POSSIBLE SEARCHES FOR CP NON-CONSERVATION IN Z BOSON DECAYS

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A B S T R A C T

We investigate how the decays $Z \rightarrow \ell^+ \ell^- \gamma$ ($\ell = e, \mu, \tau$), $Z \rightarrow 2$ jets + γ , $Z \rightarrow 3$ jets can be used to search for possible new sources of CP violation beyond the standard model (SM). CP tests can be made with these decays using CP-odd angular correlations. Within the SM these correlations are unobservably small. We parametrize the CP-violating interactions which may affect these decays by an effective Lagrangian L_{CP} and calculate the CP-odd correlations in terms of the parameters of L_{CP} .

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INTRODUCTION

Soon large numbers of Z bosons will be produced at the e^+e^- colliders SLC and LEP1. In particular, LEP1 is expected to provide several millions of Z bosons per year. The decays of the Z bosons will allow for a detailed study of interactions at a distance scale which is not easily accessible otherwise. It should be natural to look also for CP violation in Z decays. One may use the Z resonance to search for CP -violating effects which are predicted by the 3 generation standard model (SM) where CP violation is induced by the charged weak quark currents¹. The expected CP violation for $B\bar{B}$ mesons (for recent reviews see Refs. 2, 3, 4) produced at the Z peak may offer some hope to find CP -odd effects. For instance, a CP -odd asymmetry which may serve this purpose was proposed recently⁵. Here we are concerned with effects due to possible new sources of CP violation. Several proposals for CP tests in this context have already been made^{6,7,8}. We have investigated⁹ in a systematic way whether and how the main Z decays, i.e. the decays $Z \rightarrow l^+l^-$, $Z \rightarrow l^+l^-\gamma$ ($l = e, \mu, \tau$), $Z \rightarrow 2 \text{ jets}$, $Z \rightarrow 2 \text{ jets} + \gamma$, $Z \rightarrow 3 \text{ jets}$ can be used for CP tests. The main results will be presented in the following. We always consider the situation where polarization of the final states is not observed. Appropriate observables are CP -odd correlations among the momentum direction vectors of the outgoing states, i.e. CP -odd angular correlations. Within the SM these correlations turn out to be unobservably small. Rather than investigating specific "non-standard" models of CP violation we describe the CP -violating interactions which may affect the above-mentioned decays by a local effective Lagrangian L_{CP} , including operators of mass dimension $d \leq 6$. This allows for a fairly model-independent analysis. The correlations are then calculated by using this Lagrangian; i.e., our results will depend on the parameters of L_{CP} .

It is straightforward to show⁹ that tests of CP and/or T invariance cannot be made with the 2-body decay modes $Z \rightarrow l^+l^-$, $Z \rightarrow 2 \text{ jets}$ if no polarizations of the outgoing particles are measured. (Polarization measurements are thought to be feasible for τ leptons.) Therefore, we confine ourselves here to the 3-body decay modes stated above.

THREE BODY DECAYS OF THE Z BOSON

We discuss in some detail how the discrete symmetries CP , T , and CPT can be tested in the decays $Z \rightarrow l^+l^-\gamma$. This analysis also applies to $Z \rightarrow 2 \text{ jets} + \gamma$ where the flavour of the jets is tagged. Then we give some results of a similar analysis⁹ of $Z \rightarrow 2 \text{ jets} + \gamma$, $Z \rightarrow 3 \text{ jets}$ where no flavour identification is made. As stated above we always consider the situation where no polarizations of the outgoing particles are observed. For the CP -odd correlations introduced below it is however crucial that the Z bosons are polarized to a certain degree. This will always be the case for Z bosons produced at SLC and LEP. Let us first consider the spin density matrix of the

Z in its rest system. In the Cartesian basis

$$|Z, i\rangle \equiv |Z(\mathbf{p} = 0), \hat{\mathbf{e}}_i\rangle \quad (i = 1, 2, 3) \quad (1)$$

where $\hat{\mathbf{e}}_i$, being the i -th Cartesian unit vector, is the polarization vector of the state (1) the spin density matrix ρ_{ij} can be expanded as follows:

$$\begin{aligned} \rho_{ij} &= \frac{1}{3}\rho_{ij} + \frac{1}{2i}\epsilon_{ijk}s_k - s_{ij}, \\ s_{ij} &= s_{ji}, \quad s_{ii} = 0, \end{aligned} \quad (2)$$

where s_k, s_{ij} describe the vector and the tensor polarization of the Z , respectively. Z bosons produced in e^+e^- , $p\bar{p}$ or pp collisions are in general polarized to a certain degree. For instance, if the Z 's are produced by unpolarized e^+e^- beams the vector and tensor polarization of the Z in the c.m. system are, respectively:

