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Hadronic Production of Electroweak Vector Boson Pairs at Large Transverse Momentum

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ABSTRACT

Production of electroweak vector boson pairs accompanied by a large transverse momentum jet at hadron colliders is calculated to lowest order QCD with emphasis on the p_T and invariant mass distribution of the gauge boson pair. Analytic expressions for the matrix elements of all possible processes, including decays of W - and Z -bosons into massless fermion final states, are presented. It is found that these processes represent an important source for gauge boson pair production at the LHC and SSC and also constitute a background to new physics signals at these colliders, such as Higgs tagging or pair production of a fourth charged lepton.

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1. Introduction

Although many aspects of the standard electroweak gauge theory (SM) have now been verified, crucial parts of the model are not yet tested experimentally. Most notably the Higgs sector and the vector-boson self-interactions which are uniquely given by the gauge theory structure of the SM still await experimental confirmation.

Since the Higgs sector is responsible for the spontaneous symmetry breaking in the SM, the search for the Higgs particle is one of the most important tasks for future colliders. If its mass M_H is below 80 GeV the Higgs boson can be detected via the process $e^+e^- \rightarrow ZH$ at LEP200.¹⁾ For larger values of M_H the Higgs must be looked for at the LHC (pp collisions at $\sqrt{s} = 17$ TeV) or SSC (pp collisions at $\sqrt{s} = 40$ TeV). Studying the production of electroweak vector boson pairs is the most promising way to search for the Higgs boson at these colliders. If M_H is greater than twice the W -mass M_W , the Higgs will decay almost exclusively into a W or Z pair.²⁾ For $M_H < 2M_W$ it will mainly decay into $q\bar{q}$ states where the QCD background is overwhelming. However in this case one can consider rare decay modes of the Higgs boson proceeding via quark loops like $H \rightarrow \gamma\gamma$ or $H \rightarrow Z\gamma$.³⁾

In all these cases the continuum production of vector boson pairs is an important background. In the first approximation vector boson pairs are produced via $q\bar{q}$ annihilation. For a more complete knowledge, higher order QCD corrections involving real or virtual gluons have to be considered. Virtual and soft real gluon corrections mainly contribute to the so-called K factor which for some vector boson pair production processes has been calculated in ref. 4.

Hard QCD corrections on the other hand lead to the topologically distinct signature of a vector boson pair accompanied by one or more jets with large transverse momentum. Processes such as $q\bar{q} \rightarrow ZZg$ or $qg \rightarrow ZZq$ will therefore be a potentially important background *e.g.* for Higgs production at large p_T .^{5,6)} The transverse momentum spectrum of the vector boson pair can also be considered as a test of perturbative QCD.

In this paper we consider the production of vector boson pairs accompanied by a large p_T jet for all possible combinations, i.e. $\gamma\gamma$, $Z\gamma$, ZZ , $W^\pm\gamma$, W^+W^- and $W^\pm Z$. Throughout the whole paper we shall assume the SM to be valid. Using helicity amplitude techniques we present analytic expressions for the matrix elements, including the decay of W - and Z -bosons into massless fermion-antifermion final states (Section 2). Our matrix elements are therefore especially convenient when properties of, and correlations among final decay products are of interest. We then apply the matrix elements obtained to a detailed study of the transverse momentum and invariant mass distribution of the vector

boson pair (Section 3). In particular we shall consider the possibility that the jet is emitted in the forward direction while the boson pair is produced centrally. In this configuration vector boson pair plus jet production may be a background which is relevant for Higgs tagging.^{5,7)} If the jet produced in forward direction is not detected, and thus fakes missing p_T , $pp \rightarrow W^+W^-j$, $W^\pm Zj$ and ZZj may be very important background processes for pair production of a fourth generation charged lepton with mass bigger than M_W .⁸⁾ A pronounced feature of $W^\pm\gamma$ production in $q\bar{q}$ annihilation is the appearance of radiation zeros in all contributing helicity amplitudes at one value of the photon center of mass scattering angle and hence in the differential cross section.⁹⁾ In Section 3 we shall also address the question what happens to this radiation zero if the $W^\pm\gamma$ pair is accompanied by a high p_T jet. Finally, in Section 4 we shall summarize our results.

2. Matrix Elements

In order to compute the production cross section for electroweak vector boson pairs plus a jet at hadron colliders, the matrix elements of the process

$$q\bar{q} \rightarrow V_1 V_2 g \tag{2.1}$$

and the crossed processes

$$qg \rightarrow V_1 V_2 q, \quad \bar{q}g \rightarrow V_1 V_2 \bar{q} \tag{2.2}$$

have to be calculated. Here $V_1 V_2$ generically stands for all possible vector boson pair combinations

$$V_1 V_2 = \gamma\gamma, Z\gamma, ZZ, W^\pm\gamma, W^+W^-, W^\pm Z. \tag{2.3}$$

Since the incoming quarks are effectively massless, the spinor technique of ref. 10 may be used to calculate the helicity amplitudes for (2.1) and (2.2). In this method massive vector bosons decay automatically into massless fermion-antifermion pairs so that one obtains the matrix elements for the full process including the decay of the W - and Z -bosons at no extra cost. Thus they are particularly convenient when properties of the final state particles are studied. Moreover, since amplitudes are evaluated directly the resulting expressions are, especially for complicated Feynman diagrams, much simpler than with the conventional method where the squared matrix element is calculated.

To facilitate the calculation we introduce the following expression for momentum conservation:

$$b_1 p_1^\mu + b_2 p_2^\mu + b_3 p_3^\mu + b_4 p_4^\mu + b_5 p_5^\mu = 0 . \quad (2.4)$$

Here p_1 (p_2) denotes the quark (anti-quark) momentum, p_3 is the gluon momentum, p_4 and p_5 are the momenta of the final state vector bosons V_1 and V_2 , and

$$b_i = \begin{cases} +1 & \text{if } p_i \text{ is incoming,} \\ -1 & \text{if } p_i \text{ is outgoing.} \end{cases} \quad (2.5)$$

If we include the coefficients b_i in the matrix elements, all possible crossings between quarks and gluons are automatically accounted for. For $b_1 = b_2 = 1$, $b_3 = b_4 = b_5 = -1$ they apply to $q\bar{q}$ annihilation, and for $b_1 = b_3 = 1$, $b_2 = b_4 = b_5 = -1$ ($b_2 = b_3 = 1$, $b_1 = b_4 = b_5 = -1$) to quark (antiquark) gluon fusion. In cases where V_1 and/or V_2 are massive, $a\bar{b}$ and $c\bar{d}$ denote the massless fermion-antifermion states into which they decay:

$$\begin{aligned} p_4^\mu &= p_a^\mu + p_b^\mu , \\ p_5^\mu &= p_c^\mu + p_d^\mu . \end{aligned} \quad (2.6)$$

If one of the electroweak vector bosons is a photon its momentum vector will be assigned to p_4 .

The spinor technique of ref. 10 is of course not the only one that can be used to evaluate the helicity amplitudes for $V_1 V_2 j$ production. Another frequently used method is the one developed by Hagiwara and Zeppenfeld¹¹⁾ which is particularly convenient for massive fermions. As long as one is only interested in massless fermions and vector bosons in the final state the technique of ref. 10 is the most economical and fastest numerically of those presently available.

All matrix elements in this method are expressed in terms of the spinor products

$$s_{ij} = \bar{u}_+(p_i) u_-(p_j) = -s_{ji} \quad (2.7)$$

and

$$t_{ij} = \bar{u}_-(p_i) u_+(p_j) = s_{ji}^* \quad (2.8)$$

where $u_\pm(p_i)$ denotes the positive (negative)-helicity spinor of a massless fermion with momentum p_i . The absolute square value of s_{ij} is related to the vector product of p_i and

p_j by

$$2 \cdot (p_i p_j) = |s_{ij}|^2 . \quad (2.9)$$

For a numerical evaluation of the helicity amplitudes an explicit form of s_{ij} is needed. A convenient choice is¹⁰⁾

$$s_{ij} = (p_i^y + ip_i^z) \sqrt{\frac{p_j^0 - p_j^x}{p_i^0 - p_i^x}} - (p_j^y + ip_j^z) \sqrt{\frac{p_i^0 - p_i^x}{p_j^0 - p_j^x}} \quad (2.10)$$

where

$$p_i^\mu = (p_i^0, p_i^x, p_i^y, p_i^z) \quad (2.11)$$

and similarly for p_j^μ . For the polarization vector of massless spin 1 bosons we shall use

$$\begin{aligned} \epsilon_\lambda^\mu &= N_k \bar{u}_\lambda(k) \gamma^\mu u_\lambda(p_2) , \\ N_k &= 2 [(kp_2)]^{-1/2} , \end{aligned} \quad (2.12)$$

where k^μ is the vector boson momentum and $\lambda = \pm 1$ its helicity. Instead of p_2 we could have chosen any other lightlike vector q^μ which is not collinear to k^μ .¹⁰⁾ Different choices of q correspond to different gauge choices for the polarization vector. For $q = p_2$ (or $q = p_1$), however, many terms in the helicity amplitudes vanish and one obtains rather compact expressions for the matrix elements.

The spin and color averaged matrix element square for (2.1) and (2.2) is given by

$$|\overline{\mathcal{M}}|^2 = \frac{1}{4} C \mathcal{G}^2 \sum_{\lambda_1 \dots \lambda_{V_2} = \pm} |\mathcal{M}(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2})|^2 . \quad (2.13)$$

Here the factor $1/4$ arises from initial state spin averaging,

$$C = \begin{cases} \frac{4}{9} & \text{for } q\bar{q} \text{ annihilation} \\ \frac{1}{6} & \text{for } qg \text{ (}\bar{q}g\text{) fusion} \end{cases} \quad (2.14)$$

is the color averaging factor and $\mathcal{G}$ is an overall coupling constant factor which depends on the process under consideration. Finally, $\mathcal{M}(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2})$ is the helicity amplitude, where λ_1 and λ_3 denote the helicity of the quark associated with p_1 and of the gluon, respectively. $\lambda_{V_1}(\lambda_{V_2})$ either stands for the helicity of $V_1(V_2)$ if it is a photon or for the helicity of one of the decay fermions if $V_1(V_2) = W^\pm, Z$. Since we only deal with massless fermions the helicity of any given antifermion is determined by the helicity of its fermion partner, e.g. $\lambda_1 = \lambda_2$ and need not to be specified in $\mathcal{M}$.

