CERN-TH.5216/88

GLUONS AND THE SPIN OF THE PROTON

Gérard Clément

Laboratoire de Physique Théorique
Université de Nice

and

Jacqueline Stern

CERN - Geneva

A B S T R A C T

The recent observation that constituent quark helicities carry very little of the spin of the proton, in contrast with naive quark model expectations, can be understood by taking into account non-perturbative aspects of QCD, such as chiral symmetry breaking and confinement. We compute the non-strange quark contribution $\Delta u + \Delta d$ to the proton spin in a chiral field-theoretical model of quark and gluon confinement. Our mean field computations include self-consistently the effects of gluon exchange between constituent quarks. When the parameters of the model, including the effective strong fine structure constant α_s , are adjusted to reproduce as far as possible the experimental mean nucleon-delta mass and the relative nucleon-delta mass splitting, we obtain $\Delta u + \Delta d \approx 0.25$, which is only one third of the $\alpha_s = 0$ value. We show that this implies that all the proton spin is in fact orbital angular momentum.

A straightforward analysis of recent measurements of the polarized proton structure function [1] leads to the conclusion [2,3] that very little, or none, of the proton spin is carried by quark helicities:

$$\Delta q = \Delta u + \Delta d + \Delta s = \langle p | \psi^\dagger \sigma_3 \psi | p \rangle = -0.03 \pm 0.21, \quad (1)$$

the positive contribution of the non-strange quarks [3]

$$\Delta u + \Delta d = 0.21 \pm 0.14 \quad (2)$$

being balanced by a negative strange quark contribution Δs . Now, where does then the proton spin come from? An obvious possibility is orbital angular momentum [4,5]. A computation [6], in an SU(2) chiral quark-meson model, of the contributions of the non-strange quark helicities $\Delta u + \Delta d$ and mean orbital angular momentum $2\langle L_z \rangle$ to the proton spin, led to results consistent with the "experimental" determination (2).

It was suggested in reference [6] that the agreement with experiment should be improved by taking into account the effects of gluons. Now, gluons enter in this problem in three different ways, leading to contributions which we shall term the direct, anomalous, and indirect gluon contributions to the proton spin.

First, the sum rule

$$\Delta q + 2\langle L_z \rangle_q + 2\Delta g + 2\langle L_z \rangle_g = 1 \quad (3)$$

shows that gluons may contribute directly to the proton spin, both through their helicity Δg , and their angular momentum $\langle L_z \rangle_g$ [3,7].

These contributions were investigated in reference [3], in the framework of an extended Skyrme model [8,9], where the chiral field U interacts with a scalar gluonium field χ . The authors of reference [3] gene-

rated the physical proton from the (spinless) Skyrme soliton by a rigid non-relativistic rotation, and found that $\Delta q = 0$, while if the parameters of the model are adjusted so that gluons carry 50% of the proton momentum, they carry about 36% of the orbital angular momentum. However, we feel that this last result is not convincing because if, following reference [9], we rotate the Skyrme soliton in isospace, then the scalar gluonium field does not contribute to the angular momentum.

Second, there is an anomalous [10] gluon contribution to the matrix element (1) of the spin operator [7,11], so that in the left-hand side of equations (1) and (2), $\Delta u, \Delta d, \Delta s$ should be replaced by the q^2 -independent combinations [7]

$$\Delta \tilde{u} = \Delta u - \frac{\alpha_s}{2\pi} \Delta q, \quad (4)$$

etc... The effect of the anomaly is thus to relax the experimental constraint (2) for the contribution of the non-strange quark helicities, which is replaced by the relation between the quark and gluon contributions

$$\Delta u + \Delta d = 0.21 + 2 \frac{\alpha_s}{2\pi} \Delta q \pm 0.14. \quad (5)$$

Finally, it has been proposed [6,12] that gluon exchange between constituent quarks may affect the relative contributions of quark helicities and orbital angular momentum, and thus contribute indirectly to the proton spin. It is this last contribution which we shall compute in an $SU(2) \times SU(2)$ chiral field-theoretical model of quark and gluon confinement [13], treated in the mean field approximation. The

fact that $SU(2) \times SU(2)$ is a better symmetry than $SU(3) \times SU(3)$ leads us to expect that our 2-flavour computations shall give substantially correct values for the non-strange contribution $\Delta u + \Delta d$. Radiative corrections are absent from our treatment, so that there are only exchanged gluons; we shall show that these do not contribute directly to the proton spin. However, comparison of our results with experiment should allow for the anomalous radiative correction (5), and therefore give an estimate for the unknown $(\alpha_s/2\pi)\Delta g \approx \Delta u + \Delta d - 0.21$. In fact, we shall show that, in the framework of the model, the data favours $(\alpha_s/2\pi)\Delta g \approx 0$, implying $\Delta g \approx 0$.

