CAN SPACE TIME BE PROBED BELOW THE STRING SIZE?

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Abstract

Strings may be explored, through a scattering process at Planckian energies, down to distances of the order of the string size. We show that this is possible if the energy is not so extreme to cause a gravitational instability and when the scattering angle approaches some critical value from below. Above this angle, the distance starts increasing, thus departing from the usual position-momentum uncertainty relation, and in no instance is the resolution smaller than the string length. This suggests that below the Planck scale the very concept of space-time changes meaning.

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1. In a preceding paper⁽¹⁾ we have studied ultra high energy scattering in string theory. Our motivation was to investigate how the theory tackles the inconsistencies of quantum gravity at the Planck scale. We were able to recognize nontrivial classical and quantum effects originated by the large impinging energy. The soft short distance behaviour of string theory entered both in regularizing quantum effects (i.e., loop contributions) and in providing a meaningful classical limit. Here we wish to address the question to which extent short distances themselves may be explored.

In usual field theories, the short distance behaviour of products of local operators provide information on the fundamental degrees of freedom and test possible inconsistencies. Moreover, they determine features of physical quantities, as hard processes, high temperatures etc. String theory deals with physical amplitudes, so that local operators, and Green's functions are unnatural and no proper definition is yet available. Our insight on short distances must therefore come from the study of those hard processes which supposedly are sensitive to them, like fixed-angle high energy scattering⁽²⁾ and high-temperature⁽³⁾ behaviour.

We shall use our explicit calculation⁽¹⁾ to study how the theory reacts in approaching sizeable angles, and we shall find that it is not really possible to test distances shorter than the characteristic string length $\lambda_s = \sqrt{\hbar \alpha'}$, the only dimensional parameter of the theory. Qualitatively speaking, this is due to the softness of the string (i.e., its lack of hard structures) which generates a different realization of the physics of the "would be" hard processes.

Of course, in order to exhibit this feature, we must explore distances of order λ_s . But our loop resummation generates dynamically another scale, the gravitational radius $R(E) \sim (G_N E)^{1/(D-3)}$ and the approach towards λ_s depends on whether or not $R(E) > \lambda_s$. If $R(E) > \lambda_s$, we find new contributions at distances of order $R(E)$ which indicate a classical gravitational instability⁽⁴⁾ that we would like to assign to black-hole formation.

For $R(E) < \lambda_s$, these contributions are irrelevant (no black-holes with radius smaller than the string length!) so that no obstacle arises in the analysis of short distances. We shall find that larger momentum transfers do not always correspond to shorter distances. The analysis of the angle-distance relationship suggests instead a modification of the uncertainty relation at the Planck scale leading to the notion of a minimal observable length of the order of the string size.

2. Let us first recall that the high energy regime investigated in I is one of large effective coupling $g^2\alpha'$'s $(\sqrt{\alpha'})^{-(D-4)}$, but small loop expansion parameter $g^2(\sqrt{\alpha'})^{-(D-4)}$. The latter assumption ($g^2 \ll 1$, in the units $\alpha' = \hbar = 1$ that we shall often use) is a natural one in string theory, because g^2 - which is related to Newton's constant by $16\pi G_N = \alpha' g^2$ - also determines the gauge coupling. Its smallness allows a hierarchy of high energy behaviours in the loop expansion. The leading one gives powers of g^2 's - and can be interpreted as multigraviton exchange at large distances - while subleading terms are depressed by extra powers of g^2 .

We note that $R(E)^{D-3} \sim g^2 E = g\sqrt{g^2 s}$ may be larger or smaller than 1 (i.e., λ_s^{D-3}) depending on the competition between a small and a large number.