$$\begin{aligned} \mathbf{s} &= \frac{2g_{Ve}g_{Ae}}{(g_{Ve})^2 + (g_{Ae})^2}\hat{\mathbf{p}}_+, \\ s_{ij} &= \frac{1}{2}(\hat{p}_{+i}\hat{p}_{+j} - \frac{1}{3}\delta_{ij}), \end{aligned} \quad (3)$$

where $\hat{\mathbf{p}}_+ = \mathbf{p}_+/|\mathbf{p}_+|$ and $\mathbf{p}_+$ is the e^+ c.m. momentum. The neutral current couplings g_{Ve}, g_{Ae} of the electron are given in the SM by $g_{Ve} = -1/2 + 2\sin^2\theta_w$, $g_{Ae} = -1/2$. Using $\sin^2\theta_w = .23$ and choosing $\hat{\mathbf{p}}_+ = \hat{\mathbf{e}}_3$ then:

$$\begin{aligned} \mathbf{s} &= .16\hat{\mathbf{p}}_+, \\ s_{ij} &= \text{diag}(-1/6, -1/6, 1/3). \end{aligned} \quad (4)$$

Consider now the decays

$$Z \rightarrow l^+(k_+) + l^-(k_-) + \gamma(k) \quad (5)$$

where $l = e, \mu, \tau$. We shall always work in the rest system of the Z , i.e., we have $\mathbf{k}_+ + \mathbf{k}_- + \mathbf{k} = 0$. It is very convenient to analyze (5) in terms of its decay matrix. It is defined in the Cartesian polarization basis (1) by

$$\begin{aligned} R_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) &= \Gamma_{l^+l^-\gamma}^{-1} \\ &\sum_{r,s,\epsilon} \langle l^+(\mathbf{k}_+, r), l^-(\mathbf{k}_-, s), \gamma(\mathbf{k}, \epsilon) | \mathcal{T} | Z, i \rangle^* \\ &\langle l^+(\mathbf{k}_+, r), l^-(\mathbf{k}_-, s), \gamma(\mathbf{k}, \epsilon) | \mathcal{T} | Z, j \rangle, \end{aligned} \quad (6)$$

where r, s are the spin indices of $l^\pm$ and ϵ is the polarization vector of the photon. The matrix R_{ij} is normalized such that

$$\frac{1}{3} \int d\Gamma R_{ii}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) = 1, \quad (7)$$

where $d\Gamma$ is the usual phase space measure. In computing the decay width $\Gamma_{l+l-\gamma}$ and the integral (7) energy and angular cuts appropriate for a given experiment have to be taken into account. Cuts are also necessary on theoretical grounds in order to avoid the usual infrared and collinear singularities. We will assume such cuts to be C - and P -blind; i.e. the cuts should apply equally well to l^+ and l^- and equally well to particles of momenta $\mathbf{k}$ and $-\mathbf{k}$. In this way no CP -bias will be introduced by the cuts.

The following decomposition of R_{ij} can be made:

$$\begin{aligned} R_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) &= \delta_{ij} a(E_+, E_-) \\ &+ \frac{1}{i} \varepsilon_{ijl} B_l(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) \\ &- C_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}), \end{aligned} \quad (8)$$

where $E_{\pm} = k_{\pm}^0$ and $C_{ij} = C_{ij}$, $C_{ii} = 0$. Using $\hat{\mathbf{k}}_{\pm} = \mathbf{k}_{\pm}/|\mathbf{k}_{\pm}|$ and the vector $\hat{\mathbf{n}} = (\mathbf{k}_+ \times \mathbf{k}_-)/|\mathbf{k}_+ \times \mathbf{k}_-|$ which is orthogonal to the decay plane as a basis we can decompose $\mathbf{B}$ and the traceless symmetric tensor C_{ij} as follows:

$$\begin{aligned} \mathbf{B}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) &= \hat{\mathbf{k}}_+ b_1(E_+, E_-) \\ &+ \hat{\mathbf{k}}_- b_2(E_+, E_-) + \hat{\mathbf{n}} b_3(E_+, E_-), \end{aligned} \quad (9)$$

$$\begin{aligned} C_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) &= (\hat{k}_{+i} \hat{k}_{+j} - \frac{1}{3} \delta_{ij}) c_1(E_+, E_-) \\ &+ (\hat{k}_{-i} \hat{k}_{-j} - \frac{1}{3} \delta_{ij}) c_2(E_+, E_-) \\ &+ [\hat{k}_{+i} \hat{k}_{-j} + \hat{k}_{-i} \hat{k}_{+j} - \frac{2}{3} (\hat{\mathbf{k}}_+ \cdot \hat{\mathbf{k}}_-) \delta_{ij}] c_3(E_+, E_-) \\ &+ (\hat{k}_{+i} \hat{n}_j + \hat{n}_i \hat{k}_{+j}) c_4(E_+, E_-) \\ &+ (\hat{k}_{-i} \hat{n}_j + \hat{n}_i \hat{k}_{-j}) c_5(E_+, E_-). \end{aligned} \quad (10)$$

Since R is hermitean a, b_i, c_j ($i = 1, 2, 3, j = 1, \dots, 5$) are real functions.

