CHARGED MULTIPLICITY DISTRIBUTIONS IN RAPIDITY BINS

FOR pp COLLISIONS AT $\sqrt{s} = 31, 44$ AND 62 GeV

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ABSTRACT

The multiplicity distributions of charged secondaries in proton-proton interactions at $\sqrt{s} = 31, 44$ and 62 GeV have been measured with high statistics using the Split Field Magnet (SFM) detector at the CERN ISR. The multiplicity distributions for narrow bins in rapidity depend on the rapidity variable. The validity of KNO scaling is discussed. The negative binomial function fits well the charged and negative multiplicity data for full phase space and for central rapidity windows. The energy and the rapidity dependence of the negative binomial parameters, $\bar{n}$ and K , are presented. We also analyze their implications for a cascade model in terms of the average number of "clans" and of the average number of particles per "clan".

1. INTRODUCTION

In this paper we present results on the multiplicity distributions of charged particles produced in proton-proton collisions with high statistics at centre of mass energies $\sqrt{s} = 31, 44$ and 62 GeV. The data were obtained at the CERN ISR using the Split Field Magnet detector [1] operated with a minimum bias trigger.

If charged particles were produced randomly and independently, their multiplicity distributions might be expected to obey the Poisson distribution, which should become broader as the energy and thus the average multiplicity increases. However, it is well known that the distributions deviate considerably from a Poisson distribution and that differences from a Poisson distribution may be regarded as a source of information on the underlying production processes. For example any correlation between the produced particles results in broader distributions. Recently it has been argued that the experimental multiplicity data follow the negative binomial function [2,3].

The hypothesis of Feynman scaling led to KNO scaling [4], that is to the hypothesis that at sufficiently high energies one should have

$$\bar{n} \sigma_n / \sigma_{in} = \psi(z) \quad (1)$$

where $z = n/\bar{n}$, $\bar{n}$ is the average charged multiplicity, σ_n is the topological cross section for the production of n charged hadrons, σ_{in} is the inelastic cross section and ψ is a scaling function whose form is independent of energy. Many experimental results on multiplicity distributions in hadron-hadron collisions have been analysed in the context of KNO scaling. Recent experimental results from the CERN $\bar{p}p$ collider have shown that KNO scaling is violated [2], that is that there is an energy dependence of the multiplicity distribution, with the exception of the central region, $|y| < 1.5$, for which KNO scaling seems to hold. Other authors argue that one should use a modified KNO scaling (KNO-G) which seems to be self consistent at all energies [5,6].

Various models of low p_t hadron production in hadron-hadron collisions have led to specific predictions of multiplicity distributions and KNO scaling violations [6-10].

We shall present the experimental charged multiplicity distributions in sect. 3. Fits to the negative binomial function are discussed in sects 4 and 5; their implications for a cascade model in terms of the average number of "clans" and of the average number of charged particles per "clan" are considered in sect. 6. Conclusions are drawn in sect. 7.

2. EXPERIMENTAL

The data were recorded with the Split Field Magnet (SFM) detector at the CERN Intersecting Storage Rings (ISR). The detector was used with a minimum bias trigger, which required at least one candidate charged track in the spectrometer. The trigger accepted about 95% of the inelastic cross section, and about 90% of the triggers were good events. The events were analyzed with the standard SFM off-line analysis program chain. We required a "successful" vertex fit originating from at least two charged tracks in the beam overlap region. Furthermore we required that the momentum of each track be determined with a percentage error, $\Delta p/p$, smaller than 100%. This ensures that the sign of each charged particle is properly determined.

The samples of pp interactions at $\sqrt{s} = 31, 44$ and 62 GeV contained about 1,400,000, 590,000 and 815,000 events, respectively (table 1). These samples are almost one order of magnitude larger than those previously available from this experiment [11].

For the analysis of the multiplicity distributions it is customary to remove single-diffractive events, that is events in which one of the incident protons, but not both, appears to undergo diffractive dissociation [11]. We assume that an event is of the single diffractive type when either a single positively charged track is found in one of the two rapidity hemispheres with a Feynman x larger than 0.7 or if no track at all is found in one hemisphere (in this case the forward-scattered proton remains inside the beam pipe). Moreover the event is considered as single-diffractive if it has a total charged multiplicity smaller than 10. These cuts remove about 15% of the events, at each energy and at low multiplicities. More details about this procedure may be found in ref. [11].

