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**RECENT ISSUES ON THE MODULI SPACE OF
2d SUPERCONFORMAL FIELD THEORIES**

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ABSTRACT

Properties of the moduli space (flat directions) of internal 2d superconformal field theories related to compactifications of different superstrings with space-time supersymmetry are reported. The relation of these results to the structure of the non-linear σ -models of different 4D supergravities is discussed.

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1. INTRODUCTION

Recently, much attention has been devoted to the classification of 2d superconformal field theories, in connection with their relevance to superstrings compactified in a target space-time with dimension $D < D_{\text{crit}} = 10$ [1, 2].

In this respect two main ingredients have been shown to be very powerful tools: namely, the enhanced symmetries [3] of the ‘internal’ superconformal field theory, and the (target) space-time supersymmetry properties of the string scattering amplitudes [4–8].

The latter property, in the (2d) σ -model language, is related to the effective field equations for the massless background fields coming from the enforcement of superconformal invariance [9, 10] (vanishing β -functions) in σ -model perturbation theory.

In the search for candidate superstring models for a unified theory of particle interactions, the most promising ones are—with our present understanding—those that exhibit $N=1$ space-time supersymmetry in $D=4$. In string theory, space-time supersymmetry is not only a phenomenological input related to the scale hierarchies at low energies, but it seems also to be needed for the mathematical consistency of the theory in string-loop perturbation theory.

Of course, in the end we would like to destabilize the supersymmetric vacua to get a realistic model for particle physics, but we would like to do this by preserving a vanishing cosmological constant and a softly broken global supersymmetry. To understand a true string vacuum with the desired properties is a challenging problem in string theory, and it may be related to non-perturbative string physics. In fact, contrary to point-field theory [11, 12], in string theory it seems at present impossible [13] to have spontaneous supersymmetry breaking with continuous parameters, i.e. to have a non-supersymmetric vacuum continuously connected to a nearby supersymmetric one.

In superstring theories the number of space-time supersymmetries is related to the space-time dimension D and to properties of the ‘internal’ superconformal field theory.

More specifically, in $D=4$ we have an ‘internal’ conformal field theory with $C_{\text{tot}} = 9$ for the supersymmetric (left) sector and $C_{\text{tot}} = 22$ for the non-supersymmetric (right) sector. Here we refer to local world-sheet supersymmetry, which can be either $(1,0)$ (heterotic) or $(1,1)$ (type II strings), and C_{tot} is the contribution of the 2d conformal representations to the conformal anomaly (central charge). The internal conformal field theory is generally referred to as ‘compactified dimensions’, for obvious reasons.

There is a variety of ways of realizing the internal conformal field theory, many of them being equivalent owing to the peculiar properties of 2d bosons and fermions.

For our analysis, what is relevant is whether the internal conformal field theory is realized in such a way that the space-time supersymmetry charges come from one (left- or right-moving) sector, or from both. For example, we may have $N=2$ space-time supersymmetry with a $(2,2)$ internal conformal field theory for type II superstrings, or a $(4,4)$ internal conformal field theory for heterotic superstrings [7, 14]. In the first case the two space-time supercharges come one from the left and one from the right sector, whilst in the second case they both come from one sector. If we use the first $(2,2)$ superconformal system to compactify a heterotic (rather than type II) superstring, we reduce the space-time supersymmetry from $N=2$ to $N=1$. On the other hand, if we use a $(4,4)$ system for type II rather than for heterotic superstrings, we may increase the number of supersymmetries from $N=2$ to $N=4$.

In the case of smooth manifolds, these systems are related to three- and two-dimensional complex manifolds, which are respectively Kählerian and hyper-Kählerian [15]. They are known as Calabi-Yau manifolds [16]. In the two-dimensional case, the compact smooth manifold is a K_3 surface [17].

The distinct features of these two kinds of compactifications lie in the symmetries of the low-energy effective theory that are encoded into properties of the corresponding 2d σ -model.

In particular, the 'manifold' of the flat directions, related to the moduli space of the superconformal field theory [18], should not depend on the string theory on which we are compactifying. This argument, if correct, implies a relation between the structure of the effective supergravity Lagrangian for the moduli with $N=1$ and $N=2$ space-time supersymmetry for $(2,2)$ systems, and an analogous relation between $N=2$ and $N=4$ space-time supersymmetry for the $(4,4)$ systems [7, 8].

In the orbifold limit [19, 20], two orbifolds that correspond to the two previous cases are, for example, $T_6/\mathbb{Z}_3$ and $T_4/\mathbb{Z}_2$.

