CERN-TH.5152/88

ELC (European Lattice Collaboration)
RECENT RESULTS AND PROSPECTS IN THE COMPUTATION OF WEAK AMPLITUDES IN
LATTICE QCD

M.B. Gavela*)
Departamento de Física Teórica, Universidad Autónoma de Madrid, Spain

L. Maiani and S. Petrarca
Istituto di Fisica "G. Marconi", Università degli Studi di Roma "La Sapienza"
and INFN, Sezione di Roma, Italy

G. Martinelli
CERN - Geneva

O. Pène
LPTHE, 91405 Orsay, France

and

C.T. Sachrajda
University of Southampton, Southampton SO9 5NH, U.K.

Presented by G. Martinelli at the
Ringberg Workshop on Hadronic Matrix Elements and Weak Decays
Ringberg Castle, Germany
18-22 April 1988

*) On leave of absence from LPTHE, Orsay, France

ELC (European Lattice Collaboration)
RECENT RESULTS AND PROSPECTS IN THE COMPUTATION OF WEAK AMPLITUDES IN
LATTICE QCD

M.B. Gavela

Departamento de Fisica Teorica, Universidad Autonoma de Madrid, Spain,
On leave of absence from LPTHE, Orsay, France

L. Maiani and S. Petrarca

Istituto di Fisica "G. Marconi", Università degli Studi di Roma
"La Sapienza" and INFN, Sezione di Roma, Italy

G. Martinelli

Theoretical Physics Division, CERN, 1211 Geneva 23, Switzerland

O. Pène

LPTHE, 91405 Orsay, France

and

C.T. Sachrajda

University of Southampton, Southampton SO9 5NH, U.K.

1. INTRODUCTION

The solution and the understanding of the $\Delta I = \frac{1}{2}$ rule rely upon the calculation of the hadronic matrix elements of the operators which enter into the effective weak Hamiltonian. A precise determination of the matrix elements of weak operators would also allow accurate predictions for ϵ , ϵ' and heavy meson decays and mixings to be compared with the experimental results.

The evaluation of the matrix elements of weak operators is intrinsically a non-perturbative problem. A few years ago it was proposed to compute these matrix elements by numerical simulation of QCD on the lattice^{1,2}. Only QCD on the lattice offers the possibility of a first principle calculation without any extra model-dependent assumption and with no free parameters. In the following, I will briefly review the theoretical aspects of the lattice approach and the techniques used to compute the matrix elements of weak operators (Sections 2 and 3). Then I will report the main results and the current activity of our

collaboration (Section 4) with particular emphasis on the practical and theoretical difficulties still present in actual numerical simulations. In the conclusion I will try to foresee the possible developments of our activity for the near future.

2. THEORETICAL ASPECTS OF THE RENORMALIZATION OF THE WEAK HAMILTONIAN ON THE LATTICE

The effective weak Hamiltonian responsible for non-leptonic decays has the form:

$$H_{eff} = \sum_n C_n \left(\frac{M_W}{\mu} \right) O_n(\mu) \quad (2.1)$$

where $O_n(\mu)$ are local renormalized operators. The Wilson coefficients $C_n(M_W/\mu)$ are specified once the renormalization conditions for the operators $O_n(\mu)$ are given. The matrix elements of the operators $O_n(\mu)$ have to be calculated with a non-perturbative technique. We compute them by numerical simulation of QCD on a lattice. The key problem in the lattice approach is the construction, using the bare lattice operators, of the renormalized operators appearing in Eq. (2.1). Local, composite four-fermion operators suffer in fact on the lattice from two different problems: severe ultra-violet divergences and bad chiral behaviour. These two problems are both related to the famous "fermion species doubling" which is encountered when one regularizes a theory with fermions on the lattice. In the case of Wilson fermions a complete theoretical analysis^{3,4} has shown that in principle it is possible, by suitable combinations of bare lattice operators, to obtain the renormalized, finite operators O_n which enter into the effective Hamiltonian of Eq. (2.1). In the following, I briefly describe the problem and its solution.

In the Wilson formulation of QCD on the lattice⁵, in order to avoid the "fermion species doubling", one adds to the Lagrangian an "irrelevant" operator of the form

$$X = a \sum_{\mu} \bar{\psi} D_{\mu}^2 \psi \quad (2.2)$$

where a is the lattice spacing. At tree level and in the limit $a \rightarrow 0$, one can show that this term removes the unwanted extra fermion species present on the lattice. Moreover, it does not affect the correct continuum limit of the theory since its on-shell matrix elements are of order $O(a)$. At finite lattice spacing, however, the operator in Eq. (2.2) breaks explicitly the chiral symmetry even in the limit of zero bare quark mass. As a consequence the chiral properties of the theory can be recovered only when $a \rightarrow 0$. The problem becomes

more complicated beyond the tree level and suitable subtractions must be applied to the bare lattice operators in order to obtain the correct continuum limit.

A very simple example can be given even in the free massless quark case. Let us consider the (8,1) operator:

$$O_{\alpha} = (\bar{\Psi}_L \frac{\lambda_{\alpha}}{2} \gamma_{\mu} \Psi_L) (\bar{\Psi}_R \gamma^{\mu} \Psi_R) \quad (2.3)$$

where $\phi_{R,L} = [(1 \pm \gamma_5)/2]\phi$ and λ_{α} is one of the SU(3) Gell-Mann matrices acting in flavour space. On the lattice the operator in Eq. (2.3) can mix with an operator of dimension three, δO_{α} , through the diagram of Fig. 1:

Fig. 1 The insertion of the Wilson operator of Eq. (2.1) in the loop mixes the four-fermion operator (2.3) with a two-fermion operator by flipping the helicity of the quarks.

$$\delta O_{\alpha} = \frac{1}{a^3} (\bar{\Psi}_L \frac{\lambda_{\alpha}}{2} \Psi_R + \bar{\Psi}_R \frac{\lambda_{\alpha}}{2} \Psi_L) \quad (2.4)$$

Notice that δO_{α} is a $(3, \bar{3})$ operator and that the coefficient of the mixing is, for dimensional reasons, cubically divergent in a^{-1} . In the continuum, for massless quarks, the mixing of O_{α} with the operator in Eq. (2.4) cannot occur because the original operator cannot flip the helicity of the fermion fields. On the lattice the mixing is due to the Wilson term (2.1) in the quark propagator which acts as a mass term when inserted in the loop of the diagram of Fig. 1. The above example shows that a bare operator which is naively expected to belong to a certain chiral representation, (8,1) in this case, is really a mixture of (8,1) and $(3, \bar{3})$ operators and its matrix elements diverge as powers of a^{-1} on two quark states. Clearly, an operator which truly belongs to the (8,1) representation can be obtained by a suitable combination of O_{α} and δO_{α} .

In the following we will not discuss the general theory of the renormalization of lattice quark composite operators (a detailed analysis can be found in Refs. 3 and 4) but we restrict ourselves to a few specific examples relevant for kaon systems, i.e., the $\Delta S = 2$ operator:

$$O^{\Delta S=2} = \bar{s} \gamma^\mu (1-\gamma_5) d \quad \bar{s} \gamma_\mu (1-\gamma_5) d \quad (2.5)$$

and the two (8,1) operators responsible for $\Delta T = \frac{1}{2}$ transitions:

$$\begin{aligned} O^- &= [(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L) - (\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L)] - [u \leftrightarrow c] \\ O^+ &= \frac{1}{5} [(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L) + (\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L) + \\ & 2(\bar{s}_L \gamma^\mu d_L)(\bar{d}_L \gamma_\mu d_L) + 2(\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu s_L)] - \\ & [(\bar{s}_L \gamma^\mu d_L)(\bar{c}_L \gamma_\mu c_L) + (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu d_L)] \end{aligned} \quad (2.6)$$

The operator in Eq. (2.5) cannot mix with lower dimension operators since there are no two-quark operators with $\Delta S = 2$ (or $\Delta T=3/2$). As a consequence the renormalized operator $O^{\Delta S=2}$ has the form^{6,7}:

$$O^{\Delta S=2}(\mu) = Z_{LATT}^{\Delta S=2}(\mu a, g_0) [O^{\Delta S=2} + \sum_i C_i (\bar{s} \Gamma_i d)(\bar{s} \Gamma_i d)] \quad (2.7)$$

where the coefficients $C_i = C_i(g_0)$ are finite, depend only on g_0 (the lattice bare QCD coupling constant) and can be safely computed in perturbation theory. They are different from zero because of the Wilson term of Eq. (2.1). The corresponding operators $O_i = (\bar{s} \Gamma_i d)(\bar{s} \Gamma_i d)$ (Γ_i are Dirac matrices) respect vector symmetries and CPS, are parity conserving but belong to chiral representations different from the (27,1).

