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SPONTANEOUS COMPACTIFICATION ON S^{2n} AS SOLUTION TO THE
GENERALIZED EINSTEIN-YANG-MILLS-HIGGS SYSTEM

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A B S T R A C T

Generalized Lagrangians of Yang-Mills-Higgs systems, bounded from below by topological integrals, are discussed. The solutions are constructed and spontaneous compactifications with S^{2n} -spheres as internal spaces are performed.

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1. - INTRODUCTION

Spontaneous compactification mechanism has been first proposed by Cremmer and Scherk [1] in 1977. They have investigated the Yang-Mills-Higgs fields interacting with Einstein-Hilbert gravity. The space-time they used was extended to six dimensions by adjoining an extra S^2 . Then they found the solutions to the full system of equations on $V_4 \times S^2$, which admitted the Minkowskian space-time in four dimensions, so that there was no cosmological term in the gravitational equations on V_4 .

Subsequently, Cremmer and Scherk extended their scheme replacing S^2 by S^n , considering in more detail an example with S^4 , where the field configurations were those of the Belavin-Polyakov-Schwarz-Tyupkin monopole projected onto S^4 . This is not really analogous to the monopole solution [2], since in this case the sphere S^2 is embedded in R^3 , while in the S^4 -generalization the sphere S^4 is embedded in R^5 , where it is impossible to find stable instantons of the Yang-Mills system. Indeed, Salam et al. have shown that the compactification with S^2 was stable [3,4], while in another article [5] they have carried this analysis to the case of compactification on S^4 .

In fact, compactification over S^n , $n > 2$, of the σ -model Einstein system is otherwise known to be unstable, and this fact has been pointed out by Omero and Percacci [6] in their paper concerned with the extension of the compactification scheme using the coset spaces G/H other than S^n , and employing the dynamics of sigma models in place of the Yang-Mills-Higgs fields. The sigma model coupled to gravity has been first considered by Fubini, de Alfaro and Furlan [7] on S^4 , and by Gava, Jengo, Omero and Percacci [8] on CP^2 . Later, Percacci and Ranjbar-Daemi [9] have extended this compactification mechanism over a homogeneous space G/H for the Einstein-Yang-Mills systems, in which the Yang-Mills field was in fact the composite field of the sigma model ($A = g^{-1}dg$).

The problem of stability of solutions to these extended schemes [6,9] has been studied quite in detail, and for the monopole [1,3,4] as well as for the instanton-type [5,10] compactifications the results are affirmative. In both these cases the Yang-Mills-Higgs (respectively, pure Yang-Mills) systems are endowed with a topological lower energy bound.

As pointed out by Horvath et al. [4], one could expect stability in the case of compactification over S^2 , because of the topological invariant (identified with the magnetic charge) that characterizes this solution. It is therefore desirable that an extension of the scheme proposed by Cremmer and Scherk [1,2] be formulated, where one employs a non-Abelian Higgs model capable of supporting field

configurations on S^n ($n > 2$) characterized by a topological invariant similar to the magnetic charge.

This is precisely the aim of the present work, where we propose such an extension using certain non-Abelian Higgs Lagrangians [11] on R^{2n+1} , which are capable of yielding finite energy field configurations endowed with a topological charge. Then we can employ the spherically-symmetric field configuration on S^{2n} embedded in R^{2n+1} in order to perform the compactification of the generalized Einstein-Yang-Mills-Higgs system [12] on $V_4 \times S^{2n}$. A comprehensive review on compactification can be found in Duff et al. [10], especially in the context of supergravity. In the present work, we are concerned with the original non-supersymmetric systems considered in [1] and [2].

