = \frac{1}{T} \left\{ \psi_0^M (X_0^+ - X_0^i)^M + \sum_{n=1}^{\infty} \frac{\pi n T}{\sinh \pi n T} [\psi_n^i (X_n^+ - X_n^i e^{\pi n T}) + \psi_{-n}^+ (X_n^+ e^{\pi n T} - X_n^i)] \right\} \quad (30)$$

where

$$\begin{aligned} \psi^{i,t}(\sigma) &= \sum_{n \in \mathbb{Z}} \psi_n^{i,t} e^{i\pi n \sigma} \\ \bar{\psi}^{i,t}(\sigma) &= \sum_{n \in \mathbb{Z}} \psi_n^{i,t} e^{-i\pi n \sigma} \end{aligned} \quad (30')$$

After choosing the pointlike boundary conditions the expression (30) turns to:

$$\frac{1}{T} \psi_0^M (x_0^f - x_0^i)^M = \frac{\psi_0^M (\Delta x)^M}{T} \quad (31)$$

In accordance with the quantization rules one should make the following change after the computation of the path integral:

$$\psi_0^M \rightarrow \frac{1}{\sqrt{2}} \delta^M, \quad \psi_n^M \rightarrow \delta_{11} \psi_n^M \quad (32)$$

where δ^M are ten-dimensional Dirac matrices and $\delta_{11} = \prod_M \delta^M$.

For the pointlike boundary conditions from (25) it can be obtained:

$$\begin{aligned} & \int_0^\infty \frac{dT}{T^5} \prod_{n=1}^\infty \left(\frac{1+e^{-\pi n T}}{1-e^{-\pi n T}} \right)^4 \frac{\psi_0^M \Delta x^M}{T} e^{-\frac{(\Delta x)^2}{2T}} = \\ & = - \text{const.} \delta^M \frac{\partial}{\partial (x_0^f)^M} \int_0^\infty \frac{dT}{T^5} \prod_{n=1}^\infty \left(\frac{1+e^{-\pi n T}}{1-e^{-\pi n T}} \right)^4 e^{-\frac{(\Delta x)^2}{2T}} \quad (33) \end{aligned}$$

Using in (33) Fourier transformation, restoring the factor $(2\pi\alpha')^{-1}$ and taking into account, that:

$$\prod_{n=1}^\infty \left(\frac{1+e^{-\pi n T}}{1-e^{-\pi n T}} \right)^4 = \sum_{N=0}^\infty C_N e^{-\pi N T} \quad (34)$$

where $C_0=1$, $C_1=8$, $C_2=40$, $C_3=160$, $C_4=552$ etc., one will get for the propagator:

$$\begin{aligned} & \int \frac{d^{10}p}{(2\pi)^{10}} e^{ip\Delta x} \frac{1}{2\pi\alpha'} \left(\frac{1}{p^2} + 8 \frac{1}{p^2 + \frac{1}{\alpha'}} + \right. \\ & \left. - 40 \frac{1}{p^2 + \frac{2}{\alpha'}} + 160 \frac{1}{p^2 + \frac{3}{\alpha'}} + 552 \frac{1}{p^2 + \frac{4}{\alpha'}} + \dots \right) \quad (35) \end{aligned}$$

The first term in the expansion (35) corresponds to the massless Dirac particle, the other propagating modes satisfy some nonlocal equations of motion.

Let us compare the expression (35) with the result of the operator calculations. The propagator of the open fermionic string in the operator formalism has the following form:

$$F_0 \int_0^\infty dT \exp(-TL_0) \quad (36)$$

where

$$F_0 = \frac{1}{\sqrt{2}} \not{p} + \frac{1}{\sqrt{\alpha'}} \sum_{n \neq 0} \alpha_{-n}^i \psi_n^i$$

$$L_0 = p^2 + \frac{1}{\alpha'} N, \quad N = \sum_{n=1}^{\infty} (\alpha_{-n}^i \alpha_n^i + n \psi_{-n}^i \psi_n^i) \quad (37)$$

For comparison with (35) the matrix element of the (36) in pointlike boundary conditions should be computed, therefore the pointlike boundary conditions in the operator language have to be defined [22]. The conditions (27) will have the form:

$$\hat{X}_n^j |x, \alpha\rangle \gg = 0, \quad n > 0$$

$$\hat{X}_n^j = \frac{\alpha_n^j - \alpha_{-n}^j}{n}, \quad j = 1, \dots, 8 \quad (38)$$

where index j runs all the D-2 physical degrees of freedom. The states, satisfying (38), are:

$$|x, \alpha\rangle \gg = |x\rangle \otimes \exp\left[\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^i \alpha_{-n}^i\right] |0\rangle \otimes \hat{\Psi} |u_\alpha\rangle \quad (39)$$

where $\alpha_n^i |0\rangle = 0, \quad n > 0; \quad [\alpha_n^i, \alpha_m^j]_- = n \delta_{n+m,0} \delta^{ij},$

