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A SUPERSYMMETRIC $SU(4) \times O(4)$ MODEL

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A B S T R A C T

A simple $N = 1$ supersymmetric model is described, based on the group $G = SU(4) \times SU(2)_L \times SU(2)_R$. It can in principle be derived from the four-dimensional fermionic superstring and has the following attractive features: there are no large Higgs representations, the coloured triplets which mediate proton decay become naturally heavy, while the mass relations $m_e = m_d$ at M_{GUT} are retained. The right-handed neutrinos acquire large masses whilst their left-handed partners remain essentially massless.

Nowadays, superstring theories appear to be the only candidates for a consistent unification of all the fundamental forces of Nature [1]. In particular, string theories constructed directly in four dimensions [2,3] offer an attractive framework for model building. An important issue is the exploration of the possible groups that can make contact with the successful standard electroweak gauge model [4]. One of the main difficulties in obtaining it through a Grand Unified group is the absence of Higgses in the adjoint or any higher self-conjugate representations, which seems to be a general property in string theories [5].

A particularly elegant GUT model which has overcome the above problem is based on the flipped $SU(5) \times U(1)$ gauge group. It has been worked out both in a minimal grand unified version [6] and in a non-minimal variant [7] using the fermionic formulation of the four-dimensional superstrings [3]. The model enjoys remarkable features: the gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ using only the $\underline{10}$ and $\overline{\underline{10}}$ representations while one simultaneously obtains a natural doublet-triplet splitting and a heavy mass for the right-handed neutrino. Furthermore it has a natural embedding in an $SO(10)$ symmetry.

In this letter we would like to explore in the same context the $SU(4) \times SU(2)_R \times SU(2)_L$ gauge group [8], which is the only remaining possible $SO(10)$ subgroup that can lead to the standard model without using adjoint Higgs representations. We will see that this model shares similar nice features with the previously explored $SU(5) \times U(1)$ model. In fact one needs only two incomplete Higgs multiplets of the $\underline{16}$ and $\overline{\underline{16}}$ representations of $SO(10)$ to break the gauge symmetry as opposed to some large $\underline{54}$ and $\underline{126}$ Higgs fields used in the context of the previous $SO(10)$ GUT [9,10]. Although incomplete multiplets look quite unnatural in the old GUT scheme, their appearance is a usual phenomenon in string theories, since the fictitious $SO(10)$ symmetry is broken by "generalized" G.S.O. projections [3,7].

We now present the particle content of the model with its transformation properties under the $SU(4) \times SU(2)_R \times SU(2)_L$ symmetry. The sixteen fermion fields [the standard 15 $SU(5)$ fermions plus the right-handed neutrino N^c] belong to the following representations:

$$\begin{aligned} F &= (4, 1, 2) \equiv (u, d, \ell) \\ \bar{F} &= (\bar{4}, 2, 1) \equiv (u^c, d^c, \ell^c) \end{aligned} \tag{1}$$

where $\ell = (\nu, e)$ is the standard left-handed lepton doublet while $\ell^c = (\nu^c, e^c)$ are the right-handed leptons, which appear also in a doublet under $SU(2)_R$. Notice, however, that the assignments of F and $\bar{F}$ representations in (1) are those which appear when the spinorial $\underline{16}$ representation of $SO(10)$ decomposes under $SU(4) \times SU(2)_R \times U(1)_L$:

$$16 \rightarrow (4, 1, 2) + (\bar{4}, 2, 1) \quad (2)$$

Next, we enumerate the Higgs representations of the model. There are two Higgses H and $\bar{H}$, which acquire large v.e.v.s, with the following transformation properties

$$\begin{aligned} H &\equiv (4, 2, 1) \equiv (\bar{u}_H^c, \bar{d}_H^c, \bar{E}_H^c, \bar{N}_H^c) \\ \bar{H} &\equiv (\bar{4}, 2, 1) \equiv (u_H^c, d_H^c, E_H^c, N_H^c) \end{aligned} \quad (3)$$