In I we have analyzed in this regime the scattering amplitudes of two low mass closed superstring states, each with the typical structure

$$|\alpha\rangle_{ij,kl\dots} = \sum c_{nm\dots} a_n^{k\dagger} a_m^{\ell\dagger} \dots |\varepsilon_{ij}\rangle, \quad M_\alpha \ll E = \sqrt{s} \quad (1)$$

where ε denotes the ground state (graviton) polarization tensor. The leading behaviour of the N-loop amplitudes (having (N-1) handles) in D physical (uncompactified) dimensions was explicitly computed and resummed to give

$$\hat{\mathcal{S}}(\underline{b}, E) = \sum_{N=0}^{\infty} \frac{[2i\hat{\mathcal{J}}(\underline{b}, E)]^N}{N!} = \exp 2i\hat{\mathcal{J}}(\underline{b}, E) \quad (2)$$

where $\underline{b}$ is the (D-2)-dimensional impact parameter of the scattering process. Therefore,

$$\frac{1}{s} A_{fi}(s, t) = \varepsilon_a \cdot \varepsilon_d \varepsilon_b \cdot \varepsilon_c \frac{2}{i} \int d^{D-2} \underline{b} e^{i\underline{b} \cdot \underline{q}} \left(\mathcal{S}_{fi}(\underline{b}, E) - \delta_{fi} \right) \quad (3)$$

where $t = -q^2$ and $\mathcal{S}_{fi}$ is the matrix element of (2) between $li = |\alpha_a, \alpha_b\rangle$ and $lf = |\alpha_c, \alpha_d\rangle$.

The S-matrix has thus an eikonal form in terms of the operator δ , explicitly given in terms of the tree amplitude for graviton scattering by

$$\hat{\mathcal{J}}(\underline{b}, E) = \int_0^{2\pi} \frac{d\sigma_u d\sigma_d}{(2\pi)^2} : a_{tree}(\underline{b} + \underline{\hat{X}}_u(\sigma_u) - \underline{\hat{X}}_d(\sigma_d), E) : \quad (4)$$

$$a_{tree}(s, t) = [s(\varepsilon_a \cdot \varepsilon_d)(\varepsilon_b \cdot \varepsilon_c)]^{-1} A_{tree}^{grav} \xrightarrow{s \rightarrow \infty} 2g^2 s \frac{\Gamma(-\epsilon/2)}{\Gamma(1+\epsilon/2)} \left(-i\frac{s}{2}\right)^t \quad (5)$$

$$\hat{X}(\sigma) = i \sum_{n \neq 0} \frac{1}{n} \left(\alpha_n e^{in\sigma} + \tilde{\alpha}_n e^{-in\sigma} \right), \quad (6)$$

where $\hat{X}(\sigma)$ is the non-zero mode string position operator, and $u(d)$ in (4) refers to the upper (lower) string in the scattering process (Fig. 1). This can be viewed as a rescattering series of towers of string states excited by $\hat{X}_u(\hat{X}_d)$ through reggeized graviton exchange.

Disregarding, for the time being, the operator shift in Eq.(4), the eikonal function is

$$\delta(b, s) \equiv \hat{\delta} \Big|_{\hat{X}=0} = \frac{\pi g^2 s}{(4\pi)^{\frac{D}{2}}} \int_{-\infty}^{+\infty} \frac{dy}{y^{\frac{1}{2}D-1}} e^{-\frac{b^2}{4y}} F\left(\frac{s}{2}, \frac{s}{2}; 1; e^{-y}\right), \quad (7)$$

where F is the hypergeometric function and the y -contour avoids the origin from below. For large s and real b Eq.(7) simplifies to

$$\delta(b, s) \xrightarrow{\frac{b^2}{Y} > 0} \frac{g^2 s}{16\pi} \frac{1}{(4\pi Y)^{\frac{D}{2}-2}} \int_0^1 d\rho \rho^{\frac{D}{2}-3} \exp\left(-\frac{b^2}{4Y} \rho\right), \quad (8)$$

where $\bar{Y} = \log(-is) \equiv Y - i\pi/2$, with the detailed behaviour

$$\text{Re } \delta \simeq \frac{g^2 s}{8\pi} \begin{cases} 1/\Omega_{D-4} b^{D-4}, & (b \gg \sqrt{Y}) \\ \frac{1}{(4\pi Y)^{\frac{D}{2}-2}} \left[\frac{1}{D-4} - \frac{b^2}{4Y(D-2)} + \dots \right], & (b \ll \sqrt{Y}), \end{cases} \quad (9a)$$

$$\text{Im } \delta \simeq \frac{\pi g^2 s}{8 (4\pi Y)^{\frac{D}{2}-2}} \exp\left(-\frac{b^2}{4Y}\right), \quad \left(\Omega_D \equiv 2\pi^{D/2}/\Gamma\left(\frac{D}{2}\right)\right), \quad (9c)$$

whose form is shown in Fig. 2.

