CERN-TH.5183/88

BUBBLE NUCLEATION FOR FLAT POTENTIAL BARRIERS

Lars Gerhard Jensen

CERN, 1211 Geneva 23, Switzerland

and

Paul Joseph Steinhardt

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

ABSTRACT

We have studied false vacuum decay for effective potentials in which the false vacuum is separated from the true vacuum by a “flat” potential barrier. By flat, we mean that, near the top of the barrier, the potential varies quartically, rather than quadratically with the field (to leading order). We have discovered several new types of bubble solutions. One type reduces in the flat space-time limit to the mathematical solution introduced by Lee and Weinberg to describe “tunnelling without barriers.” Based on our analysis, though, we propose a significantly different interpretation of the solution. We numerically study these solutions (plus the Hawking-Moss solution) for a toy model to examine how the dominant tunnelling mode may change as a function of parameters. We propose a variant of the new inflationary scenario based on these results.

1. INTRODUCTION

During its early evolution, the universe underwent several phase transitions, such as the GUT transition, the quark-gluon phase transition, and perhaps the inflationary phase transition. For each transition, there is an order parameter, typically the set of scalar Higgs fields or some bose condensate of fermions, whose expectation value changes during the phase transition. For simplicity of discussion, we will assume that the order parameter is given by a singlet scalar field, ϕ . The effective potential energy density is a function of the expectation value of ϕ .

If the transition is first order, the universe may supercool below the critical temperature. In this circumstance, it is trapped in a metastable (false) minimum of the potential, prevented from evolving to the stable (true) minimum by a potential barrier separating the two minima. The transition occurs by the scalar field tunnelling through the potential barrier from the false to the true vacuum (see Figure 1). The tunnelling rate is proportional to e^{-B} , where B is the action for a “bubble solution” which describes the spontaneous decay of a region of space-time from false to true vacuum.^[1] The bubble solution satisfies the semi-classical, Euclideanized equations of motion. Different bubble solutions correspond to different “tunnelling modes.”

In curved space-time, two tunnelling modes are well known: (i) The Coleman-DeLuccia (CD) mode^[1] in which the bubble solution (as a function of radial distance from the bubble centre) has a core of true phase ($\phi \approx \phi_-$) separated by a thin wall ($\phi \approx \phi_{TOP}$) from an exterior of metastable phase ($\phi \approx \phi_+$). After nucleation, the bubble wall grows radially outward, converting false to true vacuum. (ii) The Hawking-Moss (HM) mode,^[2] which can be interpreted as the limiting case of “bubbles” with a very thick wall — $\phi \approx \phi_{TOP}$ everywhere.^[3]

Most previous analyses of bubble nucleation have been restricted to cases in which the potential barrier varies quadratically (to leading order) with the field near the top of the barrier. In this letter we reconsider the equations governing the tunnelling process for potential barriers which vary quartically or higher-order in the field. For these “flatter” potentials, we find that many new tunnelling modes exist.

One goal of the paper is to examine the new modes which reduce to the ones previously introduced by Lee and Weinberg (LW) in the limit of flat space.^[4] Mathematically, the

flat-space LW solution interpolates from near the true vacuum at the bubble core to the top of the barrier at large distances. (Normally, one expects the solution to approach the false phase at large distances.) In their paper, they interpret the mode as “tunnelling without barriers” — they propose that the solution describes tunnelling directly from the top of the barrier, rather than beginning from the false vacuum. They apply the solution to the new inflationary universe model in which it had been presumed that the universe “slow-rolls” from the top of a barrier which separates symmetry-breaking minima. They argue that the new solution is a tunnelling mode that competes and interferes with the usual “slow-rollover” mechanism in some cases.

In our analysis, we show that this interpretation is untenable when extended to curved space. By showing that the LW solution collapses onto the conventional CD solution for some choices of parameters and by taking proper account of the boundary conditions for the LW solution, it is shown that the LW solution should be ascribed the conventional interpretation as tunnelling from false to true vacuum. Hence, the LW solution never competes with slow-roll in the new inflationary scenario, which operates in a curved space background.