$|x\rangle$ is the eigenstate of the ten-dimensional coordinate operator; $\hat{\Psi}$ is the operator constructed from the operators

$\psi_{-n}^j, \quad n > 0$ (30'). When only the part of quantum numbers is

fixed in the initial and final states it means that the trace over the others should be taken. In the present case the trace is taken over two-dimensional fermions. It is the fact which corresponds to the choice of the antiperiodic boundary conditions in the previous section. Calculating the matrix element of the operator (36) between the initial and final states such as given by the formula (39), one obtains:

$$\langle\langle x_f, \alpha | F_0 \int_0^\infty dT \exp(-TL_0) | x_i, \beta \rangle\rangle = (\mathcal{P})_{\alpha\beta} \sum_{N=0}^{\infty} \frac{d_N}{p^2 + m_N^2} \quad (40)$$

where $\alpha' m_N^2 = N$, ($N=0,1,\dots$) is the spectrum of the open Ramond string. The weight number d_N characterizes the contribution of the each mass level to (40) and is the sum of norms of the physical states at each mass level. The norms of the two-dimensional bosonic states are to be computed directly, using the expansion of (39) over the mass levels:

$$\begin{aligned} \exp \left[\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^i \alpha_{-n}^i \right] |0\rangle &= \left(1 + \frac{1}{2} \alpha_{-1}^i \alpha_{-1}^i + \right. \\ &\left. + \frac{1}{4} \alpha_{-2}^i \alpha_{-2}^i + \frac{1}{8} \alpha_{-1}^i \alpha_{-1}^i \alpha_{-1}^j \alpha_{-1}^j + \dots \right) \end{aligned} \quad (41)$$

The contribution of the two-dimensional fermionic states to the d_N factor is simply their number at each mass level. It is the consequence of the fact that the trace over fermions is computed.

Hence, for the lowest mass level one obtains:

$$\begin{aligned} d_0 &= 1 \\ d_1 &= 8 \quad (\psi_{-1}^i | \mathcal{R} \rangle, \quad i = 1, \dots, 8) \end{aligned}$$

$$d_2 = 8 + 28 + 4 = 40$$

$$(\psi_{-2}^i | \Omega \rangle, \psi_{-1}^i \psi_{-1}^j | \Omega \rangle, \frac{1}{2} \alpha_{-1}^i \alpha_{-1}^j | 0 \rangle)$$

$$d_3 = 8 + 64 + 56 + 32 = 160$$

$$(\psi_{-3}^i | \Omega \rangle, \psi_{-2}^i \psi_{-1}^j | \Omega \rangle, \psi_{-1}^i \psi_{-1}^j \psi_{-1}^k | \Omega \rangle,$$

$$\psi_{-1}^i | \Omega \rangle \otimes \frac{1}{2} \alpha_{-1}^j \alpha_{-1}^k | 0 \rangle)$$

$$d_4 = 8 + 64 + 28 + 224 + 70 + 4 + 10 + 32 + 112 = 552$$

etc

$$\text{where } | \Omega \rangle: \psi_n^i | \Omega \rangle = 0, \quad n > 0$$

Comparing (42) and (34), one can easily see that $d_N = c_N$. Then the operator result (40) coincides with the path integral result (35).

4. The Open Neveu-Schwarz String

The results of the previous sections can be easily generalized to the case of the open Neveu-Schwarz string. Unlike (3) the Neveu-Schwarz sector is singled out by the conditions:

$$\psi_1(0, \tau) = \varepsilon \psi_2(0, \tau)$$

(43)

$$\psi_1(1, \tau) = -\varepsilon \psi_2(1, \tau)$$

where $\varepsilon = \pm 1$, but the signs for $\sigma = 0$ and $\sigma = 1$ are opposite. The fact that the components of the two-dimensional fermion are connected by boundary conditions of opposite sign (43) leads to the conclusion that there is no natural reduction of the Neveu-Schwarz string to the one-dimensional case (6). Therefore it seems that the action (6) cannot describe the bosonic excitations (this conclusion differs from that of [17]).

