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AN OFF-SHELL FORMULATION OF $N = 4$ SUPERSYMMETRIC YANG-MILLS THEORY IN TWISTOR HARMONIC SUPERSPACE

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ABSTRACT

Twistor-like harmonic variables which parametrize the coset space $SO(1,4)/SO(1,2) \times SO(2)$ are introduced. With their help the on-shell constraints for $N = 4$, $d = 5$ supersymmetric Yang-Mills theory are rewritten as conditions for flatness in the harmonic directions of superspace. A Chern-Simons off-shell action leading to those equations is proposed. There are indications that the off-shell theory might be finite, despite the fact that the on-shell one seems non-renormalizable.

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1. The problem of finding off-shell unconstrained formulations of the extended supersymmetric Yang-Mills (SYM) theories has quite a long history. As is clear now, the key to this problem is provided by the use of twistor-like (harmonic) variables. The close relationship between the superspace constraints for SYM theories and the twistor approach to self-dual Yang-Mills equations of Penrose and Ward [1,2] has been pointed out by Witten on two occasions. First, in 1978 [3] he proposed a twistor interpretation of the constraints for $N = 1, 2, 3$, $d = 4$ SYM theories (with the noticeable exception of the case $N = 4$) as integrability conditions along light-like lines in twistor space. Later, in 1986 [4] he gave a similar interpretation of the $N = 1$, $d = 10$ SYM theory. In both cases, however, those *interpretations* did not lead to a *solution* of the constraints (for $N = 1, 2$, $d = 4$) or to an *off-shell formulation* of the theory (for $N = 3$, $d = 4$ or $N = 1$, $d = 10$). Rosly [5] suggested an alternative interpretation of the $N = 2$, $d = 4$ SYM constraints, this time using twistor-like variables associated with the internal symmetry group $SU(2)$, rather than the Lorentz group $SL(2, \mathbb{C})$. Similar ideas formed the basis of the independently developed harmonic superspace approach to $N = 2$, $d = 4$ SUSY [6]. There a decisive step forward was made by obtaining *solutions* to the constraints and *off-shell formulations* for all $N = 2$, $d = 4$ SUSY theories. All that progress was based on the use of harmonic variables parametrizing the quotient $SU(2)/U(1)$.

Further, a generalization of that approach to the $N = 3$, $d = 4$ SYM theory [7] permitted one to breach the " $N = 3$ barrier" [8] and find an off-shell formulation of that theory. The key new idea was to replace the on-shell constraints on the spinor derivatives by an equivalent set of conditions for flatness in the harmonic directions of superspace. The harmonic variables parametrized the space $SU(3)/U(1) \times U(1)$, so there were *three* such (complex) directions and, correspondingly, exactly *three* harmonic curvatures which were required to vanish on shell. This allowed one to write down an *off-shell action* in the form of a Chern-Simons term, from which the above equations could be derived.

The case of $N = 4$, $d = 4$ SYM (or the physically equivalent $N = 1$, $d = 10$) proved more difficult. A straightforward guess for the case $N = 4$, $d = 4$ could be to use the harmonic space $SU(4)/SU(3) \times U(1)$, which has 3 complex dimensions too, so one could try to write down a Chern-Simons action. Unfortunately, the corresponding harmonic superspace is essentially complex, and there is no way to reproduce the specific reality condition on the

scalars of the $N = 4$, $d = 4$ SYM theory [3,9].* The problem of reality is automatically solved in the $N = 1$, $d = 10$ version of the theory, where one deals with Majorana-Weyl spinors [4]. However, an attempt [11] to use a harmonic superspace based on the "light-cone" quotient $SO(1,9)/SO(1,1) \times SO(8)$ (thus extending the interpretation of the constraints given by Witten [4]) failed. The reason is quite obvious: that quotient has 8 (complex) dimensions, so there are 8 harmonic gauge superfields, from which one can form 28 curvatures. Clearly, the vanishing of those 28 curvatures on shell cannot be obtained from an action by varying just 8 gauge superfields.**