2. - LAGRANGIANS ADMITTING TOPOLOGICAL BOUNDS

We proceed now with the construction of the simplest possible generalized Yang-Mills-Higgs model in R^{2n+1} . As in Refs. [12] and [13], we start from a Yang-Mills system defined in $(2n+2)$ dimensions, in order to derive the Yang-Mills-Higgs system in R^{2n+1} . This dimensional reduction has nothing to do with the subsequent compactification scheme, it serves merely as a model-building device in order to obtain supplementary Higgs fields.

We start with the $(2n+2)$ -dimensional space, supposing it has the topology of $R^{2n+1} \times S^1$, let us call this space M^{2n+2} . In this space, we shall consider a generalized Yang-Mills Lagrangian [11,12], involving only the gauge curvature tensor F_{MN} , with $M, N = 1, 2, \dots, 2n+2$. In what follows, we shall use small Latin indices, i, j , to denote the $(2n+1)$ variables.

It is convenient to start with the following inequality:

$$T \left(F_{M_1 M_2} - \frac{(\kappa^2)^{n-1}}{(2n)!} \varepsilon_{M_1 M_2 N_1 \dots N_{2n}} F_{N_1 \dots N_{2n}} \right)^2 \geq 0 \quad (2.1)$$

where $F_{N_1 N_2 \dots N_{2n}}$ is the totally antisymmetrized product of n curvature two-forms, and κ is a dimensional constant, $[\kappa] = \text{cm}$.

Before proceeding further, let us note that there is nothing unique about our choice of the inequality (2.1). We could have chosen any other even partition of $(2n+2)$ indices in order to define the Hodge dual in (2.1), e.g.,

$$\text{Tr} \left(\overline{F}_{M_1 \dots M_{2r}} - \frac{(x^2)^{n-2r+1}}{(2(n-r+1))!} \varepsilon_{M_1 \dots M_{2r} N_1 \dots N_{2(n-r+1)}} \overline{F}_{N_1 \dots N_{2(n-r+1)}} \right)^2 \geq 0 \quad (2.2)$$

which would even have the attractive property of exhibiting no dimensional constant κ if $n = 2r-1$, but since we have no hope of finding an explicit solution anyway [14], we will choose the simplest case when $r = 1$, i.e. (2.1). This has the advantage that we can illustrate our construction in the simplest possible case where $n = 2$, i.e., the compactification over the sphere S^4 embedded in R^5 .

Developing the square in (2.1) we can rewrite it as

$$\begin{aligned} & \text{Tr} \left[\overline{F}_{M_1 M_2}^2 + \frac{(x^2)^{2(n-1)}}{((2n)!)^2} \overline{F}_{M_1 M_2 \dots M_{2n}}^2 \right] \geq \\ & \geq 2 \frac{(x^2)^{n-1}}{(2n)!} \varepsilon_{M_1 M_2 \dots M_{2n+2}} \text{Tr} \left[\overline{F}_{M_1 M_2} \overline{F}_{M_3 \dots M_{2n+2}} \right] \end{aligned} \quad (2.3)$$

We shall use the term on the left-hand side of this inequality (2.3) as the Lagrangian of the generalized Yang-Mills system, which in the stationary case in Minkowskian metric, or in Euclidean space, is bounded from below by the right-hand side, which is nothing else but the $(n+1)^{\text{th}}$ Chern-Pontryagin density. This inequality guarantees the existence of instantons [11] in R^{2n+2} ; in the present work, we shall be interested rather in the space $R^{2n+1} \times S^1$, which enables us to introduce, in a natural way, the analogue of the Higgs field. For this particular configuration, we use the ansatz for $A_M = (A_i, A_{2n+1}, 0)$, so that the only non-vanishing components of F_{MN} will be

$$F_{MN} = (\overline{F}_{ij}, \overline{F}_{i, 2n+1}) \quad (2.4)$$

if all the functions are S^1 -invariant (do not depend on the S^1 -co-ordinate x^{2n+2}).