To make our results more transparent, we break up the helicity amplitude into a product of helicity coupling factors and subamplitudes,

$$\mathcal{M}(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2}) = \sum_i \mathcal{C}_i(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2}) \mathcal{M}_i(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2}) \quad (2.15)$$

where the sum is over the contributing Feynman diagrams. The helicity coupling factors, $\mathcal{C}_i(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2})$, are diagram and process dependent, however the subamplitudes, $\mathcal{M}_i(\lambda_1, \lambda_3, \lambda_{V_1}, \lambda_{V_2})$, are common to several processes.

We are now ready to present our helicity amplitudes. This we shall do separately for the six possible vector boson pair combinations (see (2.3)). $V_1 V_2 j$ production processes can be classified according to whether they do or do not involve W -bosons. For processes without W -bosons ($V_1 V_2 = \gamma\gamma, Z\gamma, ZZ$) Feynman diagrams involving triple gauge boson vertices do not contribute. Fig. 1a shows the diagrams which are relevant for $\gamma\gamma j$, $Z\gamma j$ and ZZj production. Processes with W -bosons ($V_1 V_2 = W^\pm\gamma, W^+W^-, W^\pm Z$), on the other hand, also depend on the $WW\gamma$ and/or WWZ couplings and, besides the diagrams of Fig. 1a, those shown in Fig. 1b also contribute.

(i) $\gamma\gamma j$

For this process the coupling constant factor is given by

$$\mathcal{G}^2 = \frac{1}{2}(4\pi)^3 e_q^4 \alpha^2 \alpha_s. \quad (2.16)$$

Here α is the electromagnetic coupling constant, e_q denotes the charge of the quarks in units of $e = \sqrt{4\pi\alpha}$, α_s is the strong coupling constant and the factor $1/2$ takes into account that the two photons in the final state are identical particles. In this case the helicity coupling factor is,

$$\mathcal{C}_i(\lambda_1, \lambda_3, \lambda_4, \lambda_5) = 1, \quad i = 1, \dots, 6. \quad (2.17)$$

For $\gamma\gamma j$ it is sufficient to calculate the helicity amplitudes for only one of the diagrams of Fig. 1a. All other contributions can be obtained from those by a cyclic permutation of the indices 3, 4 and 5. Furthermore, since fermions couple vectorlike to photons and gluons one only needs to compute the amplitudes for $\lambda_1 = -1$. Complex conjugation then gives the matrix elements for $\lambda_1 = +1$,

$$\mathcal{M}_i(+, \lambda_3, \lambda_4, \lambda_5) = \mathcal{M}_i^*(-, -\lambda_3, -\lambda_4, -\lambda_5). \quad (2.18)$$

All necessary information about $\gamma\gamma j$ production is thus contained in the amplitudes

$$\begin{aligned}
\mathcal{M}_1(-, \lambda_3, -, -) &= +4 \frac{N_{\gamma_1} N_{\gamma_2} N_g}{(p_1 p_5)} \delta_{\lambda_3, -} s_{12}^2 s_{34}^* s_{15}^* \frac{1}{b_2 b_5} , \\
\mathcal{M}_1(-, \lambda_3, +, -) &= +4 \frac{N_{\gamma_1} N_{\gamma_2} N_g}{(p_1 p_5)} \delta_{\lambda_3, -} s_{12} s_{32}^* s_{14} s_{15}^* \frac{1}{b_2 b_5} , \\
\mathcal{M}_1(-, \lambda_3, -, +) &= +8 \frac{N_{\gamma_1} N_{\gamma_2} N_g}{(p_1 p_5)} \delta_{\lambda_3, -} [b_5(p_2 p_5) + b_1(p_1 p_2)] s_{34}^* s_{15} \frac{1}{b_1 b_2 b_5} , \\
\mathcal{M}_1(-, \lambda_3, +, +) &= -4 \frac{N_{\gamma_1} N_{\gamma_2} N_g}{(p_1 p_5)} \delta_{\lambda_3, -} s_{32}^2 s_{15} s_{34} \frac{b_3}{b_1 b_2 b_5} ,
\end{aligned} \tag{2.19}$$

which correspond to the first diagram in Fig. 1a. N_{γ_1} , N_{γ_2} and N_g in eq. (2.19) denote the normalization constants for the polarization vectors of the photons and the gluon (see eq. (2.12)). The complete helicity amplitudes for left-handed quarks are obtained by summing (2.19) over the cyclic permutations of the indices 3, 4 and 5.

Due to the large permutation symmetry present in $\gamma\gamma j$, the squared matrix element can be written in a very compact form. For $q\bar{q} \rightarrow \gamma\gamma j$ we find

$$|\overline{\mathcal{M}}|^2 = \frac{8}{9} \mathcal{G}^2 (p_1 p_2) \frac{\sum_{i=3}^5 [(p_1 p_i)^2 + (p_2 p_i)^2] (p_1 p_i) (p_2 p_i)}{\prod_{i=3}^5 (p_1 p_i) (p_2 p_i)} , \tag{2.20}$$

which agrees with the result for $e^+e^- \rightarrow \gamma\gamma\gamma$ in ref. 12, taking into account that the overall constant for $q\bar{q} \rightarrow \gamma\gamma j$ is different from the one for $e^+e^- \rightarrow \gamma\gamma\gamma$. A similar expression can also be derived for $qg \rightarrow \gamma\gamma q$ and $\bar{q}g \rightarrow \gamma\gamma\bar{q}$.

(ii) ZZj

Here the overall coupling constant is

$$\mathcal{G}^2 = (4\pi)^5 \alpha_Z^4 \alpha_s , \tag{2.21}$$

where

$$\alpha_Z = \frac{\alpha}{\sin^2 \theta_W \cos^2 \theta_W} \tag{2.22}$$

is the Z -boson fermion coupling constant. θ_W in eq. (2.22) denotes the Weinberg angle. Note that $\mathcal{G}^2$ in eq. (2.21) includes already the coupling constants for the subsequent decay

of the Z -bosons into the final states $a\bar{b}$ and $c\bar{d}$. The helicity coupling factor is given by,

$$C_i(\lambda_1, \lambda_3, \lambda_a, \lambda_c) = c_{\lambda_1}^{q2} c_{\lambda_a}^a c_{\lambda_c}^c D_{ab}^Z D_{cd}^Z, \quad i = 1, \dots, 6 \quad (2.23)$$

where

$$c_{\lambda_f}^f = \begin{cases} T_3^f - e_f \sin^2 \theta_W & \text{for } \lambda_f = - \\ -e_f \sin^2 \theta_W & \text{for } \lambda_f = + \end{cases} \quad (2.24)$$

is the coupling of the Z -boson to fermions with helicity λ_f . T_3^f denotes the third component of the weak isospin. The factor

$$D_{ij}^Z = [(p_i + p_j)^2 - M_Z^2 + iM_Z \Gamma_Z]^{-1} \quad (2.25)$$

is the propagator for the Z -boson decaying into the final state $i\bar{j}$. M_Z and Γ_Z denote the mass and the total width of the Z .

In order to determine the helicity amplitudes for ZZj production including the subsequent decay of the Z -bosons into massless fermions it is sufficient to calculate the matrix elements for negative fermion helicities, *i.e.* $\mathcal{M}_i(\lambda_1 = -, \lambda_3, \lambda_a = -, \lambda_c = -)$. The amplitudes for $\lambda_a = +(\lambda_c = +)$ can be obtained by exchanging the indices a and b (c and d) in $\mathcal{M}_i(-, \lambda_3, -, -)$. This follows from the fact that the decay current $\bar{u}_{\lambda_i}(p_i)\gamma_\mu u_{\lambda_i}(p_j) = \bar{u}_{-\lambda_i}(p_j)\gamma_\mu u_{-\lambda_i}(p_i)$ and so, exchanging p_i, p_j is equivalent to reversing the helicity λ_i . The matrix elements for $\lambda_1 = +1$ are given by

$$\mathcal{M}_i(+, \lambda_3, \lambda_a, \lambda_c) = \mathcal{M}_i^*(-, -\lambda_3, -\lambda_a, -\lambda_c). \quad (2.26)$$

