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ELECTROMAGNETIC CORRECTIONS TO THE EFFECTIVE HAMILTONIAN
FOR STRANGENESS CHANGING DECAYS AND ϵ'/ϵ

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A B S T R A C T

The effective Hamiltonian for $\Delta S = 1$ decays is calculated in the leading logarithmic approximation including electromagnetic corrections to $O(\alpha \log M^2)$. New terms, neglected in previous analyses, appear in the theoretical expression for ϵ'/ϵ : they give a minor contribution, except for calculations done in the large N approximation. The isospin breaking corrections are also discussed.

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1. - INTRODUCTION

In recent years, there has been a certain debate on the rôle of electromagnetic and isospin breaking corrections in the theoretical evaluation of the CP violating amplitude ϵ' . Since the result of the CERN experiment NA31 [1] gives for the first time an indication of a non-zero ϵ' , this issue is now of considerable interest.

The reason why the electromagnetic corrections could be relevant is twofold. On the one hand they give a contribution to the imaginary part of the $\Delta I = 3/2$ decay amplitude

$$A_2 = A [K^0 \rightarrow \pi\pi (I=2)]$$

and, recalling the expression for ϵ'

$$\epsilon' \simeq \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \frac{1}{\text{Re } A_0} \left(\text{Im } A_2 - \frac{\text{Re } A_2}{\text{Re } A_0} \text{Im } A_0 \right) \quad (1)$$

one sees that a contribution to $\text{Im} A_2$ is not suppressed by the small factor

$$\omega = \frac{\text{Re } A_2}{\text{Re } A_0} \simeq 0.045 \quad (2)$$

as it is the normal (gluonic penguin) term [2] that contributes to $\text{Im} A_0$ only^{*}). The same is true for the isospin breaking corrections.

On the other hand, the electromagnetic penguin (EMP) diagram (Fig. 1) induces in the effective Hamiltonian terms containing operators (Q_7 and Q_8 in the notation of Ref. [3] that we will follow in this paper) that transform under the chiral $SU(3) \times SU(3)$ group as $(8_L, 8_R)$. In chiral perturbation theory, the matrix elements of these operators are expected to be bigger than those of the normal penguin operator Q_6 , transforming as $(8_L, 1_R)$. This is a consequence of a chiral suppression [3,4] in the matrix elements of Q_6 that is not present for the EMP operators.

Separating the two types of contributions to $\text{Im} A_2$,

$$\text{Im } A_2 = (\text{Im } A_2)_{em} + (\text{Im } A_2)_{IB} \quad (3)$$

^{*}) I am using here the so-called Kobayashi-Maskawa phase convention (as opposed to the Wu-Yang phase convention in which $\text{Im} A_0 = 0$).

and defining

$$\Omega_{\substack{em \\ (IB)}} = (\text{Im } A_2)_{\substack{em \\ (IB)}} / (\omega \text{Im } A_0) \quad (4)$$

one obtains the usual parametrization [5] of the corrections that we want to discuss

$$\epsilon' = \epsilon'_{(0)} \left(1 - \Omega_{em} - \Omega_{IB} \right) \quad (5)$$

The isospin breaking correction Ω_{IB} , due to π^0 - η and π^0 - η' mixing, is known to be positive and non-negligible: we will also discuss it later on.

After some conflicting results [3,5-7], the conclusion that the electromagnetic correction Ω_{em} is negative was reached. An important difference still remains in its numerical estimates: the results obtained in lattice QCD calculations [7-9] are much larger than the $N \rightarrow \infty$ (N = number of colours) estimate of Ref. [6].

In this paper we reconsider the effective Hamiltonian in the presence of electromagnetism and show that there are other contributions to $(\text{Im} A_2)_{em}$ that were neglected in the previous analyses.

In fact, once electromagnetic corrections are introduced, they generate an imaginary part not only for the coefficients of the EMP operators

$$Q_7 = 4 \bar{s}_L \gamma_\mu d_L \left(\bar{u}_R \gamma^\mu u_R - \frac{1}{2} \bar{d}_R \gamma^\mu d_R - \frac{1}{2} \bar{s}_R \gamma^\mu s_R \right) \quad (6)$$

$$Q_8 = -8 \left(\bar{s}_L u_R \bar{u}_R d_L - \frac{1}{2} \bar{s}_L d_R \bar{d}_R d_L - \frac{1}{2} \bar{s}_L s_R \bar{s}_R d_L \right) \quad (7)$$

but also for the coefficient of the $(27_L, 1_R)$ operator

$$\begin{aligned} \frac{3}{2} Q_1 + Q_2 - \frac{1}{2} Q_3 &= 4 \left(\bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L \right) - \\ &- 2 \left(\bar{s}_L \gamma_\mu d_L \bar{d}_L \gamma^\mu d_L + \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu s_L \right) \end{aligned} \quad (8)$$

that will also give a contribution to $\text{Im} A_2$.