Denoting A_{2n+1} by ϕ , we can write

$$\overline{F}_{i, 2n+1} = D_i \phi \quad (2.5)$$

Substituting (2.4) and (2.5) into (2.3), and integrating over S^1 , we find the inequality

$$\begin{aligned}
& \text{Tr} \left[F_{i_1 i_2}^2 + \frac{4n (\kappa^2)^{2n-2}}{((2n)!)^2} F_{i_1 i_2 \dots i_{2n}}^2 \right] + \\
& + 2 \text{Tr} \left[\frac{(\kappa^2)^{2n-2}}{((2n)!)^2} F_{i_1 \dots i_{2n}}^2 + (\mathbb{D}_i \phi)^2 \right] \geq \\
& \geq 2n \cdot \frac{2 (\kappa^2)^{n-1}}{(2n)!} \text{Tr} \left[F_{i_1 i_2} \dots F_{i_{2n-3} i_{2n-2}} \mathbb{D}_{i_{2n-1}} \phi \right] \quad (2.6)
\end{aligned}$$

Following the procedure introduced in Refs. [11] and [12], we adopt the left-hand side of (2.6) as our Lagrangian density, which is automatically bounded from below by the topological charge density appearing on the right-hand side. The latter is manifestly a pure divergence, and when subject to Higgs asymptotic conditions

$$\text{Tr} [\phi^2] \xrightarrow{r \rightarrow \infty} \text{Const.} \quad (2.7)$$

it will give rise to a topological surface integral.

In fact, the simplest possible Lagrangian density is not the entire left side of (2.6), but each of the terms taken separately:

$$\mathcal{L}_1 = \text{Tr} \left[F_{i_1 i_2}^2 + \frac{4n (\kappa^2)^{2n-2}}{((2n)!)^2} F_{i_1 \dots i_{2n-2}}^2 \right] \quad (2.8)$$

or

$$\mathcal{L}_2 = \text{Tr} \left[\frac{(\kappa^2)^{2n-2}}{((2n)!)^2} F_{i_1 \dots i_{2n}}^2 + (\mathbb{D}_i \phi)^2 \right] \quad (2.9)$$

This is because in our case both of these densities happen to be bounded from below by the topological charge density on the right-hand side of (2.1). It is easy to check that the Euler-Lagrange equations for both $\mathcal{L}_1$ and $\mathcal{L}_2$ are solved by the respective Bogomolnyi duality conditions:

$$F_{i_1 i_2} = \frac{(\kappa^2)^{n-1}}{(2n-1)!} F_{j_1 \dots j_{2n-1}, 2n} \varepsilon_{j_1 \dots j_{2n-1} i_1 i_2}, \quad (2.10)$$

or

$$D_i \phi = \frac{(\kappa^2)^{n-1}}{(2n)!} F_{j_1 \dots j_{2n}} \varepsilon^{i j_1 \dots j_{2n}}, \quad (2.11)$$

and the Bianchi identities.

Here we shall choose the second term as our Lagrangian density,

$$\mathcal{L}_0 = \text{Tr} \left[F_{i_1 \dots i_{2n}}^2 + (\kappa^2)^{2n-2} (2n)! (D_i \phi)^2 \right] \quad (2.12)$$

although this choice is by no means compulsory. Our preference is based on the fact that we anticipate the addition of a Higgs potential $V(\phi)$ to $\mathcal{L}_0$:

$$\mathcal{L} = \mathcal{L}_0 + \lambda V(\phi) \quad (2.13)$$

with the usual symmetry-breaking Higgs potential

$$V(\phi) = \text{Tr} \left[(\phi^2 - \eta^2)^2 \right] \quad (2.14)$$

This Lagrangian is also bounded from below by the topological density on the right-hand side of (2.6), for any $\lambda > 0$. The relation between $\mathcal{L}$ and $\mathcal{L}_0$ is the same as that between the 't Hooft-Polyakov system and the Prasad-Sommerfield one.