Furthermore, one has only to calculate three of the six diagrams of Fig. 1a, due to the permutation symmetry among the Z -bosons. If one computes, for example, the second, fourth and sixth graph, the others are then obtained by

$$\mathcal{M}_i = \mathcal{M}_{i+1}(a \leftrightarrow c, b \leftrightarrow d, b_4 \leftrightarrow b_5), \quad i = 1, 3, 5, \quad (2.27)$$

where $\mathcal{M}_i$ denotes the subamplitude corresponding to the i th diagram in Fig. 1a. Full information about ZZj production with subsequent decay of the Z -bosons into massless

fermions is thus contained in the amplitudes

$$\begin{aligned}
\mathcal{M}_2(-, \lambda_3, -, -) &= +8 \frac{N_g}{z_{14}} \delta_{\lambda_3, -} s_{c3}^* s_{b1} (b_4 s_{db} s_{ab}^* + b_1 s_{d1} s_{a1}^*) \frac{1}{b_2} , \\
\mathcal{M}_4(-, -, -, -) &= -8 \frac{N_g}{z_{14} z_{25}} s_{c2}^* s_{b1} (b_2 s_{d2} s_{32}^* + b_5 s_{dc} s_{3c}^*) (b_4 s_{2b} s_{ab}^* + b_1 s_{21} s_{a1}^*) , \\
\mathcal{M}_4(-, +, -, -) &= +8 \frac{N_g}{z_{14} z_{25}} s_{c2}^{*2} s_{b1} s_{dc} (b_4 s_{3b} s_{ab}^* + b_1 s_{31} s_{a1}^*) b_5 , \\
\mathcal{M}_6(-, -, -, -) &= -4 \frac{N_g}{z_{25}} \frac{1}{(p_1 p_3)} s_{c2}^* s_{21} s_{31}^* s_{b1} (b_2 s_{d2} s_{a2}^* + b_5 s_{dc} s_{ac}^*) \frac{1}{b_3} , \\
\mathcal{M}_6(-, +, -, -) &= -4 \frac{N_g}{z_{25}} \frac{1}{(p_1 p_3)} s_{c2}^* s_{31} (b_2 s_{d2} s_{a2}^* + b_5 s_{dc} s_{ac}^*) \left(\frac{1}{b_1} s_{b3} s_{23}^* + \frac{1}{b_3} s_{b1} s_{21}^* \right) ,
\end{aligned} \tag{2.28}$$

where the denominator functions z_{14} and z_{25} are given by

$$\begin{aligned}
z_{14} &= [b_1 p_1 + b_4(p_a + p_b)]^2 , \\
z_{25} &= [b_2 p_2 + b_5(p_c + p_d)]^2 .
\end{aligned} \tag{2.29}$$

Considering the complexity of the process these amplitudes have a rather compact form.

(iii) $Z\gamma j$

Since all necessary details have already been discussed for $\gamma\gamma j$ and ZZj , our discussion will be rather brief for this process. The overall coupling constant for $Z\gamma j$, including the decay of the Z -boson into the massless fermion pair $c\bar{d}$ takes the form

$$\mathcal{G}^2 = (4\pi)^4 e_q^2 \alpha_Z^2 \alpha \alpha_s \tag{2.30}$$

and the helicity coupling factor is given by

$$C_i(\lambda_1, \lambda_3, \lambda_4, \lambda_c) = c_{\lambda_1}^q c_{\lambda_c}^\varepsilon D_{cd}^Z, \quad i = 1, \dots, 6 . \tag{2.31}$$

As for ZZj it is sufficient to calculate the amplitudes for the left-handed fermions only, *i.e.* $\mathcal{M}_i(\lambda_1 = -, \lambda_3, \lambda_4, \lambda_c = -)$ (λ_4 denotes the helicity of the photon). The matrix elements for $\lambda_c = +$ are then obtained by exchanging the indices c and d as in the ZZj case. The $\lambda_1 = +$ amplitudes are given by

$$\mathcal{M}_i(+, \lambda_3, \lambda_4, \lambda_c) = \mathcal{M}_i^*(-, -\lambda_3, -\lambda_4, -\lambda_c) . \tag{2.32}$$

For the subamplitudes corresponding to first, second and fourth graph in Fig. 1a we find:

$$\begin{aligned}
\mathcal{M}_1(-, \lambda_3, -, -) &= +8 \frac{N_\gamma N_g}{z_{15}} \delta_{\lambda_3, -} s_{34}^* s_{d1} (b_1 s_{12} s_{c1}^* + b_5 s_{d2} s_{cd}^*) \frac{1}{b_2}, \\
\mathcal{M}_1(-, \lambda_3, +, -) &= +8 \frac{N_\gamma N_g}{z_{15}} \delta_{\lambda_3, -} s_{32}^* s_{d1} (b_1 s_{14} s_{c1}^* + b_5 s_{d4} s_{cd}^*) \frac{1}{b_2}, \\
\mathcal{M}_2(-, \lambda_3, -, -) &= +4 \frac{N_\gamma N_g}{(p_1 p_4)} \delta_{\lambda_3, -} s_{14}^* s_{12} s_{c3}^* s_{d1} \frac{1}{b_2 b_4}, \\
\mathcal{M}_2(-, \lambda_3, +, -) &= +4 \frac{N_\gamma N_g}{(p_1 p_4)} \delta_{\lambda_3, -} s_{c3}^* s_{14} \left(\frac{1}{b_4} s_{12}^* s_{d1} + \frac{1}{b_1} s_{42}^* s_{d4} \right) \frac{1}{b_2}, \\
\mathcal{M}_4(-, -, -, -) &= -4 \frac{N_\gamma N_g}{z_{25}} \frac{1}{(p_1 p_4)} s_{12}^2 s_{14}^* s_{c2}^* (b_5 s_{dc} s_{c3}^* + b_2 s_{d2} s_{23}^*) \frac{1}{b_4}, \\
\mathcal{M}_4(-, -, +, -) &= -8 \frac{N_\gamma N_g}{z_{25}} \frac{b_4(p_2 p_4) + b_1(p_1 p_2)}{(p_1 p_4)} s_{14} s_{c2}^* (b_2 s_{d2} s_{23}^* + b_5 s_{dc} s_{c3}^*) \frac{1}{b_1 b_4}, \\
\mathcal{M}_4(-, +, +, -) &= -4 \frac{N_\gamma N_g}{z_{25}} \frac{1}{(p_1 p_4)} s_{c2}^2 s_{dc} s_{14} \left(\frac{1}{b_4} s_{12}^* s_{13} + \frac{1}{b_1} s_{42}^* s_{43} \right) b_5, \\
\mathcal{M}_4(-, +, -, -) &= -4 \frac{N_\gamma N_g}{z_{25}} \frac{1}{(p_1 p_4)} s_{c2}^2 s_{dc} s_{14}^* s_{12} s_{13} \frac{b_5}{b_4},
\end{aligned} \tag{2.33}$$

where z_{15} is defined by an expression analogous to the one for z_{25} (see eq. (2.29)). The result for $\mathcal{M}_{3,5,6}$ is obtained by exchanging the indices 3 (gluon) and 4 (photon) in (2.33). The sum $\sum_i \mathcal{C}_i \mathcal{M}_i$ finally gives the complete matrix elements for $\lambda_1 = \lambda_c = -1$. This concludes our discussion of those processes which receive only contributions from the diagrams in Fig. 1a. We shall now turn to reactions which involve W -bosons and thus depend on the triple gauge boson vertex.

(iv) $W^\pm \gamma j$

Including the decay of the W into its final state $c\bar{d}$, the overall coupling constant for $W^\pm \gamma j$ is

$$\mathcal{G}^2 = \frac{1}{4} (4\pi)^4 \alpha_W^2 \alpha \alpha_s, \tag{2.34}$$

where

$$\alpha_W = \frac{\alpha}{\sin^2 \theta_W} \tag{2.35}$$

is the W -boson fermion coupling constant. Since the W couples only to left-handed fermions we can drop λ_1 and λ_c from the argument of $\mathcal{M}$ and retain only the helicities of

the gluon (λ_3) and the photon (λ_4):

$$\mathcal{M}_i(\lambda_3, \lambda_4) = \mathcal{M}_i(-, \lambda_3, \lambda_4, -) , \quad (2.36)$$

where i now runs from 1 to 8 and includes the diagrams depending on the triple gauge boson vertex shown in Fig. 1b. The non-zero helicity coupling factors are,

$$\begin{aligned} C_i(-, \lambda_3, \lambda_4, -) &= e_{q'} D_{cd}^W , & i &= 1, 3, 5 \\ C_i(-, \lambda_3, \lambda_4, -) &= e_q D_{cd}^W , & i &= 2, 4, 6 \\ C_i(-, \lambda_3, \lambda_4, -) &= (e_q - e_{q'}) D_{cd}^W D_{45} , & i &= 7, 8 \end{aligned} \quad (2.37)$$

where D_{cd}^W is the W propagator and q' is defined through $q\bar{q}' \rightarrow W^\pm \gamma g$. D_{cd}^W is given by eq. (2.25) with all indices Z replaced by W and

$$D_{45} = [(b_4 p_4 + b_5(p_c + p_d))^2 - M_W^2 + iM_W \Gamma_W]^{-1} . \quad (2.38)$$

The matrix elements corresponding to the diagrams of Fig. 1a ($i = 1, \dots, 6$) are simply given by the $Z\gamma j$ helicity amplitudes for left-handed fermions (see eq. (2.33)). The sub-amplitudes $\mathcal{M}_{7,8}$, which are associated with the diagrams shown in Fig. 1b, can be written in terms of

$$\begin{aligned} \mathcal{M}_7(\lambda_3, -) &= -4 N_\gamma N_g \delta_{\lambda_3, -} \left[2b_5(s_{c4}^* s_{c2} + s_{d4}^* s_{d2}) s_{c3}^* s_{d1} - 2b_4 s_{4c}^* s_{4d} s_{43}^* s_{21} \right. \\ &\quad \left. + (b_4 s_{43}^* s_{41} - b_5 s_{c3}^* s_{c1} - b_5 s_{d3}^* s_{d1}) s_{4c}^* s_{2d} \right] \frac{1}{b_2} , \\ \mathcal{M}_7(\lambda_3, +) &= -4 N_\gamma N_g \delta_{\lambda_3, -} \left[2b_5(s_{2c}^* s_{4c} + s_{2d}^* s_{4d}) s_{c3}^* s_{d1} - 2b_4 s_{4c}^* s_{4d} s_{23}^* s_{41} \right. \\ &\quad \left. + (b_4 s_{43}^* s_{41} - b_5 s_{c3}^* s_{c1} - b_5 s_{d3}^* s_{d1}) s_{2c}^* s_{4d} \right] \frac{1}{b_2} , \\ \mathcal{M}_8(-, -) &= -2 \frac{N_\gamma N_g}{(p_1 p_3)} s_{21} s_{31}^* \left[2b_5(s_{c4}^* s_{c2} + s_{d4}^* s_{d2}) s_{c2}^* s_{d1} + b_3 s_{32}^* s_{31} s_{4c}^* s_{2d} \right. \\ &\quad \left. + 2b_4(s_{41} s_{2d} - s_{21} s_{4d}) s_{42}^* s_{4c} \right] \frac{1}{b_3} , \\ \mathcal{M}_8(-, +) &= -2 \frac{N_\gamma N_g}{(p_1 p_3)} s_{21} s_{31}^* s_{c2}^* \left[2b_5(s_{2c}^* s_{4c} + s_{2d}^* s_{4d}) s_{d1} \right. \\ &\quad \left. - (2b_4 s_{42}^* s_{41} + b_3 s_{32}^* s_{31}) s_{4d} \right] \frac{1}{b_3} , \end{aligned} \quad (2.39)$$