As will be shown in Section 3, the electromagnetic correction to ϵ' can be written as follows

$$\Omega_{em} = - \frac{\sqrt{2} \alpha}{\omega} \left\{ c_6 B_6 Z + c_5 B_5 \frac{1}{N} Z + \dots \right\}^{-1} \cdot \left[\tilde{c}_8 B_8 \left(Y - \frac{X}{2N} \right) + \tilde{c}_7 B_7 \left(\frac{Y}{N} - \frac{X}{2} \right) + \frac{\tilde{c}_1 + \tilde{c}_2}{3} B_{3/2} \left(1 + \frac{1}{N} \right) X \right] \quad (9)$$

To explain the symbols used in Eq. (9), we note:

- i) the coefficients c_i are defined, in terms of the basis of operators Q_i used in Refs. [3,7], by the relations

$$J_m H = - \frac{G_F}{\sqrt{2}} J_m (V_{ts}^* V_{td}) \sum_i c_i Q_i \quad (10)$$

and $\tilde{c}_i = c_i / \alpha \approx 137 c_i$;

- ii) the vacuum insertion approximation for the matrix elements amounts to setting the parameters B_i equal to one;
iii) with $f_\pi = 130$ MeV and

$$\nu = \frac{M_R^2}{m_u + m_d} = \frac{M_{K^0}^2}{m_s + m_d} \quad (11)$$

one has

$$\begin{aligned} X &= f_\pi (M_K^2 - M_\pi^2) \\ Y &= f_\pi \nu^2 \simeq 12 X \left(\frac{150 \text{ MeV}}{m_s} \right)^2 \\ Z &= f_\pi \nu^2 4 \frac{M_K^2 - M_\pi^2}{\Lambda_\chi^2} \simeq Y \end{aligned} \quad (12)$$

In Eq. (12), $\Lambda_\chi \simeq 1$ GeV is the chiral symmetry breaking scale.

The contribution omitted in the previous analyses is the last term in Eq. (9). Also the X contributions in the matrix elements of Q_7 and Q_8 were up to now neglected. Since $Y \gg X$ and, as we will see, $2(\tilde{c}_1 + \tilde{c}_2)/(3\tilde{c}_7)$ is near to one, these contributions are in fact of minor importance in a calculation with $N = 3$.

However, in a calculation in the large N limit, as performed in Ref. [6], these terms - that are of the same order in the $1/N$ expansion as the $\tilde{c}_8 Y$ term - must be kept and may become important, given that the $N \rightarrow \infty$ calculation [6,7] gives $\tilde{c}_8 \ll \tilde{c}_7$.

In any case, new terms are present in the complete one-loop effective Hamiltonian, and we present in the following their calculation.

The paper is organized as follows. In Section 2 we discuss the strong and electromagnetic leading logarithmic corrections to obtain the effective Hamiltonian for strangeness changing decays: we will see that the operator basis used in the literature [3] is not complete when the number of active flavours is larger than three (the complete anomalous dimension matrices are given in Appendix A). The effective Hamiltonian is then used in Section 3 to discuss the electromagnetic corrections to the $\Delta I = 3/2$ decay amplitude and in particular to its imaginary part and to ϵ' . In Section 4 we briefly discuss the isospin breaking corrections to A_2 , both for its real and for its imaginary parts. Conclusions are given in Section 5. A comment on the calculation technique used in previous works is contained in Appendix B.