The existence of finite energy solutions to the Euler-Lagrange equations of the density $\mathcal{L}$ (2.13) hinges on the fact that $\mathcal{L}$ is a positive definite density whose integral is bounded from below by the topological surface integral of the expression

$$\Omega_{i_{2n-1}} \sim \varepsilon_{i_1 \dots i_{2n-1}} \text{Tr} F_{i_1 i_2} \dots F_{i_{2n-3} i_{2n-2}} \phi \quad (2.15)$$

which by Gauss' theorem can be transformed into the integral of the right-hand side of (2.1).

For this to be true, the asymptotic conditions (2.7) for the Higgs field must be satisfied, and furthermore, it was shown in Ref. [11] that the integral (2.7) can be expressed as the winding number of the vector $\hat{\phi} = \phi^\alpha / \|\phi\|$, where α is the index labelling the Lie algebra of the gauge group.

3. - COMPACTIFICATIONS WITH S^{2n} MONOPOLES

The field configurations we propose here are the spherically symmetric ones, analogous to the 't Hooft-Polyakov monopole exploited in Ref. [1]. For this we have to start with the choice of the gauge group, which once again, will be inspired by the analogy with the 't Hooft-Polyakov case.

To this end, we define the spinor representations of $SO(2n+2)$, the gauge group for which the generalized Yang-Mills system corresponding to the Lagrangian

$$\text{Tr} \left[F_{M_1 M_2}^2 + 2 \frac{(\kappa^2)^{2n-2}}{(2n)!} F_{M_1 \dots M_{2n}}^2 \right] \quad (3.1)$$

is known [12] to possess instanton field configurations [e.g., chiral $SO(4) = SU(2)$ for the Yang-Mills system in four dimensions]. The representation of the $SO(2n+2)$ is given by the matrices:

$$\Sigma_{\hat{\mu}\hat{\nu}} = -\frac{1}{4} [\Gamma_{\hat{\mu}}, \Gamma_{\hat{\nu}}] \quad (3.2)$$

which will be divided into the following subsets:

$$\Sigma_{\hat{\mu}\hat{\nu}} = \left(\Sigma_{ab} ; \Sigma_{a,2n+1} ; \Sigma_{a,2n+2} ; \Sigma_{2n+1,2n+2} \right) \quad (3.3)$$

$$\Sigma_{ab} = -\frac{1}{4} [\Gamma_a, \Gamma_b] ; \Sigma_{2n+1,2n+2} = \frac{i}{2} \Gamma_{2n+1} \quad (3.4)$$

$$\Sigma_{a,2n+1} = \frac{1}{2} \Gamma_{2n+1} \Gamma_a ; \Sigma_{a,2n+2} = \frac{i}{2} \Gamma_a \quad (3.5)$$

where the indices vary as follows: $\hat{\mu}, \hat{\nu} = 1, 2, \dots, 2n+2$; $a, b = 1, 2, \dots, 2n$; Γ_a are the γ -matrices in $2n$ dimensions, and Γ_{2n+1} the corresponding chirality matrix.

Next, we select the following subsets of $\{\Sigma_{\hat{\mu}\hat{\nu}}\}$:

$$\begin{aligned}\Sigma_{\hat{ij}} &= \left(\Sigma_{ab}, \Sigma_{a, 2n+1} \right) \\ \Sigma_{i, 2n+2} &= \left(\Sigma_{a, 2n+2}, \Sigma_{2n+1, 2n+2} \right)\end{aligned}\quad (3.6)$$

where the index i runs over $1, 2, \dots, 2n+1$, labelling the vector components in R^{2n+1} , the space in which we seek our monopoles.

Our ansatz for the spherically symmetric fields can be made then as follows:

$$A_i = \frac{(1-K(r))}{er^2} \Sigma_{ij} x^j \quad (3.7)$$

and

$$\phi = \frac{H(r)}{er^2} \Sigma_{i, 2n+2} x^i \quad (3.8)$$

$K(r)$, $H(r)$ being unknown functions, e the coupling constant.