$$\begin{aligned}
\mathcal{M}_8(+, -) &= -4 \frac{N_\gamma N_g}{(p_1 p_3)} s_{31} \left[b_5 (s_{c4}^* s_{c2} + s_{d4}^* s_{d2}) (b_1 s_{21}^* s_{d1} + b_3 s_{23}^* s_{d3}) s_{c2}^* \right. \\
&\quad \left. + b_4 s_{42}^* s_{4c}^* [(b_1 s_{41} s_{21}^* + b_3 s_{43} s_{23}^*) s_{2d} \right. \\
&\quad \left. - 2(b_1(p_1 p_2) + b_3(p_2 p_3)) s_{4d}] \right] \frac{1}{b_1 b_3} , \\
\mathcal{M}_8(+, +) &= -4 \frac{N_\gamma N_g}{(p_1 p_3)} s_{31} s_{c2}^* \left[b_5 (s_{2c}^* s_{4c} + s_{2d}^* s_{4d}) (b_1 s_{21}^* s_{d1} + b_3 s_{23}^* s_{d3}) \right. \\
&\quad \left. - b_4 (b_1 s_{41} s_{21}^* + b_3 s_{43} s_{23}^*) s_{42}^* s_{4d} \right] \frac{1}{b_1 b_3} .
\end{aligned} \tag{2.39 contd.}$$

Terms arising from the $p^\mu p^\nu / M_W^2$ piece of the s-channel W propagator have been disregarded in eq. (2.39) since they cancel in the sum of the subamplitudes $\mathcal{M}_{7,8}$ due to gauge invariance.

(v) $W^+ W^- j$

For this process one finds

$$\mathcal{G}^2 = \frac{1}{16} (4\pi)^5 \alpha_W^4 \alpha_s \tag{2.40}$$

for the overall coupling constant, including the subsequent decay $W^+ \rightarrow a\bar{b}$ and $W^- \rightarrow c\bar{d}$. Since all final state fermions are lefthanded, we can drop λ_a and λ_c from the argument of $\mathcal{M}$ and write

$$\mathcal{M}_i(\lambda_1, \lambda_3) = \mathcal{M}_i(\lambda_1, \lambda_3, -, -) . \tag{2.41}$$

The non-zero helicity coupling factors are,

$$\begin{aligned}
C_i(-, \lambda_3, -, -) &= D_{ab}^W D_{cd}^W , & i &= 1, \dots, 6 \\
C_i(\lambda_1, \lambda_3, -, -) &= D_{ab}^W D_{cd}^W D_{\lambda_1} , & i &= 7, 8
\end{aligned} \tag{2.42}$$

with

$$D_{\lambda_1} = -2 \left(\frac{e_q}{z_{45}} \sin^2 \theta_W + \frac{c_{\lambda_1}^q}{z_{45} - M_Z^2 + i M_Z \Gamma_Z} \right) , \tag{2.43}$$

$$z_{45} = (b_4(p_a + p_b) + b_5(p_c + p_d))^2 . \tag{2.44}$$

The term proportional to e_q in eq. (2.43) arises from photon and the $c_{\lambda_1}^q$ term from Z exchange.

The subamplitudes $\mathcal{M}_i$, $i = 1, \dots, 6$ corresponding to the diagrams in Fig. 1a are simply those for ZZj (see eqs. (2.27) - (2.29)). Only $\lambda_1 = -$ states give a non-zero result.

Moreover, if we identify the W^+ with V_1 in Fig. 1a, diagrams 2, 4 and 6 contribute only for up-type quarks, and diagrams 1, 3 and 5 only for down-type quarks. For the subamplitudes $\mathcal{M}_{7,8}$ of the diagrams displayed in Fig. 1b and left-handed quarks we obtain

$$\begin{aligned}
\mathcal{M}_7(-, \lambda_3) &= +4 N_g \delta_{\lambda_3, -} \left[2b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) s_{c3}^* s_{d1} - 2b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) s_{a3}^* s_{b1} \right. \\
&\quad \left. + [2b_4(s_{a3}^* s_{a1} + s_{b3}^* s_{b1}) + b_2 s_{23}^* s_{21}] s_{ca}^* s_{db} \right] \frac{1}{b_2}, \\
\mathcal{M}_8(-, -) &= -2 \frac{N_g}{(p_1 p_3)} s_{31}^* s_{21} \left[2b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) s_{a2}^* s_{b1} - 2b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) s_{c2}^* s_{d1} \right. \\
&\quad \left. + [2b_5(s_{c2}^* s_{c1} + s_{d2}^* s_{d1}) + b_3 s_{32}^* s_{31}] s_{ca}^* s_{db} \right] \frac{1}{b_3}, \\
\mathcal{M}_8(-, +) &= -4 \frac{N_g}{(p_1 p_3)} s_{31} \left[b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) (b_1 s_{21}^* s_{b1} + b_3 s_{23}^* s_{b3}) s_{a2}^* + b_5 s_{ca}^* s_{db} \right. \\
&\quad \left[(b_1 s_{21}^* s_{c1} + b_3 s_{23}^* s_{c3}) s_{c2}^* + (b_1 s_{21}^* s_{d1} + b_3 s_{23}^* s_{d3}) s_{d2}^* \right] \\
&\quad \left. - b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) (b_1 s_{21}^* s_{d1} + b_3 s_{23}^* s_{d3}) s_{c2}^* \right] \frac{1}{b_1 b_3}. \tag{2.45}
\end{aligned}$$

For right-handed quarks one finds

$$\begin{aligned}
\mathcal{M}_7(+, \lambda_3) &= +4 N_g \delta_{\lambda_3, +} \left[2b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) s_{c1}^* s_{d3} - 2b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) s_{a1}^* s_{b3} \right. \\
&\quad \left. + [2b_4(s_{a1}^* s_{a3} + s_{b1}^* s_{b3}) + b_2 s_{21}^* s_{23}] s_{ca}^* s_{db} \right] \frac{1}{b_2}, \\
\mathcal{M}_8(+, +) &= -2 \frac{N_g}{(p_1 p_3)} s_{21}^* s_{31} \left[2b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) s_{a1}^* s_{b2} - 2b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) s_{c1}^* s_{d2} \right. \\
&\quad \left. + [2b_5(s_{c1}^* s_{c2} + s_{d1}^* s_{d2}) + b_3 s_{31}^* s_{32}] s_{ca}^* s_{db} \right] \frac{1}{b_3}, \\
\mathcal{M}_8(+, -) &= -4 \frac{N_g}{(p_1 p_3)} s_{31}^* \left[b_4(s_{ac}^* s_{ad} + s_{bc}^* s_{bd}) (b_1 s_{a1}^* s_{21} + b_3 s_{a3}^* s_{23}) s_{b2} + b_5 s_{ca}^* s_{db} \right. \\
&\quad \left[(b_1 s_{c1}^* s_{21} + b_3 s_{c3}^* s_{23}) s_{c2} + (b_1 s_{d1}^* s_{21} + b_3 s_{d3}^* s_{23}) s_{d2} \right] \\
&\quad \left. - b_5(s_{ca}^* s_{cb} + s_{da}^* s_{db}) (b_1 s_{c1}^* s_{21} + b_3 s_{c3}^* s_{23}) s_{d2} \right] \frac{1}{b_1 b_3}. \tag{2.46}
\end{aligned}$$

As in eq. (2.39), terms arising from the $p^\mu p^\nu / M_Z^2$ part of the Z propagator have been disregarded in eqs. (2.45) and (2.46). Due to gauge invariance they cancel once the sum of $\mathcal{M}_{7,8}$ is taken.

The complete helicity amplitudes for $W^+ W^- j$ are given by $\sum_i \mathcal{C}_i \mathcal{M}_i$ (see eq. (2.15)). Note that due to our choice for the polarization vector of the gluon (eq. (2.12)) only $\mathcal{M}_8$ contributes for right-handed quarks and gluon helicity $\lambda_3 = -1$. This and the fact that

many helicity subamplitudes vanish due to the $V - A$ coupling of the W -bosons makes the numerical evaluation of the matrix elements for W^+W^-j particularly fast.