Looking at the universal sector of superstrings compactified on these systems (the identity operator of the internal conformal field theory), it is easy to see [7] that the antisymmetric tensor $b_{\mu\nu}$ and the dilaton ϕ belong to a hypermultiplet in the first case (for type II strings [21]) and to a vector multiplet in the second case (for heterotic strings [22]). This example of distinct features propagates to the other massless modes as well.

In the present report we will give some general results about $(2,2)$ superconformal systems, which are more interesting since they give $N=1$ space-time supersymmetry [3] in $D=4$ in the compactification of heterotic superstrings [22].

The correspondence of different superstrings compactified on the same superconformal system is discussed in Section 2. In Section 3 we describe the relation between the Kähler metric of the moduli that correspond to deformations of the Kähler class [$h_{(1,1)}$ forms], and topological data on the corresponding Calabi-Yau manifold (intersection matrices). In Section 4 we discuss the specific features of moduli space, which use results of $N=2$, $D=4$ supergravity such as the concept of dual quaternionic manifolds. A summary of the results discussed in this review is given in Section 5.

2. COMPACTIFICATION OF SUPERSTRINGS ON (2,2) SUPERCONFORMAL FIELD THEORIES

In this section we will consider superstring compactifications on a generic $(2,2)$ superconformal system. We will try to give a general classification of the moduli of $(2,2)$ superconformal field theories by making extensive use of the $N=2$ space-time supersymmetry that such systems exhibit when they are used to compactify type II superstrings [7, 8].

Let us remember that the massless modes of an abstract $(2,2)$ superconformal system corresponds to $h_{(1,1)}$ and $h_{(2,1)}$ forms, related to deformations of the Kähler class and of the complex structure, respectively. The scalar background fields corresponding to these massless degrees of freedom belong to supermultiplets of different types, depending on whether we are compactifying a heterotic or a type II (A and B) superstring. In the case of heterotic strings, the scalar massless degrees of freedom must necessarily be associated with Wess-Zumino (chiral) supermultiplets with an underlying Kählerian (Hodge) structure. The Kähler manifold K corresponding to the $h_{(1,1)}$ (A) and $h_{(2,1)}$ (B) harmonic forms will be of the form [8]

$$K = K_A \times K_B \quad (2.1)$$

There is an additional universal sector (corresponding to the identity operator of the superconformal field theory) related to $b_{\mu\nu}$ and ϕ degrees of freedom that parametrize an $SU(1,1)/U(1)$ manifold [4] (with curvature -1). Therefore, for a generic $N=1$ theory on the $(2,2)$ system we get the following Kähler manifold:

$$K_{N=1} = K_A \times K_B \times \frac{SU(1,1)}{U(1)} \quad (2.2)$$

If we use the same superconformal field theory to compactify a type II string, the manifolds K_A and K_B [with complex dimensions $h_{(1,1)}$ and $h_{(2,1)}$, respectively] will appear as Kähler manifolds associated with the $N=2$ vector multiplet scalars for type IIA [$h_{(1,1)}$] and type IIB [$h_{(2,1)}$] theories [7, 8].

The important point now, is that the Kählerian structure in $N=2$ supergravity is restricted.

The Kähler potential associated with a $N=2$ admissible Kähler manifold can be written, locally, in terms of a holomorphic function $F(z)$ as follows [23]:

$$K = -\log Y \quad (2.3)$$

with

$$Y = F + F^* - \frac{1}{2} (F_{,i} - F_{,i}^*) (z_i - z_i^*) \quad (2.4)$$

Therefore, all information about the moduli of (2,2) superconformal systems relies on the holomorphic functions $F_{A,B}(z)$ related to the ($N=2$) Kählerian structure alluded to above. What are the general properties of the $F_A(z)$ and $F_B(z)$ holomorphic functions?

For vector multiplets corresponding to deformations of the Kähler class [$h_{(1,1)}$ forms], the function $F_A(z)$ can be completely determined [8] by exploiting a Peccei–Quinn symmetry [14] of the underlying 2d superconformal field theory. The space–time scalar fields associated with the internal torsion two-form b_{ij} enter into the σ -model 2d Lagrangian as follows:

$$i \int_{S_2} \sum_a b^a(x) \omega_{ij}^a(x, \bar{x}) \varepsilon^{\alpha\beta} \partial_\alpha x^i \partial_\beta \bar{x}^j \quad (2.5)$$

Now, neglecting non-perturbative corrections coming from world-sheet instantons, the Lagrangian is invariant under a shift

$$\delta b^a(x) = C^a \quad (2.6)$$

since the forms $\omega_{ij}^a dx^i \wedge d\bar{x}^j$ are closed.