We recognize the characteristic power behaviour of $\text{Re } \delta$ for large b , and its infrared singularity for $D = 4$, which are due to the long-range graviton exchange. The finite string size (i.e., the exponential q^2 -fall off of Eq.(5)) is responsible for the smoothening of $\text{Re } \delta$ for small b , as well as for the absorptive part ($\text{Im } \delta$), due to the excitation of heavy intermediate string states.

In I we have also worked out the effects of the string excitations introduced by the operators $\hat{X}_u$ and $\hat{X}_d$ in Eq.(4). These are important even for b larger than the string size ($b > \sqrt{Y}$) because they provide for the dominant absorptive contribution even when $\text{Im } \delta(b, E)$ as given by (9c) has died out. The quadratic expansion in $\hat{X}$ of $\hat{\delta}$ yields

$$\hat{S}^{(2)} = e^{2i\delta} \exp \left[i\Delta_{\perp} : \hat{X}_{\perp}^2 : + i\Delta_{\parallel} : \hat{X}_{\parallel}^2 : + (u \leftrightarrow d) \right], \quad \left(\hat{X}^2 = \int_0^{2\pi} \frac{d\sigma}{2\pi} \hat{X}^2(\sigma) \right), \quad (10)$$

where $\Delta_{\perp}$ and $\Delta_{\parallel}$ are the eigenvalues of the matrix $\partial^2 \delta / \partial b_i \partial b_j$ in the directions perpendicular and parallel to $\underline{b}$ respectively. For $b \gg \sqrt{Y}$,

$$\frac{\Delta_{\parallel}}{D-3} = -\Delta_{\perp} = \frac{g_s^2}{4\Omega_{D-2}} b^{D-2}. \quad (11)$$

Higher powers of $\hat{X}$ in the expansion of $\delta(\underline{b} + \hat{X}_u - \hat{X}_d)$ imply corrections involving higher derivatives of $\delta(b)$, thus from Eq.(9) powers of b^{-2} for $b \gg \sqrt{Y}$ and of Y^{-1} for $b < \sqrt{Y}$.

Postponing the discussion of the dependence of Eq.(10) on initial and final states, we find the ground state matrix element to be

$$\begin{aligned} \langle 0 | S | 0 \rangle &\equiv e^{2i\delta_{el}} = e^{2i\delta} e^{-i\gamma\Delta_{\parallel}} \Gamma(1-i\Delta_{\parallel}) \left[e^{-i\gamma\Delta_{\perp}} \Gamma(1-i\Delta_{\perp}) \right]^{2(D-3)} \\ | \langle 0 | S | 0 \rangle | &= e^{-2\text{Im } \delta_{el}} \underset{b \gg \sqrt{Y}}{\sim} \exp \left[-2\pi(D-3) |\Delta_{\perp}(b, s)| \right]. \end{aligned} \quad (12)$$

This shows, by Eq.(11), that $\text{Im } \delta_{el}$ - also shown in Fig. 2 - is sizeable up to $b \approx b_D \sim (g_s^2 s)^{(D-2)^{-1}} \gg \lambda_s$.

3. We are now able to perform the Fourier transform of Eq.(3) in order to discuss the momentum transfer dependence of the amplitude, so as to uncover the distance-angle relationship we are after.

We notice first that the naive relation $q \sim b^{-1}$ is valid only if the eikonal is small. This implies values of q so small that $b^{-1} \sim q < (g_s^2 s)^{-1/(D-4)}$. Here the amplitude is perturbative, the graviton pole is unchanged, and the total cross section ($\sim s^{-1} \text{Im } A(q=0)$) shows the characteristic infrared infinity for $4 \leq D \leq 6$ and is power behaved for $D > 6$.

