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BOSON-FERMION TRANSMUTATION IN 2+1 DIMENSIONS

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ABSTRACT

Renewed interest in three-dimensional field theory is spurred by the planar character of high T_C superconducting antiferromagnets. We prove Polyakov's conjecture that a $\mathbf{CP}^1$ complex scalar field in three space-time dimensions, coupled to an external gauge field with a particular Chern-Simons term, is equivalent, at low momenta, to one neutral fermion. We find that this fermion has a (strong) Thirring potential. This is the whole content, at long wavelengths, of the Hubbard model. If the gauged group is Abelian, a few decoupled Goldstone bosons survive as well. The dressing of a scalar field to become fermionic is similar to a Higgs mechanism.

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where the constraint $\det z = 1$ is taken care of, and the three real fields ξ_i are dynamical. Technically, we use a $\mathbf{CP}^1$ representation where we separate the Goldstone modes from the fields relevant at short distance. This distinction is reasonable because we want to find the leading behavior at low p .

The Lagrangian (1) is then rewritten as $\mathcal{L}_z + \theta \mathcal{L}_{CS}$, where

$$\mathcal{L}_z = \frac{\text{Tr}}{2} \frac{1}{\gamma_0} |D_\mu z|^2 \quad (5)$$

is the kinetic term for the unitary matrix field z minimally coupled to an external gauge field A_μ with coupling e , $D_\mu = \partial_\mu - ieA_\mu$. We are free to gauge the full $\text{SU}(2)$ acting on the left, or only one of its subgroups $\text{SO}(3)$ or $\text{U}(1)$. In the first two cases, $A_\mu = A_{\mu i} \sigma^i$, whereas in the Abelian case $A_\mu = A_\mu \sigma^3$. Explicitly,

$$\mathcal{L}_z = \frac{1}{\gamma_0} \left\{ (\partial_\mu \xi_i)^2 + e^2 A^2 + A_\mu^i J_i^\mu \right\} \quad (6)$$

where the bosonic current is

$$J_i^\mu = -2e\partial^\mu \xi_i \begin{cases} i = 3 & \text{Abelian} \\ i = 1, 2, 3 & \text{non-Abelian} \end{cases} \quad (7)$$

The incorporated constraint generates a crucial tree-level mass term for the gauge field instead of the quartic $z^\dagger A^2 z$ coupling.

On the other hand, the CS term is a topological invariant for the $\mathbf{CP}^1$ model:

$$\mathcal{L}_{CS} = \frac{\text{Tr}}{16\pi^2} \epsilon_{\lambda\mu\nu} A^\lambda F^{\mu\nu} = \frac{1}{8\pi^2} \epsilon_{\lambda\mu\nu} A^\lambda \partial^\mu A^\nu \quad (8)$$

where we have assumed that A_μ is Abelian. The non-Abelian case includes also a cubic $e \epsilon_{\lambda\mu\nu} A_1^\lambda A_2^\mu A_3^\nu$ term. $\mathcal{L}_{CS}$ is just the intersection number on S^3 of inverse images of two points on S^2 .

The vacuum functional we are interested in reads

$$Z = \int [d\xi_i][dA] e^{i \int \mathcal{L}_z + \theta \mathcal{L}_{CS}} \quad (9)$$

Polyakov sketched how to carry through the $[dA]$ integration in the Landau expression for the z propagator and argued that the dressed propagator was, at low momenta, fermionic[1]. We prove this result by exploiting the well-established fact that, at low momenta, the CS term can be traded off for a minimally coupled fermion[7, 15, 16], if and only if the coefficient of the CS term is $\theta = \pm\pi$:

$$\int [d\bar{\psi}][d\psi][dA] e^{i \int d^3x \bar{\psi}(iD^\mu \gamma_\mu \pm m)\psi} = \int [dA] e^{i \int d^3x \pm \pi \mathcal{L}_{CS} + \mathcal{O}(\partial^2)} \quad (10)$$

The recent discovery of superconductivity at high temperature in antiferromagnetic layered materials has prompted a renewed interest in 2+1 dimensional field theories[1], where pairing and condensation mechanisms different from those in BCS theory should operate. The pairing might occur between electrons (or holes) and neutral fermionic magnons[2,3]. Now magnons are in principle bosonic, and they become fermionic only at low momentum[1,3,4]. Just how this occurs and what the effective field theory at low momenta is, are the issues addressed in this letter.

The starting point is the Hubbard model[5], *i.e.*, a $\mathbf{CP}^1$ non-linear sigma model minimally coupled to an $SO(3)$ gauge field with a CS (Chern-Simons-Wess-Zumino-Witten-Novikov) term[6]. We shall also consider the simpler case where only a $U(1)$ is gauged, the results are very similar.

Theories with CS terms have been extensively studied[7,8,9,10] and soliton vortices are known to appear[11] and behave with anomalous spin[12,13]. A certain boson-fermion transmutation, akin to the well-known two-dimensional bosonization, seems nevertheless to take place even in the sector of the theory with a zero winding number[14]. The idea is that the originally bosonic magnons get dressed by the gauge field with CS term and become true fermions in the limit of low momentum[1,3]. We prove this by considering the vacuum generating functional of the Hubbard model. At low momenta, for some interesting values of the original parameters, the theory is fully described by a neutral fermion with strong Thirring self-interaction.