Next we give the constraints on R which arise if CP, T , and CPT invariance holds. The decay mode (5) and the decays (18), (19), (20) considered below allow for CP tests because the Z boson states (1) in the Z rest system are eigenstates of CP — as are the final states in these decays. CP invariance implies

$$R_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) = R_{ij}(-\mathbf{k}_-, -\mathbf{k}_+, -\mathbf{k}). \quad (11)$$

It is well known that final state interactions can fake T and CPT violations even if the basic interactions involved conserve these symmetries. However, since the SM gauge couplings are small at the scale of the Z mass m_Z and the couplings of possible new CP -violating interactions should also be small for phenomenological reasons, we neglect in the following final state rescattering effects in the decays (5) and (18), (19), (20) below. (See also the comment below.) Then we obtain from CPT invariance

$$R_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) = R_{ji}(\mathbf{k}_-, \mathbf{k}_+, \mathbf{k}), \quad (12)$$

whereas from T invariance we have

$$R_{ij}(\mathbf{k}_+, \mathbf{k}_-, \mathbf{k}) = R_{ji}(-\mathbf{k}_+, -\mathbf{k}_-, -\mathbf{k}). \quad (13)$$

A more general discussion is given in Refs. 9,10. The constraints on the functions a, b_i, c_j due to (11), (12), and (13) are collected in Table 1. According to the work of Ref. 11, QED final state interactions – respectively, QCD final state interactions in $Z \rightarrow q\bar{q}$ gluon – do not generate to one-loop approximation non-zero functions b_3, c_4, c_5 .

Suppose now that a CP -violating but CPT -conserving interaction affects the decays (5). We see from Table 1 that in this case the CP -symmetry relations involving $a, b_1, b_2, c_1, c_2, c_3$ can only be violated if final state interactions are present. This implies for instance that in a calculation where both for the CP -conserving and the CP -violating amplitudes only Born diagrams are considered only the CP relations for b_3, c_4, c_5 of Table 1 can be violated. This is exemplified by explicit calculations below.

Integral observables which are sensitive to CP violation in the decays (5) can be easily constructed. Examples are:

- i) Asymmetry in the distribution of $\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-$: If rotational invariance holds for the cuts then using (2) and (8) one obtains

$$\langle \hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_- \rangle = I \mathbf{s} \quad (14)$$

where

$$\begin{aligned} I &= \frac{1}{3} \int d\Gamma |\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-| b_3(E_+, E_-) \\ &= \frac{1}{6} \int d\Gamma \sqrt{1 - (\hat{\mathbf{k}}_+ \hat{\mathbf{k}}_-)^2} \\ &\quad \cdot [b_3(E_+, E_-) + b_3(E_-, E_+)]. \end{aligned} \quad (15)$$

- ii) Asymmetry in the distribution of the tensor $\hat{k}_{+i}\hat{n}_j - \hat{k}_{-i}\hat{n}_j + (i \leftrightarrow j)$: Again for cuts respecting rotational invariance we get

$$\langle \hat{k}_{+i}\hat{n}_j - \hat{k}_{-i}\hat{n}_j + (i \leftrightarrow j) \rangle = K s_{ij} \quad (16)$$

where (2) and (8) were used and

$$K = \frac{2}{5} \int d\Gamma (1 - \hat{\mathbf{k}}_+ \cdot \hat{\mathbf{k}}_-) [c_4(E_+, E_-) - c_5(E_-, E_+)]. \quad (17)$$

Experimental observation of $I \neq 0$ and/or $K \neq 0$ would be evidence for CP -violation. Observables which require CP -violating final state interactions for being non-zero can also be constructed⁹.

The above analysis and the observables (14), (16) also apply to the decays $Z \rightarrow 2 \text{ jets} + \gamma$, where the jets arising from the fragmentation of a quark and an antiquark are tagged.

$$Z \rightarrow \bar{q}(\hat{\mathbf{k}}_+) + q(\hat{\mathbf{k}}_-) + \gamma(\hat{\mathbf{k}}) \rightarrow 2 \text{ jets} + \gamma \quad (18)$$

Here $q = u, d, s, c, b$. According to the UA1 lower limit¹² on the mass of the t quark, $m_t > 41\text{GeV}$ (see Ref. 13) 3-body decays of the Z involving $t\bar{t}$ are not very likely and we do not consider them in the following.