(vi) $W^\pm Z j$

Finally, we briefly discuss the $W^\pm Z j$ case. Here, the overall coupling constant is

$$G^2 = \frac{1}{4}(4\pi)^5 \alpha_Z^2 \alpha_W^2 \alpha_s, \quad (2.47)$$

where, as usual, the couplings for the subsequent decay $W \rightarrow a\bar{b}$ and $Z \rightarrow c\bar{d}$ are already included. Since the W couples exclusively to left-handed quarks and leptons, only amplitudes with $\lambda_1 = \lambda_a = -1$ do not vanish. Identifying the W with V_1 we find that the non-zero helicity coupling factors for $W^\pm Z$ production are,

$$\begin{aligned} C_i(-, \lambda_3, -, \lambda_c) &= c_-^q c_{\lambda_c} D_{ab}^W D_{cd}^Z, & i = 1, 3, 5 \\ C_i(-, \lambda_3, -, \lambda_c) &= c_-^{q'} c_{\lambda_c} D_{ab}^W D_{cd}^Z, & i = 2, 4, 6 \\ C_i(-, \lambda_3, -, \lambda_c) &= \pm \cos^2 \theta_W c_{\lambda_c}^c D_{ab}^W D_{cd}^Z \tilde{D}^W, & i = 7, 8 \end{aligned} \quad (2.48)$$

where q' is defined through $q\bar{q}' \rightarrow W^\pm Z g$ and

$$\tilde{D}^W = (z_{45} - M_W^2 + iM_W \Gamma_W)^{-1}. \quad (2.49)$$

The subamplitudes $\mathcal{M}_i$, $i = 1, \dots, 6$, for the diagrams of Fig. 1a are obtained from eqs. (2.27) - (2.29) and those for the diagrams of Fig. 1b ($\mathcal{M}_i$, $i = 7, 8$) from eq. (2.45). As usual, the complete helicity amplitudes are then the sum of the subamplitudes. The matrix elements for $\lambda_c = +$ are obtained from the $\lambda_c = -$ ones by interchanging the indices c and d as in the ZZj case.

This completes our discussion of helicity amplitudes for $V_1 V_2 j$ production in hadronic collisions. We have checked the matrix elements presented above in various ways. The calculation which was done by hand was repeated using the algebraic manipulation program SCHOONSCHIP. Independently we also calculated the matrix element squares for (2.1) and (2.2) without the decay of W - and Z -bosons employing conventional trace techniques and compared the result with the corresponding one obtained with the helicity amplitude method, after integrating over the phase space of the W - and Z decay products. For randomly chosen phase space points the two approaches agreed to better than 1%.

For some of the processes our results can be compared with existing literature. As already mentioned, for $q\bar{q} \rightarrow \gamma\gamma g$ we reproduce the result of ref. 12 for $e^+e^- \rightarrow \gamma\gamma\gamma$, taking

into account the difference in the overall coupling constant for the two processes. For ZZj production we can compare the cross sections obtained from our helicity amplitudes with the results given in ref. 5. We agree with them at the few per cent level. For $W^\pm\gamma j$, finally, a cross-check is provided by the radiation zero present in $W^\pm\gamma$ production.⁹⁾ The amplitude for $q\bar{q} \rightarrow W\gamma g$ factorizes in the soft gluon limit and is thus proportional to the $q\bar{q} \rightarrow W\gamma$ amplitude. Hence the radiation zero has to be present in the $W\gamma g$ amplitude if the gluon energy goes to zero. We have verified that this is indeed the case for our matrix elements.

3. Cross Sections

Using the helicity amplitudes presented in the last Section it is now straightforward to compute cross sections for V_1V_2j production at hadron colliders. Since the rates for vector boson pair production, apart from $\gamma\gamma$ and $W\gamma$, are too small to be observable at the CERN $\bar{p}p$ -collider ($\sqrt{s} = 0.63$ TeV) and at the Tevatron ($\sqrt{s} = 1.8$ TeV) we shall concentrate in the following on V_1V_2j production at the planned Large Hadron Collider LHC (pp collisions at $\sqrt{s} = 17$ TeV) and at the Superconducting Super Collider SSC (pp collisions at $\sqrt{s} = 40$ TeV).

To simplify the computations somewhat W - and Z -bosons will be treated in the narrow width approximation. Taking into account the finite width of W and Z would require the inclusion of final fermion state gluon and photon bremsstrahlung, in order to preserve color and electromagnetic gauge invariance. Since finite width effects are expected to be small, we shall ignore them in the following.

In Section 3.1 we discuss distributions for the electroweak boson pair with emphasis on the transverse momentum and invariant mass distribution. In Section 3.2 we consider the possibility that the jet in V_1V_2j is produced non-centrally. Section 3.3, finally, gives a brief description of leptonic signatures for V_1V_2j .

3.1 Electroweak Boson Pair Distributions

For a first look at the transverse momentum and invariant mass distribution of the vector boson pair it is convenient to consider V_1 and V_2 as final state particles rather than their decay products. Due to the infrared and collinear singularities present, cuts have to be made in order to arrive at a finite cross section. In processes involving photons

($\gamma\gamma j$, $Z\gamma j$ and $W^\pm\gamma j$) a p_T cut of

$$p_{T\gamma} > 50 \text{ GeV} \quad (3.1)$$

and an angular separation cut of

$$\cos \Theta_{\gamma j} < 0.9 , \quad (3.2)$$

where $\Theta_{\gamma j}$ is the angle between the jet and the photon, will be applied. Both cuts are also physically well motivated. If the transverse momentum of the photon is small, Vjj production with one of the jets being misidentified as a photon may cause severe background problems. Such misidentification may arise due to fluctuations in the calorimetry or because the jet hadronizes, with a leading π^0 carrying away most of its energy.¹³⁾ The angular separation cut ensures that the photon is not embedded in the jet, which is a necessary requirement for photon identification. For $\gamma\gamma j$ production we also require that

$$\cos \Theta_{\gamma\gamma} < 0.9 . \quad (3.3)$$

Such an isolation cut is unavoidable in a realistic study because collinear photons cannot be separated in a detector.

In order to arrive at sensible results for the p_T spectrum of the vector boson pair additional cuts have to be made. To the extent that the partons inside the proton have no transverse momentum, the vector boson pairs produced in $q\bar{q}$ annihilation have no transverse momentum in lowest order. If gluons are emitted or absorbed in the process, the boson pair will generally have transverse momentum. For small values of p_{TVV} , multiple gluon emission and/or absorption is important. Only for sufficiently large transverse momentum the first order processes $q\bar{q} \rightarrow V_1 V_2 g$, $qg \rightarrow V_1 V_2 q$ and $\bar{q}g \rightarrow V_1 V_2 \bar{q}$ dominate. Sensible results for the p_{TVV} spectrum from first order processes thus require a suitable cut on the transverse momentum. A full calculation of the p_{TVV} spectrum would require the inclusion of virtual and multiple gluon processes, which is beyond the scope of the present paper. For the simplest case of double photon production such a calculation was carried out in ref. 14.

The cut which we shall apply to the transverse momentum of the vector boson pair is

$$p_{TVV} > \varepsilon \cdot m_{VV} , \quad (3.4)$$

where ε is a parameter to be discussed below.¹⁵⁾ If m_{VV} can be determined, this cut can easily be implemented in an actual experiment. In order to roughly simulate the

performance of realistic detectors we require in addition that $p_{TVV} > 100$ GeV. This admittedly conservative cut will ensure that the jet balancing the vector boson pair is actually observed in the detector. Finally, we shall impose a rapidity cut of $|y| < 2.5$ on all final state particles in this subsection. Of course, all these cuts are somewhat arbitrary if one does not simulate a definite detector.

The meaning of the cut (3.4) is illustrated in Fig. 2 for the ZZ case. Here we compare the invariant mass spectrum for ZZ pairs at the SSC for the lowest order process $q\bar{q} \rightarrow ZZ$ (dashed line) and the first order processes for various values of ε . The solid curve shows m_{ZZ} for $\varepsilon = 0$, *i.e.* when a fixed p_{TZZ} cut of 100 GeV only is applied. The dotted (dash-dotted) line displays the results when eq. (3.4) with $\varepsilon = 0.2(0.3)$ is applied in addition to the $p_{TZZ} > 100$ GeV cut. The SM parameters used in Fig. 2 and all subsequent figures are $\alpha = \alpha(M_W) = 1/128$, $M_W = 81$ GeV, $\sin^2 \theta_W = 0.23$ and $M_Z = M_W / \cos \theta_W$. We use the parton distribution functions of Duke and Owens, set I.¹⁶⁾ The momentum scale of α_s and the parton distributions was chosen to be $\hat{s}/4$, where $\hat{s}$ is the center of mass energy.

From Fig. 2 one observes that without the ε -cut the differential cross section of the lowest and the first order processes become comparable for large m_{ZZ} . Since $d\sigma/dm_{ZZ}$ for a fixed value of m_{ZZ} is dominated by the low p_{TZZ} region, one always enters a region where the main contribution to $d\sigma/dm_{ZZ}$ originates from the soft region $p_{TZZ} \ll m_{ZZ}$, where multiple gluon processes become important, for sufficiently large m_{ZZ} . Hence, for a fixed p_{TVV} cut the results for the invariant mass m_{VV} of the vector boson pair from first order processes become unreliable. The precise value of m_{VV} where this happens depends of course on the value of the p_T cut.

In the soft region terms of order

$$\frac{1}{p_{TVV}^2} \alpha_s^n (p_{TVV}^2) \ln^m \left(\frac{m_{VV}^2}{p_{TVV}^2} \right), \quad m \leq 2n - 1,$$

become large. A cut of the form (3.4) with a sufficiently large value of ε will thus guarantee that these terms are under control for all powers of α_s . This is clearly visible in Fig. 2. The cut (3.4) mainly suppresses the cross section at large m_{ZZ} and the result for $d\sigma/dm_{ZZ}$ from the first order processes (dotted and dash-dotted line) is of $O(\alpha_s)$. The precise value of ε is of course somewhat arbitrary and could only be fixed if the full calculation including multiple gluon processes is done. The dash-dotted and dotted curves in Fig. 2, corresponding to $\varepsilon = 0.3$ and $\varepsilon = 0.2$, indicate the typical variation of the m_{VV} spectrum with ε . In the following we shall always use $\varepsilon = 0.3$.