A careful analysis of the causes of failure in the two attempts described above shows the way out. One has to look for a version of the theory which combines the nice features of the above cases (the number 3 of harmonic directions with manifest reality). It turns out that the right choice is $N = 4$, $d = 5$. Indeed, there both the Lorentz ($SO(1,4)$) and internal symmetry ($SO(5)$) groups possess symplectic structures, so a symplectic Majorana condition on the spinor derivatives can be imposed. On the other hand, bringing (not breaking!) the Lorentz symmetry $SO(1,4)$ down to $SO(1,2) \times SO(2)$ (or $SO(1,1) \times SO(3)$) by means of harmonic variables provides a harmonic space with the right dimension 3.*** All this allows us to write down, for the first time, an *off-shell action for the $N = 4$, $d = 5$ SYM theory*. It has a number of unusual features, the most striking one being the following. There are indications that the off-shell theory obtained might be *finite*, although the original on-shell one seems non-renormalizable! If this is true, an entirely new possibility might open up for attacking the old disease of quantum gravity.

In the present paper we shall present the main ideas of the construction. Prior knowledge of the basics of harmonic superspace, in particular of the treatment of the $N = 3$, $d = 4$ SYM case [6,7] could be helpful. In a future article [10] we shall give a more detailed

* Note that the above construction can be carried out in the case of $N = 4$, $d = 4$ *self-dual* complex SYM theory, where one can indeed write down an off-shell action [10].

** Recently it was claimed [12] that this problem has a solution. There one uses two more generations of harmonic variables (besides the light-cone ones), a number of ghost fields (in a classical action), and the action is non-local in the space-time variables.

*** The attentive reader might ask the question: Why not use the similar coset space $SO(5)/SO(2) \times SO(3)$ for the internal symmetry group, rather than tamper with the Lorentz group? There is a good reason for that, which is explained in section 5.

explanation, as well as some simplified examples to help the reader familiarize him(her)self with some of the most unusual features of the formulation.

2. The $N = 4$, $d = 5$ SYM on-shell theory can be defined by its superspace constraints. They are similar to those of $N = 4$, $d = 4$ SYM [3,9], and, like them, imply the equations of motion of the theory:

$$\{\Delta_{\alpha i}, \Delta_{\beta j}\} = \Omega_{ij} \Delta_{\alpha\beta} + C_{\alpha\beta} W_{ij}(x, \theta). \quad (1)$$

Here

$$\Delta_{\alpha i} = D_{\alpha i} + A_{\alpha i}(x, \theta) \quad (2)$$

are the spinor covariant derivatives with gauge connections $A_{\alpha i}$, $\Delta_{\alpha\beta} = -\Delta_{\beta\alpha}$ ($C^{\alpha\beta} \Delta_{\alpha\beta} = 0$) is the 5-vector (space-time) covariant derivative, $W_{ij} = -W_{ji}$ is the field-strength superfield,

$$\Omega_{ij} = -\Omega_{ji} \ (\Omega^2 = 1) \text{ and } C_{\alpha\beta} = -C_{\beta\alpha} \ (C^2 = 1) \quad (3)$$

are the symplectic matrices for the internal (SO(5)) and Lorentz (SO(1,4)) groups. They can be used to raise and lower both types of spinor indices, which are four-component ones (hence $N = 4$). Spinors like $\Delta_{\alpha i}$ are assumed to satisfy the symplectic Majorana condition [13]:

$$\bar{\Delta}_{\alpha i} \equiv \Delta^{\alpha i} = C^{\alpha\beta} \Omega^{ij} \Delta_{\beta j}. \quad (4)$$