The main difference of the integration measure between the sectors of Neveu-Schwarz and Ramond is the fact that in the gauge $\bar{\chi} = \bar{\rho}\zeta$, $\chi = \rho\zeta$, where $\rho = \text{const}$ and $\bar{\rho} = \text{const}$, now from the boundary conditions of the (43) type, instead of (10') one has:

$$\rho = \bar{\rho}, \quad \rho = -\bar{\rho}, \quad \text{i.e.} \quad \rho = \bar{\rho} = 0 \quad (44)$$

Hence, in Neveu-Schwarz sector there is no integration over supermodular Grassmann parameter and that is why the averaging of the supercurrent will not appear. Carrying out the calculations similar to that of the sect.2 for the propagator of the open Neveu-Schwarz string one obtains:

$$\int_0^\infty dT \left(\frac{\det \hat{\Delta}_{-1}}{T} \right)^{1/2} \left(\det \hat{\Delta}_{-1/2} \right)^{-1/2} (\det -\square_0)^{-5} \times \\ \times (\det \bar{\partial}_{1/2} \det \partial_{1/2})^5 \exp \{ -S_\alpha \} \quad (45)$$

Calculating the determinants in (45), one has:

$$\int_0^\infty \frac{dT}{T^5} \left[\frac{\Omega_{NS}(T)}{\eta(T)} \right]^4 \exp \{ -S_\alpha \} \quad (46)$$

where
$$\Omega_{NS}(T) = \prod_{n=0}^\infty \prod_{m=0}^\infty \left[\frac{\pi^2 (2m+1)^2}{T^2} + \frac{\pi^2 (2n+1)^2}{4} \right]^2$$

and the function $\eta(T)$ is defined in (22a). Regularizing and renormalizing the double infinite products in (46), for the point-

like boundary conditions the result will be:

$$\int_0^\infty \frac{dT}{T^5} e^{\frac{\pi T}{2}} \left[\frac{\prod_{n=0}^\infty (1 + e^{-\pi T(n+\frac{1}{2})})^2}{\prod_{n=1}^\infty (1 - e^{-2\pi n T})} \right]^4 e^{-\frac{(\Delta X)^2}{2T}} \quad (47)$$

Representing the infinite product in (47) as the sum and restoring the factor $(2\pi\alpha')^{-1}$, one obtains:

$$\int \frac{d^{10}p}{(2\pi)^{10}} e^{i p \Delta X} \frac{1}{2\pi\alpha'} \left[\frac{1}{p^2 - \frac{1}{2\alpha'}} + 8 \frac{1}{p^2} + \right. \\ \left. + 28 \frac{1}{p^2 + \frac{1}{2\alpha'}} + 54 \frac{1}{p^2 + \frac{1}{\alpha'}} + 138 \frac{1}{p^2 + \frac{3}{2}\frac{1}{\alpha'}} + \dots \right] \quad (48)$$

i.e. the sum of the propagating scalar field Green functions with some weight factors. The first term corresponds to the tachyon ($m^2 = -\frac{1}{2\alpha'}$). The weight numbers in (48) can be easily computed as in the section 3 by operator methods. Let us calculate the factors written in the expression (48):

$$\begin{aligned} d_{-1/2} &= 1 \\ d_0 &= 8 \quad (\psi_{-1/2}^j | \Omega \rangle, j=1, \dots, 8) \\ d_{1/2} &= 28 \quad (\psi_{-1/2}^i \psi_{-1/2}^j | \Omega \rangle) \\ d_1 &= 8 + 56 = 64 \\ & \quad (\psi_{-3/2}^i | \Omega \rangle, \psi_{-1/2}^i \psi_{-1/2}^j \psi_{-1/2}^k | \Omega \rangle) \end{aligned} \quad (49)$$

$$d_{3/2} = 4 + 64 + 70 = 138$$

$$(\frac{1}{2}\alpha_{-1}^i \alpha_{-1}^i |0\rangle, \psi_{-3/2}^i \psi_{-1/2}^j |12\rangle, \psi_{-1/2}^i \psi_{-1/2}^j \psi_{-1/2}^k \psi_{-1/2}^l |12\rangle),$$

$$\text{where } |12\rangle: \psi_s^i |12\rangle = 0, \quad s = 1/2, 3/2, \dots$$

and the indices of d_r are integer and half-integer as the spectrum of the Neveu-Schwarz string $\alpha' m_r^2 = r$, $r = -1/2, 0, 1/2, 1, \dots$. It should be stressed that in expression (48) the propagating fields are space-time bosons and in the expression (35) the propagating fields are space-time fermions.