Contrary to the previous case the two operators in Eqs. (2.6) can also mix with operators of dimension five and three or, more precisely, with $\bar{s} \sigma_{\mu\nu} F_{\mu\nu} d$, $\bar{s} \sigma_{\mu\nu} \tilde{F}_{\mu\nu} d$, $\bar{s} d$ and $\bar{s} \gamma_5 d$. On dimensional grounds, the coefficients of the operators of dimension less than six are expected to diverge as powers of a^{-1} . However, it is simple to show that the coefficient of $\bar{s} \sigma_{\mu\nu} \tilde{F}_{\mu\nu} d$ is finite because of CPS symmetry and that the coefficient of $\bar{s} \sigma_{\mu\nu} F_{\mu\nu} d$ can be made finite by the GIM cancellation when we allow the charm quark to propagate in the loop in diagrams like the one shown in Fig. 1 (this implies that the condition $m_c a \ll 1$ must be satisfied). On the other hand the coefficients of the scalar and pseudoscalar densities are always divergent.

In summary the renormalized (8,1) operators $O^\pm$ have the form:

$$\begin{aligned} O^\pm(\mu) &= Z_{LATT}^\pm(\mu a, g_0) [O^\pm + \delta_6 O^\pm + \delta_5 O^\pm + \\ & \frac{(m_c - m_u)}{a^2} C_s^\pm (\bar{s} d) + \frac{(m_c - m_u)(m_s - m_d)}{a} C_P^\pm \bar{s} \gamma_5 d] \end{aligned} \quad (2.8)$$

where $\delta_6 0^\pm$ and $\delta_5 0^\pm$ are the corrections due to the operators of dimension six (as for the $\Delta S = 2$ case) and five whose coefficients are finite and can be computed perturbatively^{*}). The coefficients of the pseudoscalar and scalar densities in Eq. (2.8), being power divergent, cannot be computed in perturbation theory. The reason is that any non-perturbative contribution $\sim e^{-(1/2\beta_0 g_0^2)}$ to power divergent coefficients will not disappear even when $g \rightarrow 0$, i.e., in the continuum limit.

The overall renormalization constants $Z_{\text{LATT}}^{\Delta S=2}$ and $Z_{\text{LATT}}^\pm$ are completely determined, including the finite terms, by the requirement that the renormalized operators $O(\mu)$ obey, for some value of $\mu \ll a$, the same renormalization conditions on quark states as the continuum operators of the effective Hamiltonian in Eq. (2.1).

A few comments are necessary at this point.

- (i) As shown by the explicit one-loop calculations of Refs. 6 and 7 and by the general theoretical analysis of Ref. 4, the off-diagonal operators in Eq. (2.7) as well as the operators in $\delta_6 0^\pm$ in Eq. (2.8) are all parity even. This result is true also non-perturbatively and is a consequence of an exact symmetry as discussed in Ref. 9.
- (ii) The parity violating off-diagonal operators of dimension five and three, i.e., $\bar{s} \sigma_{\mu\nu} \tilde{F}_{\mu\nu} d$ and $\bar{s} \gamma_5 d$, have coefficients which vanish for $m_s = m_d$.
- (iii) Non-perturbative methods to fix the coefficients of the operators of dimension three, $C_{S,P}^\pm$, have been developed and will be discussed below.

For practical reasons, we have to rely on perturbation theory to fix the coefficients of the operators of dimension six and five. As it will appear from the discussion of the lattice calculation of the kaon B-parameter, a certain systematic error is entailed in the procedure of constructing the renormalized operators using lowest-order perturbation theory and neglecting terms of order a .

As for the non-perturbative determination of the coefficients $C_{P,S}^\pm$ in Eq. (2.8), they can be fixed by requiring that the matrix elements of $0^\pm$ obey the standard soft pion theorems.

At lowest order in chiral symmetry breaking, soft pion theorems imply the following relations between matrix elements:

^{*}) The explicit expressions for $\delta_6 0^\pm$ and $\delta_5 0^\pm$ as well as the computation of the mixing coefficients can be found in Refs. 6-8.

$$\langle 0 | O_{\mu}^{\pm} | K^0 \rangle = i (m_K^2 - m_{\pi}^2) \delta^{\pm}$$

$$\langle \pi^{\pm}(p) | O_{\mu}^{\pm} | K^{\pm}(k) \rangle = -\frac{m_K^2}{f} \delta^{\pm} + \gamma^{\pm} (p \cdot k)$$

$$\langle \pi^+ \pi^- | O_{\mu}^{\pm} | K^0 \rangle = \frac{i}{f} (m_K^2 - m_{\pi}^2) \gamma^{\pm} \quad (2.9)$$

(all the $K \rightarrow \pi\pi$ matrix elements being taken for vanishing four-momentum transfer of the operator).

On the other hand, for small masses and momenta we have:

$$\langle 0 | O^{\pm} + \delta_6 O^{\pm} + \delta_5 O^{\pm} | K^0 \rangle = i \delta_4^{\pm}$$

$$\langle \pi^{\pm}(p) | O^{\pm} + \delta_6 O^{\pm} + \delta_5 O^{\pm} | K^{\pm}(k) \rangle = \delta_2^{\pm} + \gamma_2^{\pm} (p \cdot k) \quad (2.10)$$

where $O^{\pm} + \delta_6 O^{\pm} + \delta_5 O^{\pm}$ are the bare lattice operators in Eqs. (2.6) added with the corrections due to the operators of dimension six ($\delta_6 O^{\pm}$) and five ($\delta_5 O^{\pm}$) whose coefficients are taken from perturbation theory.

Similarly:

$$\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle = i \delta_P$$

$$\langle \pi^{\pm}(p) | \bar{s} d | K^{\pm}(k) \rangle = \delta_S + \gamma_S (p \cdot k) \quad (2.11)$$

Then by a suitable linear combination of $O^{\pm} + \delta_6 O^{\pm} + \delta_5 O^{\pm}$ with $\bar{s}d$ and $\bar{s}\gamma_5 d$ we can enforce the relations in (2.9). This implies:

$$\gamma^{\pm} = Z_{LATT}^{\pm}(\mu a, g_0) \left[\gamma_2^{\pm} - \frac{\delta_2^{\pm} \gamma_S}{\delta_S} \right] + O(m_K^2) \quad (2.12)$$

In Eq. (2.12) the $O(m_K^2)$ terms depend on $\delta^{\pm}$ but are of higher order in the chiral expansion.