The $\mathbf{CP}^1$ model in three Euclidean dimensions (or Minkowskian, just use the metric when raising and lowering indices) is simply

$$\mathcal{L}_{\mathbf{n}} = \frac{1}{\gamma_0} (\partial_\mu n_i)^2 \quad , \quad (1)$$

where γ_0 is some running normalization parameter and the vector field $\mathbf{n} \in S^3$, *i.e.*, $\mathbf{n} \in \mathbf{R}^3$ with the constraint $\mathbf{n} \cdot \mathbf{n} = 1$. Usually, one swaps the $\mathbf{n}$ field for two complex scalar fields, $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ via the identifications

$$z^\dagger \sigma^i z = n^i \quad (2)$$

with the restriction

$$z^\dagger z = 1 \quad . \quad (3)$$

The Lagrangian must then include an extra term of the form $\lambda(z^\dagger z - 1)$ to account for the constraint.

It is much more convenient, however, to write z as a 2×2 unitary matrix:

$$z = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix} = e^{i\xi_i \sigma^i} \quad (4)$$

where the coupling of the fermion ψ to the gauge field A is g .

Hence at low momenta and with $\theta = \pm\pi$, we may write

$$Z \approx Z_{\xi\psi} = \int [d\bar{\psi}][d\psi][d\xi_i][dA] e^{i \int [\mathcal{L}_z + \bar{\psi}(i\gamma^\mu D_\mu \pm m)\psi]} \quad (11)$$

and the integration over A is straightforward. For extra care, one can add a covariant gauge-fixing term $\frac{1}{2\alpha}(\partial \cdot A)^2$. The resulting action includes kinetic terms for the ξ_i and ψ fields and a term of the form $\frac{2}{e^2}(J_z + J_\psi)^2$ which, suprisingly, cancels the kinetic terms for the ξ_i fields coupled to the gauge field. One can now define a new fermionic field

$$\chi = \begin{cases} e^{i\xi_i\sigma^i}\psi & \text{non-Abelian} \\ e^{i\xi_3\sigma^3}\psi & \text{Abelian} \end{cases} \quad (12)$$

in terms of which $Z_{\xi\psi}$ reads simply

$$Z_{\xi\psi} = Z_\chi = \int [d\bar{\chi}][d\chi] e^{i \int \{\bar{\chi}(i\gamma^\mu \partial_\mu \pm m)\chi - \frac{\gamma_0}{4}(\bar{\chi}\gamma^\mu\chi)^2\}} \quad (13)$$

provided $e/g = -1$. This is the effective action for the Hubbard model at low energy and the main result of this letter. In the Abelian case, the factor

$$\prod_{i=1,2} \int [d\xi_i] e^{i \int \frac{1}{\gamma_0}(\partial_\mu \xi)^2} \quad (14)$$

has been absorbed into the overall normalization.

The physical implications of this result are that, for momenta smaller than the mass parameter m , the theory is well described by one massive fermion *neutral* with respect to the original gauge field. In the Abelian case, two free massless Goldstone bosons survive as well (one for each ungauged continuous symmetry). Some sort of Higgs mechanism has taken place whereby the degrees of freedom in the bosonic current have eaten up the CS gauge field and thus changed statistics. The strength of the Thirring self-interaction is proportional to γ_0 . The $\mathbf{CP}^1$ model has two phases, depending on whether γ_0 is larger or smaller than a critical value[14, 17]. We are working throughout (like Polyakov[1]) in the phase with $\gamma_0 > \gamma_{\text{cr}}$, and the renormalization flow for γ_0 is such that γ_0 increases as the momentum decreases (it is both asymptotically free and confining). Hence at low momenta, the Thirring self-interaction grows. This might cause problems for the advertised pairing between χ and electrons (or holes).

The whole mechanism relies, nevertheless, on the lowest-order equivalence between Z_{CS} and Z_ψ . Beyond the CS term, a kinetic-like $F_{\mu\nu}^2$ term is induced, whose coefficient[14] is $+\frac{1}{24\pi m}$ (for both $\pm m$) instead of the standard $-\frac{1}{4}$. This coefficient undergoes the additive correction $\frac{c}{16\pi^2 m}$ if we add to $\mathcal{L}_\psi$ a Thirring term with coefficient $\frac{c}{m}$. It is thus possible

to fine-tune the coefficient of F^2 to zero. The effect of this adjustment is a change in the strength of the χ 's Thirring self-coupling. We have not investigated the behaviour of this corrected coefficient under scale changes nor whether the fine-tuning survives radiative corrections. At sufficiently low momenta, however, the CS term dominates over the kinetic term, so spin transmutation will take place at long wavelengths regardless of the corrections to terms with more derivatives.

A related problem is the dressing of fermions minimally coupled to a CS gauge field to turn into bosons. The supersymmetrization of the z field yields for a fermionic matrix field ϕ the constraint $\det \phi = 1$, so that, essentially, $\bar{\phi}\phi = 1$ and $\bar{\phi}\gamma_\mu\phi$ is unconstrained. Using the low-energy equivalence between the CS term and a minimally coupled fermion, the spin transmutation mechanism in terms of the current algebra should be the same, pointing towards some sort of generalized Fermi-Bose equivalence in three dimensions. The effective low-energy theory should contain a neutral scalar $\bar{\psi}\phi$ and a neutral vector $\bar{\psi}\gamma_\mu\phi$ with self-couplings to be determined.

Another point in need of further clarification is the fate of a $\mathbf{CP}^1$ model with coefficient $\theta \neq \pm\pi$. As Balachandran *et al.* have found, the value of θ in the gauged $SO(3)$ (Hubbard) model is quantized: $\theta = 0, \pm\pi/2, \pm\pi$. It is of interest, then, to analyze the case $\theta = \pm\pi/2$ to complement the work of this letter. One could perhaps conjecture that it should correspond to an effective low-energy theory with “half” a fermion. In three dimensions there is no chirality, but one can nevertheless consider the Siegel projection operator familiar to supermembrane practitioners, $P = \frac{1}{2}(1 \pm H^{\mu\nu\rho}\Gamma_{\mu\nu\rho})$ with H the induced Kalb-Ramond field strength. These and related issues are under further study.

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