The second goal of this paper is to analyze the tunnelling solutions for a sample potential and determine numerically which modes dominate as a function of the parameters. We especially consider the choice of parameters relevant for inflationary cosmology. We find that the HM mode dominates in the inflationary regime. We discuss an interesting variant of the new inflationary scenario which requires less fine-tuning of parameters than the original picture.

In Section 2, we introduce the equations necessary to describe the tunnelling process, and, in Section 3, we prove the existence of new tunnelling modes for flat potential barriers. In Section 4, we present a toy model and describe the numerical analysis of the tunnelling modes. In Section 5, we discuss the interpretation of the Lee-Weinberg solution in the curved space limit using both numerical and analytical arguments. In Section 6, we describe how the dominant tunnelling mode varies as a function of the parameters for our toy model, based on our numerical analysis. In Section 7, we discuss the application of our results to inflationary cosmology.

2. THE COLEMAN-DELUCCIA EQUATIONS

The equations describing nucleation of bubbles in the presence of gravity have been derived by Coleman and DeLuccia.^[1] They are obtained by analytic continuation into Euclidean space of the equations of motion of the scalar field and the Robertson-Walker scale factor ρ . The Euclidean analog of the Robertson-Walker line element is

$$ds^2 = d\xi^2 + \rho^2(\xi)d\Omega_3^2 \quad (2.1)$$

where $d\Omega_3$ is the element of length on the 3-sphere, ρ is the (Euclidean) scale factor, and $\xi^2 \equiv \tau^2 + r^2$. The Euclidean action is^[6]

$$I = \frac{M_P^2}{16\pi} \int_Y (R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V) \sqrt{g} d^4x + \frac{M_P^2}{8\pi} \int_{\partial Y} K \sqrt{h} d^3x \quad (2.2)$$

where M_P is the Planck mass, Y is the manifold described by (2.1), R is the scalar curvature, $V(\phi)$ is the effective potential of the scalar field, ∂Y is the boundary of Y , and K is the trace of the extrinsic curvature on ∂Y . The last term in (2.2) is necessary, in general, to obtain the correct action; in this paper however, we shall only be interested in de Sitter space. In this case, ∂Y vanishes and we can ignore this term. (For *anti*-de Sitter space this term does not vanish.)

Using Eq. (2.1) the equations of motion derived from the action (2.2) are

$$\begin{aligned} \ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} &= V'(\phi) \\ \dot{\rho}^2 &= 1 + \frac{8\pi}{3M_P^2} (\frac{1}{2} \dot{\phi}^2 - V) \rho^2 \end{aligned} \quad (2.3).$$

The action reduces to

$$I(\phi, \rho) = 4\pi^2 \int_0^{\xi_{max}} d\xi (\rho^3 V(\phi) - \frac{3M_P^2}{8\pi} \rho) \quad (2.4).$$

The boundary conditions on ϕ are determined by demanding that the “friction” term $3\dot{\rho}\dot{\phi}/\rho$ be finite. They are $\rho(0) = 0$, $\dot{\phi}(0) = 0$ and $\dot{\phi}(\xi_{max}) = 0$, where ξ_{max} is defined as the first zero of ρ after $\xi = 0$.

The bubble nucleation rate per unit volume per unit time is related to the Euclidean action through $\Gamma \propto e^{-B}$ where $B = I(\phi, \rho) - I(\phi_+, \rho_+)$.^[1] Here (ϕ, ρ) is the bubble solution to (2.3) and (ϕ_+, ρ_+) is the false vacuum solution to (2.3). We notice that in the flat space limit, *i.e.* in the limit of $M_P \rightarrow \infty$, the equation of motion (2.3) reduces to

$$\ddot{\phi} + \frac{3}{\xi} \dot{\phi} = V'(\phi) \quad (2.5)$$

and $\xi_{max} = \infty$.