2. - ONE-LOOP EFFECTIVE HAMILTONIAN

In the six-quark standard model and using the unitarity of the Cabibbo-Kobayashi-Maskawa matrix V , the uncorrected effective Hamiltonian, appropriate at a scale $M \approx m_W$, can be written

$$H^{(0)} = H_c^{(0)} + H_t^{(0)} \quad (13)$$

with

$$H_q^{(0)} = - \frac{G_F}{\sqrt{2}} V_{qs}^* V_{qd} O_q \quad (14)$$

$$O_q = 4 \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L - (u \rightarrow q) \quad (15)$$

The effect of QCD corrections has been studied since many years [10-14]. The effective Hamiltonian at a scale $\mu \ll M$ is obtained, as is well known, expanding both $H^{(0)} = H(M)$ and $H(\mu)$ in a complete basis of four-fermion operators

$$H^{(0)} = \sum_i c_i^{(0)} O_i$$

$$H = \sum_i c_i O_i \quad (16)$$

The coefficients c_i obey a renormalization group equation ($t = \log(M^2/\mu^2)$)

$$\left(-\frac{\partial}{\partial t} + \beta_s(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) c_i + c_\kappa \Gamma_{\kappa i} = 0 \quad (17)$$

where the matrix Γ is defined by^{*)}

$$O_i^{(unrenorm.)} = \left(\delta_{ij} + \log \Lambda^2 \Gamma_{ij} \right) O_j \quad (18)$$

In terms of the running coupling constant $\alpha_s(t)$, defined by

$$\begin{cases} \frac{d\alpha_s}{dt} = \beta_s[\alpha_s(t)] \\ \alpha_s(0) = \alpha_s \end{cases} \quad (19)$$

the solution of Eq. (17) can be written as

$$c_i(t, \alpha_s) = c_k[t=0, \alpha_s(t)] \left[U_s(t, 0) \right]_{ki} \simeq c_k^{(0)} \left[U_s(t, 0) \right]_{ki} \quad (20)$$

In Eq. (20), we have used the definition

$$U_s(t_2, t_1) = \bar{T} \exp \int_{t_1}^{t_2} dt' \Gamma[\alpha_s(t')] \quad (21)$$

where $\bar{T}$ is an anti- t ordering operator (t decreases from left to right in a matrix product).

In the one-loop (leading logarithmic) approximation, calling $N (=3)$ the number of colours and f the number of active flavours (quarks with mass much less than μ), one has

$$\beta_s(\alpha_s) = - \frac{b_s}{4\pi} \alpha_s^2 = - \frac{11N - 2f}{12\pi} \alpha_s^2 \quad (22)$$

$$\Gamma_{ij}(\alpha_s) = - \frac{1}{4\pi} \alpha_s (\gamma_s)_{ij} \quad (23)$$

and therefore, with a slight abuse of notation,

$$U_s(t, 0) = \left(\frac{\alpha_s(t)}{\alpha_s} \right)^{\frac{1}{b_s} \gamma_s} = \left(\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)} \right)^{\frac{1}{b_s} \gamma_s} \quad (24)$$

Since both the function $\beta(\alpha_s)$ and the anomalous dimension matrix γ_s depend explicitly on the number f of active flavours, one must successively use Eq. (20) to obtain the coefficients at the scale m_t , m_b , m_c and μ . We will assume $m_c = 1.5$ GeV, $m_b = 4.5$ GeV and will present results for $m_t \approx m_w$ and $m_t = 40$ GeV -

^{*)} The matrix Γ is related to the anomalous dimension matrix γ of Refs. [13,3,7] by $\Gamma = -\frac{1}{2}\gamma$.

probably too small a value [15,16] - for illustration purposes. We have therefore to introduce the running coupling constants for the effective theories with f active flavours (in leading logarithmic approximation)

$$\alpha_{s,f}(Q^2) = \frac{4\pi}{b_s(f) \log(Q^2/\Lambda_f^2)} \quad (25)$$

where the different Λ_f are so related to respect the continuity of α_s at the thresholds. For ease of writing, we will use the notation $\alpha'_s(Q^2) \equiv \alpha_{s,5}(Q^2)$, $\alpha''_s(Q^2) \equiv \alpha_{s,4}(Q^2)$ and $\alpha'''_s(Q^2) \equiv \alpha_{s,3}(Q^2)$. The numerical results will be given for $\Lambda_4 = 100$ MeV and $\Lambda_3 = 300$ MeV.

It is not worth giving here the complicated final expression for the QCD corrected effective Hamiltonian that can be found in Ref. [13] and can also be obtained in the limit $\alpha \rightarrow 0$ from formulae to be given later. The anomalous dimension matrices are reported in Appendix A, in the same operator basis defined by Gilman and Wise [13], namely

$$\begin{aligned} Q_1 &= 4 \bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu u_L \\ Q_2 &= 4 \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L \\ Q_3 &= 4 \bar{s}_L \gamma_\mu d_L \bar{q}_L \gamma^\mu q_L \\ Q_4 &= 4 \bar{s}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu d_L \\ Q_5 &= 4 \bar{s}_L \gamma_\mu d_L \bar{q}_R \gamma^\mu q_R \\ Q_6 &= -8 \bar{s}_L q_R \bar{q}_R d_L \end{aligned} \quad (26)$$

with $q_{L(R)} = (1 \mp \gamma_5)q/2$. A sum over all active flavours is implied in the definition of the operators Q_i , $i > 3$, that are mixed with the others through the gluonic penguin diagrams.