Both the Yang-Mills potential A_i and the Higgs field ϕ take their values in the spinorial representation of $SO(2n+1)$. In the case of $n = 1$, where we start from the four-dimensional $SO_{\pm}(4) = SU(2)$ Yang-Mills gauge field and construct the 't Hooft-Polyakov system, it has a $SU(2)$ -valued connection A_i and $SU(2)$ -valued Higgs field ϕ .

This completes, in principle, the description of the monopoles in R^{2n+1} . For the $n = 3$ case the rigorous proof of their existence has been given in [15].

Let us now give a more detailed example in the case when $n = 2$, i.e., when the index i takes on five values $(1, 2, \dots, 2n+1)$. Our Lagrangian is then

$$\mathcal{L} = \text{Tr} \left(F_{ij}^2 + 4! \kappa^4 (D_m \phi)^2 \right) \quad (3.9)$$

In flat space, the Euler-Lagrange equations are:

$$\begin{aligned}-4! \left\{ F_{kl}, D_i F_{ijkl} \right\}_+ + 4! \cdot 2 \kappa^4 [\phi, D_j \phi] &= 0 \\ (D_i \phi = \partial_i \phi + [A_i, \phi], \text{ etc.})\end{aligned}\quad (3.10)$$

and

$$D_m D^m \phi = 0 \quad (3.11)$$

The right-hand side of the last equation should be replaced by $\lambda(\delta V/\delta\phi)$ if the term λV is added to the Lagrangian. It is easy to show how the generalized Bogomolnyi self-duality conditions solve the Euler-Lagrange equations (3.10) and (3.11). If we put

$$F_{ijkl} = \kappa^2 \varepsilon_{ijklm} D_m \phi \quad (3.12)$$

then the second equation, (3.11), becomes

$$D^m D_m \phi = \frac{1}{4! \kappa^2} D^m (\varepsilon_{mijkl} F_{ijkl}) = 0 \quad (3.13)$$

by virtue of the Bianchi identities for F_{ij} ; the first equation (3.10) can be written as:

$$-\left\{ F_{kl}, \kappa^2 \varepsilon_{ijklm} D_i (D_m \phi) \right\}_+ + 2\kappa^4 [\phi, D_j \phi] = 0 \quad (3.14)$$

because of the antisymmetry in the indices, i, m , we have

$$\varepsilon_{ijklm} D_i D_m \phi = \frac{1}{2} \varepsilon_{ijklm} [F_{im}, \phi] \quad (3.15)$$

so that now

$$-\frac{\kappa^2}{2} \left\{ F_{kl}, \varepsilon_{ijklm} [F_{im}, \phi] \right\}_+ + 2\kappa^4 [\phi, D_j \phi] = 0 \quad (3.16)$$

More explicitly,

$$\begin{aligned} & -\varepsilon_{ijklm} (F_{kl} F_{im} \phi - F_{kl} \phi F_{im} + F_{im} \phi F_{kl} - \phi F_{im} F_{kl}) + \\ & + 4\kappa^2 [\phi, D_j \phi] = 0 \end{aligned} \quad (3.17)$$

which is equivalent with

$$-\frac{1}{3!} \varepsilon_{ijklm} [F_{imkl}, \phi] + 4\kappa^2 [\phi, D_j \phi] = 0 \quad (3.18)$$

but $F_{imkl} = \kappa^2 \epsilon_{imkl n} D_n \phi$ by virtue of (3.12), so that

$$-\frac{\kappa^2}{3!} \epsilon_{ijklm} \epsilon_{imkl n} [D_n \phi, \phi] + 4\kappa^2 [\phi, D_j \phi] = 0 \quad (3.19)$$

as $\epsilon_{ijklm} \epsilon_{imkl n} = 4! \delta_{jn}$, (3.19) becomes an identity.