Finally we briefly discuss the decays $Z \rightarrow 3 \text{ jets}, 2 \text{ jets} + \gamma$ where no flavour identification of the jets is made. Consider

$$Z \rightarrow \text{jet}(\mathbf{k}_1) + \text{jet}(\mathbf{k}_2) + \text{jet}(\mathbf{k}_3) \quad (19)$$

and

$$Z \rightarrow \text{jet}(\mathbf{k}_1) + \text{jet}(\mathbf{k}_2) + \gamma(\mathbf{k}_3). \quad (20)$$

In order to use these decays for CP tests the jets have to be ordered according to a CP -blind criterion. Several such criteria can be given^{8,9}. In the calculations below we have chosen for definiteness

$$|\mathbf{k}_1| \geq |\mathbf{k}_2| \geq |\mathbf{k}_3| \quad (21)$$

for the decay (19) and

$$|\mathbf{k}_1| \geq |\mathbf{k}_2| \quad (22)$$

for the decay (20). Of course, the remarks made above on the CP -blindness of the cuts apply also here.

A decay-matrix formalism analogous to the one outlined above can be set up for (19) and for (20). The essential difference compared to (5) and (18) is that the jets in (19), (20) are to be treated as identical particles in phase space. Details are given in Ref. 9 where also the constraints arising from CP, T , and CPT invariance are derived. It turns out that appropriate observables being sensitive to CP violation in the decays (19) or (20) are the tensors

$$\hat{k}_{ai}\hat{n}_j + \hat{k}_{aj}\hat{n}_i \quad (a = 1, 2, 3), \quad (23)$$

where $\hat{\mathbf{k}}_a = \mathbf{k}_a/|\mathbf{k}_a|$ and $\hat{\mathbf{n}} = (\mathbf{k}_1 \times \mathbf{k}_2)/|\mathbf{k}_1 \times \mathbf{k}_2|$. For cuts respecting rotational invariance the expectation values of the 3 tensors (23) take the following form:

$$\langle \hat{k}_{ai}\hat{n}_j + (i \leftrightarrow j) \rangle = H_a s_{ij} \quad (a = 1, 2, 3) \quad (24)$$

where

$$H_a = \frac{2}{5} \int d\Gamma [(\hat{\mathbf{k}}_a \cdot \hat{\mathbf{k}}_1)c_4(E_1, E_2) + (\hat{\mathbf{k}}_a \cdot \hat{\mathbf{k}}_2)c_5(E_1, E_2)] \quad (25)$$

and the functions c_4, c_5 in (25) are specific to the respective decay matrices associated with (19) and (20). If CP is a good symmetry then⁹ $c_4 = c_5 = 0$. That is, the correlations (24) provide genuine CP tests for the decays (19) and (20) – like the correlations (14) and (16) for the decays (5) and (18). These correlations cannot be faked by final state rescattering arising from CP -conserving interactions. The above CP tests – and others⁸ – can be applied also to corresponding e^+e^- reactions away from the Z peak and to $p\bar{p}$ collisions.

CP-VIOLATING EFFECTIVE LAGRANGIAN

Which interactions could generate CP -violating effects in the 3-body decays considered above? Consider the SM Born diagram Fig. 1a which represents these decays at the level of the elementary particles of the SM: $Z \rightarrow \bar{f}f\gamma$ ($f = \text{lepton or quark}$) and $Z \rightarrow \bar{q}qG$ ($G = \text{gluon}$). CP -violating interactions could affect the following 1-particle irreducible vertices contributing to these decays: $ff\gamma$ respectively qqG (Fig. 1b), ffZ (Fig. 1c), $ffZ\gamma$ respectively $qqZG$ (Fig. 1d). (Flavour-off-diagonal vertices are irrelevant on phenomenological grounds.) Within the SM there is no CP violation in the lepton sector if the neutrino masses are zero. As to the quark sector of the SM it is straightforward to show that the charged weak quark currents do not generate any CP -violating contribution to the above vertices to one-loop approximation. Therefore, within the SM, one expects the CP -odd correlations introduced above for the decays (18), (19) and (20) to be at most of order

$$\left(\frac{g_w^2}{4\pi}\right)^2 c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta \leq 10^{-7}. \quad (26)$$

In (26) g_w is the SU(2) gauge coupling and $c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta$ ($s_i = \sin \vartheta_i, c_i = \cos \vartheta_i$) is the combination of mixing angles¹ to which all CP violation in the SM is proportional¹⁴. That is, the CP -odd correlations discussed above are sensitive to possible new forms of CP violation only. Within the gauge theory context many extensions of the SM – e.g., left-right symmetric models, supersymmetric models, SU(2)×U(1) models with several Higgs doublets – which provide such new sources were discussed in the literature. (For reviews, see Ref. 2,3). Typically these models generate CP -violating contributions to the above vertices already to 1-loop approximation. However, an analysis of these models is in general hampered by the occurrence of many unknown parameters. It seems therefore appropriate for our purposes to use the method of effective Lagrangians in order to keep the analysis rather model-independent.