Recently, the flat-space equation has been studied by Lee and Weinberg^[4] for the case of very flat (quartic) potentials, such as might be relevant for inflationary cosmology.^[6] The potential they considered had *no* false vacuum, just a “slow rollover”, flat region interpolating between ϕ_{TOP} , a point of unstable equilibrium, and the true vacuum.

The consideration of flat potentials was motivated by the new inflationary scenario. In the conventional scenario, the universe is trapped at $\phi = \phi_{TOP}$ during supercooling by a temperature-dependent energy barrier which just disappears for small temperatures. Assuming that there is no bubble solution to drive the transition (since there is no barrier), small fluctuations drive the field slightly away from ϕ_{TOP} towards the true vacuum. Then, the field evolves via the semi-classical equations of motion to the true vacuum — the so-called “slow rollover.”

However, Lee and Weinberg argue that, even though there is no barrier separating ϕ_{TOP} from the true vacuum, there may be a bubble solution after all! They exhibit a solution to the equations of motion which interpolates directly from $\phi = \phi_{TOP}$ to near the true minimum. Such a tunnelling mode could compete with slow rollover, if their interpretation is correct. If the Lee-Weinberg (LW) solution dominates, the tunnelling mode interferes with the inflationary scenario since the tunnelling would fill the universe with small bubbles which skip the slow-roll process and, hence, do not inflate. The bubble walls result in unacceptable inhomogeneities, as in the old inflationary scenario.^[7]

One of the goals of this paper is to re-examine the possible bubble solutions, especially the LW mode, in *curved space*, the correct condition relevant for inflationary cosmology.

3. NEW SOLUTIONS FOR FLAT POTENTIAL BARRIERS

Consider a double-well potential with a metastable state ϕ_+ and a stable state ϕ_- (see Figure 1). We choose the maximum of the barrier that separates ϕ_+ and ϕ_- to lie at $\phi_{TOP} = 0$. Suppose that the expansion of the potential near $\phi = 0$ is given by:

$$V(\phi) = V(0) - \frac{\lambda}{4}\phi^4 + M^4\tilde{V}\left(\frac{\phi}{M}\right) \quad (3.1),$$

where $\tilde{V}$ contains higher order terms in ϕ and M determines the mass scale of $\tilde{V}$. The potential satisfies our notion of a “very flat” potential because there are no quadratic terms in the expansion. Although the potential is non-renormalizable it can be viewed as an effective field theory for a renormalizable theory with a cutoff, *e.g.* supergravity cutoff at the Planck scale.

Substituting (3.1) in (2.3), we find the bubble equations:

$$\begin{aligned} \ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} &= -\lambda\phi^3 + M^3\tilde{V}'\left(\frac{\phi}{M}\right) \\ \dot{\rho}^2 &= 1 - H_0^2\rho^2 + \frac{8\pi}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{\lambda}{4}\phi^4 - M^4\tilde{V}\left(\frac{\phi}{M}\right)\right)\rho^2 \end{aligned} \quad (3.2),$$

where $H_0^2 = 8\pi V(0)/3M_P^2$ is the Hubble scale at $\phi \approx 0$. This system of equations has several solutions in general.

Clearly, the choice ($\phi \equiv 0$, $\rho = H_0^{-1} \sin H_0\xi$) is a valid solution to the bubble equations. This choice corresponds to the HM solution in which $\phi = \phi_{TOP}$ everywhere in space (within a causal horizon volume^[3]).

To see that additional solutions exist, first consider small ϕ (that is, restricted to be near the top of the barrier) where we can neglect the last term in each of eqs. (3.2). We may then solve the Einstein equation for ρ , $\rho = H_0^{-1} \sin H_0\xi$. The equation for ϕ becomes:

$$\ddot{\phi} + 3H_0 \cot H_0\xi \dot{\phi} = -\lambda\phi^3 \quad (3.3)$$

with boundary conditions $\dot{\phi}(0) = \dot{\phi}(\pi/H_0) = 0$. We notice that (3.3) is analogous to the equation of motion for a Newtonian particle with position ϕ moving in a potential $\lambda\phi^4/4$ under the influence of a frictional force, with initial conditions that the velocity at “time”

$\xi = 0$ vanishes. To have a solution which satisfies the boundary conditions, we also require that the particle come to rest at “time” $\xi = \pi/H_0$. This condition will only be satisfied for special choices of the initial position, $\phi(0) = \phi_0$, which differ for each of the different tunnelling modes.