After these cuts, we were left with about 900,000, 350,000 and 500,000 events at the three energies considered, 31, 44 and 62 GeV, respectively.

For part of the present analysis, no correction for geometrical acceptance was made. It was checked with our previous data [11] and with the new data as shown in fig. 1 that this does not bring important corrections when the data are analyzed in the KNO form (that is $\bar{n}P_n = \bar{\sigma}_n/\sigma_{in}$ versus z), which is essentially a self-normalizing form and thus removes, at least at first order, acceptance effects.

For other analyses, in particular for the fits to the Negative Binomial (NB) function, we need data corrected for geometrical and reconstruction acceptances. The data were corrected with the following procedure.

A single track generator in the detector simulation program was tuned to reproduce some observed features of particle production, that is the p_t and y dependences of the inclusive data. At least 10^6 single particles per energy were generated and tracked through the detector, allowing for interactions and decays. Hits in the proportional wire chambers were generated. From them, considering also measurement errors, tracks were reconstructed using the standard SFM reconstruction programs. The ratio between the number of tracks found in a given cell in phase space and the corresponding number of generated tracks is the efficiency for that cell. This efficiency is a combination of geometrical efficiency and track reconstruction efficiency.

The efficiencies were later used in a second Monte-Carlo program to obtain the matrix P whose elements P_{mn} represent the probability that an event with a true charged multiplicity n is observed with a charged multiplicity m . For this purpose we used a generator of events with multiple charged tracks, which reproduces the measured p_t distributions. Using the already determined acceptances, the elements P_{mn} were obtained.

Once we know the matrix P , the final procedure is the following: one starts with a negative binomial distribution, determined by two initial parameter values for $\bar{n}_1$ and k_1 . This allows to compute for each n the

parent multiplicity T_n , which convoluted with the matrix P gives the observed multiplicity: $O_m = \sum_n P_{mn} T_n$. The O_m elements are compared with the measured uncorrected values O'_m . An optimization is then performed, varying the two initial parameters of the parent binomial distribution until the χ^2 is minimal. We obtained rather good values of χ^2/DOF . We checked with our previous data [11] the consistency of the method and a reasonable independency from the choice of the NB function for the correction (the previous data did not make this assumption).

The contamination due to V^0 decays into charged particles was corrected by normalizing the two parameters $\bar{n}$ and k of the negative binomial function of our data at one energy (44 GeV) to those computed from a best fit through the data compilation [2,12-34] for the whole phase space. The same normalization factor was then applied to correct the other two energies as described in sect. 5.

3. CHARGED MULTIPLICITY DISTRIBUTIONS

Fig. 2 shows the uncorrected multiplicity distributions in the KNO form, for different bins of the modulus of the c.m. pseudorapidity variable (0.5-1, 1.5-2, 2.5-3, 3-3.5, 3.5-4) for pp collisions at $\sqrt{s} = 31$ and 62 GeV. We recall that the pseudorapidity is defined as $\eta = -\ln \tan \theta/2$, where θ is the c.m. polar angle of each track. The modulus of the pseudorapidity is appropriate since the pp system is forward-backward symmetric. It is observed that the curves for $|\eta| < 3$ at $\sqrt{s} = 62$ GeV superimpose one another, while the data for larger rapidities are considerably different, in particular at the largest values of z . The same qualitative trend is observed at $\sqrt{s} = 31$ GeV. Moreover the data at the two energies are identical in the central region, for $|\eta| < 1.5$. It is worth pointing out that events with no particle in the selected $\Delta\eta$ region are retained. If the events are not retained the distributions change slightly.