In Figs. 3 and 4 we show the invariant mass distribution of the vector boson pair for processes with and without photons, using the cuts specified above. The maximum of the cross section for $\gamma\gamma j$ is a purely artificial effect of the transverse momentum cut on the photons and the jet and is approximately located at

$$m_{\gamma\gamma} \approx \sum_i p_{Ti}^{cut}. \quad (3.5)$$

In all other cases the maximum is a threshold effect caused by the W and/or Z mass. For $W\gamma j$ and WZj the sum of the cross sections for W^+ and W^- is displayed.

Figures 5 and 6 show the p_T spectrum of the vector boson pair for processes with and without photons, respectively. For $W\gamma j$ and WZj we calculated again the sum of the cross sections for W^+ and W^- . For processes which do not depend on the triple gauge boson vertex ($\gamma\gamma j$, $Z\gamma j$ and ZZj) the p_T distribution is found to drop somewhat faster than for processes which do. The p_{TVV} spectrum also turns out to be very insensitive to the value of ε . This can be easily understood by noting that for a fixed value of p_{TVV} (3.4) implies an upper bound for the allowed vector boson pair mass. Since the high invariant mass tail contributes only very little to the integrated cross section, it is clear that the transverse momentum distribution of the electroweak boson pair is relatively insensitive to ε .

In Table 1 we list the total V_1V_2j production cross sections within our cuts and a minimum transverse momentum of the vector boson pair of 100, 200 and 300 GeV at the LHC and the SSC, and compare them with the rates for the corresponding lowest order $2 \rightarrow 2$ process $q\bar{q} \rightarrow V_1V_2$. The cuts imposed on V_1 and V_2 in the $2 \rightarrow 2$ process are, of course, the same as in V_1V_2j . Table 1 demonstrates that the V_1V_2j production cross sections for $p_{TVV}^{min} = 100$ GeV are generally large so that these processes represent an important source for gauge boson pair production at the LHC and SSC. The cross section is dominated by the quark gluon fusion processes; $q\bar{q}$ annihilation contributes only 10-30% to the total rate. Even for $p_{TVV}^{min} = 300$ GeV the event rate is still sizeable, typically of $O(10\%)$ of the one for $p_{TVV}^{min} = 100$ GeV. By comparison with the rate for the corresponding $2 \rightarrow 2$ process we see that the requirement that the electroweak vector boson be produced in association with a large transverse momentum jet reduces the cross section typically by a factor 4 to 5.

For $W^\pm\gamma$ the rate of the first order process, $W^\pm\gamma j$, is for $p_{TW\gamma}^{min} = 100$ GeV even bigger than the one of $q\bar{q} \rightarrow W^\pm\gamma$. This anomalous behavior can be traced back to the radiation zero⁹⁾ present in the differential cross section of $W^\pm\gamma$ production in $q\bar{q}$ annihilation and

which suppresses the total rate substantially. Such an effect is not present in the other gauge boson pair production processes. For $u\bar{d} \rightarrow W^+\gamma$ the radiation zero occurs at $\cos\theta^* = -1/3$, where θ^* is the center of mass scattering angle of the photon relative to the u -quark direction. Since one does not know which of the two incoming protons to associate with the u - and d -quark, only $|\cos\theta^*|$ is measurable in pp collisions and the radiation zero is considerably washed out. In order to eliminate the strong peaking of the differential cross section at $\cos\theta^* = \pm 1$ arising from the collinear singularity it is convenient to study the center of mass rapidity distribution $d\sigma/d|y^*|$ of the photon,¹⁷⁾ with

$$y^* = \frac{1}{2} \ln \frac{1 + \cos\theta^*}{1 - \cos\theta^*} . \quad (3.6)$$

The resulting distribution for $W^+\gamma$ is shown in Fig. 7 (solid line). A $p_{T\gamma} > 50$ GeV cut and a rapidity cut of $|y| < 2.5$ on the W and the photon are applied. The pronounced dip at $|y^*| = 0$ is due to the radiation zero. As demonstrated by the dashed line in Fig. 7, no such dip is present in $W^+\gamma j$ for $p_{TW\gamma} > 100$ GeV. Instead the y^* distribution peaks for small rapidity values and it is thus not surprising that the total $W\gamma j$ rate is larger than the cross section for the lowest order process $q\bar{q} \rightarrow W\gamma$. Only for $q\bar{q} \rightarrow W\gamma g$ and $p_{TW\gamma} \rightarrow 0$ would the radiation zero be present. However, as we have pointed out above one cannot make reliable predictions in this limit since virtual and multiple gluon corrections become important.

In contrast to $W\gamma$ the center of mass and the $W\gamma$ rest frame do not coincide for $W\gamma j$, and instead of y^* one could also consider the photon rapidity $y_{W\gamma}^*$ in the $W\gamma$ rest frame. It turns out that the $y_{W\gamma}^*$ distribution is very similar to the one of the center of mass rapidity y^* and hence is not shown here.

At this point it has to be emphasized that all cross sections listed in Table 1 are total rates which do not include any branching ratios for W and Z decays. If one requires leptonic decays for W and Z identification - the hadronic decay modes of W and Z will be swamped by the QCD background¹⁸⁾ - the observable rates will be much smaller with the exception of $\gamma\gamma j$. The leptonic signatures for $V_1 V_2 j$ will be discussed somewhat more in detail in Section 3.3.

Since the UA1 and UA2 Collaboration have recently reported the observation of photon pair production at the CERN $p\bar{p}$ collider¹⁹⁾ it is interesting to calculate the rate for $\gamma\gamma j$ production also for CERN collider and Tevatron energies. Using the UA1/UA2 photon cuts of $p_{T\gamma} > 12$ GeV and $|y_\gamma| < 3$ ¹⁹⁾ we find for $p_{Tj} = p_{T\gamma\gamma} > 15$ GeV and $|y_j| < 3$ a total $\gamma\gamma j$ rate of 1.8 pb in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV. Similarly, we obtain a $\gamma\gamma j$ cross

section of 3.9 pb at Tevatron energies ($\sqrt{s} = 1.8$ TeV). Here we have used cuts reported from preliminary CDF results on jet²⁰⁾ and prompt photon production:²¹⁾ $p_{T\gamma} > 15$ GeV, $|y_\gamma| < 4$, $p_{Tj} = p_{T\gamma\gamma} > 20$ GeV, $|y_j| < 4$. UA1/UA2 and CDF may thus be able to observe photon pair production associated with a high p_T jet in the near future. We remark, however, that the precise value of the $\gamma\gamma j$ production rate depends quite strongly on the cut on the photon p_T .

Incidentally this is also true for the LHC and SSC. So far we have required that $p_{T\gamma} > 50$ GeV in $\gamma\gamma j$, $Z\gamma j$ and $W^\pm\gamma j$ production. Table 2 shows how the total cross sections change if this cut is replaced by $p_{T\gamma} > 100$ GeV (all other cuts are left unchanged). For $Z\gamma j$ and $W^\pm\gamma j$ the rates drop by about a factor 2 whereas for $\gamma\gamma j$ they are reduced by a factor 4 to 5.

3.2 Non-central Jet Production in $pp \rightarrow V_1 V_2 j$

Recently it has been suggested that, using forward detectors covering the rapidity range $3 < |y| < 5$, it may be possible to detect a heavy SM Higgs boson produced via WW -fusion in the $H \rightarrow WW \rightarrow \ell\nu jj$ decay channel^{5,7)} ($\ell = e, \mu$) by tagging the two W -radiating quarks emerging with $p_T \sim M_W$. Without quark tagging the $H \rightarrow WW$ signal would be swamped by the Wjj background.¹⁸⁾

Since the Higgs boson decays isotropically, the signal for $H \rightarrow WW \rightarrow \ell\nu jj$ consists of the $\ell\nu$ and a jet pair all in the central region, and of two tagging jets in the forward region. Similarly, the decay $H \rightarrow ZZ \rightarrow \ell^+\ell^- jj$ results in a pair of tagging jets, two centrally produced jets, and in a charged lepton pair. To avoid misidentification between central and tagging jets a rapidity gap has to be required between the two jet types which we have chosen to be 0.5 (between 2.5 for the central and 3.0 for the forward region). Roughly 30% of the time, however, one of the tagging jets happens to have a rapidity between 2.5 and 3.0, or bigger than 5.0 and thus is lost.⁷⁾ In this case, where only one of the tagging jets is observed, W^+W^-j , $W^\pm Zj$ and ZZj produced with the jet in the rapidity range $3 < |y_j| < 5$ and the boson pair decaying into two leptons plus two jets in the central region are, together with $Wjjj$ production, potentially dangerous background processes for Higgs tagging. If the Higgs signal would be large one simply could afford to ignore the portion of the cross section where one of the tagging jets is lost. Since this is not the case one would like to use such events as well and it is important to know how large the background from $V_1 V_2 j$ production is.

In Fig. 8 we compare the invariant mass and the p_T distribution of the vector boson

pair in $pp \rightarrow W^+W^-j$ and $pp \rightarrow ZZj$ for jets produced in the central and in the forward direction at $\sqrt{s} = 40$ TeV. The curves for $W^\pm Zj$ are almost degenerate with the W^+W^-j lines and are not displayed to avoid overburdening the figure. In all cases we applied (3.4) with $\epsilon = 0.3$ as well as a p_{Tj} cut of 100 GeV and a rapidity cut of $|y| < 2.5$ on the vector bosons. It is obvious that the total cross section for the jet in forward direction is much smaller than for a centrally produced jet, typically a factor 20-30. Quark gluon and $q\bar{q}$ annihilation contribute about equally to the cross section. The m_{VV} as well as the p_{TVV} spectrum drops for $3 < |y_j| < 5$ considerably faster than for $|y_j| < 2.5$.