To determine what choices of ϕ_0 will satisfy the boundary conditions, we use a variant of the undershoot-overshoot argument that has been applied to quadratic potentials.^[8,3] An *overshoot* solution to (3.2) corresponds to initial conditions $(\phi(0), \dot{\phi}(0), \rho(0)) = (\phi_0, 0, 0)$ such that there exist a ξ_0 with $0 < \xi_0 \leq \infty$, for which $\rho(\xi_0) = 0$ but $\dot{\phi}(\xi) \neq 0$ for all $0 < \xi < \xi_0$. On the other hand, if there exist a $\xi_1 > 0$ for which $\dot{\phi}(\xi_1) = 0$ and $\rho(\xi) \neq 0$ for all $0 < \xi < \xi_1$, then we call the solution an *undershoot* solution.

First, consider the case where we release the particle at zero velocity at a point $\phi > 0$ but *very close to zero*, i.e. $\phi \lesssim H_0/\sqrt{\lambda}$. The time scale for ϕ to begin moving is $t_\phi \approx \sqrt{|\phi/V'(\phi)|} \sim 1/\sqrt{\lambda\phi^2} \gtrsim H_0^{-1}$. We should compare this to the time scale for evolution of the scale factor; $\rho = H_0^{-1} \sin H_0\xi$; for ρ , the time scale is $t_\rho \approx H_0^{-1}$. Thus, any release point $0 < \phi_0 \lesssim H_0/\sqrt{\lambda}$ causes an overshoot solution. This is different from the case for quadratic potentials where release near the top of the barrier can result in an undershoot solution.^[3]

Using eqs. (3.2) once again, we may apply a similar timing argument to show that, if ϕ_0 is chosen *close to the true vacuum*, then the result is also an *overshoot* solution. For quadratic potentials, this limit also corresponds to an overshoot solution.

For the quadratic potential, there is undershoot for small ϕ (near ϕ_{TOP}) and overshoot for large ϕ (near the true vacuum). Thus, it is clear that a solution must exist for intermediate ϕ at the boundary between undershoot and overshoot. For the flatter potentials, one has overshoot in both limits, and, hence, a solution may not exist.

However, we have more information if we consider the flat space limit of the bubble equations. For potentials of the type shown in Figure 1, it is known^[8] that undershoot solutions exist for some range of ϕ in the flat space limit. Hence, it follows that, for M/M_P small enough, t_ϕ is smaller than t_ρ : there must exist a range of ϕ_0 corresponding to undershoot solutions. By our previous arguments, these values of ϕ_0 must lie in between the two overshoot regimens near ϕ_{TOP} and ϕ_- . These undershoot solutions may lie in one continuous span of ϕ_0 , or there may be several spans of ϕ_0 corresponding to undershoot solutions with overshoot solutions in between.

The bubble solutions themselves lie at the boundary between undershoot and overshoot solutions. Having shown that choosing ϕ_0 in a region close to the top of the barrier or close to the true vacuum causes overshoot solutions, we conclude that there must be an even number of non-trivial solutions to (3.2), since each undershoot span between these two overshoot regions gives rise to two solutions to (3.2) plus boundary conditions. (In addition, there is the trivial HM solution.)

The results contrast with the case of potential barriers which vary quadratically with the field (to leading order). As noted above, when non-trivial bubble solutions exist for the quadratic potential barrier, the region near the top of the barrier is an undershoot region, rather than overshoot region.^[3] Thus, there is one non-trivial solution (the CD solution) at the boundary. Since the quartic potential barriers can result in two or more non-trivial solutions, they require tunnelling modes which are not present for quadratic potential barriers.