Then the field strength W_{ij} in (1) comes out real:

$$\bar{W}_{ij} \equiv W^{ij} = \Omega^{ik} \Omega^{jl} W_{kl}. \quad (5)$$

Thus one avoids the additional reality constraint of the $N = 4$, $d = 4$ theory [3]. A peculiarity of the constraint (1) concerns the number of scalar physical fields contained in W_{ij} . As it stands, W_{ij} corresponds to 6 scalars (its θ^0 components), whereas one would expect only 5 in $N = 4$, $d = 5$ SYM (the sixth one present in the $N = 4$, $d = 4$ case is obtained from the fifth component of the gauge field $A_\mu(x)$ upon dimensional reduction). In other words, one would be tempted to impose the additional constraint $\Omega^{ij} W_{ij} = 0$. Actually, this is not necessary, since the Bianchi identities for (1) have the corollary [10]:

$$\Delta_{\alpha i} (\Omega^{jk} W_{jk}) = 0,$$

so $\Omega^{jk}W_{jk}$ must be a covariant constant. Assuming standard field behavior at infinity, we conclude that this constant must vanish.

Another consequence of the Bianchi identities is the commutation relation:

$$[\Delta_{\alpha i}, \Delta_{\beta \gamma}] = \frac{2}{5}C_{\alpha\beta}\Delta_{\gamma}^k W_{ik} - \frac{2}{5}C_{\alpha\gamma}\Delta_{\beta}^k W_{ik} + \frac{1}{5}C_{\beta\gamma}\Delta_{\alpha}^k W_{ik}. \quad (6)$$

3. The $N = 4$, $d = 5$ SYM constraints in their original form (1) do not have a clear geometric meaning. Our first goal will be to rewrite them in the form of integrability conditions, which will subsequently allow us to solve them. To do this we introduce auxiliary harmonic variables. This step resembles very much the trick due to Ward in the treatment of the $d=4$ self-dual Yang-Mills equations [2]. One can say [14] that the twistor variables of Penrose-Ward parametrize the quotient $SU(2)/U(1)$, where $SU(2)$ is part of the Euclidean Lorentz group $SO(4) \sim SU(2) \times SU(2)$ in $d=4$. The twistor variables used by Witten in his interpretation of the $N = 1, 2, 3$, $d = 4$ SYM constraints [3] can be associated with the quotient $SO(1,3)/SO(1,1) \times SO(2)$ (similar objects have appeared elsewhere [15] under the name "ambitwistors"). In our case the Lorentz group is $SO(1,4) \sim USp(1,1)$, and the subgroup we are interested in can be chosen either as $SO(1,1) \times SO(3)$ or as $SO(1,2) \times SO(2)$. The two choices are essentially equivalent, the former corresponds to passing to a "covariant light-cone frame", the latter to "covariant dimensional reduction" from $d=5$ to $d=3$. We shall define our twistor-like (or harmonic) variables to parametrize the coset space

$$\{u\} \sim \frac{SO(1,4)}{SO(1,2) \times SO(2)} \sim \frac{USp(1,1)}{SL(2, R) \times U(1)}. \quad (7)$$

In more detail, the variables u form a 4×4 matrix:

$$\begin{pmatrix} u^{+a\alpha} \\ u^{-a\alpha} \end{pmatrix} \in USp(1,1), \quad (8)$$

where the index α is a $USp(1,1) \sim SO(1,4)$ spinor (four component) one, $\pm$ is a $U(1) \sim SO(2)$ index (charge), and a is an $SL(2, R) \sim SO(1,2)$ spinor (two component) index. The definition of a $USp(1,1)$ matrix implies a number of algebraic conditions on $u^{\pm a\alpha}$ (see [10] for a detailed derivation):

$$\begin{aligned} \bar{u}^{\pm a\alpha} &= \epsilon_{ab} C_{\alpha\beta} u^{\mp b\beta} \\ u^{\pm a\alpha} C_{\alpha\beta} u^{\pm b\beta} &= 0 \end{aligned}$$

$$u^{\pm a\alpha} C_{\alpha\beta} u^{\mp b\beta} = \epsilon^{ab}$$

$$u^{+a\alpha} \epsilon_{ab} u^{-b\beta} + u^{-a\alpha} \epsilon_{ab} u^{+b\beta} = C^{\alpha\beta}. \quad (9)$$