For q not so small, the eikonal is large and the angle-distance relation is determined by a saddle point in $\underline{b}$ at

$$q_i = -2 \left. \frac{\partial \text{Re } \mathcal{J}}{\partial b_i} \right|_{\underline{b} = \underline{b}_s}, \quad \theta \equiv \frac{|q|}{\sqrt{s}} = -\frac{2}{\sqrt{s}} \left. \frac{\partial \text{Re } \mathcal{J}}{\partial b} \right|_{b = b_s} \quad (13)$$

The quadratic integration in $\underline{b}$ around $\underline{b}_s$ yields

$$\frac{1}{s} A_{s_i}(s, t) = \epsilon_a \cdot \epsilon_d \cdot \epsilon_b \cdot \epsilon_c \cdot \frac{2}{i} \left(\frac{i\pi}{\Delta_\perp} \right)^{\frac{D-3}{2}} \left(\frac{i\pi}{\Delta_\parallel} \right)^{\frac{1}{2}} e^{iq \cdot \underline{b}_s} \mathcal{J}_{fi}(b_s, E) \quad (14)$$

Since $\text{Re } \delta$ (Fig. 2) is a smooth function of b^2/Y for small b , and decreases as a power for large b , its b -derivative has the form depicted in Fig. 3. Eq.(13) shows therefore that there are 2 or no real saddle points depending on whether or not $\theta < (R(E)/\sqrt{Y})^{D-3}$.

For small angles, it is the larger saddle point $b_s > \sqrt{Y}$ that dominates the amplitude^(*), having both a larger coefficient and smaller absorption. Eq.(13) yields in this case, in the c.m. frame,

$$\theta = \frac{D-2}{D-3} \left(\frac{R(E)}{b_s} \right)^{D-3} = \frac{8\pi g_N \sqrt{s}}{\Omega_{D-2} b_s^{D-3}} \xrightarrow{D=4} \frac{4 g_N \sqrt{s}}{b_s} \quad (15)$$

which corresponds to the classical Einstein deflection in an Aichelburg-Sexl metric⁽⁵⁾. Deflections as Eq.(13) are indeed⁽⁶⁾ those experienced by test particles in a shock wave background metric with a profile described by δ . Similarly, the quantization of the superstring in such a metric yields⁽⁷⁾ precisely the same S-matrix we obtained in our infinite genus flat metric calculation.

By introducing Eq.(15) into Eq.(14) we obtain the small angle behaviour of the elastic ground state amplitude

$$\frac{1}{s} A_{el}(s, t) \simeq e^{i\phi} \left(-\frac{2\pi i b_s}{q} \right)^{\frac{D-2}{2}} \exp\left(-2\pi(D-3) \frac{q}{b_s}\right) \quad (16)$$

(*) The second solution $b_s < \sqrt{Y}$ corresponds to the classical trajectory in which the two strings overlap and the deflection angle decreases with the impact parameter.

where ϕ embodies the phases occurring in (14) and (12).

The validity of our results, and therefore the angular region we can explore, is subject to consistency conditions implicit in our approach. First, for the very validity of the large- s hierarchy we need that the momentum transfer carried by each exchanged graviton be small in Planck's units. This implies, more precisely

$$\langle q_{\text{single}} \rangle \equiv \frac{q}{\langle N \rangle} \ll \frac{1}{\sqrt{Y}} , \quad (17)$$

where $\langle N \rangle - 1$ is the average genus, which in our multiloop resummation is $\langle N \rangle \sim \delta \sim qb_+$. Therefore

$$\langle q_{\text{single}} \rangle \equiv \frac{q}{\langle N \rangle} \sim 1/b_+(q, E) , \quad (18)$$

so that (17) implies

$$b_+ > \lambda_s \sqrt{Y} , \quad \theta < \left(\frac{R(E)}{\lambda_s \sqrt{Y}} \right)^{D-3} . \quad (19)$$