In constructing the relevant CP -violating effective Lagrangian L_{CP} we restrict ourselves to operators of mass dimension $d \leq 6$. This is justified if the mass scale Λ_{CP} characterizing possible new interactions is markedly larger than the Z mass m_Z . Only flavour-diagonal neutral current interactions are considered. The Lagrangian L_{CP} can be represented by various sets of independent operators. The following form⁹

is convenient for our purposes:

$$\begin{aligned}
L_{CP}(x) = & \sum_{\psi=q,l} \left\{ -\frac{i}{2} d_\psi \bar{\psi}(x) \sigma^{\mu\nu} \gamma_5 \psi(x) F_{\mu\nu}(x) \right. \\
& - \frac{i}{2} \tilde{d}_\psi \bar{\psi}(x) \sigma^{\mu\nu} \gamma_5 \psi(x) [\partial_\mu Z_\nu(x) - \partial_\nu Z_\mu(x)] \\
& + [f_{V\psi} \bar{\psi}(x) \gamma^\nu \psi(x) + f_{A\psi} \bar{\psi}(x) \gamma^\nu \gamma_5 \psi(x)] Z^\mu(x) F_{\mu\nu}(x) \} \\
& + \sum_q \left\{ -\frac{i}{2} d'_q \bar{q}(x) T^a \sigma^{\mu\nu} \gamma_5 q(x) G_{\mu\nu}^a(x) \right. \\
& + [h_{Vq} \bar{q}(x) T^a \gamma^\nu q(x) + h_{Aq} \bar{q}(x) T^a \gamma^\nu \gamma_5 q(x)] Z^\mu(x) G_{\mu\nu}^a(x) \}
\end{aligned} \tag{27}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c. \tag{28}$$

In (27), (28) $\psi(x)$ denotes a quark field $q(x)$ (colour indices being suppressed) or a lepton field $l(x)$; $A_\mu(x)$, $Z_\mu(x)$, $G_\mu^a(x)$ is the photon, Z , and gluon field, respectively; g_s is the QCD coupling, and T^a are the generators of $SU_c(3)$ satisfying $[T^a, T^b] = if_{abc} T^c$. The interaction Hamiltonian corresponding to (27) is odd under CP . The dimension 5 operators with the couplings $d_\psi, \tilde{d}_\psi, d'_q$ correspond to electric dipole, weak dipole, and chromoelectric dipole interactions, respectively. These operators are fermion chirality flipping. The dimension 6 operators in (27) describe fermion-chirality-conserving contact interactions. Anomalous¹⁵ CP -violating $Z\gamma\gamma$, $ZZ\gamma$, and ZGG interaction vertices could also exist. However, since these vertices must vanish if all gauge bosons are on-shell, such CP -violating interactions are described (up to $d \leq 6$) by the $Zff\gamma$ and $ZqqG$ operators of (27). (For comprehensive lists of CP -conserving and violating $d \leq 6$ operators investigated in other contexts see Ref. 16.)

The real parameters $d_\psi, \tilde{d}_\psi, d'_q, f_{V\psi}, f_{A\psi}, h_{Vq}$, and h_{Aq} correspond to constant real form factors and vertex functions, respectively; i.e., possible imaginary parts of these form factors and vertex functions are neglected. Higgs models of CP violation^{17,18} - for recent investigations see Ref. 19 - are especially interesting for our considerations. Because Higgs couplings are proportional to fermion masses, sizeable CP -violating form factors and 4-particle vertex functions involving heavy flavours - i.e. the τ lepton, the c or b quarks - might be generated if this source of CP violation exists in nature.