The harmonic functions [6] can carry external indices of the subgroup $SL(2, R) \times U(1)$, but they are scalars of the big group $USp(1, 1)$. For example,

$$f^{+a}(u) = u^{+a\alpha} f_{\alpha} + u^{+a\alpha} u^{+b\beta} u_b^{-\gamma} f_{\alpha\beta\gamma}^1 + u^{+b\alpha} u_b^{+\beta} u^{-a\gamma} f_{\alpha\beta\gamma}^2 + \dots \quad (10)$$

The expansion in (10) goes over all those independent irreducible products of $u^{\pm a\alpha}$ (with (9) taken into account), which have the same external index $+a$. The coefficients are irreducible tensors (spinors) of $USp(1, 1)$ (i.e. of the original $d=5$ Lorentz group). The coset space (7) has 6 real or 3 complex dimensions, so there are 3 independent covariant harmonic derivatives, compatible with the defining conditions (9):

$$D^{++ab} = u^{+a\alpha} \partial / \partial u_b^{-\alpha} + u^{+b\alpha} \partial / \partial u_a^{-\alpha} = D^{++ba}. \quad (11)$$

Together with their complex conjugates

$$D^{--ab} = \bar{D}^{++ab}, \quad (12)$$

they form the algebra of $USp(1, 1)$:

$$[D^{++ab}, D^{++cd}] = 0 \quad (13a)$$

$$[D^{++ab}, D^{--cd}] = \epsilon^{cb} D_0^{ad} + \epsilon^{db} D_0^{ac} + \epsilon^{ca} D_0^{bd} + \epsilon^{da} D_0^{bc} + (\epsilon^{ad} \epsilon^{bc} + \epsilon^{ac} \epsilon^{bd}) D_0, \quad (13b)$$

where $D_0^{ad} = D_0^{da}$ are the generators of $SL(2, R)$, and D_0 is the generator of $U(1)$.

4. The harmonic variables introduced above will allow us to rewrite the constraints (1) in the form of integrability conditions. Indeed, multiplying (1) by $u^{+a\alpha} u^{+b\beta}$ and using (9) we obtain:

$$\{\Delta_i^{+a}, \Delta_j^{+b}\} = -\frac{1}{2} \Omega_{ij} \epsilon^{ab} \Delta_x^{++}, \quad (14a)$$

where

$$\Delta_i^{+a} = u^{+a\alpha} \Delta_{\alpha i}, \quad \Delta_x^{++} = u^{+a\alpha} u_a^{+\beta} \Delta_{\alpha\beta}. \quad (15)$$

In addition, from (6) we derive:

$$[\Delta_i^{+a}, \Delta_x^{++}] = 0. \quad (14b)$$

Equation (14) is a projection of the original constraint, which looks as though there were no Yang-Mills fields (no curvatures). Then we can say that (14) is the integrability condition for a harmonic superfield $\Phi(x, \theta, u)$ to satisfy the following *analyticity condition*:

$$\Delta_i^{+a}\Phi = \Delta_x^{++}\Phi = 0. \quad (16)$$

To get a better understanding of what (16) actually means, let us look at the realization of SUSY in superspace:

$$\delta x_{\alpha\beta} = i(\epsilon_\alpha^i \theta_{i\beta} - \epsilon_\beta^i \theta_{i\alpha} - \frac{1}{2} C_{\alpha\beta} \epsilon^{i\gamma} \theta_{i\gamma}), \quad \delta \theta_{\alpha i} = \epsilon_{\alpha i}. \quad (17)$$