5. Closed Strings

The results of the previous sections can be generalized to the case of closed strings. The world-sheet corresponding to the closed string propagator is the cylinder. The structure of the integration measure over metrics on the cylinder differs from the integration measure over the metrics on the strip only by the conformal Killing vector contribution. The contribution of the two-dimensional bosons and bosonic ghosts to the integration measure on the cylinder was computed in the original paper [1]. Left and right fermion sectors are independent on the cylinder. The boundary conditions can be imposed independently in each sector - the periodicity or antiperiodicity when the field is transported along one of the simplest non-contractible loop. Therefore the contribution of the two-dimensional fermions to the integration measure on the cylinder is the product of the left and right sectors contributions, unlike [23]. There are altogether three different closed string sectors: NSxNS, NSxR or RxNS and RxR. In Ramond (left and right) sectors one can impose on the gravitino the gauge $\bar{\chi} = \bar{\rho} \bar{\zeta}$ or $\chi = \rho \zeta$ and correspondingly $\bar{\chi} = 0$ or

$\chi = 0$ in Neveu-Schwarz sectors. In the case of closed strings unlike (12b) the antighost β and $\bar{\beta}$ zero modes are present or absent independently.

As a result for the closed string propagators it can be obtained:

(i) NS x NS sector

$$\int_0^\infty \frac{dT}{T^5} \left[\frac{\det -\square^{(-)}}{P(T)} \right]^4 \exp \{-S_c\} \quad (50a)$$

$$\det -\square^{(-)} = \prod_{n=0}^\infty \prod_{m=0}^\infty \left[\frac{\pi^2 (2m+1)^2}{T^2} + \pi^2 (2n+1)^2 \right]^4$$

(ii) NS x R sector

$$\int_0^\infty \frac{dT}{T^5} \frac{[\det -\square^{(+)} \det -\square^{(-)}]^2}{[P(T)]^4} \left\{ \frac{1}{T} \int \psi_\alpha \partial X_\alpha \right\} e^{-S_c} \quad (50b)$$

$$\det -\square^{(+)} = \prod_{n=1}^\infty \prod_{m=0}^\infty \left[4\pi^2 n^2 + \frac{\pi^2 (2m+1)^2}{T^2} \right]^4 \prod_{m=0}^\infty \left[\frac{\pi^2 (2m+1)^2}{T^2} \right]^2$$

(iii) R x R sector

$$\int_0^\infty \frac{dT}{T^5} \left[\frac{\det -\square^{(+)}}{P(T)} \right]^4 \left\{ \frac{1}{T^2} \int \bar{\psi}_\alpha \partial X_\alpha \int \psi_\alpha \partial X_\alpha + \right.$$

$$\left. + \frac{1}{2T^2} \int (\psi_\alpha \bar{\psi}_\alpha) \right\} \exp \{-S_c\}$$

In the formulae (50):

$$P(T) = \prod_{n=1}^\infty \prod_{m=1}^\infty \left[\frac{\pi^2 n^2}{T^2} + 4\pi^2 m^2 \right]^2$$

Only some particular cases of the expressions (50) will be considered below. Fixing the pointlike boundary conditions and calculating the infinite products we get:

(i) NS x NS sector

$$\int_0^\infty \frac{dT}{T^5} e^{2\pi T} \frac{\prod_{n=0}^\infty (1 + e^{-\pi(2n+1)T})^{16}}{\prod_{n=1}^\infty (1 - e^{-4\pi n T})^8} e^{-\frac{(\Delta X)^2}{2T}} \quad (52)$$

(ii). NS x R sector

$$\int_0^\infty \frac{dT}{T^5} e^{\pi T} \prod_{n=1}^\infty \left(\frac{1 + e^{-\pi(2n+1)T}}{1 - e^{-2\pi n T}} \right)^8 \left(\frac{\psi_0 \Delta X}{T} \right) e^{-\frac{(\Delta X)^2}{2T}} \quad (53)$$

Expanding the infinite products in (52) and (53) into the infinite sums one obtains the representation for the closed string propagators as the sum of propagating string modes. It can be pointed out that in the expression (52) the propagating fields are space-time bosons and in the expression (53) they are space-time fermions.

The expressions (52) and (53) contain infrared singularities - tachyons. However it is known [24] that the physical spectrum of the NSxR sector is tachyonfree due to the constraint $L_0 - \tilde{L}_0 = 0$. When computing the bosonic closed string propagator [1] for pointlike boundary conditions the constraint $L_0 - \tilde{L}_0 = 0$ is ensured automatically. But when the trace over two-dimensional fermions is taken in the closed fermionic string this constraint is spoilt owing to the two-dimensional fermionic contribution. This is the reason for the presence of tachyon in (53).