The quantum numbers of $\bar{H} = (\bar{4}, 2, 1)$ coincide with those of the second piece on the right-hand side of Eq. (2) while $H = (4, 2, 1)$ is the corresponding piece of the decomposition of the $\underline{16}$ of $SO(10)$ under the $SU(4) \times SU(2)_R \times SU(2)_L$ symmetry

$$\bar{16} \rightarrow (\bar{4}, 1, 2) + (4, 2, 1) \quad (4)$$

The Higgs field which will provide masses for the fermions and realize the symmetry breaking of $SU(2)_L \times U(1)_Y$ arises from the decomposition of the $\underline{10}$ representation of $SO(10)$

$$10 \rightarrow (6, 1, 1) + (1, 2, 2) \equiv D_6 + h \quad (5)$$

Thus, together with the required Higgs $h \equiv (1, 2, 2)$ there appears a sextet field which, after the symmetry breaking, should receive a large mass. Finally, the model employs $n_g + 1$ singlet fields (n_g is the number of generations)

$$\phi_m \equiv (1, 1, 1) \quad ; \quad m: 1, \dots, 4 \quad (6)$$

where only one of them acquires a non-zero v.e.v.

We are ready now to write the most general superpotential, invariant under the Z_2 symmetry $\bar{H} \leftrightarrow -\bar{H}$:

$$\begin{aligned} \mathcal{W} &= \lambda_1^{ij} F_i \bar{F}_j h + \lambda_2^{im} \bar{F}_i H \phi_m \\ &+ \lambda_3 H H D_6 + \lambda_4 \bar{H} \bar{H} D_6 \\ &+ \lambda_5 h h \phi_m + \lambda_6^{mnq} \phi_m \phi_n \phi_q \end{aligned} \quad (7)$$

After writing all the possible trilinear terms in (7) we are ready to discuss the gauge symmetry breaking.

The Higgs fields H and $\bar{H}$ when acquiring v.e.v.'s $\langle H \rangle \equiv \langle N_H \rangle$, $\langle \bar{H} \rangle \equiv \langle \bar{N}_H^c \rangle \sim M_{\text{GUT}}$ *) break $SU(4) \times SU(2)_R \times SU(2)_L$ symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Notice that $SU(4)$ breaks to $SU(3)_C \times U(1)_{B-L}$. The u_H^c , $\bar{u}_H^c$, E_H^c , $\bar{E}_H^c$ fields and the gauge boson corresponding to some combination of the B-L generator and the I_{3R} generator of $SU(2)_R$ become massive, with masses of the order M_{GUT} ; the orthogonal combination of the latter, forms the $U(1)_Y$ of the standard model. The remaining coloured triplets d_H , $\bar{d}_H^c$ receive masses through the λ_3 and λ_4 couplings of the superpotential in the following way.

The sextet field $D_6 \equiv (6, 1, 1)$, after the breaking of the $SU(4) \times SU(2)_R$ symmetry decomposes into two coloured triplets, as follows:

$$D_6 = D_3 + \bar{D}_3 \equiv (3, 1, 1) + (\bar{3}, 1, 1) \quad (8)$$

When the Higgses H and $\bar{H}$ acquire v.e.v.'s the λ_3 and λ_4 terms become

$$\lambda_3 \langle N_H^c \rangle d_H^c D_3 + \lambda_4 \langle \bar{N}_H^c \rangle \bar{d}_H^c \bar{D}_3 \quad (9)$$

Thus d_H^c and $\bar{d}_H^c$ combine with the D_3 and $\bar{D}_3$ respectively to form massive states of order $M_{\text{GUT}} \sim \langle N_H^c \rangle$, $\langle \bar{N}_H^c \rangle$. Therefore, there is no danger of fast proton decay. We now proceed to analyze the rest of the Yukawa couplings in the superpotential (7).