The method explained above requires the computation of the $\langle \pi | O | K \rangle$ matrix elements at different values of the transferred momentum $q^2 = (p-k)^2$ in order to disentangle δ from γ in Eqs. (2.10) and (2.11). Equation (2.12) allows the determination of the $\Delta T = \frac{1}{2} K \rightarrow 2\pi$ amplitude at lowest order in chiral perturbation theory:

$$\frac{A(K_S \rightarrow \pi^+ \pi^-)}{m_K} = 2 G_F \sin \theta_c \cos \theta_c \frac{m_K^2 - m_\pi^2}{m_K f} [C(\mu) \gamma^- + C^+(\mu) \gamma^+] \quad (2.13)$$

The procedure described above cannot be used for the "penguin" operators relevant for CP violation (i.e., ϵ'/ϵ). In fact, short of putting the top quark on the lattice (this would correspond to $m_t a \ll 1$), the "penguin" operators need two non-perturbative subtractions ($\bar{s} \sigma_{\mu\nu} F_{\mu\nu} d$ and $\bar{s} d$) for the parity-conserving part. On the other hand, they still require only one subtraction in the parity-violating sector. So in principle one can use the $K \rightarrow 0$ amplitude to fix the coefficient of $\bar{s} \gamma_5 d$ (e.g., by imposing the $\langle 0 | O^\pm | K^0 \rangle = 0$) and compute directly the amplitude $K \rightarrow 2\pi$ of the renormalized operator.

A very nice alternative procedure has been recently proposed in Ref. 9. The idea is to evaluate the form factor $K \rightarrow 2\pi$ at the point $p_K^2 = p_\pi^2 = m_K^2 = m_\pi^2 = m^2$.

In this limit, because of CPS symmetry, the coefficient of $\bar{s} \gamma_5 d$ (as well as the coefficient of $\bar{s} \sigma_{\mu\nu} \tilde{F}_{\mu\nu} d$) vanishes and the Cabibbo/Gell-Mann¹⁰ theorem is evaded because the operator carries some energy. Moreover all the counter-terms of dimension six, being parity conserving^{6,7}, cannot contribute to the matrix element. Thus we have:

$$\langle \pi^+ \pi^- | O^\pm(\mu) | K^0 \rangle = \langle \pi^+ \pi^- | Z_{LATT}^\pm O^\pm | K^0 \rangle = i \frac{2m^2 \gamma^\pm}{f} \quad (2.14)$$

With this method $\gamma^\pm$ is directly obtained by the lattice operator (times the factor $Z_{LATT}^\pm$) without any further subtraction.

3. PRACTICAL METHODS TO COMPUTE THE OPERATOR MATRIX ELEMENTS ON THE LATTICE

Let us take the following two-point correlation function on the lattice:

$$G_5(m_t) = \sum_{\vec{M}} \langle P_5(\vec{M}, m_t) P_5^\dagger(0, 0) \rangle e^{i \vec{q} \cdot \vec{M}} \quad (3.1)$$

where the four-vector $n = (\vec{n}, n_t)$ labels the points on a Euclidean, hypercubic lattice. $\vec{q}$ is the spatial momentum and P_5 the pseudoscalar density:

$$P_5(\vec{M}, m_t) = \bar{\psi}(\vec{M}, m_t) \gamma_5 \psi(\vec{M}, m_t) \quad (3.2)$$

At large time distances the correlation in Eq. (3.1) is dominated by the lowest lying state (the pion or the kaon for example):

$$G_5(m_t) \rightarrow \frac{|\langle 0 | P_5 | PS \rangle|^2}{(2 E_{PS} a)} e^{-(E_{PS} a) m_t} = \frac{Z_{PS}}{(2 E_{PS} a)} e^{-(E_{PS} a) m_t} \quad (3.3)$$

where $E_{PS} = \sqrt{\vec{q}^2 + m_{PS}^2}$ and m_{PS} is the mass of the lightest pseudoscalar particle (e.g., $m_{PS} = m_{\pi, K}$). Heavier states of energy E'_{PS} are suppressed at large n_t as $\exp[-(E'_{PS} - E_{PS})a]n_t$. The measurement of the exponential decay of the correlation with time at $q = 0$ gives thus the mass of the lightest state.

To compute f_π and f_K a very convenient way is to take the ratio¹¹:

$$\frac{G_{A_0}(m_t)}{G_5(m_t)} = \frac{\sum_{\vec{m}} \langle A_0(\vec{m}, m_t) P_5^+(0, 0) \rangle}{\sum_{\vec{m}} \langle P_5(\vec{m}, m_t) P_5^+(0, 0) \rangle} \rightarrow \frac{(f_{PS} a)(m_{PS} a)}{Z_A Z_{PS}^{1/2}} \quad \text{at large } m_t \quad (3.4)$$

where $f_{PS} = f_{\pi, K, \dots}$, $A_0 = \bar{\psi} \gamma_0 \gamma_5 \psi$, Z_A is the axial current renormalization constant on the lattice³ and Z_{PS} , defined in Eq. (3.3), can be measured from the pseudoscalar-pseudoscalar correlation function.

We now consider the more complicated case of three-point functions involving two sources of pseudoscalar particles (e.g., $\bar{\psi} \gamma_5 \psi$) and a local operator whose matrix elements we want to compute. Three-point correlations are needed for the calculation of the kaon B-parameter, i.e., the matrix element $\langle K^0 | (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma_\mu d_L) | \bar{K}^0 \rangle$, and the K- π matrix elements of the weak Hamiltonian. They have the general form:

$$G(m_{t_1}, m_{t_2}) = \sum_{\vec{m}_1, \vec{m}_2} \langle P_5(\vec{m}_1, m_{t_1}) O(0, 0) P_5(\vec{m}_2, m_{t_2}) \rangle e^{i\vec{q}_1 \cdot \vec{m}_1 + i\vec{q}_2 \cdot \vec{m}_2} \quad (3.5)$$

where $O \sim (\bar{\psi} \Gamma \psi)(\bar{\psi} \Gamma \psi)$ or $\bar{\psi} \Gamma \psi$, Γ being one of the Dirac matrices.

For $\Delta S = 2$ (or $\Delta T = 3/2$) transitions only the "eight" diagram in Fig. 2 contributes. As shown in the figure all the quark lines stem from the origin and we can use directly the quark propagators $S_{\alpha\beta}^{AB}(n, 0)$ ($A, B = \text{colour}$; $\alpha, \beta = \text{spin}$) calculated for hadron spectroscopy. The calculation of the three-point correlation for $\Delta T = \frac{1}{2}$ operators is considerably more difficult because of the presence of the so-called "eye" graphs of Fig. 3. The same problem is present for quark bilinear operators ($\sim (\bar{\psi} \Gamma \psi)$) as shown in Fig. 4.

Fig. 2 "Eight" graph. All the quark propagators stem from the origin where the four-quark operator is.

Fig. 3 "Eye" graph. One quark propagator goes from the generic point n_1 to any other point n_2 .

Fig. 4 Diagram for the insertion of a two-quark operator.

Techniques have been developed to compute the diagrams in Figs. 3 and 4^{12,13}.

For large time distances ($n_{t1}=n_{t_K} \rightarrow -\infty$; $n_{t2}=n_{t_\pi} \rightarrow +\infty$) the correlation in Eq. (3.5) is dominated by the matrix element of the operator sandwiched between the lightest pseudoscalar mesons:

$$G(m_{t_K}, m_{t_\pi}) \rightarrow \frac{Z_K^{1/2} Z_\pi^{1/2}}{(2E_K a)(2E_\pi a)} \langle \pi(p) | O | K(k) \rangle e^{-(E_K a)m_{t_K} - (E_\pi a)m_{t_\pi}} \quad (3.6)$$

$Z_{K,\pi}$ and the meson energies $E_{K,\pi}$ can be found by studying the two-point correlation function in Eq. (3.3). Equation (3.6) is then used for the determination of $\langle \pi | O | K \rangle$ (or $\langle 0 | O | \pi K \rangle$).