If we identify $b^a(x) = \text{Re } z^a$ in Eqs. (2.3) and (2.4), it follows that the Peccei–Quinn symmetry requires the function F_A to be of the form [8]

$$F_A(z) = i d_{abc}^A z^a z^b z^c \quad (2.7)$$

up to an additive constant. The functional arbitrariness is therefore reduced to numerical d_{abc} coefficients—which, in principle, can be determined for any given superconformal field theory—by computing a three-point vertex correlation (two spin-one and one scalar) for massless fields belonging to vector supermultiplets.

For the undetermined additive constant in $F_A(z)$, we will see later from string considerations that it is actually zero. We note that cubic F functions for the $N=2$ Kähler metric were introduced some years ago [23] to generalize to the $N=2$ case, $N=1$ flat potential models [24]. Also, they are naturally obtained if one considers matter coupling in 5D rather than in 4D supergravity [25].

We now pass to the moduli sector that corresponds to deformations of the complex structure [$h_{(2,1)}$ forms]. In this case we cannot use the same argument as we did before. However, we may

notice that a Peccei–Quinn symmetry exists also for this sector, but at the level of hypermultiplets rather than vector multiplets.

Therefore, by exploiting the general parametrization of a quaternionic manifold in $N=2$ supergravity—as, for example, dictated by harmonic superspace [26]—we may infer the general structure for the quaternionic metric for a type IIA string. In Section 4 it will be shown that such a quaternionic metric should also be associated with a Kähler metric, encoded in the holomorphic function $F_B(z)$, of the associated Kähler manifold for vector multiplets of type IIB strings. The determination of this function will be given elsewhere.

3. KÄHLER MANIFOLDS FROM INTERSECTION MATRICES

In the previous section we have shown that the Kähler potential associated with complex fields in K_A , i.e. those massless modes corresponding to deformations of the Kähler class, has the general form

$$K_A = -\ln Y_A \quad (3.1)$$

with

$$Y_A = -\frac{i}{4} d_{abc} (z^a - z^{*a})(z^b - z^{*b})(z^c - z^{*c}) \quad (3.2)$$

This result is obtained by plugging Eq. (2.7) into Eq. (2.4). In Ref. [8] it was also shown that an explicit formula for the coefficients d_{abc} in Eq. (2.7) can be obtained in terms of the three-point functions of the underlying superconformal field theory.

Here we want to observe further that the relevant three-point functions are just the same as those defining the Yukawa coupling [27, 28] [for the families in heterotic strings corresponding to $h_{(1,1)}$ massless modes], hence they are just proportional to the intersection matrix [27]

$$g_{abc} = \int_{K_3} \omega_a \wedge \omega_b \wedge \omega_c \quad (3.3)$$

where $\{\omega_a\}$ is a (real) basis for $H^{1,1}(K)$.

The simplest way to realize that Eq. (3.3) is related (modulo an irrelevant normalization) to the coefficients d_{abc} is to look at the effective couplings in the field theory limit [29].

In the 4D Lagrangian there is a term proportional to [23]

$$\text{Re } z^a F_{\mu\nu}^b F_{\rho\sigma}^c \varepsilon^{\mu\nu\rho\sigma} \quad (3.4)$$

whose coefficient is precisely d_{abc} .

In the case of compactification on a smooth Calabi–Yau manifold the coefficients for this coupling can be obtained by Kaluza–Klein techniques. The couplings we are looking for, containing an $\epsilon_{\mu\nu\rho\sigma}$ symbol, should come from a term [29]

$$\int_{M_4 \times K_3} B \wedge F \wedge F \quad (3.5)$$

of the 10D Lagrangian. Here $F = dA$ is the field strength of the three-form A , and B is a two-form of 10D type-IIA supergravity.

Then, in terms of the $(1,1)$ harmonic functions ω^a , and the space-time (4D) scalars ϕ^a and vectors A_μ^a , the relevant coupling reads:

(3.6)

Comparing this equation with the coefficients d_{abc} , we get the identification [29]

$$\int_{M_4} d^4x \phi^a F_{\mu\nu}^b F_{\rho\sigma}^c \varepsilon^{\mu\nu\rho\sigma} \int_{K_3} \omega_a \wedge \omega_b \wedge \omega_c \quad (3.7)$$

In Ref. [27] a kinetic matrix for the space-time scalars corresponding to (1,1) forms was given. We will now compare our results with the one given in Ref. [27].