A very crude estimate of the WWj , WZj and ZZj background in the region $p_{Tj} > 100$ GeV to Higgs tagging where one of the tagging jets is lost is obtained by integrating $d\sigma/dm_{VV}$ at $m_{VV} = M_H$ over twice the Higgs width Γ_H . It turns out that for the range of M_H which we have explored ($300 \text{ GeV} \leq M_H \leq 700 \text{ GeV}$) signal and background are roughly comparable. However, since we have imposed (3.4) which suppresses $d\sigma/dm_{VV}$ for large values of m_{VV} (compare Fig. 2) we certainly underestimate the background which in reality may well be a factor 2 or 3 higher. Nevertheless our estimate demonstrates that the V_1V_2j background cannot be neglected in a detailed study of Higgs tagging where only one of the tagging jets is observed.

If a forward detector covering the $|y| > 2.5$ region is not feasible at the LHC or SSC there is another interesting new physics signal where V_1V_2j production with a non-centrally produced jet would be an important background. In this case jets produced with a rapidity of more than 2.5 would not be detected and only a vector boson pair with a large unbalanced transverse momentum would be observed. For W^+W^- this is precisely the signal one expects for pair produced charged heavy leptons L with mass $m_L > M_W$.⁸⁾ If L is accompanied by a massless neutrino ν_L it will predominantly decay into $W\nu_L$, and $L\bar{L}$ production will result in a W pair plus missing transverse momentum $p_{T,\text{miss}}$ which is the sum of the transverse momenta carried away by the neutrinos. To calculate the fake $p_{T,\text{miss}}$ distribution from W^+W^-j one just has to compute the p_{TWW} spectrum for $|y_j| > 2.5$. For transverse momenta less than 500 GeV the resulting $d\sigma/dp_{TWW}$ is about a factor 2 to 3 bigger than the one shown in Fig. 8b for $3 < |y_j| < 5$ and at least one order of magnitude above the $L\bar{L}$ signal.⁸⁾ To reject this background a cut on the opening angle of the W decay products in the transverse plane might be helpful.²²⁾

3.3 Leptonic Signatures for $V_1 V_2 j$

So far we have studied properties of the vector boson pair in $V_1 V_2 j$, regarding W - and Z -bosons as final state particles. Experimentally, of course, W and Z have to be discovered through their decays. In order to demonstrate how the matrix elements of Section 2 can be used when properties of the final state fermions from W and Z decay are of interest, we finally discuss the invariant mass and the so-called cluster transverse mass spectrum of vector boson pairs when realistic cuts are imposed on the decay products of W and Z .

Because it is uncertain whether W^+W^- , $W^\pm Z$ and ZZ pairs can be identified when one of them decays hadronically¹⁸⁾ one probably has to require both vector bosons to decay leptonically, more precisely into electrons and muons, in order to obtain a clean signal for W^+W^-j , $W^\pm Zj$ and ZZj . Similarly, the $W(Z)$ in $W\gamma j(Z\gamma j)$ probably has to decay leptonically in order to overcome the QCD background.

The experimental signal for $Z\gamma j$ thus consists of a charged lepton pair, an isolated, high p_T , electromagnetic cluster and a high p_T jet. Similarly, two charged lepton pairs, each with an invariant mass of about M_Z , and a high p_T jet are the signal for ZZj . In Fig. 9 we show the $Z\gamma$ and ZZ invariant mass spectrum at the SSC when the leptonic Z decay with branching fraction

$$\text{BR}(Z \rightarrow e^+e^-, \mu^+\mu^-) = 0.069 \quad (3.7)$$

(corresponding to a top-quark mass of 50 GeV) is included and cuts are imposed on the charged final state leptons. The cuts which we apply on the Z decay products are $p_{T\ell} > 25$ GeV and $|y_\ell| < 2.5$ ($\ell = e, \mu$). Photon and jet cuts are the same as in Section 3.1 (see eqs. (3.1) - (3.4), with $\varepsilon = 0.3$ and $p_{Tj} > 100$ GeV). For comparison, we have also included the $\gamma\gamma$ mass spectrum from $\gamma\gamma j$. The cuts on the final state leptons reduce the cross section by 15 - 35% compared with the case where a rapidity cut of 2.5 is applied to the Z -bosons.

The signal for $W\gamma j$ consists of an isolated, high p_T , electromagnetic cluster and an isolated charged lepton in association with missing p_T and a hard jet. In W^+W^-j two neutrinos are emitted which combine to form the $p_{T,\text{miss}}$ vector. In addition to missing transverse momentum W^+W^-j is signalled by two oppositely charged leptons and a high p_T jet. $W^\pm Zj$, finally, leads to a charged lepton pair invariant mass $\sim M_Z$, a third charged lepton, $p_{T,\text{miss}}$ and a large transverse momentum jet. Since the neutrino from $W \rightarrow \ell\nu$

is not observed, the $W^\pm\gamma$, $W^\pm Z$ and W^+W^- invariant mass cannot unambiguously be determined at hadron colliders. In this case the cluster transverse mass

$$m_T^2(C, p_{T,\text{miss}}) = \left(\sqrt{\mathbf{p}_{TC}^2 + m_C^2} + |\mathbf{p}_{T,\text{miss}}| \right)^2 - (\mathbf{p}_{TC} + \mathbf{p}_{T,\text{miss}})^2, \quad (3.8)$$

where $\mathbf{p}_C$ and m_C are the total momentum and invariant mass of the cluster C , is a useful quantity. In Fig. 10 we show the m_T distribution of the gauge boson pair for the three $V_1 V_2 j$ processes involving charged weak bosons at the SSC, including the leptonic branching ratio (3.7) for the Z and

$$\text{BR}(W \rightarrow e\nu, \mu\nu) = 0.193 \quad (3.9)$$

for the W . In $W\gamma j$ the cluster C consists of the photon and the charged lepton ($\mathbf{p}_C = \mathbf{p}_\gamma + \mathbf{p}_\ell$), in W^+W^-j of the two opposite charge leptons ($\mathbf{p}_C = \mathbf{p}_\ell + \mathbf{p}_{\ell'}$), and in WZj of the three charged leptons ($\mathbf{p}_C = \mathbf{p}_{\ell'} + \mathbf{p}_{\ell+} + \mathbf{p}_{\ell-}$). A $p_{T,\text{miss}} > 25$ GeV cut is applied, together with the charged lepton, photon and jet cuts of Fig. 9. Due to the two neutrinos emitted in the W^+W^- decay the m_T distribution drops for W^+W^-j faster than for $W\gamma j$ and WZj . There is no kinematical threshold in m_T for $W\gamma$ and W^+W^- whereas $m_T > M_Z$ in WZ . The cuts on the decay products of W and Z reduce the rates by 40 - 60% compared with the case where a rapidity cut of 2.5 is imposed on the weak bosons.

4. Discussion and Conclusions

We have presented, in the last section, some numerical results for the production of vector boson pairs associated with a high p_T jet and discussed two cases where $V_1 V_2 j$ production may contribute substantially to the background of new physics signals. Mostly we have concentrated on the transverse momentum and invariant mass spectrum of the gauge boson pair. All calculations were performed using the helicity amplitudes presented in Section 2. Our phenomenological discussion in Section 3 is, of course, far from being complete. In particular, properties of, and correlations among the final state fermions from W and Z decay were only briefly addressed.

In addition to the usual situation where one assumes that all particles are produced centrally we also discussed the case where the jet in $V_1 V_2 j$ is produced outside the central rapidity range $|y| < 2.5$. In this configuration some of the vector boson pair production processes may contribute to the Higgs tagging background, notably when one of the tagging

jets is not identified. If forward detectors covering the rapidity range $|y| > 2.5$ are not feasible at the LHC or SSC, W^+W^-j with the jet produced at rapidity $|y_j| > 2.5$ would also fake the $W^+W^-p_{T,\text{miss}}$ signal of a pair produced charged heavy lepton with mass $m_L > M_W$.

The results presented in this paper can be viewed as a first step towards a complete calculation of the transverse momentum spectrum of all possible vector boson pair production processes. Figures 5 and 6 show the high p_T tail of the gauge boson pair transverse momentum distribution for the LHC and SSC, which can be reliably calculated from the first order processes (2.1) and (2.2) and which may thus serve as a test for perturbative QCD. As emphasized in Section 3, a calculation of the low p_T part requires the inclusion of virtual and higher order corrections. The cross sections listed in Table 1 demonstrate that V_1V_2j production is an important source for vector boson pairs at future hadron colliders, in particular for $W^\pm\gamma$ where the rate for the lowest order process is suppressed due to the radiation zero present in this reaction.⁹⁾ If the $W\gamma$ pair is accompanied by a jet with a $p_T > 100$ GeV, no trace of this radiation zero is left and the cross section for the first order processes is even larger than the one for $q\bar{q} \rightarrow W\gamma$. However, also for the other combinations of gauge boson pairs production with an associated hard jet constitutes a substantial fraction of the lowest order cross section.