The model we shall use is defined by the lagrangian [14,13]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} K F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} [i\gamma^\mu D_\mu - f(\sigma - i\vec{E} \cdot \vec{\phi} \gamma^5)] \psi \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{\lambda^2}{4} (\sigma^2 + \vec{\phi}^2 - F_\pi^2 + \frac{m_\pi^2}{\lambda^2})^2 \\ & + F_\pi m_\pi^2 \sigma, \end{aligned} \quad (6)$$

where D_μ is the gauge covariant derivative

$$D_\mu \psi = \partial_\mu \psi + ig_s \frac{\lambda_a}{2} A_\mu^a \psi \quad (7)$$

and the dielectric function K [15] depends on the chiral invariant $\sigma^2 + \vec{\phi}^2$, and goes to zero as the chiral fields $(\sigma, \vec{\phi})$ go to the bottom $(F_\pi, 0)$ of the potential well, thereby confining the gluonic field $F_{\mu\nu}^a$.

Let us first show that, in the stationary mean field approximation, the direct gluonic contribution to the proton spin is zero. To simplify the discussion, we shall treat the gluonic field in the abelian approximation. The total angular momentum tensor of the gluonic field is [16]

$$J^{\psi} = \int d^3x \left[(x^i D_a^k \partial_j A_a^k - x^j D_a^k \partial_i A_a^k) + (D_a^i A_a^j - D_a^j A_a^i) \right] \quad (8)$$

with $D_a^k = K E_a^k$. The first part of this expression looks like the orbital contribution, the second like the spin contribution. However, this decomposition is meaningful only for physical (transverse) gluons. The canonical fields for these are A_a^k , transverse in the Coulomb gauge $\partial_k A_a^k = 0$, and the transverse projection $D_{aT}^k = D_a^k - (\partial_k \partial_\ell / \Delta) D_a^\ell$. It is easy to show by integration by parts that transverse projection conserves the total angular momentum (8):

$$J^{\dot{ij}} = J_T^{\dot{ij}}. \quad (9)$$

Now, if gluons are unconfined ($K = 1$) and in a static, abelian field configuration, $\vec{D}_a = -\vec{\nabla} V_a$ is longitudinal and so both the transverse gluonic spin and orbital angular momentum are zero. In the case of a variable dielectric function K , the proof is less general and relies on spherical symmetry, which is implicitly assumed in the mean field approach: the radial vector field $\vec{D}_a$ is necessarily longitudinal, $\vec{D}_a = \vec{\nabla} \phi_a$, where ϕ_a depends only on r , and the conclusion is the same as before. Obviously, this argument is model independent.

Localized stationary solutions, representing a hadron, of the field equations deriving from a chiral lagrangian such as (6) are usually obtained in the mean field approximation by assuming that the N constituent quarks are in the hedgehog state

$$q_0 = \begin{pmatrix} u(r) \chi_h \\ i v(r) \vec{\sigma} \cdot \vec{e} \chi_h \end{pmatrix} \quad (10)$$

with $(\vec{\sigma} + \vec{e}) \chi_h = 0$. The mean colour fields $\vec{E}_a, \vec{B}_a$, and the associated potentials $V_a, \vec{A}_a$ are related to the N quark mean fields q^i by the

classical field equations

$$\begin{aligned}\vec{\nabla} \cdot (\kappa \vec{E}_a) - g_s f_{abc} \kappa \vec{E}_b \cdot \vec{A}_c &= g_s \sum_{i=1}^N q^{i+} \frac{\lambda_3}{2} q^i, \\ \vec{\nabla}_\perp (\kappa \vec{B}_a) - g_s f_{abc} \kappa \vec{E}_b V_c + g_s f_{abc} \kappa \vec{B}_b \wedge \vec{A}_c &= \\ &= g_s \sum_{i=1}^N q^{i+} \frac{\lambda_3}{2} \vec{\alpha} q^i.\end{aligned}\quad (11)$$

If all the quarks are in the same state, the total colour charges of hadrons (colour singlets) are zero, as well as the chromoelectric fields $\vec{E}_a$. Moreover, if the quarks are in the hedgehog state (3), which is such that $\chi_k^\dagger \vec{\sigma} \chi_k = 0$, the colour currents of the various quarks are (independently) zero, and so the chromomagnetic fields $\vec{B}_a$ are zero. So at this stage the localized solutions of the theory are identical to those obtained in the case $g_s = 0$ [14].