The $R \times R$ sector should be considered a little more carefully. Restoring in (50c) the normalization of X^M variables as in (1) one obtains:

$$\int_0^\infty \frac{dT}{T^5} \prod_{n=1}^\infty \left(\frac{1+e^{-2\pi n T}}{1-e^{-2\pi n T}} \right)^8 \left\{ \frac{(\bar{\psi}_0 \Delta X)(\psi_0 \Delta X)}{T} + \right. \\ \left. + \frac{1}{T} (\psi_0 \bar{\psi}_0) \right\} \exp \left[-\frac{(\Delta X)^2}{2T} \right] \quad (54)$$

where the second term in the curl brackets is the consequence of the presence of the χ^2 -term in (1) and (8). This second term is necessary to get the result:

(iii) $R \times R$ sector.

$$\bar{\psi}_0^M \frac{\partial}{\partial (X_0^+)^M} \psi_0^N \frac{\partial}{\partial (X_0^+)^N} \int_0^\infty \frac{dT}{T^5} \prod_{n=1}^\infty \left(\frac{1+e^{-2\pi n T}}{1-e^{-2\pi n T}} \right)^8 e^{-\frac{(\Delta X)^2}{2T}} \quad (55)$$

The results of this section can be compared with the operator results given, for example, in Ref. [26]. It can be seen that they differ by the absence of $\delta(L_0 - \tilde{L}_0)$ in (52), (53) and (55), which ensure the contribution only of the physical closed string spectrum. This δ -function is restored in the one-loop statistical sum, where its appearance is the result of the integration over the real part of the ^{complex} modular parameter. But for the propagator the world-sheet has the single real modular parameter. However the expressions for closed string propagators are useful when studying the ultraviolet singularities connected with the causality properties of strings.

In the conclusion of this section let us notice that in the

limit $\alpha' \rightarrow 0$ the $R \times R$ sector of the closed fermionic string corresponds to the $N=2$ one-dimensional supergravity. It is well-known that only the closed strings contain graviton in the spectrum and can consistently interact with the external gravitational fields. For the open strings it is impossible to define consistently such interaction. It can explain the fact that the quantum Hamiltonian coincides with the classical one for the $N=2$ one-dimensional supergravity in a curved space. For the $N=1$ case there appear some corrections, connected with the operator ordering [27]. However, in the first order of the perturbation theory the path integral with the classical action (without corrections) for Dirac particle ($N=1$ theory) gives the result for Green function, coinciding with the expansion of the solution of the corresponding Dirac equation for Green function in the curved space (see Ref. [29]).

6. Discussion

The obtained results allow^{us)} to analyze the properties of the propagators in the fermionic string theory. The most important point is the structure of the singularities of the expressions for the propagators. The infrared singularities are present due to the tachyons in string spectra. Apart from the infrared tachyon singularities these expressions possess the ultraviolet singularities. One can observe them in the following way. The calculation of the double infinite products, leading to the results (33), (45), (52), (53) and (55) can be carried out changing the order of taking the double product [25]. Then it can be easily seen that the main ultraviolet singularity has the following form:

$$\lim_{\epsilon \rightarrow 0} \int dT \exp \left[\frac{\pi}{T} - \frac{(\Delta X)^2}{2T \cdot 2\pi\alpha'} \right] = \lim_{\epsilon \rightarrow 0} \int dT \exp \left[\frac{1}{2T} \left(2\pi - \frac{(\Delta X)^2}{2\pi\alpha'} \right) \right] \quad (56)$$

The integral in (56) diverges exponentially, when:

$$(\Delta x)^2 < 4\pi^2 \alpha' \quad (57)$$

It is important, that:

$$\ell = 2\pi \sqrt{\alpha'} \quad (58)$$

is the common fundamental length for all (open and closed, bosonic and fermionic) strings in the critical dimensions. The singularities of (56) are connected with the mass spectrum of string theories, increasing as a linear exponent of mass. It follows that string theory can be classified to the non-localizable theories when we consider its causal properties. Green function of a non-localizable theory can have a singularity in the spacelike domain but the singular point should be not farther from the light cone than ℓ [28,30].

When the ends of the propagator are sewed the ultraviolet singularities should vanish. The reason is the appearance of modular invariance (in the case of closed strings) and sum over spin structures. From this point of view it would be interesting to understand how the restriction of integration domain only to the modular figure appears.

Let us, finally, point out that Green functions of strings can be useful in studying motion of strings in external fields. The dependence of a propagator upon the fermionic and ghost boundary values can be interesting from the point of view of string field theory and the theory of Riemann surfaces with boundaries.

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