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TABLE 1

Cross sections for $V_1 V_2$ production from $q\bar{q}$ annihilation, and $V_1 V_2 j$ production for a minimum p_{TVV} of 100, 200 and 300 GeV at a) the LHC and b) the SSC. In addition eq. (3.4) with $\varepsilon = 0.3$ is imposed on p_{TVV} in the $V_1 V_2 j$ case. W , Z and the jet must have rapidity $|y| < 2.5$ while photons are required to have $p_T > 50$ GeV, $|y| < 2.5$ and to be separated from each other and the jet by $\cos \Theta_{\gamma\gamma} < 0.9$ and $\cos \Theta_{\gamma j} < 0.9$.

a) LHC cross sections ($\sqrt{s} = 17$ TeV)

process	σ [pb]	p_{TVV}^{min} process	100 GeV σ [pb]	200 GeV σ [pb]	300 GeV σ [pb]
$\gamma\gamma$	2.92	$\gamma\gamma j$	0.51	0.12	0.04
$Z\gamma$	4.43	$Z\gamma j$	0.85	0.21	0.08
ZZ	6.62	ZZj	0.85	0.22	0.08
$W^\pm\gamma$	4.51	$W^\pm\gamma j$	5.16	1.85	0.79
W^+W^-	41.8	W^+W^-j	8.97	2.93	1.17
$W^\pm Z$	16.8	$W^\pm Zj$	7.19	2.62	1.12

b) SSC cross sections ($\sqrt{s} = 40$ TeV)

process	σ [pb]	p_{TVV}^{min} process	100 GeV σ [pb]	200 GeV σ [pb]	300 GeV σ [pb]
$\gamma\gamma$	5.16	$\gamma\gamma j$	1.37	0.37	0.15
$Z\gamma$	8.24	$Z\gamma j$	2.35	0.73	0.30
ZZ	13.00	ZZj	2.43	0.84	0.36
$W^\pm\gamma$	8.66	$W^\pm\gamma j$	15.52	6.55	3.18
W^+W^-	78.9	W^+W^-j	27.3	11.0	5.00
$W^\pm Z$	33.0	$W^\pm Zj$	22.5	10.4	5.10

TABLE 2

Total rates for $\gamma\gamma j$, $Z\gamma j$ and $W^\pm\gamma j$ production at the LHC ($\sqrt{s} = 17$ TeV) and the SSC ($\sqrt{s} = 40$ TeV) for $p_{T\gamma} > 50$ GeV and $p_{T\gamma} > 100$ GeV. A cut of 100 GeV in addition to eq. (3.4) with $\epsilon = 0.3$ on the transverse momentum of the gauge boson pair is imposed. W , Z , photons and jets must have a rapidity $|y| < 2.5$. Furthermore, photons are required to be separated from each other and the jet by $\cos \Theta_{\gamma\gamma} < 0.9$ and $\cos \Theta_{\gamma j} < 0.9$.

process	$\sqrt{s} = 17$ TeV, σ [pb]		$\sqrt{s} = 40$ TeV, σ [pb]	
	$p_{T\gamma} > 50$ GeV	$p_{T\gamma} > 100$ GeV	$p_{T\gamma} > 50$ GeV	$p_{T\gamma} > 100$ GeV
$\gamma\gamma j$	0.51	0.10	1.37	0.30
$Z\gamma j$	0.85	0.37	2.35	1.14
$W^\pm\gamma j$	5.16	2.24	15.52	7.46

FIGURE CAPTIONS

1. Feynman graphs contributing to $q\bar{q} \rightarrow V_1 V_2 g$, $qg \rightarrow V_1 V_2 q$ and $\bar{q}g \rightarrow V_1 V_2 \bar{q}$. The diagrams given in b) contribute only to processes with a $W^\pm$ in the final state. V_1 and V_2 generically denote the electroweak vector bosons.
2. ZZ invariant mass spectrum at the SSC. The dashed curve is for $q\bar{q} \rightarrow ZZ$. The solid, dotted and dash-dotted lines show the results for ZZj and three different p_{TZZ} cuts: $p_{TZZ} > 100$ GeV and no ε -cut (solid), $p_{TZZ} > 100$ GeV and $\varepsilon = 0.2$ (dotted), and $p_{TZZ} > 100$ GeV and $\varepsilon = 0.3$ (dash-dotted). Rapidity cuts $|y| < 2.5$ are imposed on the Z -bosons and the jet.
3. Invariant mass spectrum of the electroweak boson pair for $W^\pm \gamma j$ (dash-dotted line), $Z \gamma j$ (dashed line) and $\gamma \gamma j$ (solid line) at a) the LHC and b) the SSC. For $W^\pm \gamma j$ the sum of the $W^+ \gamma j$ and $W^- \gamma j$ cross section is shown. m_{VV} generically denotes the invariant mass of the boson pair. A cut of 100 GeV in addition to eq. (3.4) with $\varepsilon = 0.3$ is imposed on p_{TVV} . Moreover, W , Z and the jet must have rapidity $|y| < 2.5$ while photons are required to have $p_T > 50$ GeV, $|y| < 2.5$ and to be separated from each other and the jet by $\cos \Theta_{\gamma\gamma} < 0.9$ and $\cos \Theta_{\gamma j} < 0.9$.
4. Invariant mass spectrum of the electroweak boson pair for $W^+ W^- j$ (solid line), $W^\pm Z j$ (dashed line) and $ZZ j$ (dash-dotted line) at a) the LHC and b) the SSC. For $W^\pm Z j$ the sum of the $W^+ Z j$ and $W^- Z j$ cross section is plotted. m_{VV} generically denotes the invariant mass of the boson pair. A cut of 100 GeV in addition to eq. (3.4) with $\varepsilon = 0.3$ is imposed on p_{TVV} . W , Z and the jet must satisfy $|y| < 2.5$.
5. Transverse momentum spectrum of the electroweak boson pair for $W^\pm \gamma j$ (dash-dotted line), $Z \gamma j$ (dashed line) and $\gamma \gamma j$ (solid line) at a) the LHC and b) the SSC. For $W^\pm \gamma j$ the sum of the $W^+ \gamma j$ and $W^- \gamma j$ cross section is shown. p_{TVV} generically denotes the transverse momentum of the boson pair. A cut of 100 GeV in addition to eq. (3.4) with $\varepsilon = 0.3$ is imposed on p_{TVV} . Moreover, W , Z and the jet must have rapidity $|y| < 2.5$ while photons are required to have $p_T > 50$ GeV, $|y| < 2.5$ and to be separated from each other and the jet by $\cos \Theta_{\gamma\gamma} < 0.9$ and $\cos \Theta_{\gamma j} < 0.9$.
6. Transverse momentum spectrum of the electroweak boson pair for $W^+ W^- j$ (solid line), $W^\pm Z j$ (dashed line) and $ZZ j$ (dash-dotted line) at a) the LHC and b) the SSC. For $W^\pm Z j$ the sum of the $W^+ Z j$ and $W^- Z j$ cross section is plotted. p_{TVV} generically denotes the transverse momentum of the boson pair. A cut of 100 GeV in addition to eq. (3.4) with $\varepsilon = 0.3$ is imposed on p_{TVV} . W , Z and the jet must satisfy $|y| < 2.5$.

7. Center of mass rapidity spectrum of the photon for $pp \rightarrow W^+\gamma$ (solid line) and $pp \rightarrow W^+\gamma j$ (dashed line) at $\sqrt{s} = 40$ TeV. A cut of 100 GeV in addition to eq. (3.4) with $\varepsilon = 0.3$ is imposed on $p_{TW\gamma}$. Moreover, the W and the jet must have rapidity $|y| < 2.5$ while the photon is required to have $p_T > 50$ GeV, $|y| < 2.5$ and to be separated from the jet by $\cos \Theta_{\gamma j} < 0.9$.
8. Comparison of a) the invariant mass spectrum and b) the transverse momentum spectrum of the electroweak boson pair for $|y_j| < 2.5$ and $3 < |y_j| < 5$ in $pp \rightarrow W^+W^-j$ (dashed line) and $pp \rightarrow ZZj$ (solid line) at $\sqrt{s} = 40$ TeV. In addition to eq. (3.4) with $\varepsilon = 0.3$ a cut of 100 GeV is imposed on p_{TWW} and p_{TZZ} . W 's and Z 's are required to have a rapidity $|y| < 2.5$.
9. $Z\gamma$ (dashed line) and ZZ (dash-dotted line) mass distribution $B \cdot d\sigma/dm_{VV}$ in $pp \rightarrow Z\gamma j \rightarrow \ell^+ \ell^- \gamma j$ and $pp \rightarrow ZZj \rightarrow \ell^+ \ell^- \ell'^+ \ell'^- j$ ($\ell, \ell' = e, \mu$) at $\sqrt{s} = 40$ TeV for $p_{T\ell, \mu} > 25$ GeV and $|y_{e, \mu}| < 2.5$. For comparison, the $\gamma\gamma$ mass spectrum from $\gamma\gamma j$ is also shown (solid line). B generically denotes the branching fraction for the leptonic Z decays: $B = 1$ in $\gamma\gamma j$, $B = 0.069$ in $Z\gamma j$ and $B = 4.7 \cdot 10^{-3}$ in ZZj . The photon and jet cuts are the same as in Fig. 3.
10. Distribution of the cluster transverse mass $B \cdot d\sigma/dm_{TVV}$ of the electroweak boson pair for $pp \rightarrow W^\pm \gamma j \rightarrow \ell^\pm j p_{T, \text{miss}}$ ($m_T(\ell\gamma, p_{T, \text{miss}})$, solid line), $pp \rightarrow W^+W^-j \rightarrow \ell^+ \ell'^- j p_{T, \text{miss}}$ ($m_T(\ell\ell', p_{T, \text{miss}})$, dashed line) and $pp \rightarrow W^\pm Z j \rightarrow \ell'^\pm \ell^+ \ell^- j p_{T, \text{miss}}$ ($m_T(\ell'\ell^+\ell^-, p_{T, \text{miss}})$, dash-dotted line) ($\ell, \ell' = e, \mu$) at $\sqrt{s} = 40$ TeV. m_{TVV} generically denotes the cluster transverse mass and B the leptonic branching fraction for W 's and Z 's: $B = 0.193$ in $W\gamma j$, $B = 0.037$ in W^+W^-j , and $B = 0.013$ in WZj . For $W\gamma j$ and WZj the sum over the two W -charges is taken. The following cuts are applied on the W and Z decay products: $p_{T\ell, \mu} > 25$ GeV, $|y_{e, \mu}| < 2.5$, $p_{T, \text{miss}} > 25$ GeV. Photon and jet cuts are the same as in Fig. 3.

Fig. 1a

Fig. 1b

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6a

Fig. 6b

Fig. 7

Fig. 8a

Fig. 8b

Fig. 9

Fig. 10