CALCULATION OF CP -ODD CORRELATIONS

The respective decay matrices R - i.e. the functions a, b_i, c_j - for the decays (5), (18), (19), and (20) can now be calculated in lowest order perturbation theory using the SM ffZ , $ff\gamma$, qqG couplings and the CP -violating couplings given by (27). The CP -violating terms in the decay matrices arise from the interference between the CP -conserving and CP -violating Born amplitudes depicted in Fig. 1. The functions

a , b_i , c_j of the various decay matrices are listed in Ref. 9. These functions allow for a calculation of the above CP -odd correlations which can be searched for in the 3-body Z decays. Instead of using the parameters appearing in (27) it is convenient to introduce the following dimensionless parameters $\hat{d}_\psi$, $\hat{f}_{V\psi}$, $\hat{f}_{A\psi}$, $\hat{h}_{Vq}$, $\hat{h}_{Aq}$ through

$$\begin{aligned}\tilde{d}_\psi &= \frac{e}{\sin\theta_w \cos\theta_w m_Z} \hat{d}_\psi, \\ f_{V\psi} &= \frac{-e^2 Q_\psi}{\sin\theta_w \cos\theta_w m_Z^2} \hat{f}_{V\psi}, & f_{A\psi} &= \frac{-e^2 Q_\psi}{\sin\theta_w \cos\theta_w m_Z^2} \hat{f}_{A\psi}, \\ h_{Vq} &= \frac{eg_s}{\sin\theta_w \cos\theta_w m_Z^2} \hat{h}_{Vq}, & h_{Aq} &= \frac{eg_s}{\sin\theta_w \cos\theta_w m_Z^2} \hat{h}_{Aq},\end{aligned}\quad (29)$$

where $\psi = l, q$; $e > 0$ is the proton charge, and Q_ψ is the electric charge of the fermion ψ in units of e . The (chromo) electric dipole form factors $d_\psi(d'_q)$ actually drop out in the calculations of the CP -violating functions of the various decay matrices - due to a spin mismatch between the respective CP -conserving and CP -violating amplitudes. Furthermore, in the CP -odd correlations given below the following SM neutral current couplings of a fermion ψ appear:

$$g_{V\psi} = T_{3\psi} - 2Q_\psi \sin^2\theta_w, \quad g_{A\psi} = T_{3\psi}, \quad (30)$$

where $T_{3\psi} = -1/2(\psi = e, \mu, \tau, d, s, b)$, $T_{3\psi} = 1/2(\psi = u, c)$. In calculations we have used $\sin^2\theta_w = .23$ and $m_Z = 92 \text{ GeV}$. For the calculations of the decay widths we took for the electromagnetic and strong couplings at the Z mass: $\alpha = 1/128$, $\alpha_s = .11$.

The necessary energy and angular cuts which we use (see below) for calculating the CP -odd correlations do not violate rotational invariance. Therefore the formulae (14), (16), and (24) hold. The results of our calculations can then be represented as follows:

i) $Z \rightarrow l^+ l^- \gamma$, $Z \rightarrow \bar{q} q \gamma \rightarrow 2 \text{ jets} + \gamma$ (tagged jets):

For a given fermion ψ , the integrals I_ψ , K_ψ determining the correlations (14) and (16), respectively, take the form:

$$I_\psi = \hat{f}_{V\psi} g_{V\psi} w_{1\psi} - \hat{f}_{A\psi} g_{A\psi} w_{2\psi} + \hat{d}_\psi g_{A\psi} w_{3\psi}, \quad (31)$$

$$K_\psi = (\hat{f}_{A\psi} g_{V\psi} - \hat{f}_{V\psi} g_{A\psi}) x_\psi, \quad (32)$$

and the numbers $w_{i\psi}$ ($i = 1, 2, 3$), x_ψ , which depend on the cuts, are given in Table 2. In calculating these numbers we used the following cuts: Denoting the minimal angle required between two particles by δ and the minimal energy required for each particle by E , we chose: $\delta_{e^+e^-} = \delta_{e\gamma} = \delta_{\tau^+\tau^-} = \delta_{\tau\gamma} = 7^\circ$, $\delta_{\mu^+\mu^-} = \delta_{\mu\gamma} = 3^\circ$, $\delta_{q\bar{q}} = \delta_{q\gamma} = 30^\circ$, $E_e = 1 \text{ GeV}$, $E_\mu = E_\tau = 3 \text{ GeV}$, $E_q = 2 \text{ GeV}$ ($q = u, d, c, s$), $E_b = 4.3 \text{ GeV}$, and $E_\gamma = 10 \text{ GeV}$. For the quark masses we used $m_u = m_d = 10 \text{ MeV}$, $m_s = 175 \text{ MeV}$, $m_c = 1.3 \text{ GeV}$, and $m_b = 4.3 \text{ GeV}$. If one lowers the minimal photon energy for instance to 1 GeV , the numbers for $w_{i\psi}$, x_ψ given in Table 2 decrease

by approximately a factor of 3 basically because the respective decay widths increase by about the same factor. (This reflects the infrared singularity in the soft photon limit.) The small numbers for $w_{3\psi}$ reflect the fact that the interference of the helicity conserving amplitude Fig. 1a with the amplitude Fig. 1c involving helicity flips results in a suppression factor m_ψ/m_Z (m_ψ = fermion mass).