The resulting hedgehog hadrons have zero mean spin and isospin, but physical hadrons may be recovered from these by the cranking procedure [9,17,18], which consists in isorotating the mean fields q and $U = \sigma - i\gamma^5 \vec{\tau} \cdot \vec{\phi}$ to

$$\begin{aligned}q' &= e^{-i\frac{\vec{E}}{2} \cdot \vec{\omega} t} q, \\ U' &= e^{-i\frac{\vec{\tau}}{2} \cdot \vec{\omega} t} U e^{i\frac{\vec{\tau}}{2} \cdot \vec{\omega} t},\end{aligned}\quad (12)$$

and extremizing the cranked energy $\langle H + \vec{\omega} \cdot \vec{I} \rangle$, where $\vec{I}$ is the field-theoretical isospin operator. The resulting cranked mean spin and isospin are then, for slow rotations, linear in $\vec{\omega}$:

$$\langle \vec{J} \rangle = -\langle \vec{I} \rangle = \mathcal{I} \vec{\omega} + O(\vec{\omega}^2), \quad (13)$$

where $\mathcal{I}$ is the hadronic moment of inertia, which may be decomposed as the sum of quark spin, quark orbital, and meson contributions [6]:

$$\mathcal{J} = \mathcal{J}_{qs} + \mathcal{J}_{qe} + \mathcal{J}_{qm} . \quad (14)$$

The various observables of a hadron of spin-isospin j depend on the cranking parameter $\omega = |\vec{\omega}|$, which is obtained by solving the physical constraint [18]

$$\langle \vec{J}^2 \rangle' = \langle \vec{I}^2 \rangle' = (\Delta \vec{I})^2(\omega) + \mathcal{J}^2 \omega^2 = j(j+1) . \quad (15)$$

Cranking does not directly affect the mean colour fields, but gives rise to non-vanishing currents which generate non-vanishing chromomagnetic fields $\vec{B}_a$ (the chromoelectric fields are of course still zero). The extremization of the cranked energy thus involves self-consistently [19] the cranked mean fields q' , U' , $\vec{B}'_a$. To carry out this extremization, we expand the cranked mean fields in powers of ω [17]

$$q' = q_0 + \omega q_1 + \dots , \quad U' = U_0 + \omega U_1 + \dots , \quad \vec{B}'_a = \omega \vec{B}_{1a} + \dots . \quad (16)$$

The corresponding expansion of the cranked energy is

$$E' \equiv \langle H + \vec{\omega} \cdot \vec{I} \rangle' = E_0 + \omega^2 E_2 + \dots . \quad (17)$$

The second-order energy may be decomposed as

$$E_2 = E_{2q} + E_{2m} + E_{2g} , \quad (18)$$

where

$$E_{2q} = \int d^3x \, N [q_1^\dagger (\hat{h}_0 - \epsilon_0) q_1 + q_1^\dagger \hat{\omega} \cdot \vec{\tau} q_0 + q_0^\dagger \hat{\omega} \cdot \vec{\tau} q_1] \quad (19)$$

($\hat{h}_0 = \vec{\alpha} \cdot \vec{p} + \beta (\sigma_0 - i \vec{\tau} \cdot \vec{\tau} \gamma^5)$ is the zeroth order Dirac hamiltonian, ϵ_0 the associated Dirac energy), E_{2m} is linear in the second-order meson fields (the first order meson fields are identically zero [17]) and so does not contribute to the variational equations, and

$$E_{2g} = \int d^3x \sum_a \sum_{i \neq j} \left[\frac{K_0}{2} \vec{B}_{ai}^a \cdot \vec{B}_{aj}^a - g_s \vec{A}_{ai}^a \langle \frac{\lambda^a}{2} \rangle_j \cdot (q_i^+ \vec{\alpha} q_0 + q_0^+ \vec{\alpha} q_i) \right]. \quad (20)$$

In writing (20) we have excluded from the sum, as in previous bag or soliton bag treatments [19-21], the chromomagnetic self-energy ($i=j$) terms; note also that, although we have not restricted ourselves to one-gluon exchange, non-linear terms are automatically absent from the second order energy (20).