4. NUMERICAL ANALYSIS OF A TOY MODEL

In the previous section we have argued that new solutions must appear when the potential is very flat (quartic) around the top of the barrier. One cannot find analytic expressions for the solutions to (3.2) for a general potential. We have opted to study the solutions numerically for a toy model with a sixth-order potential of the form $V(\phi) = \lambda M^4 U(\phi/M)$ with:

$$U(\Phi) = \frac{1}{6}\Phi^6 - \frac{\epsilon}{5}\Phi^5 - \frac{1}{4}\Phi^4 + U_0 \quad (4.1)$$

A potential V of this type has a double-well shape and is quartic around the top of the barrier separating the two vacua. U is given in terms of the asymmetry parameter ϵ , which determines the difference between the true and false vacuum energy densities, and the energy density of the barrier, U_0 . (In this section, we will only consider cases where U_0 is chosen so that the energy density of the true vacuum is non-negative. In Section 6, we further restrict to the case of zero energy density for the true vacuum.) After suitable rescaling of fields and variables, (2.3) may be written

$$\begin{aligned} \ddot{\Phi} + 3\frac{\dot{\beta}}{\beta}\dot{\Phi} &= U'(\Phi) \\ \dot{\beta}^2 &= 1 + \frac{8\pi}{3}\left(\frac{M}{M_P}\right)^2\left(\frac{1}{2}\dot{\Phi}^2 - U(\Phi)\right)\beta^2 \end{aligned} \quad (4.2).$$

Here dot indicates differentiation with respect to $\eta = M\sqrt{\lambda}\xi$, and $\Phi = \phi/M$, $\beta = M\sqrt{\lambda}\rho$. The action becomes

$$B = -\frac{2\pi^2}{\lambda} \left(\int U(\Phi) \beta^3 d\eta - \frac{3}{16\pi^2} \left(\frac{M_P}{M} \right)^4 \frac{1}{U_F} \right) \quad (4.3)$$

where U_F is the value of U at the false vacuum. A numerical analysis of (4.1-3) in the region of parameter space where $U \geq 0$ confirms our expectations (Section 3) that there is an even number of solutions, in general.

In the limit where we let any one of the three parameters U_0 , M/M_P or ϵ become large, we find that no non-trivial solutions exist. This is the strong gravity limit in which the bubble radius exceeds the de Sitter radius, *i.e.* the bubble cannot fit inside the Euclideanized de Sitter space-time geometry.^[1] As in the case of quadratic potential barriers, the HM solution is the only nucleation mode in this limit. However, contrary to the quadratic case, where the CD solution converges to the HM solution in the strong gravity limit,^[3] no such convergence occurs for quartic potential barriers.

As the three parameters become smaller, non-trivial solutions can be found. For example keeping U_0 and ϵ fixed but arbitrary, and decreasing M/M_P corresponds to approaching the flat-space limit. In the flat-space limit, it is well known that at least one non-trivial solution exists, the Coleman flat-space solution.^[8] Furthermore, for potentials that are quartic near the top of the barrier, the LW analysis suggests that two types of solutions should be present, one tunnelling towards the true vacuum (LWt) and the other tunnelling towards the false vacuum (LWf). We should expect from the above that, for small M/M_P , at least four non-trivial solutions will be present: (1) the CD solution, which converges onto the Coleman solution; (2-3) LWt and LWf which converge on the flat-space LW solutions as M/M_P tends to zero; and (4) one new solution.

Our numerical analysis confirms the existence of each of these solutions for small enough M/M_P . We find that one of these indeed does converge onto the Coleman solution and the other two converge onto the LW solutions in the flat space limit as expected. In addition, we find one new solution (NS) which converges onto the HM solution as $M/M_P \rightarrow 0$. This solution, like the HM solution, has no analog in flat space; even though it is a solution to the flat-space equations of motion, its action diverges in the flat space limit (again, like the HM solution) and, hence, is discarded. However, for non-zero M/M_P , it is a legitimate new solution.