The first term gives masses to all the sixteen fermions when the Higgs field

$$h = \begin{pmatrix} h^0 & \bar{h}^+ \\ h^- & \bar{h}^0 \end{pmatrix} \quad (10)$$

which breaks the electroweak symmetry, acquires a v.e.v.

$$\langle h \rangle = \begin{pmatrix} v & 0 \\ 0 & \bar{v} \end{pmatrix} \quad (11)$$

*) In contrast to the $SU(5) \times U(1)$ flipped model, here the D and F flatness does not necessarily imply $\langle H \rangle \equiv \langle \bar{H} \rangle$.

In fact, we have

$$\lambda_1^{ij} F_i \bar{F}_j \langle h \rangle \rightarrow m_u^{ij} (u_i u_j^c + \nu_i N_j^c) + m_d^{ij} (d_i d_j^c + e_i e_j^c) \quad (12)$$

where

$$m_u^{ij} = \lambda_1^{ij} v \quad \text{and} \quad m_d^{ij} = \lambda_1^{ij} \bar{v} \quad (13)$$

Thus we retain in this model the mass relations of previous GUTs [9], i.e.,

$$m_d = m_e \quad (14a)$$

$$m_u = m_{\nu N^c} \quad (14b)$$

The second mass relation looks disastrous but the λ_2 coupling in the superpotential takes care of it. In fact when the Higgs field H acquires a v.e.v., one gets the mass term

$$\lambda_2^{jm} \langle H \rangle N_j^c \phi_m \quad (15)$$

This term, together with (14b), and the λ_6 coupling of the superpotential which gives an additional mass for the singlets ϕ ,

$$\lambda_6 \langle \phi_m \rangle \phi_n \phi_q \rightarrow \mu \phi_n \phi_q \quad (16)$$

will provide us with a see-saw type neutrino mass matrix of the form

$$\begin{pmatrix} \nu & N^c & \phi \end{pmatrix} \begin{pmatrix} 0 & m_u & 0 \\ m_u & 0 & M_{\text{GUT}} \\ 0 & M_{\text{GUT}} & \mu \end{pmatrix} \begin{pmatrix} \nu \\ N^c \\ \phi \end{pmatrix} \quad (17)$$

The above mass matrix when diagonalized gives three light eigenvalues of order $m_u^2 \mu / M^2$ which correspond to the left Majorana neutrinos while the rest of the neutral particles (right-handed neutrinos and singlets) acquire masses of order M_{GUT} . Thus there is no conflict with neutrino oscillations and lepton number violation since the scale $m_u^2 \mu / M^2$ is too small.

We note here that the parameter μ is related to the v.e.v. of the fourth singlet ϕ_m which is responsible for the Higgs mixing, as is obvious from the coupling λ_5 of the superpotential:

$$\lambda_5 \phi_m h h \rightarrow \lambda_5 \langle \phi_m \rangle (h^0 \bar{h}^0 - \bar{h}^+ h^-) \quad (18)$$

Therefore this is of the order of the electroweak scale. The above coupling has a significant rôle to play in the superpotential, since, as is known, it is essential for the establishment of the electroweak symmetry-breaking vacuum with both $\langle h^0 \rangle$ and $\langle \bar{h}^0 \rangle = 0$, and in order to avoid an unacceptable electroweak axion. (Note that $\langle h^0 \rangle$ gives masses to the up quarks while $\langle \bar{h}^0 \rangle$ gives them to the down.)

Finally, we notice that there are no monopoles in our model since the $U(1)_{EM}$ is not contained in a simple gauge group.

In summary, we have seen that we have constructed a simple and economical model based on the smallest possible Higgs representations. The allowed trilinear superpotential terms all seem to play an important rôle in solving most of the serious phenomenological problems. This model can in principle be derived from the 4-D fermionic superstring. Since the gauge symmetry is a product of orthogonal groups, namely $SO(6) \times O(4)$, one could use only periodic or antiperiodic world sheet fermions to construct it. We will return to this issue in a future publication.

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