From the way in which Peccei-Quinn symmetry acts on the space-time fields, we get the following identification for the 'torsion' and Kähler classes:

$$[\tau] = 2 \operatorname{Re} z^a [\omega_a] \quad (3.8)$$

$$[\omega] = -2 \operatorname{Im} z^a [\omega_a] = \Upsilon^a [\omega_a] \quad (3.9)$$

from which we infer for the Kähler class J,

$$J = [\omega_0] = \langle \Upsilon^a \rangle [\omega_a] \quad (3.10)$$

The above identification is made possible by the fact that in N=2 supergravity the Kähler manifold has a preferred set of coordinates [23] (in the N=1 formulation this structure is completely hidden).

From the relation between the d_{abc} coefficients and the intersection numbers g_{abc} [Eqs. (3.6) and (3.7)], we get the following identification between Ref. [27] and our results [29]:

$$g(J, J, J) \rightarrow d_{abc} \Upsilon^a \Upsilon^b \Upsilon^c = d_{YYY} \quad (3.11)$$

$$g(J, J, F_a) \rightarrow d_{abc} \Upsilon^b \Upsilon^c = (d_{YY})_a \quad (3.12)$$

$$g(J, F_a, F_b) \rightarrow d_{abc} \Upsilon^c = (d_Y)_{ab} \quad (3.13)$$

The Kähler metric corresponding to the Kähler potential in Eq. (3.1) is [23]

$$G_{a\bar{b}} = g \frac{(d_{YY})_a (d_{YY})_{\bar{b}}}{(d_{YYY})^2} - g \frac{(d_Y)_{a\bar{b}}}{d_{YYY}} \quad (3.14)$$

which differs from the normalization given in Ref. [27] by an extra volume factor

$$24 [d_{YYY}]^{-1} = 1/2 [\text{Vol of } K_3]^{-1} \quad (3.15)$$

This extra factor is due to the Weyl rescaling, which is needed in order to get the Einstein term in canonical form. Without this conformal factor the metric would not be Kählerian.

The fact that the Kähler metric obtained from our N=2 techniques is the unique Kähler metric, conformal with the one of Ref. [27], is a further confirmation of our current space-time identification [Eq. (3.10)] of the Kähler class.

Our results can be checked in explicit examples, and they allow us to compute the non-linear σ -model of the space-time scalar fields in terms of pure topological data of an abstract (2,2) superconformal theory.

4. THE HYPERMULTIPLY SECTOR OF TYPE II SUPERSTRINGS

When (2,2) superconformal systems are used to compactify a type II rather than a heterotic string, the $h_{(2,1)}$ and $h_{(1,1)}$ forms correspond to hypermultiplets in type IIA and type IIB superstrings, respectively. This fact was discussed in Ref. [7] and further investigated in Ref. [8].

Because of $N=2$ space-time local symmetry, these degrees of freedom must be associated with a quaternionic manifold [23, 30]. Since the universal sector, containing the antisymmetric tensor $b_{\mu\nu}$, the dilaton ϕ , and two extra (Ramond) scalars (corresponding to the identity operator of the internal superconformal field theory), gives an extra hypermultiplet, we expect to have quaternionic manifolds of (real) dimension $4[h_{(2,1)} + 1]$ and $4[h_{(1,1)} + 1]$ in type IIA and type IIB strings, respectively. Since the role of vector multiplets and hypermultiplets is interchanged by flipping the chirality from $A \rightleftharpoons B$, we expect that an operation should exist that allows us to associate with any ($N=2$ admissible) Kähler manifold of complex dimension h , a quaternionic manifold of (real) dimension $4(h + 1)$. Indeed such an operation exists, and it corresponds to the notion of a ‘dual’ quaternionic manifold. This notion was fully investigated in Ref. [8].

Dual quaternionic manifolds are manifolds that can be obtained by $D=3$ duality from $N=2$, $D=4$ admissible Kähler manifolds.

As is well known from the work of Günaydin, Sierra and Townsend [31], if we dimensionally reduce an $N=2$, $D=4$ supergravity theory, with n Abelian vector multiplets, to $D=3$, we obtain an $N=4$ supergravity theory with $(n + 1)$ ‘hypermultiplets’ that parametrize a quaternionic manifold. However, the contrary is not true, i.e. not all quaternionic manifolds are dual. The amazing fact about type II superstrings is that the hypermultiplets of type IIB(A) theories live precisely in the dual quaternionic manifolds of the Kähler manifolds associated with the vector multiplets of type IIA(B) superstrings.