The study of the multiplicity distributions in the fragmentation region (defined as $|y| > |y_{\max}| - 2$) is better performed in terms of the laboratory rapidity, $y_{\text{lab}} = y_{\max} - y$. Fig. 3 shows the uncorrected multiplicity distributions in the KNO form for different intervals of the

laboratory rapidity variable. It is apparent that data in the fragmentation region at 31 and 62 GeV superimpose one another, suggesting a kind of scaling or of "universality" in the fragmentation region.

Fig. 4 shows corrected multiplicity data in an integral form, that is for absolute c.m. rapidities smaller than a maximum value ($|y_{\text{cut}}| < 1$, 2, 3, 4 and full phase space respectively). We checked that the uncorrected data have the same trends. The data at $\sqrt{s} = 31, 44$ and 62 GeV show the same trend, with the distributions becoming narrower as the rapidity bin increases. These behaviours are qualitatively the same as those observed in $\bar{p}p$ collisions at higher energies [2, 30-32].

We checked that the uncorrected data have exactly the same trend as the corrected data. But in order to obtain quantitative results we have to use the corrected data.

4. THE (NEGATIVE) BINOMIAL DISTRIBUTION

The UA5 Collaboration and other groups have shown that charged hadron multiplicity distributions are well described by Negative Binomial (NB) functions, not only for the full phase space, but also for particles produced in limited pseudorapidity intervals, centred around zero. The negative binomial function may be written in the form.

$$P(n; \bar{n}, k) = \frac{k(k+1) \dots (k+n-1)}{n!} \left(\frac{\bar{n}}{\bar{n}+k} \right)^n \left(\frac{k}{\bar{n}+k} \right)^k \quad (2)$$

where k and $\bar{n}$ are free parameters. $\bar{n}$ is the average charged hadron multiplicity and the parameter k is related to the dispersion $D = (\overline{n^2} - \bar{n}^2)^{1/2}$ by the relation

$$\frac{D^2}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{k} \quad (3)$$

The distribution (2) may be characterized by a recurrence relation between P_n , which represents the occurrence of multiplicities n in a large ensemble of collisions, and P_{n+1} . The relation is

$$\frac{P_{n+1}}{P_n} = \frac{(n+1) P_{n+1}}{P_n} = a + b n \quad (4)$$

with $a = \bar{n}k/(\bar{n} + k)$ and $b = \bar{n}/(\bar{n} + k)$.

For $k = \infty$ (corresponding to $b = 0$ and $a = \bar{n}$) the distribution (2) becomes a Poisson distribution

$$P(n) = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \quad (5)$$

which describes independent emission of n particles. In this case one has from (3) $D^2 = \bar{n}$.

For $k = 1$ (corresponding to $b = a$) the distribution (2) becomes an exponential distribution

$$P_n = \left(\frac{\bar{n}}{\bar{n} + 1} \right)^n \frac{1}{\bar{n} + 1} \quad (6)$$

which corresponds to stimulated emission.

For $k = 0$ (corresponding to $a = 0$) one has the logarithmic distribution ($n \geq 1$, $b < 1$)

$$P_n = \frac{b^n}{n} \left[\ln \frac{1}{1-b} \right]^{-1} \quad (7)$$

One has a negative binomial distribution when k is positive, in which case $D^2 > \bar{n}$. For k integer and negative the distribution becomes a normal binomial, in which case $D^2 < \bar{n}$.

5. FITS TO THE NEGATIVE BINOMIAL

We made fits to the NB function of the charged multiplicity data from the literature, for the whole phase space, for $\bar{p}p$ and pp interactions at different energies [2,12-34]. This was done in order to have a more coherent analysis.

Most of the data for $\bar{p}p$ concern non-single diffractive events. Instead most of the multiplicity data for pp interactions contain single diffractive events. In practice the difference between the two sets may be substantially reduced by the application of a cut at low multiplicities

($n_{\min}$). In our analysis this cut, which removes most of the single diffractive events, was increased as the c.m. energy increases. Thus our fits concern approximately the same value of $z_{\min} = n_{\min}/\bar{n} \approx 0.6$.

We also applied a cut at the highest multiplicity values ($n_{\max}$), at approximately $z_{\max} = n_{\max}/\bar{n} \approx 3.5$.