As we increase M/M_P we see from our numerical solution of the equations that the LWf solution converges onto the NS as M/M_P converges to some value $R_1(\epsilon, U_0)$. For M/M_P greater than $R_1(\epsilon, U_0)$ they cease to exist. For M/M_P between R_1 and some $R_2(\epsilon, U_0)$ two non-trivial solutions exist, the LWt and the CD solutions. In this parameter interval the LWt solution converges onto the CD solution and at R_2 they collapse into one solution. For M/M_P greater than R_2 no non-trivial solutions exist.

From these results, several important questions can be addressed.

5. INTERPRETATION OF THE LEE-WEINBERG SOLUTION

A curved-space interpretation of the Lee-Weinberg (LW) solution can be obtained by comparing it with the more familiar CD solution in the strong gravity limit. As M/M_P increases, the LWt solution converges onto the CD solution. The physical interpretation of the CD solution is well known: it describes a tunnelling event from the false vacuum into the true vacuum. The convergence of the LW and CD solutions in the limit of strong gravity strongly suggests that the LW solution should also be interpreted as a tunnelling solution from the false to the true vacuum. This contrasts with the Lee-Weinberg interpretation as a tunnelling solution from the top of the barrier to the true minimum (see discussion in Section 2).

A rigorous proof, valid for all non-zero M/M_P , can be obtained by determining the value of ϕ at $t = 0$ far from the bubble core. Extrapolated far from a bubble core at $t = 0$, ϕ should asymptotically approach the expectation value corresponding to the initial state. The interpretation of the LW solution, therefore, is resolved by determining whether ϕ approaches the false vacuum or ϕ_{TOP} at large distances from the core.

In curved space, though, the Euclidean bubble solution does not directly describe the field configuration far from the bubble core. The solution must first be analytically continued back to real space where it describes only a bounded region of space-time about the core in which ϕ approaches some value $\phi = \phi_F$ at $t = 0$ at the boundary. The value at the boundary, ϕ_F , equals the value of ϕ at $\xi = \xi_{max}$ in the Euclidean solution (see Section 3 for definition of ξ_{max}). The long-distance behavior is obtained by matching onto the analytically-continued solution. It can be shown that the equation which describes how ϕ varies from ϕ_F as the distance from the core, r , increases to infinity is equivalent to the real-space equation of

motion for a particle rolling down a hill whose shape is given by the real potential beginning from ϕ_F .^[7]

The key to our proof is the fact that the Euclidean LW bubble solutions extend from $\phi < \phi_{TOP}$, to the true side of the barrier, $\phi > \phi_{TOP}$. The Euclidean LWt solution, for example, extends from near the true vacuum, across the barrier top, to a point, ϕ_F , beyond ϕ_{TOP} on the false vacuum side of the potential. Hence, ϕ_F is a point of instability along the potential and lies on the false vacuum side of ϕ_{TOP} . It is, therefore, obvious what happens when we solve the real space equation of motion to determine the behavior of ϕ at large distances from the core: ϕ rolls from ϕ_F towards the false vacuum. The only interpretation for the LWt solution is one in which the initial state is the false vacuum, not the top of the barrier. We emphasize that this argument is valid for any potential barrier, quadratic or otherwise.

Since the proper initial state for the LW tunnelling mode is the false vacuum rather than the top of the barrier, the LW action approaches infinity in the flat-space limit. (Since ϕ approaches ϕ_{TOP} rather than ϕ_+ as r approaches infinity, the large r contribution to $I(\phi, \rho) - I(\phi_+, \rho_+)$ is divergent.) Hence, we further conclude that the LW solutions, when properly interpreted, can contribute to tunnelling only in curved space. Our conclusions clearly conflict with the original interpretation and use of the LW solutions.

6. DOMINANT TUNNELLING MODES FOR OUR TOY MODEL

In Section 4, we have shown that there may be five different tunnelling modes for our toy model, if the HM mode is included. In this section, we wish to examine how different modes dominate the tunnelling process as a function of parameters.