What remains to be seen is if such a solution may serve for the spontaneous compactification, i.e., to be interpreted as a part of solution to a system in $(4+2n)$ -dimensions containing also the Einsteinian gravity coupled to the Yang-Mills-Higgs system discussed above. The answer is positive: it is enough to put the ansatz on the S^{2n} -sphere parametrized by the unit vector in $2n+1$ dimensions, n^i :

$$A_i = \frac{1-K}{e R_0^2} \sum_j n_j \quad (3.20)$$

and

$$\phi = \frac{H}{e R_0^2} \sum_{i, 2n+2} n_i \quad (3.21)$$

K and H being now some constants. If μ, ν denote the four-dimensional space-time indices, the full set of equations will be now, in the total space being a direct product of $V_4 \times R^{2n+1}$ (with $N = 2n+1$):

$$\frac{1}{2} \frac{(N-1)(N-2)}{R_0^2} g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (3.22)$$

$$0 = T_{\mu j} \quad (3.23)$$

and

$$\frac{1}{2} \frac{(N-2)(N-3)}{R_0^2} g_{ij} = -8\pi G T_{ij} \quad (3.24)$$

plus the equations of motion of the fields themselves. As no fields depend on x^μ , the $T_{\mu\nu}$ -tensor is proportional to $-\frac{1}{2} g_{\mu\nu}$; with

$$\mathcal{L} = -\frac{1}{2(2n)!} \text{Tr} \left(F_{ijkl}^2 + \frac{1}{2} (D_i \phi)^2 \right) + \lambda V(\phi),$$

[Note that we include now the potential term $\lambda V(\phi)$ which will assure that T_{ij} will not vanish as in the self-dual case.]

this yields just a scale for $g_{\mu\nu}$, and the Einstein equations in V_4 become

$$R_{\mu\nu} = 0 \quad (3.25)$$

The remaining equations are (for $n = 2$, i.e., $N = 5$):

$$\begin{aligned} \frac{1}{2} \frac{(N-2)(N-3)}{R_0^2} g_{ij} &= \\ &= -8\pi G \left[g_{ij} \mathcal{L} + T_2 (\Phi_i \phi D_j \phi) + \frac{1}{4} \text{Tr} F_{iklm} F_{jklm} \right] \end{aligned} \quad (3.26)$$

By a quite straightforward, although tedious calculus, it can be shown that the right-hand side of (3.26) is proportional to g_{ij} if we choose the ansatz (3.20) and (3.21). The calculus is feasible in the case $n = 2$, because the anticommutation relations of $\Sigma_{\hat{\mu}\hat{\nu}}$ needed to compute F_{ijkl} (we recall that $F_{ijkl} = \{F_{i[j}, F_{|k|l]}\}_+$) are particularly simple in six dimensions:

$$\begin{aligned} \{\Sigma_{\hat{\mu}\hat{\nu}}, \Sigma_{\hat{\lambda}\hat{\rho}}\}_+ &= \frac{1}{2} (g_{\hat{\mu}\hat{\lambda}} g_{\hat{\nu}\hat{\rho}} - g_{\hat{\mu}\hat{\rho}} g_{\hat{\nu}\hat{\lambda}}) \mathbb{I} + \\ &+ \frac{i}{2} \varepsilon_{\hat{\mu}\hat{\nu}\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\tau}} \Sigma_{\hat{\sigma}\hat{\tau}} \end{aligned} \quad (3.27)$$

The total Lagrangian can be implemented by the Gauss-Bonnet term, which in more than four dimensions is not a pure divergence:

$$GB = R_{MNKL} R^{MNKL} - 4 R_{KL} R^{KL} + R^2 \quad (3.28)$$

This term has to be multiplied by a new dimensional constant (cm^2) when added to R ; once again, the solutions can be certainly found, and their stability would depend on the relative signs of R and GB as they appear in the Lagrangian. These problems will be dealt with in our future paper.

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