To compute $K \rightarrow 2\pi$ matrix elements we consider the four-point correlation function:

$$G(m_{t_1}, m_{t_2}, m_{t_3}) = \sum_{\vec{m}_1, \vec{m}_2, \vec{m}_3} \langle P_5(\vec{m}_1, m_{t_1}) O(0,0) P_5(\vec{m}_2, m_{t_2}) P_5(\vec{m}_3, m_{t_3}) \rangle \quad (3.7)$$

For simplicity we have put to zero all spatial momenta. All the formulae will refer to $m_K^2 = m_\pi^2 = m^2$ and $Z_{K,\pi} = Z$ in the following. At large time distances ($n_{t1} = n_{tK} \rightarrow -\infty$; $n_{t2,3} = n_{t\pi_1, \pi_2} \rightarrow +\infty$ and $n_{t\pi_1} = n_{t\pi_2}$, one has:

$$G(m_{t_K}, m_{t_{\pi_1}}, m_{t_{\pi_2}}) = \frac{Z_5^{3/2}}{(2ma)^3} \langle \pi_1 \pi_2 | O | K \rangle e^{-(ma)[|m_{t_K}| + m_{t_{\pi_1}} + m_{t_{\pi_2}}]} \quad (3.8)$$

Thus from the knowledge of Z_5 and m one is able to compute the $K \rightarrow 2\pi$ matrix elements.

4 RESULTS FROM OUR COLLABORATION

4.1. Meson decay constants

We start by presenting the results for the meson decay constants. In Table 1 the value of the pseudoscalar meson decay constants:

$$\langle 0 | A_{PS}^\mu | PS \rangle = i p^\mu f_{PS} \quad (4.1)$$

obtained on the lattice^{9,14,15} are compared with predictions from QCD sum rules^{16,17} and (when known) with their experimental values.

The systematic error quoted for f_π reported in the Table originates from the calibration of the lattice spacing with different physical masses (e.g., the ρ or proton mass). A detailed discussion can be found in Ref. 18. This error is absent if we compute the other pseudoscalar meson decay constants f_{PS} from the ratio f_{PS}/f_π . This method is equivalent to fixing the lattice spacing using f_π itself. The further advantage is that the poorly-known renormalization constant of the axial current³, Z_A , cancels in the ratio. It is interesting to notice the good agreement of f_K with its experimental value. $f_{D,DS}$ agrees well with the results from QCD sum rules. The B-meson decay constant in the Table has been obtained from f_D by extrapolating with the non-relativistic formula:

$$f \sqrt{M} = \text{const.} \quad (4.2)$$

Meson	f_{lattice}	$f_{\text{QCD-sum rules}}$	$f_{\text{experiment}}$
π	$[140 \pm 20 \pm 20(\text{syst})]$ [18] MeV		131 MeV
K	(158 ± 13) MeV [18] (173 ± 83) MeV [15]		164 MeV
D	(180 ± 30) MeV [18] (215 ± 60) MeV [15] (136 ± 35) MeV [9]	(172 ± 15) MeV [16]	< 290 MeV
D_s	(218 ± 30) MeV [18] ~ 251 MeV [15] (200 ± 33) MeV [9]	~ 220 MeV [16]	-
B_d	~ 120 MeV* [18] (90 ± 23) MeV* [9]	(187 ± 24) MeV [16]	-
B_s	~ 150 MeV* [18] (160 ± 30) MeV* [9]		-

TABLE 1
Pseudoscalar decay constants from lattice calculations (second column), QCD sum rules (third column) and their experimental value (fourth column). The results denoted by a * have been obtained from f_{D,D_s} using Eq. (4.1).

where M is the heavy meson mass. Equation (4.2) is expected to work well (up to logarithmic corrections¹⁹⁾ for really heavy quarks but it is not clear that it can already be used for the charm quark. The value of f_B in the Table is sensibly lower than the result from QCD sum rules. For more details on the heavy quark decay constant, see the talk by M.B. Gavela at this meeting.

4.2. The kaon B-parameter and the $\Delta I = 3/2$ amplitude

The simplest (and first studied^{1, 2, 9, 18, 20, 21}) weak transition amplitude concerns the matrix element of the $\Delta S = 2$ four-fermion operator:

(4.3)

which is usually expressed in terms of the B-parameter appearing on the right-hand side of Eq. (4.3). To compute $B_{\bar{K}^0 K^0}$ we extract, at several values of the pseudoscalar meson mass, m , the matrix elements $\langle K^0 | 0 | \bar{K}^0 \rangle$ and $\langle 0 | 0 | \bar{K}^0 K^0 \rangle$ from the three-point correlation functions [cf. Eq. (3.6)].

Lowest-order chiral perturbation theory predicts:

$$\begin{aligned} \langle 0 | O(\mu) | \bar{K}^0 K^0 \rangle^{\Delta S=2} &= \alpha + \beta m^2 + \gamma P_{\bar{K}^0} \cdot P_{K^0} + O(m^4) = \\ &= \alpha + (\beta - \gamma) m^2 + O(m^4) \\ \langle K^0 | O(\mu) | \bar{K}^0 \rangle^{\Delta S=2} &= \alpha + (\beta + \gamma) m^2 + O(m^4) \end{aligned} \quad (4.4)$$

when the mesons are at rest.

For the truly renormalized operator $O_{\text{REN}}^{\Delta S=2}$, α and β should vanish. However, since the subtraction has been computed only at lowest order in g_0^2 and terms of order a still play a rôle, the operator, renormalized in perturbation theory, has α and β different from zero. So we fit the matrix elements of $O^{\Delta S=2}$ to the expressions in Eq. (4.4) and take as our best estimate of the matrix element the coefficient γ . In so doing we are certainly left with a systematic error. In fact the residual unknown subtraction will not only cancel α and β but also modify γ ($\gamma_0 \rightarrow \gamma_{0\text{REN}}$ only as $a \rightarrow 0$). The measurement of the renormalization constant of the local vector current, $\bar{\psi} \gamma_\mu \psi$, on the lattice, which can be evaluated quite precisely in a non-perturbative way, strongly suggests that the systematic main effects in the renormalization of the operators are not due to an inadequate accuracy of perturbation theory but to effects of order $\Lambda_{\text{QCD}} a$, which are certainly present at finite lattice spacing²².

Actually, to evaluate the B-parameter, it is convenient to fit the matrix elements of the operator as a function of the squared matrix element of the axial current. The coefficient of the fit is the B-parameter (up to an overall renormalization constant $8/3Z_A^2$). In Fig. 5, the ratios $\langle 0 | O^{\Delta S=2} | \bar{K}^0 K^0 \rangle / Z_5$ ($Z_5 = Z_{\text{PS}}$) and $\langle K^0 | O^{\Delta S=2} | \bar{K}^0 \rangle / Z_5$ are reported as a function of $8/3(\langle 0 | A_0 | \bar{K}^0 \rangle)^2 / Z_5 = 8/3(f_K^2 m_K^2 - Z_A^2 Z_5)$ at $\beta = 6.2$ (15 configurations). These combinations of matrix elements are particularly convenient for reducing the errors since they are directly computed by taking ratios of correlation functions:

Fig. 5 $\langle 0 \rangle / Z_5$ as a function of $\frac{8}{3} [(\langle 0 | A_0 | \bar{K}^0 \rangle)^2 / Z_5] = \frac{8}{3} (f_K^2 m_K^2 / Z_A^2 Z_5)$ at $\beta = 6.2$ on the $16^3 \times 48$ lattice. The points correspond to the matrix elements $\langle 0 | O^{\Delta S=2} | \bar{K}^0 \bar{K}^0 \rangle$, i.e., $p_K \cdot p_K = -m_K^2$ on the left-hand side, and $\langle K^0 | O^{\Delta S=2} | K^0 \rangle$, i.e., $p_K \cdot p_K = m_K^2$ on the right-hand side. Six different combinations of quark masses corresponding to $K = 0.1500, 0.1510$ and 0.1520 have been used.