For our calculations [22] we used a family of dielectric functions of the form [23]

$$K = \left(1 - \frac{\sigma^2 + \vec{q}^2}{F_\pi^2} \right)^n \quad (21)$$

with $n \geq 0$. The value $n = 0$ corresponds to unconfined gluons; the requirement that the string tension, which may also be computed in such a model, be finite leads to the restriction $n > 1$ [23]. We chose for the chiral parameters the physical values $F_\pi = 93$ MeV, $m_\pi = 139.6$ MeV, leaving us with the three parameters f , $m_\sigma^2 = 2\lambda^2 F_\pi^2 + m_\pi^2$, $\alpha_s \equiv g_s^2/4\pi$ (the effective strong fine structure constant), which can be adjusted to reproduce, as far as possible, the experimental values of the mean nucleon-delta mass $\bar{E}$ (1085 MeV) and of the relative nucleon-delta mass splitting $\Delta E/\bar{E}$ (0.27). The fully quantum-corrected nucleon and delta energies are given by [18]

$$E = M + \frac{1}{2J} [j(j+1) - (\Delta \vec{I})^2] , \quad (22)$$

where M is the hedgehog mass corrected for translational fluctuations [24,18], $j(j+1)/2J$ is the cranking contribution, and $-(\Delta \vec{I})^2/2J$ the

ω -dependent correction for rotational fluctuations [17,18]. The dimensionless ratio $\Delta E/\bar{E}$ depends on $\mathcal{J}$ and thus on α_s , which we chose (as in our previous treatments of gluonic effects [13,25,18]) to fix, for given values of f and m_σ , so that

$$\frac{\Delta E}{\bar{E}}(\alpha_s) = 0.27 \quad . \quad (23)$$

The other two parameters are not completely independent because, for a given value of m_σ , there is a critical value f_c of f below which there is no mean field solution [26,22]. We chose $f = f_c(m_\sigma)$, which gives the best values [22] for the mean nucleon-delta mass; for this choice, K is a monotonous function of κ . The mean nucleon-delta mass also becomes better as m_σ decreases, until it reaches a plateau at $\bar{E} \simeq 1010$ MeV for $m_\sigma \lesssim 600$ MeV; however for $m_\sigma < 600$ MeV the isoscalar nucleon mean square radius $\langle r^2 \rangle_{I=0}$ becomes much too high to be acceptable, so that our preferred value is $m_\sigma = 600$ MeV (for which $f_c = 4.05$).

For these values, the variation with α_s of the contribution of the non-strange quark helicities to the proton spin

$$\Delta u + \Delta d = 3\omega \int d^3x (q_1^+ \sigma_3 q_0 + q_0^+ \sigma_3 q_1) \equiv 2\mathcal{J}_{q_0} \omega \quad (24)$$

is shown in fig.1 for $n = 0, 1, 2$. For $n = 0$ (unconfined gluons) and $n = 1$, the values of α_s given by equation (23) are uncomfortably large ($\alpha_s \gg 2$). For $n = 2$ [19], which leads to a finite string tension, $\Delta u + \Delta d$ varies quickly with α_s , showing that the effect of exchanged gluons (the indirect gluon contribution) is crucial. The correct relative nucleon-delta mass splitting $\Delta E/\bar{E}$ is obtained for $\alpha_s = 0.38$, corresponding to

$$\Delta u + \Delta d = 0.25 \quad (n=2, m_\sigma = 600 \text{ MeV}, f = 4.05) , \quad (25)$$

which is only one-third of the $\alpha_s = 0$ value. When n is further increased this value does not vary much, the value obtained for $n=3$, $\alpha_s = 0.05$ being $\Delta u + \Delta d = 0.20$. Similarly, $\Delta u + \Delta d$ varies slowly with m_σ , the value for $n=2$, $m_\sigma = 1200 \text{ MeV}$, $f = 3.95$, $\alpha_s = 0.05$ being $\Delta u + \Delta d = 0.11$.