The results of the fits of the pp and $\bar{p}p$ multiplicity data found in the literature are given in table 2. Most of the χ^2 values are reasonable, with few exceptions. The obtained values for $\bar{n}$ and k^{-1} are displayed in figs 6 and 7, respectively.

In table 2 and figs 6 and 7 are indicated only two of our energies, 31 and 62 GeV. This because we have used the 44 GeV data to establish the correction for events containing tracks coming from the decay of not identified V^0 s.

It is customary to express the increase with c.m. energy of the mean charged multiplicities as

$$\bar{n} = A + B \ln s + C \ln^2 s \quad (8)$$

A fit to formula (8) of the mean charged multiplicities of table 2 yields the values A, B, C given in table 4. The χ^2/DoF of the fit is not very good, but it should be reminded that the data belong to many different experiments and one should add systematic errors to the statistical uncertainties. Anyhow one has a B value almost equal to zero.

The k_{ch}^{-1} values of table 2 were fitted to the expression

$$k_{\text{ch}}^{-1} = \alpha + \beta \ln \sqrt{s} \quad (9)$$

The fit has reasonable χ^2/DoF and yields the parameters α , β given in table 5.

Negative multiplicities, n_- , may be computed from the corrected charged multiplicities, n_{ch} , using the formulae

$$\begin{aligned} n_{\text{ch}} &= 2n_- + 2 && \text{for pp} \\ n_{\text{ch}} &= 2n_- && \text{for } \bar{p}p \end{aligned} \quad (10)$$

The negative multiplicities, computed with this procedure, were fitted to NB obtaining values for $\bar{n}_-$ and k_-^{-1} (table 3 and figs 6,7). These were in turn fitted to expressions (8) and (9) obtaining the parameters given in tables 4 and 5.

For the full phase space, it may be more appropriate to apply the negative binomial function to the production of negative tracks in non-single diffractive events, since the negative multiplicity n_- can have any positive integer value. Instead the charged multiplicity distribution is restricted to even integer values. One may nevertheless use the negative binomial function for the charged distributions for empirical fittings.

The multiplicity distributions for bins in rapidity centred around zero, were corrected for acceptances using the same procedure described for the distributions for the whole phase space. We obtained the distributions shown in fig. 4. Also these distributions were fitted to the NB function, with the procedure already described. The fits have reasonable values of χ^2/DoF . The parameters $\bar{n}_{ch}$ and $\bar{n}_-$ are shown in fig. 8, while the k_{ch} and k_- are shown in fig. 9 as function of y_{cut} , the maximum absolute rapidity value for each bin.

Both $\bar{n}_{ch}$ and $\bar{n}_-$ increase with the size of the rapidity bin, almost linearly until y equals 2 or 3, that is in the central region. The inclusion of the fragmentation region brings only a further limited increase. As expected $\bar{n}_{ch}$ is slightly more than a factor of two larger than $\bar{n}_-$.

Also k_{ch} and k_- increase with y_{cut} , but the increase is small for $y < 1$, suggesting a global dependence of k versus y_{cut} of a parabolic type. k_{ch} is only a little smaller than k_- and the percentage difference decreases as the c.m. energy increases. We have checked that k_+ , the k -value for the distribution of positive particles, is equal to k_- , with the exception of the fragmentation region, where k_+ is slightly larger than k_- .

6. INTERPRETATION OF THE NEGATIVE BINOMIAL DISTRIBUTION

Cascade processes may be represented by negative binomial functions. With reference to fig. 10 cascading means that N particles, are originally emitted in the collision and each gives rise to n_c^i particles. One says that the n_c^i particles belong to a clan, whose ancestor is the i -th original particle (3).

The simplest assumption is that clans are emitted independently; thus their distribution obeys the Poisson distribution (5) (with $\bar{n} = N$). The multiplicity distribution of particles in a single clan, $P_c(n_c)$ can vary from case to case, but its average should obey a specific relation. The solution is the logarithmic distribution for $n_c \geq 1$.

$$P_c(n_c) = \frac{1}{n_c} \left(\frac{\bar{n}}{\bar{n} + k} \right)^{n_c} \frac{1}{\ln(1 + \frac{\bar{n}}{k})} \quad (12)$$

Note that

$$(n_c + 1)P_c(n_c + 1)/P_c(n_c) = b n_c \quad (13)$$

Note also that $P_c(0) = 0$ by definition, since the clan must have at least one charged particle.