$$\begin{aligned}
 & \frac{\langle 0 | O^{\Delta S=2} | \bar{K}^0 \bar{K}^0 \rangle}{Z_5} \rightarrow \frac{G(m_{t_1}, m_{t_2})}{G_K(m_{t_1}) G_K(m_{t_2})} \\
 & \frac{\langle K^0 | O^{\Delta S=2} | K^0 \rangle}{Z_5} \rightarrow \frac{G(m_t)}{G_K(m_t)} \\
 & \frac{8}{3} \frac{(\langle 0 | A_0 | \bar{K}^0 \rangle)^2}{Z_5} \rightarrow \frac{8}{3} \left(\frac{G_{A_0}(m_t)}{G_K(m_t)} \right)^2 \quad (4.5)
 \end{aligned}$$

The value of the B-parameter obtained from a linear and a quadratic fit to the points in Fig. 5 and the analogue at $\beta = 6$ are reported in Table 2. We take as our best estimate for the B-parameter the results of the quadratic fits, given the fact that terms of order m^4 are still important in the range of masses considered. To combine the results at $\beta = 6$ and $\beta = 6.2$ we must take into account that the B-parameter is a function of the scale, and use the renormalization group evolution of the corresponding operator.

B-parameter		
Linear fit	Quadratic fit	
0.81±0.16	0.75±0.20	$\beta = 6$ 30 configurations
0.88±0.20	0.60±0.20	$\beta = 6.2$ 15 configurations

TABLE 2
Results for the B-parameter obtained from a linear and a quadratic fit to the matrix elements of $O^{\Delta S=2}$.

Combining the results of the Table, one obtains:

$$B_{K^0 \bar{K}^0}(\mu) = 0.65 \pm 15 \quad \text{at } \mu \simeq 3 \text{ GeV} \quad (4.6)$$

which in terms of the renormalization group invariant B-parameter corresponds to:

$$B_{K^0 \bar{K}^0} = [\alpha_s(\mu)]^{-6/25} B_{K^0 \bar{K}^0}(\mu) = 0.87 \pm 0.20 \quad (4.7)$$

for $\Lambda_{\text{QCD}} = 200 \text{ MeV}$. Using $SU(3) \times SU(3)$ chiral perturbation theory²³ the value of $B(\mu)$ can be translated into the weak non-leptonic $\Delta T = 3/2$ amplitude $K^+ \rightarrow \pi^+ \pi^0$:

$$\frac{\langle \pi^+ \pi^0 | H_W | K^+ \rangle}{m_K} = G_F \frac{\sin \vartheta_c \cos \vartheta_c}{2} \frac{3}{4} \frac{m_K^2 - m_\pi^2}{m_K} C_4(\mu) \times \left[\frac{\langle K^0 | O^{\Delta S=2}(\mu) | \bar{K}^0 \rangle}{f_K m_K^2} \right]$$

One finds:

$$A_{3/2} = \frac{\langle \pi^+ \pi^0 | H_W | K^+ \rangle}{m_K} = [6 \pm 2 \pm 1(\text{syst})] 10^{-8} \quad (4.8)$$

to be compared with the experimental value $A_{3/2}^{\text{exp}} = 3.7 \times 10^{-8}$.

With the techniques exposed in the previous section, using four-point correlation functions, it has also been possible to compute directly the $\Delta T = 3/2$ matrix element $\langle \pi^+ \pi^0 | H_W | K^+ \rangle$. Chiral perturbation theory gives the following relations when all mesons are at rest and $m_K^2 = m_\pi^2 = m^2$:

$$\begin{aligned} \langle \pi^+ | H_W | K^+ \rangle &= g_{3/2} m^2 \\ \langle \pi^+ \pi^- | H_W | K^0 \rangle &= \frac{2i}{f} g_{3/2} m^2 \\ \langle \pi^- | H_W | \pi^- K^0 \rangle &= -\frac{6i}{f} g_{3/2} m^2 \end{aligned} \quad (4.9)$$

An explicit lattice calculation¹⁸ of the last two matrix elements has shown that the relation:

$$\frac{\langle \pi^- | H_W | K^0 \pi^- \rangle}{\langle \pi^+ \pi^- | H_W | K^0 \rangle} \simeq -3$$

is satisfied with a good approximation in the mass range $m \approx 0.6-1.0$ GeV. The values of the coupling $g_{3/2} a^2$, a being the lattice spacing, of Eq. (4.9) from $K-\pi$ and $K-\pi-\pi$ transition at $\beta = 6$ and 6.2 are reported in Table 3. From the Table we see that soft-pion theorems and lowest-order chiral relations are satisfied at the 30% level.

The results for $-1/3 \langle K^0 \pi^- | 0_4 | \pi^- \rangle$ as a function of m^2 at $\beta = 6$ and $\beta = 6.2$ are reported in Figs. 6a and 6b respectively. Only a linear fit in m^2 has been done. With the present accuracy, we cannot exclude corrections of order m^4 or effects of order a such that the matrix element does not really vanish as $m \rightarrow 0$. However, the expected chiral behaviour seems to be well reproduced by the numerical results.

Given that off-diagonal subtractions are only needed for $K-\pi$ amplitudes, and that meson masses are never very small, the $K-\pi$ and $K-\pi\pi$ comparison from Table 3 is an excellent test of our computation and of the approach to the chiral limit. Soft pion relations are indeed a very powerful way to control the

$g_{3/2} a^2$		
$K^+-\pi^+$	$K^0-\pi^+\pi^-$	
$(2.0 \pm 0.8)10^{-2}$	$(1.3 \pm 0.4)10^{-2}$	$\beta = 6$ 15 configurations
$(5.9 \pm 1.2)10^{-3}$	$(4.4 \pm 1.2)10^{-3}$	$\beta = 6.2$ 15 configurations

TABLE 3

The coupling $g_{3/2}$ defined in Eqs. (4.9) as computed from the $K-\pi$ and $K-\pi\pi$ matrix elements. Notice the good agreement between the two values both at $\beta = 6$ and $\beta = 6.2$.

Fig. 6 a,b) The quantity $g_{3/2} m^2 / f$, defined in Eqs. (4.9) and derived from the matrix element $-1/3 \langle K^0 \pi^- | O_4 | \pi^- \rangle$ is reported at $\beta = 6$ and $\beta = 6.2$ as a function of the pseudoscalar meson mass $m_K^2 = m_\pi^2 = m^2$.

renormalization of the relevant operators on the lattice. A similar test should also be performed in the more complicated case of $\Delta I = \frac{1}{2}$ transition amplitudes.

4.3. The $\Delta I = \frac{1}{2}$ amplitude²⁴

To lowest order in masses and momenta, chiral perturbation theory gives the expressions for the $K_S \rightarrow \pi^+\pi^-$ amplitude

$$\frac{A(K_S \rightarrow \pi^+ \pi^-)}{m_K} = 2G_F \sin \theta_c \cos \theta_c \frac{m_K^2 - m_\pi^2}{m_K f_\pi} [C^-(\mu) \gamma^- + C^+(\mu) \gamma^+] \quad (4.10)$$

where $\gamma^\pm$ are defined according to:

$$\langle \pi^+ \pi^- | O^\pm(\mu) | K^0 \rangle = \frac{i}{f_\pi} (m_K^2 - m_\pi^2) \gamma^\pm \quad (4.11)$$

In the same approximation, $\gamma^\pm$ appear in the following K- π and K- $\pi\pi$ matrix elements:

$$\langle \pi^+(0) | O^\pm(\mu) | K^+(\vec{q}) \rangle = \delta^\pm m^2 + \gamma^\pm E(\vec{q}) m \quad (4.12)$$

$$\langle 0 | O^\pm(\mu) | K^+(0) \pi^-(0) \rangle = \delta^\pm m^2 - \gamma^\pm m^2 \quad (4.13)$$

$$\langle \pi^+(0) \pi^-(0) | O^\pm(\mu) | K^0(0) \rangle = \frac{2i}{f_\pi} m^2 \gamma^\pm \quad (4.14)$$

$$\langle \pi^+(0) | O^\pm(\mu) | K^0(0) \pi^+(0) \rangle = -\frac{2i}{f_\pi} m^2 \gamma^\pm \quad (4.15)$$

where we have taken K and π degenerate in mass ($m_K^2 = m_\pi^2 = m^2$), $\vec{q}$ is the spatial momentum, $E(\vec{q}) = \sqrt{m^2 + \vec{q}^2}$, and $f_\pi = 131$ MeV.