It should be remarked that for $f = 3$ only five of the operators in Eq. (26) are independent, since in this case

$$Q_4 = -Q_1 + Q_2 + Q_3 \quad (27)$$

Also, it must be recalled that the operator

$$Q^{(+)} = Q_1 + Q_2 - \frac{1}{f+2} (Q_3 + Q_4) \quad (28)$$

is multiplicatively renormalized by the strong interactions. This operator, for $f = 3$, belongs to a 27 representation of the flavour group, $SU(3)$, and contains the operator with $\Delta I = 3/2$ determining the amplitude A_2 . Therefore in the final effective Hamiltonian

$$H(\mu) = H_c(\mu) + H_t(\mu) \quad (29)$$

this operator appears with the coefficient^{*)}

$$- \frac{G_F}{\sqrt{2}} \left(V_{cs}^* V_{cd} + V_{ts}^* V_{td} \right) a_+ \quad (30)$$

and the amplitude A_2 has no imaginary part (in the Kobayashi-Maskawa phase convention).

In the presence of electromagnetism, one follows the same strategy of calculation. The renormalization group equation obeyed by the coefficients is now modified to [17]

$$\left(-\frac{\partial}{\partial t} + \beta_s(\alpha_s) \frac{\partial}{\partial \alpha_s} + \beta(\alpha) \frac{\partial}{\partial \alpha} \right) c_i + c_k \Gamma_{ki} = 0 \quad (31)$$

and its solution is still given by an equation similar to Eq. (20)

$$c_i(t, \alpha_s, \alpha) = c_k[0, \alpha_s(t), \alpha(t)] \left\{ \bar{T} \exp \int_0^t dt' \Gamma[\alpha_s(t'), \alpha(t')] \right\}_{ki} \quad (32)$$

where one has introduced the running QED coupling $\alpha(t)$ defined by

$$\begin{cases} \frac{d\alpha}{dt} = \beta[\alpha(t)] \\ \alpha(0) = \alpha \end{cases} \quad (33)$$

In the one-loop approximation (e_f is the electric charge of the active fermions)

$$\beta(\alpha) = \frac{b}{4\pi} \alpha^2 = \frac{1}{3\pi} \sum_f e_f^2 \alpha^2 \quad (34)$$

*) With $N = 3$ and $m_t \approx m_w$ one has

$$a_+ = \frac{1}{2} \left(\frac{\alpha_s'(m_w^2)}{\alpha_s'(m_b^2)} \right)^{\frac{6}{23}} \left(\frac{\alpha_s''(m_b^2)}{\alpha_s''(m_c^2)} \right)^{\frac{6}{25}} \left(\frac{\alpha_s'''(m_c^2)}{\alpha_s'''(\mu^2)} \right)^{\frac{6}{27}}$$

$$\Gamma_{ij}(\alpha_s, \alpha) = -\frac{\alpha_s}{4\pi} (\gamma_s)_{ij} - \frac{\alpha}{4\pi} (\gamma)_{ij} \quad (35)$$

The matrix γ is obtained from the "infinite" part of the diagrams in Figs. 1 and 2, with a photon running in the wavy line. Now it is evident that for $\mu < m_t$ the penguin diagram generates new operators, because of the different couplings of the photon to up-type and down-type quarks. It should be remarked that graphs like those in Fig. 2 also generate new operators, if the operators Q_3, Q_4, Q_5, Q_6 are inserted in the diagrams, precisely for the same reason. This fact had been overlooked in the original calculation by Bijnens and Wise [3] and was partially corrected by Buras and Gérard [6].

To my knowledge, it had not yet been noted that the very basis of operators defined in Ref. [3] and used in all subsequent works [5-7] is not complete: more precisely, it is complete only for $\mu < m_c$ and $f = 3$.