Here we will not give the proof—it amounts to showing some equivalence between vertices of type IIA and type IIB 4D superstrings dimensionally reduced to $D=3$ space-time dimensions [8]. The above relation gives us a constructive way of building the dual quaternionic manifolds for any given $N=2$ (vector-multiplet) Kähler manifold, whose Kähler potential is specified by a given holomorphic function $F(z)$. In the case of manifolds that come from a truncation of $N=8$ supergravity [32], the classification was already given in Ref. [31]. Here, just as examples, we report the list (Table 1) of all dual quaternionic manifolds associated with ($N=2$) Kählerian symmetric spaces, which were studied by Cremmer and Van Proeyen [33].

In the application of dual quaternionic manifolds to superstrings, we can easily work out these spaces for the case of the untwisted sector of $\mathbb{Z}_N$ orbifolds [19, 20], since they can be deduced by making a suitable truncation of the $N=2$, $D=10$ supergravity Lagrangians or, analogously, by a truncation of $N=8$, $D=4$ supergravity. For all symmetric $\mathbb{Z}_N$ Coxeter orbifolds [34] reported in Table 2, we give in Tables 3 and 4 the corresponding manifolds for (the untwisted fields of) type IIA and IIB superstrings.

Tables 3 and 4 are related by the map that interchanges scalar with vector multiplets. Therefore, the quaternionic manifolds of Tables 3 and 4 are dual to the Kähler manifolds in Tables 4 and 3. In the case of heterotic superstrings compactified on the same orbifolds, the moduli space is given by Eq. (2.2). This can easily be checked by comparing Tables 3 and 4 with a direct calculation performed by Cvetič, Louis and Ovrut [35].

5. CONCLUSIONS

In the present report we have reviewed some results on the manifold structure^{*)} of scalar background fields that correspond to 'flat directions' in the σ -model approach of different superstring compactifications. These 'flat directions' are related to the moduli space of superconformal field theories, and the low-energy supergravity theories related to these massless supermultiplets describe generalized 'no-scale' models of the type introduced some years ago in the context of $N=1$, $D=4$ supergravity with sliding scalar fields [24, 36].

From a string point of view, these manifolds can be regarded as a generalization (to arbitrary superconformal systems) of the manifold $SO(6,22)/SO(6) \times SO(22)$ associated with the Lorentzian lattice introduced by Narain [1] in the context of the classification of different superstring vacua of toroidal compactifications.

Indeed the isometries of these manifolds are realized as generalized duality transformations [37, 38] both in the field theory limit and in the 2d non-linear σ -model [39].

The Lorentzian lattice of Narain is related to the non-linear σ -model of $N=4$, $D=4$ supergravity [40], and the other non-linear σ -models are generalizations thereof.

There are indications that similar considerations can be extended to more general background fields, and this could give a clue to the understanding of the general perturbative structure of effective Lagrangians coming from arbitrary superstring compactifications.

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^{*)} In string theories these manifolds are generally modded out by some discrete symmetries. Here only the local structure is reported.

Table I

The c-map for *symmetric* Kähler spaces
(of the restricted type allowed in $N = 2$ supergravity)

Kähler space	$\dim \mathbb{C}$	Quaternionic space
$\frac{U(1, n)}{U(n) \times U(1)}$ (minimal coupling)	n	$\frac{U(2, n+1)}{U(2) \times U(n+1)}$
$\frac{SU(1,1)}{U(1)} \times \frac{SO(n-1,2)}{SO(n-1) \times SO(2)}$	$n \geq 2$	$\frac{SO(n+1,4)}{SO(n+1) \times SO(4)}$
$\frac{SU(1,1)}{U(1)}$	1	$\frac{G_{2(+2)}}{SO(4)}$
$\frac{Sp(6, \mathbb{R})}{U(3)}$	6	$\frac{F_{4(+4)}}{USp(6) \times SU(2)}$
$\frac{U(3,3)}{U(3) \times U(3)}$	9	$\frac{E_{6(+2)}}{SU(6) \times SU(2)}$
$\frac{SO^*(12)}{U(6)}$	15	$\frac{E_{7(-5)}}{SO(12) \times SU(2)}$
$\frac{E_{7(-26)}}{E_6 \times SO(2)}$	27	$\frac{E_{8(-24)}}{E_7 \times SU(2)}$