The calculation on the lattice of the matrix elements on the left-hand side of Eqs. (4.12) to (4.15) allows a determination of $\gamma^\pm$, i.e., of the physical amplitude, to lowest order in chiral perturbation theory.

As discussed in Refs. 3 and 4 (see also Section 2), in the SU(3) symmetric limit, the renormalized operators $O^\pm(\mu)$ are related to the lattice operators $O^\pm$ as follows:

$$O^\pm(\mu) = Z_{\text{LATT}}^\pm(\mu a, g_0) [O_{\text{pert}}^\pm + \delta_6 O^\pm + \delta_5 O^\pm] \quad (4.16)$$

where $O_{\text{pert}}^\pm = O^\pm + \delta_6 O^\pm + \delta_5 O^\pm$ is the naive lattice operator plus the appropriate operators of dimensions six and five.

To fix $C_S^\pm$ one can impose:

$$\langle \pi^+(0) | O^\pm(\mu) | K^+(0) \rangle = 0 \quad (4.17)$$

which corresponds to $\delta^\pm = -\gamma^\pm$ *).

The smoothness assumption on the behaviour of the matrix elements as a function of m^2 made in Eqs. (4.12)-(4.15) may fail in the presence of octet scalar particles comparatively light with respect to the pseudoscalar mass m . Exchange of scalar particles as shown in Figs. 7a-c gives non-smooth contributions to the matrix elements of Eqs. (4.13)-(4.15), enhanced by the scalar propagator when $M_S^2 \approx 4m^2$. The matrix element in Eq. (4.12) is relatively unaffected, because the momentum transfer is space-like. For quark masses at which current numerical studies are done, the scalar octet mass M_S is close to $2m^+$.

Fig. 7 Feynman diagrams relative to the octet scalar contribution to $K-\pi$ (a) and $K-\pi\pi$ (b) and (c), weak transitions. The symbol $\otimes$ indicates the insertion of the weak Hamiltonian.

*) As discussed in Ref. 4, $\delta^\pm$ do not appear in the physical decay amplitude and in fact are arbitrary. Different values of $\delta^\pm$ correspond to different and equivalent subtraction prescriptions needed to define $O^\pm(\mu)$.

+) A scalar mass even smaller than $2m$ is found by high statistics preliminary results from the APE collaboration.

In such a situation the only theoretically safe way to extract $\gamma^\pm$ is from the matrix element of Eq. (4.12). By imposing the subtraction condition Eq. (4.17), one has:

$$\gamma^\pm = \frac{\langle \pi^+(0) | O^\pm(\mu) | K^+(\vec{q}) \rangle}{M[E(\vec{q}) - M]} \quad (4.18)$$

Because of the subtraction involved, the signal in the numerator of Eq. (4.18) is small and affected by large statistical fluctuations. The direct calculation of the $K\pi\pi$ amplitude needs no subtraction, is subject to smaller statistical fluctuations, but may be contaminated by spurious octet scalar contributions unless $M_S^2 \gg 4m^2$.

$K\pi$ and $K\pi\pi$ matrix elements are computed from three- and four-point correlations of the operators and pseudoscalar sources. The relevant diagrams for the calculation of $K\pi$ and $K\pi\pi$ matrix elements are given in Figs. 2, 3, 4 and 8. In the following we will denote the contribution to $\gamma^\pm$ from the eye and from the other (eight-shaped) diagrams, by $\gamma_{\text{eye}}^\pm$ and $\gamma_{\text{eight}}^\pm$ respectively.

Fig. 8 The "eight", (a) and (b), and "eye" (c), diagrams for $K\pi\pi$ four-point correlation functions.

We discuss first $K-\pi$ matrix elements.

The values of $\gamma_{eye}^{\pm}$ obtained using Eq. (4.18) are reported in Table 4 and plotted in Figs. 9a and 9b versus $(ma)^2$. Given the large errors, to extrapolate to the chiral limit we have either fitted $\gamma_{eye}^{\pm}$ to a constant taking the weighted average of the three points (I) or extrapolated linearly discarding the last point (II). The results are also given in Table 4 together with the eight-diagram contribution $\gamma_{eight}^{\pm}$.

Using Eq. (4.1) with the experimental values for m_K , m_{π} and f_{π} and using $a^{-1} = 1.8$ GeV we find the amplitude for $K_S \rightarrow \pi^+\pi^-$ reported in Table 6 where we have used $c^{(-)}(\mu) = -1.7$ and $c^{(+)}(\mu) = 0.75$ at $\mu = 1.8$ GeV. In the Table, R is the following ratio:

$$R = \frac{\langle \pi^+\pi^- | H_W | K_S \rangle}{\langle \pi^+\pi^0 | H_W | K^+ \rangle} \quad \text{exp} \sim 21.2 \quad (4.19)$$

The results for $A(K_S \rightarrow \pi^+\pi^-)$ are still compatible with zero, given that the statistical error is $\sim 100\%$. The result for γ_{eye}^+ is definitely worse than the one for γ_{eye}^- . The trend of the data is, however, encouraging. In particular they favour a positive sign for R , in agreement with the fact that $B(K_S \rightarrow \pi^+\pi^-) > 2B(K_S \rightarrow \pi^0\pi^0)$. More statistics are clearly needed.

If we use the matrix element in Eq. (4.13) to extract $\gamma_{eye}^{\pm}$ we obtain results with smaller statistical errors, corresponding to a large signal, but R comes out to be negative. These results have been presented and discussed in length in Ref. 21. The anomalous sign may be again due to the presence of a nearby scalar pole which dominates the amplitude in this range of masses.

A scalar pole may affect also the $K-\pi\pi$ calculation, where subtractions are not needed because of CPS symmetry⁹. If this is the case this method is reliable only at low pseudoscalar masses. The values of $2m^2\gamma_{eye}^{\pm}/f(m)$, reported in Table 5, have been obtained from the average of the matrix elements, Eqs. (4.14) and (4.15). $f(m)$ is the pseudoscalar meson axial coupling as a function of the quark mass. In the Table, also $2m^2\gamma_4/f(m)$, which refers to the $\Delta I = 3/2$ operator is given:

$$O_4 = (\bar{s}_L \gamma_{\mu} d_L)(\bar{u}_L \gamma^{\mu} u_L) + (\bar{s}_L \gamma_{\mu} u_L)(\bar{u}_L \gamma^{\mu} d_L) - (\bar{s}_L \gamma_{\mu} d_L)(\bar{d}_L \gamma^{\mu} d_L)$$

Assuming a constant behaviour in m^2 of all γ s, we obtain the physical amplitudes reported in Table 6.