The condition (17) automatically eliminates those subleading contributions to the eikonal which come from other string degrees of freedom (like gravitino exchanges). However in I we have identified other subleading terms, due to graviton interactions, whose expansion parameter is $(R/b)^{2(D-3)}$ for large b . They first arise at 2-loop level from the H-diagram of Fig. 4a which contains contributions of the form

$$\text{Im } a^H(s, \underline{b}) \sim g^6 s^2 \int d\underline{y}' d\underline{b}' \left(\nabla \alpha(\underline{y}', \underline{b}') \right)^2 \left(\nabla \alpha(\underline{\gamma} - \underline{y}', \underline{b} - \underline{b}') \right)^2 , \quad (20)$$

$$\alpha(s, b) \equiv a_{\text{tree}} / g^2 s .$$

For $R(E) > \sqrt{Y}$ (a situation in which there are 2 distinct saddle points in Eq.(13) at all angles) the large b correction to the eikonal was found in I from Eq.(20) to be

$$a_{\text{tree}}(s, b) (c_1 + i c_2 Y) \left(\frac{R(E)}{b} \right)^{2(D-3)} , \quad b > R(E) . \quad (21)$$

Therefore the leading eikonal is justified only if $b_s \gg R(E) > \sqrt{Y}$, i.e. by Eq.(16) $\theta \ll 1$, and of course distances much larger than the string size. For b approaching $R(E)$ these contributions of classical^(*) type cannot be neglected, and are expected to provide all powers of $(R(E)/b)^{2(D-3)}$ arising from higher loop diagrams, as those of Fig. (4b). We expect to come back⁽⁴⁾ to the analysis of this interesting regime.

A different situation arises when $R(E) < \sqrt{Y}$. The evaluation (21) still holds for $b \gg \sqrt{Y} > R(E)$. However, for $b < \sqrt{Y}$ we obtain, from Eq.(20),

$$\text{Im } a^H \sim a_{\text{tree}} Y \left(\frac{R(E)}{\lambda_s \sqrt{Y}} \right)^{2(D-3)}, \quad (b \lesssim \sqrt{Y}, R < \sqrt{Y}), \quad (22)$$

thus showing that the magnitude of a^H saturates at the string length if this overcomes the gravitational radius. The classical corrections are therefore always small for $R(E) \ll \sqrt{Y}$ and can be neglected.

On the other hand, the condition for real saddle points in Eq.(13) implies, for $R(E) < \sqrt{Y}$,

$$\theta < \theta_M \equiv \left(R(E) / \sqrt{Y} \right)^{D-3} < 1, \quad (23)$$

which ensures, by (19), that $\langle q \text{ single} \rangle < 1/\sqrt{Y}$. Therefore, for $R(E) < \sqrt{Y}$, there is a region of small, but fixed (i.e. E independent) angles for which our results, as Eq.(16), are reliable.

Let us discuss, in this case, the dependence of the amplitudes on the initial and final states, following from the quadratic expansion (10), evaluated at the saddle point. The explicit form of $\hat{S}^{(2)}$, given in I, is

$$\begin{aligned} \hat{S}^{(2)} = e^{2i\delta_{el}} \prod_{n,j} \exp \left(\frac{-i\Delta_j/n}{1-i\Delta_j/n} a_n^{j\dagger} \tilde{a}_n^{j\dagger} \right) \left(1 - \frac{i\Delta_j}{n} \right)^{-\left(a_n^{j\dagger} a_n^j + \tilde{a}_n^{j\dagger} \tilde{a}_n^j \right)} \\ \cdot \exp \left(\frac{-i\Delta_j/n}{1-i\Delta_j/n} a_n^j \tilde{a}_n^j \right), \quad (j = ||, \perp, u, d), \end{aligned} \quad (24)$$

(*) It seems at first sight astonishing that higher loop contributions may give rise to classical corrections. This is due to the fact that the string size $\lambda_s^2 \sim \alpha' \kappa$, which is a quantum parameter, enters both in the loop counting and as regulator of the short distance behaviour. The u.v. pathology of quantum gravity is transmuted by the string into a quantum generation of the classical effects!