The average number of particles in one clan is given by

$$\bar{n}_c = \frac{\bar{n}}{k} [\ln(1 + \frac{\bar{n}}{k})]^{-1} \quad (14)$$

while the average number of ancestors is given by

$$\bar{N} = \frac{\bar{n}}{\bar{n}_c} = k \ln(1 + \frac{\bar{n}}{k}) \quad (15)$$

In the context of the cascade model, $1/k$ gives a measure of the degree of aggregation and is given by

$$\frac{1}{k} = \frac{P_1(2)}{P_2(2)} \quad (16)$$

where $P_1(2)$ is the probability to have 2 particles in one clan and $P_2(2)$ is the probability to have 2 particles in two clans, one per clan.

What has been said should be valid only if one does not consider all phase space since conservation laws (as for example that for electric charge) impose constraints. If instead one considers small rapidity intervals, the conservation laws do not give constraints, but the clans should be substituted with "effective clans".

Fig. 11 shows the average number of charged ($\bar{N}_{ch}$) and of negative ($\bar{N}_-$) clans produced in pp interactions at $\sqrt{s} = 31, 44$ and 62 GeV for intervals in c.m. rapidity $|y| < y_{cut}$. The average number of effective clans increases with y_{cut} , reaching, for full phase space, a value of 6.5 clans per interaction. This number is independent of energy over the ISR energy range. The average number of clans with at least one negative particle has a rapidity dependence similar to that of $\bar{N}_{ch}$ and reaches a maximum value of 4 clans per interactions.

The average number of charged particles per clan (n_{ch}) shown in fig. 12, has only a small dependence on y_{cut} , with a small peak at $y_{cut} \sim 1.6$ at $\sqrt{s} = 31$ GeV, $y_{cut} = 2$ at 44 GeV and 2.3 at 62 GeV. There is instead a sizeable energy dependence: the mean value of charged particles per clan over the whole phase space increases from 1.5 at $\sqrt{s} = 31$ GeV to 1.7 at 44 GeV and 1.9 at 62 GeV. The y and s dependences of the average number of negative particles per clan follow the same dependences already discussed for the charged particles (fig. 12).

In fig. 13 our results are compared with those from other hadron-hadron experiments as well as with results obtained in e^+e^- collisions [35]. It is observed that in hadron-hadron collisions the average number of charged clans is essentially energy independent for energies larger than 20 GeV, thus over a very large energy range. Instead the average number of charged particles per clan grows strongly with energy. These two features are better seen in figs 14 and 15, which show $\bar{N}$ and $\bar{n}_c$ versus $\sqrt{s}$. Instead for e^+e^- collisions the clans are smaller and more numerous (fig. 13).

The cascade model with ancestors and clans could in principle be the same as the cluster model, which was used to interpret the rapidity correlations data of proton-proton interactions at ISR energies. But while clusters were small, energy independent and could be interpreted

mainly as resonances, the clans needed to explain the multiplicity data seem to be quite different. The number of clans remains constant, while the size of the clan (i.e. the number of particles per clan) grows with energy. If this is correct one would expect an increase with energy of real three particle and in general of multiparticle correlations in rapidity.

It has to be pointed out that over the ISR energy range the mean number of clans is about 6.6, the mean number of charged particles per clan is about 1.7 and both have only a minor energy dependence. These values are about equal to what was found for the mean number of clusters and for the number of particles per cluster. Thus the two models are essentially identical at ISR energies; it is only at higher energies that they become quite different.