By definition, the mode which dominates is the one with the lowest action. In order to compare the modes, one must calculate the action for each one. Unfortunately, only for the HM solution is the analytical expression known, so we have only been able to compute the actions numerically. In particular, we have calculated the action, B , for different choices of the parameters M/M_P , ϵ and U_0 , for each tunnelling mode. For each case, we have adjusted U_0 so that the true vacuum has zero energy density or, equivalently, zero cosmological constant.

A diagram plotting the dominant tunnelling mode as a function of ϵ and M/M_P is shown in Figure 2. The parameter, λ , cancels from the equations of motion; it only affects the

prefactor of B . For convenience, we have chosen $\lambda = 1/40$; this value is sufficiently small that, if the universe slow rolls from the top of the barrier, ϕ_{TOP} , it will inflate the 60 e-foldings required to solve the cosmological homogeneity, flatness, and monopole problems.^[6] (See next section.)

The dominant tunnelling mode is indicated in each region using the notation introduced in earlier sections. The previously undefined demarcation is the term “spinodal mode.” This does not refer to any of the Euclidean bubble solutions discussed in the previous sections. Rather, this region corresponds to the limit where the bubble nucleation picture breaks down altogether. Formally, the lowest-action mode is the HM solution, but the calculated action is less than unity. Hence, even small fluctuations not of the HM type can drive the universe over the barrier and initiate a slow-roll towards the true vacuum. A part of this region corresponds to the range of parameters in which the conventional new inflationary scenario applies.

From Figure 2, it can be seen that, except for the spinodal region, either the CD mode or the HM mode dominates over the entire range of parameters. Both the two LW solutions and the NS solution have higher action than the CD or HM solutions. Considering the logarithmic plot we see that the HM mode covers a much larger region of parameter space than the CD mode, contrary to the situation for the original old inflationary scenario, where the CD mode was the dominating mode. The one surprise is the strip labelled “HM-mode.” In the original Hawking-Moss analysis, this mode dominated as the barrier becomes small compared with the energy density difference between the true and false phases. Here, however, the HM mode dominates even as the energy density asymmetry (as measured by ϵ) becomes very small compared to the barrier height, provided that M/M_P is large enough. A similar phenomenon has been noted earlier by Coleman and one of us using the thin-wall approximation for quadratic potentials. We shall exploit this phenomenon in the following section.^[9]

7. IMPLICATIONS FOR INFLATIONARY COSMOLOGY

One of the chief applications of the theory of false vacuum decay is inflationary universe cosmology. The principle motivation for studying flat potential barriers in this paper derives from the fact that, should the universe somehow evolve to the barrier peak, $\phi = \phi_{TOP}$, the flat barrier can result in slow evolution to the true phase accompanied by the necessary amount of inflation to solve cosmological problems. In our toy model, for example, sufficient inflation (60 e-foldings) can occur over some range of parameters provided that $\lambda = 1/40$ or less. (Here, for simplicity, we ignore the constraint on barrier flatness imposed by density fluctuations.)

Whether an inflationary transition occurs, therefore, reduces to whether the dominant tunnelling solution is one which carries the universe to $\phi = \phi_{TOP}$, and then allows it to slow roll to the true phase. Two results have been uncovered in our analysis of tunnelling through flat potential barriers which bear on the issue of inflation. The first result, which has already been addressed in detail in Section 5, regards the interpretation of the Lee-Weinberg (LW) solution. If the original interpretation had been valid, the LW solution could compete with the slow roll even if the universe somehow evolved to ϕ_{TOP} . We have now shown that the original interpretation is incorrect, at least in the flat space limit. The LW solution is associated with tunnelling from the false vacuum — not the top of the barrier — and, hence, does not interfere with the usual slow roll scenario.

A second result relevant to inflationary cosmology can be observed in Section 6, Figure 2. An alternative to the usual new inflationary scenario emerges. In the conventional picture, the universe supercools until the barrier disappears (or nearly disappears), after which fluctuations drive the universe away from the false vacuum towards the true phase. In Figure 2, the range of parameter space corresponding to this scenario is the one marked “spinodal mode.” Note that, since the barrier disappears altogether, all regions of space become unstable to fluctuations simultaneously. (Of course, fluctuations may drive different regions towards the true phase at different rates.)