δ_{el} being defined in Eq.(12).

We notice first that (24) conserves, for each scattering string, all the fermionic modes, as well as the difference ($v_n - \tilde{v}_n$) of right and left-moving bosonic occupation numbers for each mode n . Matrix elements not satisfying these selection rules arise at subleading level, or from higher orders in the $\hat{X}$ -expansion of $\hat{\delta}$ that we have shown to be subleading.

In the diffractive absorption region $b_s < b_D$, i.e., large Δ 's, the dominant matrix elements concern in (out) states with $v_n = \tilde{v}_n$, for which

$$\hat{S}^{(2)} \xrightarrow{\Delta_j \gg 1} e^{2i\delta_{el}} \exp\left(\sum_{n < \Delta_j} a_n^{j+} \tilde{a}_n^{j+}\right) |0\rangle\langle 0| \exp\left(\sum_{n < \Delta_j} a_n^j \tilde{a}_n^j\right) \quad (25)$$

This shows a simple universal behaviour: leading matrix elements connect states which are excitations of the ground state with any number of spinless left-right pairs, and they are all equal to the elastic graviton amplitude. Other matrix elements within the above selection rules (i.e., connecting states with $v_n - \tilde{v}_n \neq 0$ and possibly higher spin) are depressed by powers of $\Delta_{//}^{-1}$ or $\Delta_{\perp}^{-1}$. Nevertheless, these (many) states with smaller amplitudes have an important role in insuring the unitarity of the $\hat{S}^{(2)}$ operator and are thus needed for the consistency of the approximation scheme.

4. Can we explore $\theta > \theta_M$, so as to reach "large" angle scattering? For $\theta > \theta_M$ the saddle points become complex, so that $\text{Re } b^2$ may be negative. In this region the behaviour of Eq.(7) simplifies for $-\text{Re } b^2 \gg Y$, and can be evaluated by a saddle point in y , yielding

$$\begin{aligned} \mathcal{J}(b, s) &\xrightarrow{-\text{Re } b^2 \gg Y} \frac{g_s^2}{y_s^{\frac{D}{2}-1}} \frac{i}{b} \exp\left(-\frac{b^2}{4y_s}\right) \quad (a) \\ y_s e^{-y_s} &= -\frac{s}{b^2} \quad (b) \end{aligned} \quad (26)$$

By replacing Eq.(26) into Eq.(13) we obtain, for $1 > \theta \gg \theta_M$ ($-\text{Re } b_s^2 \gg Y_s$),

$$\begin{aligned} \theta &\approx \frac{g_s^{2/5}}{(\sqrt{y_s})^{0.3}} \frac{i}{\sqrt{y_s}} \exp\left(-\frac{b_s^2}{4y_s}\right) \\ b_s &\approx 2i\sqrt{y_s} \left(\log \frac{\theta}{\theta_M} - \frac{i\pi}{2}\right)^{1/2}, \quad y_s \approx Y - O(\log Y) \end{aligned} \quad (27)$$

Correspondingly, we find by Eq.(14), the behaviour

$$\frac{1}{S} A_{el}(s, t) \sim \exp \left(-2q \sqrt{Y \log \frac{\theta}{\theta_M}} \right) . \quad (28)$$

Let us, again, control the magnitude of $\langle q \text{ single} \rangle$. Since $\langle N \rangle \sim i\delta \sim iqY/b_s$,

$$\langle q_{\text{single}} \rangle = \frac{q}{\langle N \rangle} \simeq \frac{b_s}{iY} \simeq \sqrt{\frac{1}{Y} \log \frac{\theta}{\theta_M}} , \quad (\theta \gg \theta_M) . \quad (29)$$

We thus find that, in this "large θ " regime b_s and $\langle q \text{ single} \rangle$ are proportional and $\langle q \text{ single} \rangle \sqrt{Y}$ is not small.