To see how this comes about, consider the contribution of the diagrams in Fig. 2 when the operator Q_3 , defined in Eq. (26), appears in the vertex. It is easy to see that these diagrams generate a mixing of Q_3 with a new operator

$$Q_9 = 6 \bar{s}_L \gamma_\mu d_L \left(\sum_q e_q \bar{q}_L \gamma^\mu q_L \right) \quad (36)$$

and the calculation gives a contribution $-4/3$ to the element γ_{39} of the matrix γ defined in Eqs. (18) and (35). Analogously, an insertion of Q_4 in the same diagrams generates another operator

$$Q_{10} = 6 \sum_q e_q \bar{s}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu d_L \quad (37)$$

and an equal contribution to $\gamma_{4,10}$. This is exactly equivalent to the fact noted by Buras and Gérard [6] that an insertion of Q_5 (Q_6) in these diagrams generates mixing with Q_7 (Q_8), with a contribution to $\gamma_{5,7}$ ($\gamma_{6,8}$) of $+4/3$, the opposite sign being due to the different chiral structure (Q_7 and Q_8 are defined in Eqs. (6) and (7) for $f = 3$).

Furthermore, it is clear that the electropenguin diagram, Fig. 1, always gives an equal contribution to $\gamma_{1,7}$ and to $\gamma_{1,9}$ since the coupling of the photon to fermions is vectorial. Therefore, for $f = 5$, a complete operator basis is given by the ten operators $Q_1, \dots, Q_{10}$.

For $f = 4$, ($q = u, d, s, c$) there is actually a relation

$$Q_{10} = -Q_3 + Q_4 + Q_9 \quad (38)$$

that reduces the number of independent operators to nine.

For $f = 3$ ($q = u, d, s$) two further relations hold, Eq. (27), and

$$Q_9 = \frac{1}{2} (3Q_1 - Q_3) \quad (39)$$

thus reducing the order of the matrix Γ to seven. For $\mu < m_c$, the operator basis can be assumed, following the current literature, as $Q_1, Q_2, Q_3, Q_5, Q_6, Q_7, Q_8$.

The calculation of the coefficients using Eq. (32) looks unnecessarily complicated. Given that $\alpha/\pi \log(m_w/1 \text{ GeV})^2 \approx 0.02$, it is reasonable to restrict the calculations of electromagnetic contributions to the terms $O(\alpha \log(m_w^2/\mu^2))$ (but keeping all orders in $\alpha_s \log(m_w^2/\mu^2)$), neglecting higher order corrections. It is easy to see that in this approximation the coefficients are given by [17]

$$\begin{aligned} C_i(t, \alpha_s, \alpha) &\simeq c_{\kappa}^{(\omega)} \left[U_s(t, 0) - \frac{1}{4\pi} \int_0^t dt' \alpha(t') U_s(t, t') \gamma U_s(t', 0) \right]_{ki} = \\ &= c_{\kappa}^{(\omega)} \left\{ \left[\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)} \right]^{\frac{\gamma_s}{b_s}} - \frac{1}{4\pi} \int_{\mu^2}^{M^2} \frac{dQ^2}{Q^2} \alpha(Q^2) \left[\frac{\alpha_s(M^2)}{\alpha_s(Q^2)} \right]^{\frac{\gamma_s}{b_s}} \gamma \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{\frac{\gamma_s}{b_s}} \right\}_{ki} \end{aligned} \quad (40)$$

where γ_s is the matrix appearing in Eq. (35), that contains as a submatrix the matrix in Eq. (23).

A repeated use of Eq. (40) allows us to obtain the final expression for the coefficients, that is given below in the case $m_t = m_w$ and $\mu \ll m_c$. We denote by $\gamma'_s(\gamma''_s, \gamma'''_s)$ and $\gamma'(Q^2, \mu^2)$ the anomalous dimension matrices for $f = 5(4, 3)$, that are given explicitly in Appendix A. Let us define furthermore the matrices

$$F'(m_t^2, Q^2) = \left[\frac{\alpha'_s(m_t^2)}{\alpha'_s(Q^2)} \right]^{\frac{1}{b'_s} \gamma'_s} \quad (41)$$

$$F'(Q^2, m_b^2) = \left[\frac{\alpha'_s(Q^2)}{\alpha'_s(m_b^2)} \right]^{\frac{1}{b'_s} \gamma'_s} \quad (42)$$

where $b'_s = b_s(f=5) = (11N-10)/3$. Analogous definitions hold for $F''(m_b^2, Q^2)$ and $F''(Q^2, m_c^2)$ (in the $f=4$ effective theory) and for $F'''(m_c^2, Q^2)$ and $F'''(Q^2, \mu^2)$ (for $f=3$).