Using the harmonic variables $u^{\pm a\alpha}$, we can choose a new (analytic) basis:

$$x_{\alpha\beta}^A = x_{\alpha\beta} + 2i\theta^{+(ai}\theta_i^{-b)}u_{a[\alpha}^+u_{b\beta]}^-,$$

$$\theta_i^{\pm a} = u^{\pm a\alpha}\theta_{\alpha i}, \quad (18)$$

where the full $N = 4$ SUSY algebra can be realized in a subspace involving only 4 of the 5 x 's and half of the θ 's, as well as the new variables u :

$$x_A^{++} = u^{+a\alpha}u_a^{+\beta}x_{\alpha\beta}^A, \quad x_A^{ab} = u^{+(a\alpha}u^{-b)\beta}x_{\alpha\beta}^A = x_A^{ba}, \quad \theta^{+ai} = u^{+a\alpha}\theta_{\alpha}^i;$$

$$\delta x_A^{++} = 2i\epsilon^{+ai}\theta_{ai}^+, \quad \delta x_A^{ab} = 2i\epsilon^{-(ai}\theta_i^{+b)}, \quad \delta \theta^{+ai} = \epsilon^{+ai}, \quad \delta u^{\pm a\alpha} = 0. \quad (19)$$

Then we conclude that the solution of the rigid (i.e. without Yang-Mills fields) analog of (16),

$$D_i^{+a}\Phi = D_x^{++}\Phi = 0, \quad (20)$$

is a superfield defined in the analytic subspace (19):

$$\Phi = \Phi(x_A^{++}, x_A^{ab}, \theta^{+ai}, u^{\pm a\alpha}). \quad (21)$$

An important property of the analytic superspace (19) is the existence of a special conjugation operation ($\sim$) [6,7], which combines complex conjugation of x, θ and u (see(9)) with reflection, $u^\pm \rightarrow \pm u^\mp$, so that:

$$\tilde{x}^{++} = x^{++}, \quad \tilde{x}^{ab} = x^{ab}, \quad \tilde{\theta}^{+ai} = \theta^{+ai}, \quad \tilde{u}^{\pm a\alpha} = \pm u^{\pm a\alpha}. \quad (22)$$

In other words, the superspace (19) is real in the sense of operation (22) (although it is complex in the usual sense), so a superfield like Φ (21) can have real coefficients in its expansion.

The fields $f(x^{++}, x^{ab}, u)$ in the θ -expansion of (21) are not ordinary fields. They are functions of 4 space-time coordinates (x^{++}, x^{ab}) and 3 harmonic variables ($u^{\pm a\alpha}$ modulo the freedom associated with $SL(2, R) \times U(1)$ transformations), altogether they depend on 7 complex (real in the sense (22)) variables. This is clearly more than the initial 5 coordinates of a field $f(x_\mu)$ in ordinary $d=5$ space-time. The conclusion is that by restricting the space-time dependence of the fields $f(x^{++}, x^{ab}, u)$ to only 4 of the 5 space-time coordinates, we do not lose any functional freedom (in particular, our fields are not necessarily on-shell ones). This point, as well as the relationship between ordinary 5-dimensional fields and the new 4+3-dimensional ones will be explained in greater detail in [10].

5. The projected constraints (14) are not only a corollary of the original one (1), they are actually equivalent to it. To see this one has to make sure that the covariant derivatives (i.e. the gauge connections therein) are just linear (or bilinear) functions of the harmonic variables, as indicated in (15). This is the same as demanding that the following commutation relations hold:

$$[D^{++ab}, \Delta_i^{+c}] = [D^{++ab}, \Delta_x^{++}] = 0. \quad (23)$$

Then one can pull the harmonic dependence out of (14), and using the properties (9) of $u^{\pm a\alpha}$ and the fact that they are arbitrary variables, one easily recovers (1). Notice that W_{ij} , obtained in this way, has still its trace, but as we explained earlier, it vanishes as a consequence of the Bianchi identities. At this point it becomes clear why we had to "harmonize" the Lorentz group $SO(1,4)$ and not the internal symmetry one $SO(5)$. Had we done otherwise, we would have obtained the relation

$$\{\Delta_\alpha^{+a}, \Delta_\beta^{+b}\} = -\frac{1}{2}C_{\alpha\beta}\epsilon^{ab}W^{++}, \quad W^{++} = u^{+ai}u_a^{+j}W_{ij}, \quad (24)$$

instead of (14). However, this is not an integrability condition (there is curvature in the RHS of (24)), and we cannot demand that $W^{++} = 0$ (since W_{ij} does not depend on $u^\pm$, that would mean $W_{ij} = 0$).