ii) $Z \rightarrow 3jets, Z \rightarrow 2jets + \gamma$:

It turns out that the tensor correlations (24) for these decays are generated only by the helicity conserving $d = 6$ interactions in (27) — like the tensor correlation (16). Therefore we put the quark masses to zero in the following. For the decay (19) the integrals H_a of the 3 correlations (24) then take the form:

$$H_a^{3jet} = y_a \sum_q \left(\hat{h}_{Aq} g_{Vq} - \hat{h}_{Vq} g_{Aq} \right) \quad (a = 1, 2, 3). \quad (33)$$

Calculating the correlations (24) for the decay (20), one gets for the integrals (25):

$$H_a^{2jet} = z_a \sum_q Q_q^2 \left(\hat{f}_{Aq} g_{Vq} - \hat{f}_{Vq} g_{Aq} \right) \quad (a = 1, 2, 3). \quad (34)$$

The numbers y_a and z_a in (33) and (34) depend on the cuts and jet-ordering criteria. For the decays (19), (20) we used the ordering criteria (21), (22), respectively. The following cuts were chosen: $E_{jet} = E_\gamma = 5 \text{ GeV}$, $\delta_{jet,jet} = \delta_{jet,\gamma} = 30^\circ$. We then obtain

$$(y_1, y_2, y_3) = (-2.1 \times 10^{-3}, 2.5 \times 10^{-3}, 2.8 \times 10^{-4}) \quad (35)$$

$$(z_1, z_2, z_3) = (-.026, .028, .011). \quad (36)$$

The respective decay widths are $\Gamma_{3jet} = 745 \text{ MeV}$, $\Gamma_{2jet+\gamma} = 8.9 \text{ MeV}$. Increasing the minimal energies and/or minimal angles leads to larger numbers y_a , z_a , but, of course, to smaller decay widths. It remains to be investigated whether other jet-ordering criteria lead, for given cuts, to stronger correlations (24).

Which level of sensitivity can be reached for the dimensionless parameters (29) by measuring our CP -odd correlations? We give only crude order-of-magnitude estimates assuming 5×10^6 Z 's and assuming the widths of the distributions of the (dimensionless) CP -odd observables to be .1. Measurements of the tensor correlations (16) and (24) should be sensitive to $\hat{f}_{VI}$, $\hat{f}_{AI}$, $\hat{f}_{Vq}$, $\hat{f}_{Aq}$, $\hat{h}_{Vq}$, $\hat{h}_{Aq} \sim O(10^{-1})$. The existing upper bounds on the electric dipole moments of e, μ , and of the neutron presumably preclude such rather large values for $\hat{f}, \hat{h}$ associated with e, μ, u, d . (A conclusion without caveats is, however, not possible.) In any case measurement of the CP -odd tensor correlations will provide new information about CP -odd vertex functions for heavy flavours. Due to helicity suppression the vector correlations (14) are insensitive to the weak dipole form factors for e, μ, u, d, s . If sizeable weak dipole form factors exist for τ, c, b — i.e., $\hat{d}_\tau, \hat{d}_c, \hat{d}_b \sim O(1)$ — detection may be possible by means of (14). However, such large values would lead to a considerable deviation of the total Z width from its SM value.

SUMMARY

We have shown that the standard 3-body decay modes of the Z boson can be used to search for new sources of CP violation. The CP -odd correlations which we introduced and calculated in a model-independent way, serve this purpose. They should be fairly easily measurable in an experiment. These CP tests can also be applied to the corresponding e^+e^- reactions away from the Z resonance and to the corresponding $p\bar{p}$ reactions, although our calculations are specific to Z boson decays. Given the expectation that several millions of Z 's will be produced at LEP1 measurements of these correlations will provide new information – at least; useful upper bounds – on some CP -violating flavour-diagonal vertex functions and weak dipole form factors for heavy flavours.