One has

$$\begin{aligned}
 c_i = c_k^{(0)} & \left\{ F'(m_t^2, m_b^2) A F''(m_b^2, m_c^2) B F'''(m_c^2, \mu^2) - \right. \\
 & - \frac{1}{4\pi} \left[\int_{m_b^2}^{m_t^2} \frac{dQ^2}{Q^2} \alpha(Q^2) F'(m_t^2, Q^2) \gamma' F'(Q^2, m_b^2) A F''(m_b^2, m_c^2) B F'''(m_c^2, \mu^2) + \right. \\
 & + F'(m_t^2, m_b^2) A \int_{m_c^2}^{m_b^2} \frac{dQ^2}{Q^2} \alpha(Q^2) F''(m_b^2, Q^2) \gamma'' F''(Q^2, m_c^2) B F'''(m_c^2, Q^2) + \\
 & \left. \left. + F'(m_t^2, m_b^2) A F''(m_b^2, m_c^2) B \int_{\mu^2}^{m_c^2} \frac{dQ^2}{Q^2} \alpha(Q^2) F'''(m_c^2, Q^2) \gamma''' F'''(Q^2, \mu^2) \right] \right\}_{ki} \quad (43)
 \end{aligned}$$

The matrices A and B are needed to project on the new basis of operators when crossing a threshold: their explicit form is also given in Appendix A.

In the actual extensive calculations, we neglected the running of α , after checking at some representative points that the electromagnetic corrections were changed by less than 5%.

The final effective Hamiltonian can be written in the form

$$H(\mu) = - \frac{G_F}{\sqrt{2}} \sum_i \left(a_i V_{cs}^* V_{cd} + b_i V_{ts}^* V_{td} \right) Q_i \quad (44)$$

the sum running (for $\mu < m_c$) over the values 1,2,3,5,6,7,8.

The coefficients $a_i(b_i)$ are obtained applying the procedure described above to the part $H_c(H_t)$ of the Hamiltonian, see Eq. (13). The "bare" values of the coefficients are $a_i^{(0)} = b_i^{(0)} = \delta_{i2}$.

With the usual phase convention for the KM matrix V, one has

$$\text{Im} (V_{cs}^* V_{cd}) = - \text{Im} (V_{ts}^* V_{td}) \quad (45)$$

Therefore, the imaginary parts of the decay amplitudes are induced by the operator

$$\text{Im } H = - \frac{G_F}{\sqrt{2}} \text{Im} (V_{ts}^* V_{td}) \sum_i c_i Q_i \quad (10)$$

where

$$c_i = b_i - a_i \quad (46)$$

Those coefficients that are relevant for the subsequent calculations are displayed in Figs. 3 and 4, as a function of the subtraction point μ , for μ values such that $\alpha_s(\mu) \leq 1$ and for two different choices of parameters: ($\Lambda_4 = 100$ MeV, $m_t \approx m_w$) and ($\Lambda_4 = 300$ MeV, $m_t = 40$ GeV). Values of the coefficients at different scales μ are needed, according to the method used in the estimate of the matrix elements (that should themselves depend on μ in such a way as to render the final physical amplitude μ independent).

In Figs. 3 and 4 the coefficient of the gluonic penguin operator Q_6 , that gives the bulk of the contributions to ε' , is reported for comparison (full lines). The other coefficients (that have been divided by $\alpha = 1/137$) determine the imaginary part of the $\Delta I = 3/2$ amplitude A_2 . $\tilde{c}_7$ (dashed lines) and $\tilde{c}_8$ (dot-dashed) are the coefficients of the operators already considered in the literature [3,5-7]. The coefficient neglected up to now (dotted line) is defined to be, according to the value of μ : $\tilde{c}_+ = \tilde{c}_1 + \tilde{c}_2$ for $\mu < m_c$, $\tilde{c}_+ = 3/2 \tilde{c}_9$ for $m_c < \mu < m_b$, and finally $\tilde{c}_+ = 3/2(\tilde{c}_9 + \tilde{c}_{10})$ for $m_b < \mu < m_t$. This coefficient happens to be bigger than $\tilde{c}_7$ and $\tilde{c}_8$, but the matrix element of its operator is small, as we will discuss in the next section.