7. DISCUSSION AND CONCLUSIONS

The main results of the present analysis of the charged multiplicity distributions at ISR energies are the following:

- (a) There is a dependence of the multiplicity distribution on the c.m. rapidity (or pseudorapidity) variable.
- (b) Differential and integral multiplicity distributions for charged and for negative particles are well represented by negative binomial functions. At a fixed energy the parameter k increases with increasing range of rapidity. The parameter k^{-1} relative to the full phase space increases as $\ln\sqrt{s}$.
- (c) For a limited range of c.m. rapidity, say $|y| < 1.5$, KNO scaling is valid, from pp interactions at $\sqrt{s} = 11$ GeV to $\bar{p}p$ interactions at S_{ppS} collider energies. Some authors argue that this is accidental and comes about from an increasing average multiplicity $\bar{n}$ and a decreasing k . Since data in the fragmentation region are qualitatively different from data in the central region, one should specify the used rapidity range when comparing data at different energies. The original KNO scaling should be applied to the whole phase space. The use of KNO-G scaling would bring a more consistent picture.

- (d) The interpretation of the negative binomial fits in terms of a cascade process yields a fixed number of clans (about 7.5) and an increase of the clan size with energy. In this picture the clans cannot be resonances, as was thought in the old cluster model.

The basic interpretation of the above findings is still rather speculative. Some of the effects may arise from the onset of gluon-gluon collisions as a new component of the inelastic cross section [15], or in general from gluon effects leading to "jetty" events [17].

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TABLE CAPTIONS

TABLE 1 Number of events before and after experimental cuts.

TABLE 2 Results of the NB fits to pp and $\bar{p}p$ charged multiplicity distributions at different energies. For $\bar{p}p$ and our pp data the distributions are relative to non-single diffractive events, while for the pp they are for inelastic events (inclusive of single diffraction). The NB fits concern multiplicities n between the $n_{\min}$ and $n_{\max}$ values quoted in the table. The minimum cut, $n_{\min}$, effectively removes single diffractive events and makes the fitted parts of the pp and $\bar{p}p$ multiplicities essentially identical. Quoted errors are mainly statistical.

TABLE 3 Results of the NB fits to pp and $\bar{p}p$ negative multiplicity distributions at different energies. The conditions are the same as in table 2.

TABLE 4 Fits of the mean charged and negative multiplicities to the formula $\bar{n} = A + B \ln s + C \ln^2 s$.

TABLE 5 Fits of the k^{-1} values to the formula $k^{-1} = \alpha + \beta \ln \sqrt{s}$.

TABLE 1

c.m. energy (GeV)	Events	
	Before cuts	After cuts
62.2	816460	515470
44.5	593370	347350
30.4	1467210	888800

TABLE 2

Experiment	Ref.	$\sqrt{s}$ [GeV]	$n_{\min}$	$n_{\max}$	k^{-1}	$\langle n \rangle$	χ^2/DoF
BROMBERG (pp)	[12]	10.7	4	14	0.087 ± 0.010	5.78 ± 0.11	2.2
BABINTSEV (pp)	[13]	11.5	4	16	0.074 ± 0.015	5.83 ± 0.20	4.1
ERWIN (pp)	[14]	13.8	4	18	0.110 ± 0.015	6.51 ± 0.18	1.8
MORSE (pp)	[15]	13.8	4	18	0.085 ± 0.015	6.51 ± 0.20	4.6
BRENNER (pp)	[16]	13.8	4	18	0.162 ± 0.012	5.99 ± 0.40	1.1
BROMBERG (pp)	[17]	13.9	4	18	0.092 ± 0.010	6.37 ± 0.10	2.1
BRENNER (pp)	[16]	18.2	4	20	0.010 ± 0.030	7.72 ± 0.30	0.4
BARISH (pp)	[18]	19.7	6	22	0.099 ± 0.020	7.98 ± 0.18	1.6
FIRESTONE (pp)	[22]	23.9	6	24	0.103 ± 0.010	8.90 ± 0.11	1.4
DAO (pp)	[23]	23.9	6	24	0.122 ± 0.020	9.07 ± 0.24	1.8
BAILLY (pp)	[24]	26.6	6	24	0.121 ± 0.012	9.40 ± 0.14	0.5
KASS (pp)	[26]	27.4	6	26	0.108 ± 0.014	9.76 ± 0.18	3.8
BROMBERG (pp)	[17]	27.6	6	26	0.124 ± 0.015	9.36 ± 0.17	1.6
KICHIMI (pp)	[25]	27.6	6	24	0.129 ± 0.025	8.91 ± 0.25	0.4
SFM (pp)	[11]	30.4	6	26	0.160 ± 0.025	9.92 ± 0.29	0.2
LEBC-MPS (pp)	[27]	38.8	6	32	0.137 ± 0.006	10.64 ± 0.11	2.3
SFM (pp)	[11]	44.5	6	30	0.166 ± 0.017	11.30 ± 0.22	0.4
SFM (pp)	[11]	52.6	8	34	0.133 ± 0.014	12.68 ± 0.24	0.3
SFM (pp)	[11]	62.2	8	36	0.135 ± 0.018	13.63 ± 0.29	1.2
UA5 (pp)	[32]	200	10	56	0.208 ± 0.028	21.29 ± 0.58	0.1
UA5 (pp)	[30]	540	12	76	0.289 ± 0.012	28.65 ± 0.25	0.6
UA5 (pp)	[32]	900	12	92	0.350 ± 0.031	34.20 ± 0.74	0.2
SFM (pp)	This	30.4	5	26	0.109 ± 0.010	9.69 ± 0.12	0.8
SFM (pp)	expt.	62.2	5	29	0.166 ± 0.012	12.80 ± 0.14	0.2