Within errors the $K \rightarrow \pi\pi$ results are quite compatible with those obtained from the $q \neq 0$ $K \rightarrow \pi$ matrix elements, and do not show any non-smooth variation

Fig. 9 γ_{eye}^- (a) and γ_{eye}^+ (b) versus $m^2 a^2$ from the $K-\pi$ matrix elements at $q \neq 0$, see Eq. (4.18) of the text.

ma	$\gamma_{eye}(q \neq 0) \times 10^3 x a^2$	$\gamma_{eye}^*(q \neq 0) \times 10^3 x a^2$	$\gamma_{eight} \times 10^3 x a^2$	$\gamma_{eight}^* \times 10^3 x a^2$
0.523 0.436 0.327	(-3 $\pm$ 9) (-8 $\pm$ 10) (-24 $\pm$ 17)	(-2 $\pm$ 11) (0 $\pm$ 13) (+11 $\pm$ 16)	---	---
chiral limit	(-8 $\pm$ 9)(I) (-20 $\pm$ 12)(II)	(+2 $\pm$ 12)(I) (+5 $\pm$ 19)(II)	-2.9 $\pm$ 1.5	3.9 $\pm$ 1.5

TABLE 4

$\gamma_{eye, eight}^{\pm}$ extrapolated to the chiral limit from K- π matrix elements.

$\frac{2m^2}{f(m)} \gamma_{eye}$ $\times 10^4 x a^3$	$\frac{2m^2}{f(m)} \gamma_{eye}^*$ $\times 10^4 x a^3$	$\frac{2m^2}{f(m)} \gamma_{eight}$ $\times 10^4 x a^3$	$\frac{2m^2}{f(m)} \gamma_{eight}^*$ $\times 10^4 x a^3$	$\frac{2m^2}{f(m)} \gamma_4$ $\times 10^4 x a^3$
-8.0 $\pm$ 5.4 -5.5 $\pm$ 3.7	5.4 $\pm$ 2.3 3.7 $\pm$ 1.7	-0.33 $\pm$ 0.28 -0.14 $\pm$ 0.24	0.51 $\pm$ 0.07 0.38 $\pm$ 0.06	0.59 $\pm$ 0.13 0.37 $\pm$ 0.11

TABLE 5

$\gamma_{eye, eight}^{\pm}$ from K- $\pi\pi$ matrix elements at two different values of the pseudoscalar meson mass.

β	SOURCE	$A(K_S \rightarrow \pi^+ \pi^-) / m_K$	$A(K^+ \rightarrow \pi^+ \pi^0) / m_K$	R
6.0	K- $\pi(q \neq 0)$	(1.2 $\pm$ 1.3) 10^{-6} (I) (2.5 $\pm$ 1.8) 10^{-6} (II)	(16.0 $\pm$ 6.0) 10^{-8}	8 $\pm$ 24(I) 16 $\pm$ 40(II)
6.2	K- $\pi\pi$	(2.1 $\pm$ 1.3) 10^{-6}	(5.9 $\pm$ 1.4) 10^{-8}	35 $\pm$ 31
Experiment		0.78 10^{-6}	3.8 10^{-8}	21.2

TABLE 6

Results from Ref. 24) obtained at two different values of the strong coupling constant $\beta = 6/g_0^2$, $\beta = 6$ and $\beta = 6.2$.

with the pseudoscalar meson mass m . Within the present data, we are unable to decide whether any contribution of the scalar particle is hidden in the errors, or is really small.

In conclusion, the method based on the calculation of $K-\pi$ matrix elements with a non-vanishing space-momentum, $\vec{q}$, supports the observed enhancement of the $\Delta I = \frac{1}{2}$ amplitude, although within very large statistical fluctuations. The eye-diagrams are the source of the enhancement, an indirect confirmation of the scheme of Ref. 25 since penguin diagrams are indeed generated by eye-diagrams. The vacuum saturation approximation gives $\gamma_{\text{eye}}^-/\gamma_{\text{eye}}^+ = -\frac{1}{2}$, while we find a larger ratio which seems to be just the continuation of the short-distance enhancement.

The $K-\pi\pi$ method gives results in agreement with the previous ones, but its validity is subject to the demonstration that the scalar pole has little influence in the present quark mass range. Finally, the $K-\pi$ timelike region method gives results incompatible with the previous ones, which may be due to the effect of the scalar pole and/or imperfect subtraction.

4.4. Electropenguin contribution to ε'/ε

The electromagnetic contribution to the CP-violating parameter ε' induced by the electropenguin diagrams has been widely discussed in recent literature²⁶. After integration of t and c quark virtual loops, the electropenguin contribution to the $I = 2$, $K \rightarrow \pi\pi$ amplitude, A_2 , is determined by the matrix elements of the following four-fermion operators:

$$O_{LR} = \frac{1}{3} \left\{ (\bar{s}_L \gamma_\mu u_L) [\bar{u}_R \gamma^\mu u_R - \bar{d}_R \gamma^\mu d_R] + (\bar{s}_L \gamma_\mu u_L) (\bar{u}_R \gamma^\mu d_R) \right\} \quad (4.20)$$

$$(O_c)_{LR} = \frac{1}{3} \left\{ (\bar{s}_L \gamma_\mu t^A u_L) [\bar{u}_R \gamma^\mu t^A u_R - \bar{d}_R \gamma^\mu t^A d_R] + (\bar{s}_L \gamma_\mu t^A u_L) (\bar{u}_R \gamma^\mu t^A d_R) \right\} \quad (4.21)$$

The operators in Eqs. (4.20) and (4.21) have pure $\Delta I = 3/2$. From the point of view of flavour $SU(3)$, they are a superposition of 27-plet and decuplet, unlike the fully left-handed operator $O^{(4)}$:

$$O^4 = (\bar{s}_L \gamma_\mu d_L) [(\bar{u}_L \gamma^\mu u_L) + (\bar{d}_L \gamma^\mu d_L)] + \quad (4.22)$$

which is a pure 27.

In a recent paper²⁷, the $K-\pi$ and $K-\pi\pi$ matrix elements of the operators in Eqs. (4.20) and (4.21) have been computed on the lattice. The results of Ref. 27 can be summarized as follows.

- (i) In the chiral limit, the $K-\pi$ matrix elements of O_{LR} and $(O_c)_{LR}$ are both very close to their vacuum saturation value, the latter being also computed on the lattice. More precisely, we find:

$$\begin{aligned} B_{LR}(a^{-1} \simeq 2.2 \text{ GeV}) &= 1.0 \pm 0.1 \\ (B_c)_{LR}(a^{-1} \simeq 2.2 \text{ GeV}) &= 0.95 \pm 0.1 \end{aligned} \quad (4.23)$$

- (ii) Chiral relations are obeyed within about 50% accuracy, similarly to what is found in Ref. 18 for $O_{LL}^{(4)}$.
- (iii) For both O_{LR} and $(O_c)_{LR}$, the matrix elements $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$, which allow for a scalar octet intermediate state, are between 5 to 10 times larger than the $\langle \pi^+ \pi^- | O | K^0 \rangle$ and $\langle \pi^- | O | \pi^- K^0 \rangle$ matrix elements, which do not. This is strongly suggestive of an important rôle of the scalar octet also in the $K \rightarrow \pi\pi$, $\Delta I = \frac{1}{2}$ amplitude. It suggests that a reliable calculation of $\Delta I = \frac{1}{2}$ CP-conserving and CP-violating $K \rightarrow \pi\pi$ amplitudes requires quark masses much closer to the chiral limit.