A different scenario can result if the choice of parameters corresponds to the strip marked HM mode. For this choice, the universe supercools in the false vacuum phase and remains trapped even as the temperature decreases to zero (or, properly speaking, the de Sitter temperature). That is, there remains a *large* barrier compared to the energy density difference

between true and false vacua.) A similar situation resulted in disaster in the old inflationary scenario.^[7] However, for the flat potential barriers, there is an interesting escape.

In the old inflationary scenario, the dominant tunnelling mode is the CD mode. The CD mode is unsuccessful cosmologically because it produces small bubbles of true phase in a background of false phase. The bubbles themselves never inflate, and unacceptable inhomogeneities result.^[7]

In our new scenario, the dominant mode is the HM mode in which a region whose radius is comparable to a causal horizon distance jumps to the top of the barrier, rather than directly to the true phase. Once the HM tunnelling has occurred, the region evolves just as in the original new inflationary scenario: fluctuations drive it from the top of the flat potential barrier towards either the true or false minimum. If the potential barrier is sufficiently flat at its peak, sufficient inflation will occur in this region, as required to solve cosmological horizon, flatness and monopole problems. As the region approaches the true minimum, say, it will reheat and, henceforth, continue to evolve according to the Friedmann-Robertson-Walker (FRW) picture. In other words, the region will evolve into a region of space-time much like our observable universe.

Regions that remain in the false phase or which fluctuate back to the false phase from the top of the barrier continue to inflate. Hence, most of the universe is in the false vacuum at any given time. This is somewhat unattractive, but acceptable. As has been done before, we can appeal to notions such as the weak anthropic principle: Only a small fraction of the universe is hospitable for life as we know it. Human beings, stars, galaxies, etc., cannot evolve in a false vacuum. We have therefore arranged matters so that all regions in the universe are either completely impossible for conscious beings (false phase), or homogeneous, flat, and monopole-free FRW universes such as the one in which we find ourselves.

If these arguments are deemed reasonable, we have an interesting variant of the inflationary scenario. The possibility extends the range of parameter space for the inflationary scenario into the region of small ϵ corresponding to a large barrier separating the false and true vacua, as illustrated in Figure 2, thus reducing the degree of fine-tuning required to achieve a successful model.

The authors wish to thank Sidney Coleman for many useful discussions. This work is supported in part by DOE Contract No. EY-C-02-3071 (PJS).

REFERENCES

1. S. Coleman and F. DeLuccia, *Phys. Rev.* **D21** (1980) 3305.
2. S. W. Hawking and I. Moss, *Phys. Lett.* **110B** (1982) 35.
3. L. G. Jensen and P. J. Steinhardt, *Nucl. Phys.* **B237** (1984) 176.
4. K. Lee and E. J. Weinberg, *Nucl. Phys.* **B246** (1986) 354.
5. G. W. Gibbons and S. W. Hawking, *Phys. Rev.* **D15** (1977) 2752.
6. A. H. Guth, *Phys. Rev.* **D23** (1981) 347;
A. D. Linde, *Phys. Lett.* **108B** (1982) 389;
A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48** (1982) 1220.
7. A. H. Guth and E. J. Weinberg, *Nucl. Phys.* **B212** (1983) 321.
8. S. Coleman, *Phys. Rev.* **D15** (1977) 2929.
9. S. Coleman and P. J. Steinhardt, to appear.

FIGURE CAPTIONS

1. $V(\phi)$ shown versus ϕ . The false vacuum at ϕ_+ is separated from the true vacuum at ϕ_- by a potential barrier.
2. A plot showing the dominant tunnelling mode for the toy model in Eq. (4.1) as a function of ϵ and $\mu \equiv 8\pi M^2/3M_P^2$ for $\lambda = 1/40$. U_0 has been adjusted so that the true vacuum energy density is zero. The dashed lines indicate the region for which a successful inflationary scenario can result (see Section 7).

Fig. 1

Fig. 2