6. Having achieved an understanding of the meaning of the constraints (14), we can now proceed to solving them. To this end we make a *generalized gauge transformation* of the superfields,

$$\Phi_A = e^{iv} \Phi, \quad (25)$$

with parameter $v(x, \theta, u)$, such that

$$A_i^{+a} = e^{-iv} D_i^{+a} e^{iv}, \quad A_x^{++} = e^{-iv} D_x^{++} e^{iv}. \quad (26)$$

We are able to do that just because (14) holds, i.e. because the superspace is *flat* in the corresponding directions. In other words, in the new *analytic* gauge frame we have:

$$\Delta_i^{+a} = D_i^{+a}, \quad \Delta_x^{++} = D_x^{++}. \quad (27)$$

Instead, the harmonic derivatives D^{++ab} acquire connections as a result of the gauge rotation (25):

$$\Delta^{++ab} = D^{++ab} + V^{++ab}, \quad V^{++ab} = e^{iv} D^{++ab} e^{-iv}. \quad (28)$$

Then the relation (23) changes its meaning:

$$\begin{aligned} [\Delta^{++ab}, D_i^{+c}] &= [\Delta^{++ab}, D_x^{++}] = 0 \Rightarrow D_i^{+c} V^{++ab} = D_x^{++} V^{++ab} = 0 \Rightarrow \\ V^{++ab} &= V^{++ab}(x^{++}, x^{cd}, \theta_i^{+c}, u_\alpha^{\pm c}). \end{aligned} \quad (29)$$

This is not all, since the commutation relation between two harmonic derivatives (13a) now becomes a non-trivial constraint on the new connections:

$$[\Delta^{++ab}, \Delta^{++cd}] = 0 \Rightarrow D^{++ab} V^{++cd} - D^{++cd} V^{++ab} + [V^{++ab}, V^{++cd}] = 0. \quad (30)$$

It is important to realize that making the gauge frame rotation (25), we have replaced the original gauge group with u -independent parameters $\tau(x, \theta)$ by a new one with analytic parameters $\Lambda(x^{++}, x^{ab}, \theta^+, u)$:

$$\Phi' = e^{i\tau} \Phi \rightarrow \Phi'_A = e^{i\Lambda} \Phi_A \Rightarrow e^{iv'} = e^{i\Lambda} e^{iv} e^{-i\tau}. \quad (31)$$

This explains why in the analytic frame the short spinor derivatives (27) are covariant. The newly introduced harmonic connection transforms under the analytic Λ gauge group, and not under the original τ one (see (28) and use $D^{++ab} \tau = 0$):

$$V^{++ab'} = e^{i\Lambda} D^{++ab} e^{-i\Lambda} + e^{i\Lambda} V^{++ab} e^{-i\Lambda}. \quad (32)$$

7. To summarize, so far we have done the following. We have solved the constraints (14) in the "pure gauge" form (26) by introducing a "bridge" $v(x, \theta, u)$ between the two gauge frames. The bridge is not an independent superfield. It can be found as a solution of the relation (28),* given any set of analytic harmonic connections V^{++ab} satisfying equation (30). The latter is in fact just the integrability condition on V^{++ab} for the existence of a bridge. We know that the constraints (14) are equivalent to the equations of motion (1), so we conclude that in the analytic gauge frame the constraint (30) is the new form of those equations.