REFERENCES

1. M. Kobayashi and T. Maskawa, *Progr. Theor. Phys.* **49**, 652 (1972).
2. J. F. Donoghue, B. Holstein, and G. Valencia, *Int. J. Mod. Phys. A* **2**, 319 (1987).
3. W. Grimus, *Fortschr. d. Physik* **36**, 201 (1988).
4. I. I. Bigi, V. A. Khoze, N. G. Uraltsev, and A. I. Sanda, SLAC preprint SLAC-PUB-4476(1987);
see also the reports given by A. Ali, D. London, B. Cox, and D. Cline at this workshop.
5. W. B. Atwood, I. Duinev, and P. Grosse-Wiesmann, SLAC preprint SLAC-PUB-4544 (1988).
6. L. Stodolsky, *Phys. Lett.* **150B**, 221 (1985).
7. W. S. Hou, N. G. Deshpande, G. Eilam, and A. Soni, *Phys. Rev. Lett.* **57**, 1406 (1986);
J. Bernabéu, A. Santamaria, and M. B. Gavela, *Phys. Rev. Lett.* **57**, 1514 (1986).
8. J. F. Donoghue and G. Valencia, *Phys. Rev. Lett.* **58**, 451 (1987).
9. W. Bernreuther, U. Löw, J. P. Ma, and O. Nachtmann, “ CP violation and Z boson decays”, Heidelberg preprint HD-THEP-88-26 (1988).
10. W. Bernreuther, U. Löw, J. P. Ma, and O. Nachtmann, “How to test CP , T , and CPT invariance in the three photon decay of polarized 3S_1 positronium”, HD-THEP-87-25, to be published in *Z. Phys. C* (1988).
11. J. G. Körner and H. Schuler, *Z. Phys. C* **26**, 559 (1985).
12. C. Albajar et al., *Z. Phys. C* **37**, 505 (1988);
D. Denegri, talk given at this workshop.
13. G. Altarelli, M. Diemoz, G. Martinelli, and P. Nason, CERN preprint CERN-TH. 4978 (1988).
14. C. Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985).
15. F. M. Renard, *Nucl. Phys. B* **196**, 93 (1982);
G. Gounaris, R. Kögerler, and D. Schildknecht, *Phys. Lett.* **137B**, 261 (1984);

- Y. Tomozawa, Phys. Lett. **139B**, 455 (1984);
V. Barger, H. Baer, and K. Hagiwara, Phys. Rev. **D3**, 1513 (1984);
A. Barroso et al., Z. Phys. **C28**, 149 (1985); **C33**, 243 (1986);
K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa,
Nucl. Phys. **B282**, 253 (1987).
16. W. Buchmüller and D. Wyler, Nucl. Phys. **B268**, 621 (1985);
C. N. Leung, S. T. Love, and S. Rao, Z. Phys. **C31**, 433 (1986).
17. T. D. Lee, Phys. Rev. **D8**, 1226 (1973).
18. S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).
19. G. C. Branco, A. J. Buras, and J. M. Gérard,
Nucl. Phys. **B259**, 306 (1985);
H. Y. Cheng, Phys. Rev. **D34**, 1397 (1986);
J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987);
G. Ecker, W. Grimus, and H. Neufeld, Phys. Lett. **194B**, 251 (1987).

FIGURE CAPTION

- Fig. 1 a) SM Born diagram representing the decays $Z \rightarrow \bar{f}f\gamma$ ($f = l, q$) and $Z \rightarrow \bar{q}qG$.
Solid lines represent fermions, the wiggly line represents a photon or a gluon,
respectively.
b),c),d): Diagrams depicting the CP -violating interactions (full circles) given by
(27).

Table 1 Constraints on the functions a , b_i , c_j of (8)-(10)
from CP invariance and, neglecting absorptive parts,
from CPT and T invariance.

CP	CPT	T
$a(E_+, E_-) = a(E_-, E_+)$	$a(E_+, E_-) = a(E_-, E_+)$	
$b_1(E_+, E_-) = -b_2(E_-, E_+)$	$b_1(E_+, E_-) = -b_2(E_-, E_+)$	
$b_3(E_+, E_-) = -b_3(E_-, E_+)$	$b_3(E_+, E_-) = b_3(E_-, E_+)$	$b_3(E_+, E_-) = 0$
$c_1(E_+, E_-) = c_2(E_-, E_+)$	$c_1(E_+, E_-) = c_2(E_-, E_+)$	
$c_3(E_+, E_-) = c_3(E_-, E_+)$	$c_3(E_+, E_-) = c_3(E_-, E_+)$	$c_4(E_+, E_-) = 0$
$c_4(E_+, E_-) = c_5(E_-, E_+)$	$c_4(E_+, E_-) = -c_5(E_-, E_+)$	$c_5(E_+, E_-) = 0$

Table 2 Decay widths for (5), (18) and coefficients appearing in the correlation integrals (31), (32). Cuts are specified in the text.

ψ	decay width (MeV)	$w_{1\psi}$	$w_{2\psi}$	$w_{3\psi}$	x_ψ
e	2.54	.018	.018	7.6×10^{-7}	.068
μ	3.15	.014	.014	1.2×10^{-4}	.056
τ	2.20	.020	.020	2.9×10^{-3}	.076
u	2.00	.029	.029	2.4×10^{-5}	.097
c	1.96	.029	.029	3.1×10^{-3}	.097
d	.64	.023	.023	1.8×10^{-5}	.075
s	.64	.023	.023	3.2×10^{-4}	.075
b	.57	.025	.024	8.1×10^{-3}	.079