TABLE 3

Experiment	Ref.	$\sqrt{s}$ [GeV]	$n_{\min}$	$n_{\max}$	k^{-1}	$\langle n \rangle$	χ^2/DoF
BROMBERG (pp)	[12]	10.7	1	6	0.050 ± 0.010	1.91 ± 0.06	2.6
BABINTSEV (pp)	[13]	11.5	1	7	0.050 ± 0.010	1.88 ± 0.08	6.7
ERWIN (pp)	[14]	13.8	1	8	0.064 ± 0.020	2.31 ± 0.08	1.6
MORSE (pp)	[15]	13.8	1	8	0.050 ± 0.010	2.25 ± 0.08	4.7
BRENNER (pp)	[16]	13.8	1	8	0.141 ± 0.070	2.11 ± 0.15	1.1
BROMBERG (pp)	[17]	13.9	1	8	0.050 ± 0.010	2.19 ± 0.06	2.0
BRENNER (pp)	[16]	18.2	2	9	0.051 ± 0.028	2.89 ± 0.12	0.4
BARISH (pp)	[18]	19.7	2	10	0.054 ± 0.030	3.01 ± 0.10	1.5
FIRESTONE (pp)	[22]	23.9	2	11	0.062 ± 0.014	3.46 ± 0.10	1.3
DAQ (pp)	[23]	23.9	2	11	0.088 ± 0.027	3.55 ± 0.10	1.8
BAILLY (pp)	[24]	26.0	2	11	0.090 ± 0.012	3.71 ± 0.08	0.5
KASS (pp)	[26]	27.4	2	12	0.071 ± 0.011	3.90 ± 0.10	3.5
BROMBERG (pp)	[17]	27.6	2	12	0.095 ± 0.021	3.69 ± 0.10	1.6
KICHIMI (pp)	[25]	27.6	2	11	0.101 ± 0.036	3.48 ± 0.11	0.4
SFM (pp)	[11]	30.4	2	12	0.143 ± 0.033	4.00 ± 0.12	0.2
LEBC-MPS (pp)	[27]	38.8	2	15	0.111 ± 0.010	4.34 ± 0.07	2.1
SFM (pp)	[11]	44.5	2	14	0.151 ± 0.021	4.68 ± 0.10	0.4
SFM (pp)	[11]	52.6	3	16	0.112 ± 0.017	5.33 ± 0.11	0.3
SFM (pp)	[11]	62.2	3	17	0.114 ± 0.018	5.81 ± 0.13	1.1
UA5 (pp)	[32]	200	4	27	0.172 ± 0.029	10.49 ± 0.30	0.1
UA5 (pp)	[30]	540	5	37	0.264 ± 0.012	14.16 ± 0.15	0.6
UA5 (pp)	[32]	900	5	45	0.330 ± 0.032	16.93 ± 0.38	0.2
SFM (pp)	This	30.4	1	14	0.091 ± 0.016	3.93 ± 0.07	0.1
SFM (pp)	expt.	62.2	1	16	0.148 ± 0.016	5.37 ± 0.10	0.2