The results on the electropenguin contribution to ϵ'/ϵ can be stated in two different ways:

- a) from a full-lattice calculation of $(\text{Im } A_2/A_2)_{\text{EMP}}$ one finds:

$$\left| \frac{\epsilon'}{\epsilon} \right|_{\text{EMP}} = (0.86 \pm 0.32) 10^{-3} \left(\frac{A}{1.05} \right)^2 \left(\frac{\lambda}{0.221} \right)^4 \left(\frac{\rho}{0.6} \right) \sin \varphi \quad (4.24)$$

where the error is purely statistical; A , λ , ρ and ϕ are the parameters of the C-K-M matrix in the Wolfenstein parametrization²⁸;

- b) writing

$$\epsilon' = -i e^{i(\delta_2 - \delta_0)} \frac{\omega}{\sqrt{2}} \frac{y_{\text{em}} A_0}{A_0} (1 + \Omega_{\text{EMP}} - \Omega_{\eta, \eta'}) \quad (4.25)$$

with $\Omega_{\eta, \eta'}$ arising from $\pi-\eta$ and $\pi-\eta'$ mixing, one gets:

$$\Omega_{\text{EMP}} \simeq 0.17 / B_6 \quad (4.26)$$

where B_6 is the B-parameter of the penguin operator O_6 ²⁶. The above value for Ω_{EMP} is of the same order of the estimated value of $\Omega_{\eta, \eta'}$, but with the opposite sign.

This result is quite different from what was obtained using the $1/N_c$ expansion. It agrees with previous lattice determinations of the B-parameter of left-right operators^{9,29}. All previous lattice calculations found a B-parameter close to unity both with Wilson⁹ and staggered fermions²⁹. In Ref. 29 they also estimated the ordinary penguin B-parameter with the result $B_6 = 0.5 \pm 0.2$. According to Eq. (4.26), and assuming $B_6 = 0.5$, this would imply that the electropenguin effects can be as large as 35%.

The statistical error on the figure reported in (4.26) is rather small, $\sim 10\%$. The result, however, is affected by systematic errors. In particular, from the comparison of the $K-\pi$ and $K-\pi\pi$ matrix elements, using the soft pion relation, one can estimate an overall error of $\sim 50\%$.

5. CONCLUSION AND PROSPECTS

The quality of the results on weak decay amplitudes from the lattice is already comparable to other theoretical approaches. To improve the results and to beat statistical and systematic effects larger computer resources are needed.

Among the systematic effects, it seems to be rather difficult, at the moment, to do a complete, non-quenched calculation with realistic quark masses. On the contrary, it will certainly be possible in the near future to compute the relevant matrix elements in the quenched approximation with a much larger statistics (about a factor ten larger than the statistics used to obtain the results presented above) and with much smaller (and more realistic) pseudoscalar meson masses ($m \sim 300\div 400$ MeV). In fact, as discussed before, the second major source of systematic errors, besides the quenching, originates from the extrapolation of the results to the chiral limit. We think that a factor of ten in statistics and pseudoscalar masses as low as 300 MeV should enable us to prove (or disprove) definitively that strong interactions do indeed provide the $\Delta I = \frac{1}{2}$ enhancement. To achieve this goal, we are planning to use the Italian dedicated parallel computer, APE³⁰, which is as powerful as a CRAY XMP48 and has a rather large memory (~ 64 MWords). On this computer, we will be able to obtain several thousand CPU hours to compute the weak Hamiltonian matrix elements.

ACKNOWLEDGEMENTS

I have found this workshop particularly fruitful for the many, heated discussions among the participants. I would like to thank the organizers who made it possible. I would also like to thank M.B. Gavela, L. Maiani, O. Pène, S. Petrarca and C.T. Sachrajda for their enjoyable and enthusiastic collaboration. Finally, I am grateful to A.J. Buras, E. de Rafael, H. Leutwyler, A. Pich, S. Sharpe and A. Soni for many useful discussions and comments.

REFERENCES

- 1) N. Cabibbo, G. Martinelli and R. Petronzio, Nucl. Phys. B244 (1984) 381.
- 2) R.C. Brower, M.B. Gavela, R. Gupta and G. Maturana, Phys. Rev. Lett. 53 (1984) 1318.
- 3) M. Bochicchio, L. Maiani, G. Martinelli, G.C. Rossi and M. Testa, Nucl. Phys. B262 (1985) 331.
- 4) L. Maiani, G. Martinelli, G.C. Rossi and M. Testa, Phys. Lett. B176 (1986) 445; Nucl. Phys. B289 (1987) 505.
- 5) K.G. Wilson, "New Phenomena in Subnuclear Physics", ed. A. Zichichi (Plenum, New York, 1977).
- 6) G. Martinelli, Phys. Lett. 141B (1984) 345.
- 7) C. Bernard, T. Draper and A. Soni, Phys. Rev. D36 (1987) 3224.
- 8) G. Curci, E. Franco, L. Maiani and G. Martinelli, Phys. Lett. B202 (1988) 363.
- 9) C. Bernard, T. Draper, G. Hockney and A. Soni, Talk given by C. Bernard at the "International Symposium: Field Theory on the Lattice", Seillac (France), Sept. 28 - Oct. 2, 1987, Nucl. Phys. B (Proc.Sup.) 4 (1988) 483.
- 10) N. Cabibbo, Phys. Rev. Lett. 12 (1964) 62;
M. Gell-Mann, Phys. Rev. Lett. 12 (1964) 155.
- 11) L. Maiani and G. Martinelli, Phys. Lett. B178 (1986) 265.
- 12) C. Bernard, in Gauge Theory on a Lattice: 1984, eds. C. Zachos et al., Nat. Tech. Information Service, Springfield, VA (1984).
- 13) G.W. Kilcup, S.R. Sharpe, R. Gupta, G. Gulnarik, A. Patel and T. Warnock, Phys. Lett. 164B (1985) 347.
- 14) O. Pène, Invited talk at the EPS-HEP Conference, Uppsala (June 1987);
M.B. Gavela, L. Maiani, G. Martinelli, O. Pène and S. Petrarca, Phys. Lett. B206 (1988) 113.
- 15) T.A. De Grand and R.D. Loft, Preprint COLO-HEP-165 (November 1987).
- 16) S. Narison, CERN Preprint TH. 4768 (1987).

- 17) C.A. Dominguez and N. Paver, Phys. Lett. B197 (1987) 423.
- 18) M.B. Gavela, L. Maiani, G. Martinelli, O. Pène, S. Petrarca, F. Rapuano and C.T. Sachrajda, Nucl. Phys. B306 (1988) 677.
- 19) M.A. Shifman and M.B. Voloshin, Sov. Nucl. Phys. 45(2) (1987) 292.
- 20) L. Maiani and G. Martinelli, Phys.Lett. B181 (1986) 344.
- 21) M.B. Gavela, L. Maiani, G. Martinelli, O. Pène and S. Petrarca, Talk given by L. Maiani at the "International Symposium: Field Theory on the Lattice", Seillac (France), Nucl. Phys. B (Proc. Suppl.) 4 (1988) 466.
- 22) G. Martinelli and C.T. Sachrajda, CERN Preprint TH. 5042 (1988).
- 23) J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Lett. B199 (1986) 442.
- 24) M.B. Gavela, L. Maiani, S. Petrarca, G. Martinelli and O. Pène, CERN Preprint TH. 5032 (1988).
- 25) M.A. Shifman, A.I. Vainshtein and V.J. Zakharov, Nucl. Phys. B120 (1977) 346; Sov. Phys. JETP 45 (1977) 670.
- 26) J. Bijneus and M.B. Wise, Phys. Lett. B137 (1984) 245;
J.F. Donoghue, E. Golowich, B.R. Holstein and J. Trampetic, Phys. Lett. B179 (1986) 361 and B188 (1987) 511;
A.J. Buras and J.-M. Gérard, Phys. Lett. B192 (1987) 156;
S. Sharpe, Phys. Lett. B194 (1987) 551.
- 27) E. Franco, L. Maiani, G. Martinelli and A. Morelli, CERN Preprint TH. 5101 (1988).
- 28) L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
- 29) R. Gupta, G. Guralnik, W. Kilcup, A. Patel and S.R. Sharpe, Phys. Lett. B192 (1987) 149.
- 30) M. Albanese et al., Computer Phys. Communications 45 (1987) 345.