5. Summarizing, the angle $\theta_M \sim (R(E)/\sqrt{Y})^{D-3}$ representing the maximal classical deflection angle is the dividing point of two different regimes. For $\theta < \theta_M$, the amplitude is dominated by the classical trajectory (at the largest impact parameter $b_s > \lambda_s \sqrt{Y}$) and $\langle q \text{ single} \rangle \sim \hbar / b_s < \hbar / \lambda_s \sqrt{Y}$ is small. The physics of this small angle scattering is an infinite genus effect and, yet, a soft process.

For $\theta > \theta_M$, no classical trajectory is available, b_s is essentially imaginary and proportional, by (29), to $\langle q \text{ single} \rangle$. This means that the string is being stretched, much as in the fixed angle, fixed genus calculation⁽²⁾, but the corresponding $\langle q \text{ single} \rangle$ is neither small, nor particularly large.

Since $q/\langle N \rangle$ is sizeable, our approach is here doubtful, because we lose the small parameter which allowed us to reduce the number of degrees of freedom we have resummed. Many subleading terms will start counting which roughly correspond to integrating over the moduli frozen by the small $\langle q \text{ single} \rangle$ asymptotic condition. This may eventually lead to the loss of the summability property that we have found for the leading terms^(*).

(*) The convergence of leading asymptotic terms in a theory in which the whole perturbative series diverges is a feature of string theory common to conventional field theories. The small angle asymptotic condition reduces analogously the number $N!$ of diagrams in the latter and the volume $N!$ of moduli space⁽⁸⁾ in the former.

On the other hand, the fact that $q/\langle N \rangle$ is not very large makes the region $\theta > \theta_M$ also doubtful for the applicability of fixed genus, fixed angle calculations. In fact, the latter show⁽²⁾ amplitudes which are exponentially damped in q^2/N , but with large coefficients which for $N \sim \sqrt{\alpha'} q$ are $O(N!)$ and overcome the damping. The big large genus contribution casts doubts over the meaning of that perturbative approach. But even if, by some mechanism, very large N are cut, $N \sim O(q)$ would dominate the sum, thus bringing unavoidably into the game soft physics.

Similar remarks hold for the dependence of the S-matrix on the scattering states. The asymptotic relations we found for $\theta < \theta_M$ are, not surprisingly, different from the ones proposed⁽⁹⁾ for large angles. For instance, our amplitude does not factorize in the quantum numbers of each scattering particle, but shows factorization for the two scattering strings, with selection rules for each of them. But, again the case $\theta > \theta_M$ is puzzling in any approach. In our method, higher orders in the X-expansion are depressed only logarithmically by inverse powers of $-b_s^2 \sim Y \log(\theta/\theta_M)$ and will become important; in the fixed genus approach $X_{cl} \sim q/N$ is not big enough, for large genus, to dominate its fluctuations in the asymptotic relations.

This discussion suggests that large angle scattering in string theory remains a challenging problem, for which we have as yet little intuition on the relevant degrees of freedom.

In conclusion the physics of large momentum transfers in string theory is characterized by two phenomena. On one hand, with increasing angle, the loop number $\langle N \rangle$ increases so fast that $q/\langle N \rangle$ may become sizeable but not large in Planck's units: the elementary scattering is never too hard. On the other hand, the change of regime at $\theta = \theta_M$ changes the relation between distance and momentum transfer from the naive one in (18) to the one in (29) thus suggesting a modification of the uncertainty relation at the Planck's scale $x \sim \hbar/q + q \alpha' Y$ and the existence of a minimal observable length, of the order of the string size $\sim \lambda_s \sqrt{Y}$.

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FIGURE CAPTIONS

Fig. 1 - Impact parameter picture of the genus $h = N-1$ contribution to asymptotic string scattering.

Fig. 2 - Qualitative impact parameter behaviour of the eikonal function $\delta(b, s)$, and of $\text{Im } \delta_{\text{el}}$ which includes the diffractive string corrections.

Fig. 3 - Qualitative plot of the derivative function $-(1/\sqrt{s}) \partial \text{Re } \delta / \partial b$ illustrating the saddle point solutions as function of θ .

Fig. 4 - (a) The H-diagram and (b) Higher order contributions giving rise to classical corrections.

Fig. 1

Fig. 2

Fig. 3

Fig. 4