The next step is to go *off shell*. This means to remove the on-shell constraint (30), so that the analytic harmonic connections V^{++ab} will become completely arbitrary. Then we should try to find an action, from which equation (30) will follow. Here comes the crucial point that our harmonic space (7) has been designed to have *3 dimensions*. Equation (30) tells us that all the harmonic curvatures must vanish on-shell, but there are exactly 3 of them, so we can write down the following *Chern-Simons off-shell action*:

$$S = \int du dx^{++} d^3 x^{ab} d^8 \theta_i^{+a} \epsilon_{(ab)(cd)(ef)} \text{Tr} [V^{++ab} D^{++cd} V^{++ef} + \frac{2}{3} V^{++ab} V^{++cd} V^{++ef}]. \quad (33)$$

The integral in (33) is over the analytic subspace (19), where the superfields V^{++ab} are defined. It involves a harmonic integration, which can be thought of as extracting the singlet part of the harmonic expansion of the integrand [6].** It is crucial that the integrand has no U(1) charge (as well as no SL(2,R) indices), otherwise the harmonic integral will vanish. Indeed, in (33) the charge of the measure (+2 from dx^{++} and -8 from $d^8 \theta^+$) exactly balances that of the Lagrangian (+6).

The action (33) is invariant under the analytic gauge group (32) up to total derivatives (which is typical for Chern-Simons invariants). The equation of motion (30) follows from (33) by straightforward variation. Once it has been derived, one can solve the relation (28) for the bridge. Then one can perform the inverse gauge rotation (25), which will

* This relationship between bridge v and prepotential V is a common feature of all gauge theories formulated in harmonic superspace [6,7]. Notice that a similar equation in the context of self-dual Yang-Mills theory is known as the Sparling equation [16].

** A more rigorous definition of the harmonic integral would require the introduction of an explicit parametrization of the space (7) and the factorization out of its infinite volume. More comments on this point can be found in [10].

produce the connections A_i^{+a}, A_x^{++} . Since (29) holds, one can extract from A_i^{+a}, A_x^{++} the connections $A_{\alpha i}(x, \theta), A_\mu(x, \theta)$, which will automatically satisfy the original constraint (1), i.e. the equations of motion for the $N = 4, d = 5$ SYM theory.

We see that contact with the familiar 5-dimensional space-time theory can be made only on shell. The off-shell action (33) lives in a strange new world with 4+3 bosonic dimensions. At this point the puzzled reader might ask the question: Where in (33) do I find my familiar Yang-Mills action

$$S_{YM} = \int d^5x \text{Tr} F^{\mu\nu} F_{\mu\nu} ? \quad (34)$$

Well, the answer is: nowhere! The only common point between the actions (33) and (34) is that *on shell* they produce the same equations of motion, but they are radically different *off shell* (actually, no off-shell action of the type (34) exists for the full $N = 4, d = 5$ SYM theory). To emphasize this crucial point even more, we turn the attention of the reader to the dimension of the gauge coupling constant g . Normally, the Yang-Mills equation of motion $\nabla^\mu F_{\mu\nu} = 0$ does not tell us anything about $[g]$. It is only when we try to imagine this equation as been obtained from an action like (34), that we have to fix the dimensions of A_μ and g . In our case they are: $[A_\mu] = \frac{3}{2}, [g] = -\frac{1}{2}$, and we reach the inevitable conclusion that the theory is non-renormalizable (just like gravity in $d=4$). However, deriving the same equation (as a part of (1)) from an action like (33) drastically changes the situation. Indeed, now everything is dimensionless: the measure (4 dx 's and 8 $d\theta$'s, the u 's have dimension 0), the harmonic derivatives D and connections V , so we must have $[g] = 0$ as well! This is an indication that the quantum theory based upon the action (33) will be renormalizable. Moreover, it has all the chances to be even finite! One way to see this would be to develop Feynman rules satisfying the conditions of the non-renormalization theorem [17], i.e. that they are manifestly supersymmetric and all the vertices are given as integrals over all the 16 θ 's. To do this requires some work, but we feel optimistic in view of the recently performed quantization [18] of the $N = 3, d = 4$ harmonic superspace SYM theory [7], which in many respects resembles the present one. In the case of a positive outcome we would have an example of how an off-shell theory may dramatically differ in its quantum properties from what could have been anticipated by just looking at the on-shell theory. Then, who knows, maybe $N = 8$ supergravity will turn out finite someday?

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