Our result (25) for our preferred values of the model parameters (which give quite good values for the other baryonic observables [22]) is consistent with the EMC data, equation (5), if $\alpha_s \Delta g \simeq 0$. Taking into account the strange quark sea contribution, which cannot be computed in our approach, but is estimated [3] to be $\Delta s = -0.24 \pm 0.07$, we thus conclude that

$$\Delta q \simeq 0 , \quad \Delta g \simeq 0 , \quad \langle L_z \rangle \simeq \frac{1}{2} \quad (26)$$

(as anticipated in reference [3] and [5]), so that all the proton spin is in fact due to orbital angular momentum. To arrive at this conclusion it was essential to take into account the effect of gluon exchange between quarks, which is both necessary to account for the observed nucleon delta mass splitting, and enhances the relative non-strange quark orbital contribution.

Acknowledgement

One of us (G.C.) wishes to thank the CERN Theory Division for the kind hospitality afforded during the completion of this work.

References

-
- [1] EM Collaboration, J. Ashman et al., Phys. Lett. 206B (1988) 364.
 - [2] J. Ellis, R. Flores and S. Ritz, Phys. Lett. 198B (1987) 393;
M. Gluck and E. Reya, Dortmund preprint DO-TH-87/14 (1987).
 - [3] J. Ellis and M. Karliner, Phys. Lett. 213B (1988) 73.
 - [4] P.G. Ratcliffe, Phys. Lett. 192B (1987) 180.
 - [5] G.P. Ramsey, J.-W. Qiu, D. Richards and D. Sivers, Argonne preprint
ANL-HEP-PR-88-16 (1988).
 - [6] J. Stern and G. Clément, Mod. Phys. Lett. A (1988).
 - [7] G. Altarelli and G.G. Ross, Phys. Lett. 212B (1988) 391.
 - [8] T.H.R. Skyrme, Proc. Roy. Soc. A260 (1961) 127; Nucl. Phys. 31
(1962) 556.
 - [9] G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B228 (1983) 433.
 - [10] S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cim. A51 (1969) 47.
 - [11] A.V. Efremov and O.V. Teryaev, Dubna preprint E2-88-287 (1988).
 - [12] F. Myhrer and A.W. Thomas, Phys. Rev. D38 (1988) 1633.
 - [13] M. Bourenane, J. Stern and G. Clément, Mod. Phys. Lett. A3 (1988)
615.
 - [14] M.C. Birse and M.K. Banerjee, Phys. Lett. 136B (1984) 284; Phys.
Rev. D31 (1985) 118;
S. Kahana, G. Ripka and V. Soni, Nucl. Phys. A415 (1984) 351.
 - [15] T.D. Lee, Phys. Rev. D19 (1979) 1802.
 - [16] J.D. Bjorken and S.D. Drell, "Relativistic Quantum Fields", Mc Graw
Hill (New York) 1965.

- [17] T.D. Cohen and M.K. Banerjee, Phys. Lett. 167B (1986) 21;
T.D. Cohen and W. Broniowski, Phys. Rev. D34 (1986) 3472.
- [18] G. Clément and J. Stern, Mod. Phys. Lett. A3 (1988).
- [19] Q. Haider and L.C. Liu, J. Phys. G : Nucl. Phys. 12 (1986) 452;
M. Bickeboller, M.C. Birse and L. Wilets, Z. Phys. A - Atomic
Nuclei 326 (1987) 89;
L.R. Dodd and A.G. Williams, Phys. Lett. B210 (1988) 10.
- [20] T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12
(1975) 2060.
- [21] D.E. Kahana and J. Milana, Nucl. Phys. A468 (1987) 493.
- [22] J. Stern and G. Clément, in preparation.
- [23] M. Bickeboller, M.C. Birse, H. Marshall and L. Wilets, Phys. Rev.
D31 (1985) 2892.
- [24] J.L. Dethier, R. Goldflam, E.M. Henley and L. Wilets, Phys. Rev.
D27 (1983) 2191.
- [25] J. Stern, M. Bourenane and G. Clément, Mod. Phys. Lett. A3 (1988).
- [26] W. Broniowski and T.D. Cohen, Nucl. Phys. A458 (1986) 452.

Figure caption

Fig.1 The non-strange quark contribution to the proton spin as a
function of α_s for $n = 0, 1, 2$ ($m_p = 600$ MeV, $\beta = 4.05$).