Comparing our results with the results obtained by Sharpe [7] (see Figs. 1, 2, 3 and 6 in his work) we notice that the coefficients c_6 , $\tilde{c}_7$ and $\tilde{c}_8$ are in perfect agreement. They also agree (except $\tilde{c}_8$) with the original calculation of Ref. [3], which was made with a somewhat different technique. This point is discussed in Appendix B.

3. - ELECTROMAGNETIC CORRECTIONS TO ε' AND TO $\text{Re}A_2$

We define the $K \rightarrow 2\pi$ decay amplitudes in terms of the effective Hamiltonian given in Eq. (44) as follows

$$A(K \rightarrow \pi\pi) = -i \langle \pi\pi | H | K \rangle \quad (47)$$

Let us recall moreover the definitions

$$A_0 = \frac{2}{3} A(K^0 \rightarrow \pi^+ \pi^-) + \frac{1}{3} A(K^0 \rightarrow \pi^0 \pi^0) \quad (48)$$

$$A_2 = \frac{\sqrt{2}}{3} \left[A(K^0 \rightarrow \pi^+ \pi^-) - A(K^0 \rightarrow \pi^0 \pi^0) \right] \quad (49)$$

To obtain the decay amplitudes, an estimate of the matrix elements of the operators Q_i is needed. We give in the following the values of the matrix elements in the vacuum insertion approximation (VIA), that it is customary to assume as reference values, parametrizing the uncertainty with a multiplicative factor called B-parameter. We will reintroduce the B-parameters only at the end of our discussion, for ease of writing.

It is convenient first to define combinations of operators having definite ΔI : we consider the case $f = 3$ appropriate for $\mu < m_c$. Anyhow, since the matrix elements are evaluated in the valence approximation, the further terms containing c-quark operators present for $f = 4$ give no contribution: Eqs. (27), (38) and (39) hold approximately for the VIA matrix elements also for $f > 3$.

We define the operators

$$\begin{aligned} O_- &= 4 (\bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L - \bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu u_L) = Q_2 - Q_1 \\ Q'_3 &= 4 \bar{s}_L \gamma_\mu d_L (\bar{u}_L \gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L) \\ Q_7^{(1/2)} &= 2 [\bar{s}_L \gamma_\mu d_L (\bar{u}_R \gamma^\mu u_R - \bar{s}_R \gamma^\mu s_R) - \bar{s}_L \gamma_\mu u_L \bar{u}_R \gamma^\mu d_R] \\ Q_8^{(1/2)} &= -4 [\bar{s}_L u_R \bar{u}_R d_L - \bar{s}_L s_R \bar{s}_R d_L - \bar{s}_L d_R \bar{u}_R u_L] \end{aligned} \quad (50)$$

that, together with Q_5 and Q_6 , induce $\Delta I = \frac{1}{2}$ transitions.

Their VIA matrix elements are (N is the number of colours)

$$\langle \pi^+ \pi^- | O_- | K^0 \rangle = i \left(1 - \frac{1}{N} \right) X$$

$$\langle \pi^+ \pi^- | Q'_3 | K^0 \rangle = i \frac{1}{N} X = \langle \pi^+ \pi^- | Q_3 | K^0 \rangle$$

$$\begin{aligned}
\langle \pi^+ \pi^- | Q_5 | K^0 \rangle &= -i \frac{1}{N} Z \\
\langle \pi^+ \pi^- | Q_6 | K^0 \rangle &= -i Z \\
\langle \pi^+ \pi^- | Q_7^{(1/2)} | K^0 \rangle &= i \left(\frac{2Y}{N} + \frac{Z}{2N} + \frac{X}{2} \right) \\
\langle \pi^+ \pi^- | Q_8^{(1/2)} | K^0 \rangle &= i \left(2Y + \frac{Z}{2} + \frac{X}{2N} \right)
\end{aligned}$$

(51)

and for any of them

$$\begin{aligned}
\langle \pi^0 \pi^0 | O^{(1/2)} | K^0 \rangle &= \langle \pi^+ \pi^- | O^{(1/2)} | K^0 \rangle \\
\langle \pi^+ \pi^0 | O^{(1/2)} | K^+ \rangle &= 0
\end{aligned}$$

(52)

In Eq. (51), we used the definitions given in Eqs. (11) and (12) for X, Y, Z, that we repeat here:

$$\begin{aligned}
X &= f_\pi (M_K^2 - M_\pi^2) \\
Y &= f_\pi v^2 \simeq 12 X \left(\frac{150 \text{ MeV}}{m_s(\mu)} \right)^2 \\
Z &= Y \ 4 \ \frac{M_K^2 - M_\pi^2}{\Lambda_X^2}
\end{aligned}$$

(12)

We notice that X is independent of the renormalization scale μ , but Y and Z are not. The μ dependence in the coefficients and in the B-parameters must compensate for this, to give a μ -independent matrix element of the effective Hamiltonian.