TABLE 4

Multiplicity distribution	$A \pm \Delta A$	$B \pm \Delta B$	$C \pm \Delta C$	χ^2/DoF
Charged	2.51 ± 0.61	-0.16 ± 0.15	0.178 ± 0.009	3.2
Negative	0.78 ± 0.31	-0.27 ± 0.08	0.106 ± 0.005	3.2

TABLE 5

Multiplicity distribution	$\alpha \pm \Delta\alpha$	$\beta \pm \Delta\beta$	χ^2/DoF
Charged	-0.065 ± 0.013	0.055 ± 0.034	0.9
Negative	-0.104 ± 0.012	0.058 ± 0.031	0.9

FIGURE CAPTIONS

- Fig. 1 Charged multiplicity distributions in the KNO form for non-single diffractive events for pp collisions at $\sqrt{s} = 31, 44$ and 62 GeV. The black and open points refer to data uncorrected and corrected for geometrical and track reconstruction acceptances, respectively.
- Fig. 2 Charged multiplicity distributions in the KNO form for different intervals of the modulus of the c.m. pseudorapidity variable for minimum bias events in pp interactions at (a) $\sqrt{s} = 31$ and (b) $\sqrt{s} = 62$ GeV (data uncorrected).
- Fig. 3 Charged multiplicity distributions in the KNO form for different intervals of the lab. rapidity variable for pp interactions at 31 and 62 GeV (data uncorrected).
- Fig. 4 Corrected integral charged multiplicity distributions in the KNO form for bins in c.m. absolute rapidity from 0 to a maximum value (1-5) for pp interactions at (a) $\sqrt{s} = 31$ GeV, (b) 44 and (c) 62 GeV.
- Fig. 5 Charged multiplicity distributions for the full phase space for pp interactions at $\sqrt{s} = 62$ GeV. The black points are the measured uncorrected data. The solid line is the parent negative binomial distribution, corrected for acceptances. The open points are obtained from the convolution of the NB with the matrix elements P_{mn} (see text).
- Fig. 6 Compilation versus $\sqrt{s}$ of the parameter $\bar{n}$ obtained from NB fits of the charged and negative multiplicities for the whole phase space.
- Fig. 7 Compilation versus $\sqrt{s}$ of the parameter k^{-1} obtained from NB fits of the charged (data points and solid line) and negative (dotted-dash line) multiplicities for the whole phase space.

FIGURE CAPTIONS (Cont'd)

- Fig. 8 The parameters $\bar{n}_{ch}$ and $\bar{n}_-$ of the negative binomial distribution for various c.m. rapidity intervals, $|y| < 1, 1.25, \dots, 4$, and for the whole phase space for pp interactions at 31, 44 and 62 GeV.
- Fig. 9 The parameters (a) k_{ch} and (b) k_- of the NB for the same conditions as in fig. 8.
- Fig. 10 Illustration of a cascade process: first are produced N clans, each of which gives rise to n_c particles.
- Fig. 11 Average number of charged $\bar{N}_{ch}$ and negative $\bar{N}_-$ clans for different intervals in c.m. rapidity, $|y| < y_{cut}$, for pp interactions at 31, 44 and 62 GeV.
- Fig. 12 Average number of charged and negative particles per clan for different intervals in c.m. absolute rapidity $|y| < y_{cut}$ for pp interactions at 31, 44 and 62 GeV.
- Fig. 13 Average number of (a) charged clans and of (b) charged particles per clan in different experiments for different intervals in c.m. rapidity.
- Fig. 14 Compilation of the average number of clans in the whole phase space versus c.m. energy. The lines are only meant to guide the eye.
- Fig. 15 Compilation of the average number of particles (charged and negatives) per clan in the whole phase space versus c.m. energy. The lines are only meant to guide the eye.

Fig. 1

Fig. 2(a)

Fig. 2(b)

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15