Table II

Symmetric $\mathbb{Z}_N$ Coxeter orbifolds

Case	Lattice	$\mathbb{Z}_N$	(a, b, c)	N	χ	$h_{1,1}$	$h_{2,1}$	$h_{1,1}^{(0)}$	$h_{2,1}^{(0)}$
1	$[\mathrm{SU}(3)]^3$	$\mathbb{Z}_3$	$1/3 (1, 1, -2)$	27	72	36	0	9	0
2	$\mathrm{SU}(3) \times (\mathrm{G}_2)^2$	$\mathbb{Z}_6$	$1/6 (1, 1, -2)$	3	48	29	5	5	0
3	$[\mathrm{SU}(4)]^2$	$\mathbb{Z}_4$	$1/4 (1, 1, -2)$	16	48	31	7	5	1
4	$\mathrm{SU}(3) \times \mathrm{SO}(8)$ $\mathrm{SU}(2) \times \mathrm{SU}(6)$	$\mathbb{Z}_6$	$1/6 (1, 2, -3)$	12	48	35	11	3	1
5	$\mathrm{SO}(9) \times \mathrm{SO}(4)$ $\mathrm{SO}(8) \times \mathrm{SO}(4)$	$\mathbb{Z}_8$	$1/8 (1, 2, -3)$	8	48	28	4	3	1
6	$\mathrm{SO}(4) \times \mathrm{F}_4$	$\mathbb{Z}_{12}$	$1/12 (1, 4, -5)$	4	48	31	7	3	1
7	$\mathrm{SO}(5) \times \mathrm{SO}(9)$ $\mathrm{SO}(5) \times \mathrm{SO}(8)$	$\mathbb{Z}_8$	$1/8 (1, 3, -4)$	4	48	27	3	3	0
8	$\mathrm{SU}(3) \times \mathrm{SO}(8)$ $\mathrm{SU}(3) \times \mathrm{F}_4$ E_6	$\mathbb{Z}_{12}$	$1/12 (1, 4, -5)$	3	48	27	3	3	0
9	$\mathrm{SU}(7)$	$\mathbb{Z}_7$	$1/7 (1, 2, -3)$	7	48	24	0	3	0

Table III

Scalars' σ -models for the UNTWISTED sector of type IIA
string compactified on the orbifolds of Table II

Case	Vector multiplets' manifold	$\dim \mathbb{C}$	Jordan algebra	Hypermultiplet manifold	dim.
1	$U(3,3)/[U(3) \times U(3)]$	9	$J_3^{\mathbb{C}}$	$SU(2,1)/U(2)$	4
2	$\frac{SU(1,1)}{U(1)} \times \frac{SO(2,4)}{SO(2) \times SO(4)}$	5	$\mathbb{R} + Q(4)$	$SU(2,1)/U(2)$	4
3	$\frac{SU(1,1)}{U(1)} \times \frac{SO(2,4)}{SO(2) \times SO(4)}$	5	$\mathbb{R} + Q(4)$	$U(2,2)/[U(2) \times U(2)]$	8
4, 5, 6	$[SU(1,1)/U(1)]^3$	3	$\mathbb{R} + \mathbb{R} + \mathbb{R}$	$U(2,2)/[U(2) \times U(2)]$	8
7, 8, 9	$[SU(1,1)/U(1)]^3$	3	$\mathbb{R} + \mathbb{R} + \mathbb{R}$	$SU(2,1)/U(2)$	4

Table IV

Same as Table III, for type IIB

Case	Vectors' manifold	$\dim \mathbb{C}$	Hypermultiplet manifold	dim.	Jordan algebra
1	-	0	$E_{6(+2)}/[SU(2) \times SU(6)]$	40	$J_3^{\mathbb{C}}$
2	-	0	$SO(4, 6)/[SO(4) \times SO(6)]$	24	$\mathbb{R} + Q(4)$
3	$SU(1, 1)/U(1)$	1	$SO(4, 6)/[SO(4) \times SO(6)]$	24	$\mathbb{R} + Q(4)$
4, 5, 6	$SU(1, 1)/U(1)$	1	$SO(4, 4)/[SO(4) \times SO(4)]$	16	$\mathbb{R} + \mathbb{R} + \mathbb{R}$
7, 8, 9	-	0	$SO(4, 4)/[SO(4) \times SO(4)]$	16	$\mathbb{R} + \mathbb{R} + \mathbb{R}$

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