Therefore the VIA estimate could only be correct at one particular (and unknown) μ value, if any: according to common wisdom, this should be a rather low one, hopefully still allowing for the use of coefficients evaluated perturbatively.

A short comment is needed on the matrix element of the gluonic penguin operator Q_6 . The expression that we use

$$Z = f_\pi v^2 \frac{M_\pi^2 - M_K^2}{\Lambda_\chi^2} \quad (53)$$

has been obtained in analyses [18,19] of the momentum dependence of the matrix elements of scalar and pseudoscalar densities in chiral perturbation theory. It satisfies the constraint to vanish in the exact SU(3) limit [20] and at the same time is essentially in agreement (this time in the limit $M_\pi^2 \ll M_K^2$) with the original estimate of Ref. [11]. A very detailed analysis may be found in Ref. [19]. In our numerical calculations we will assume the chiral symmetry breaking scale parameter to be $\Lambda_\chi \simeq 1$ GeV and therefore

$$Z \simeq Y \quad (54)$$

One should notice, however, that in the chiral limit Z vanishes and Y does not.

The $\Delta I = 3/2$ operators are

$$\begin{aligned} O^{(3/2)} &= 4 \left[\bar{s}_L \gamma_\mu d_L (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L \right] = \\ &= 2 Q_1 + Q_2 - Q_3' \end{aligned} \quad (55)$$

$$\begin{aligned} Q_7^{(3/2)} &= 2 \left[\bar{s}_L \gamma_\mu d_L (\bar{u}_R \gamma^\mu u_R - \bar{d}_R \gamma^\mu d_R) + \bar{s}_L \gamma_\mu u_L \bar{u}_R \gamma^\mu d_R \right] = \\ &= Q_7 - Q_7^{(1/2)} \end{aligned} \quad (56)$$

$$\begin{aligned} Q_8^{(3/2)} &= -4 \left[\bar{s}_L u_R \bar{u}_R d_L - \bar{s}_L d_R \bar{d}_R d_L + \bar{s}_L d_R \bar{u}_R u_L \right] = \\ &= Q_8 - Q_8^{(1/2)} \end{aligned} \quad (57)$$

and their VIA matrix elements are given by

$$\langle \pi^+ \pi^- | O^{(3/2)} | K^0 \rangle = i \left(1 + \frac{1}{N} \right) X \quad (58)$$

$$\langle \pi^+ \pi^- | Q_7^{(3/2)} | K^0 \rangle = i \left(\frac{Y}{N} - \frac{X}{2} \right) \quad (59)$$

$$\langle \pi^+ \pi^- | Q_8^{(3/2)} | K^0 \rangle = i \left(Y - \frac{X}{2N} \right) \quad (60)$$

Moreover,

$$\begin{aligned} \langle \pi^0 \pi^0 | O^{(3/2)} | K^0 \rangle &= -2 \langle \pi^+ \pi^- | O^{(3/2)} | K^0 \rangle \\ \langle \pi^+ \pi^0 | O^{(3/2)} | K^+ \rangle &= \frac{3}{\sqrt{2}} \langle \pi^+ \pi^- | O^{(3/2)} | K^0 \rangle \end{aligned} \quad (61)$$

and similar relations from $Q_7^{(3/2)}$ and $Q_8^{(3/2)}$.

From Eqs. (10), (44), (46), (47), (48) and (49) and the matrix elements we obtain, reinserting the B-parameters:

$$\begin{aligned} \text{Im } A_0 &= - \frac{G_F}{\sqrt{2}} \text{Im} (V_{ts}^* V_{td}) \cdot \\ &\cdot \left\{ - \left(c_6 B_6 + \frac{1}{N} c_5 B_5 \right) Z + \frac{2c_2 - c_4}{3} B_- \left(1 - \frac{1}{N} \right) X + \right. \\ &+ \left(c_3 B_3 + \frac{c_1 + c_2}{3} B'_3 \right) \frac{1}{N} X + c_7 B_7 \left(\frac{2Y}{N} + \frac{Z}{2N} + \frac{X}{2} \right) + \\ &\left. + c_8 B_8 \left(2Y + \frac{Z}{2} + \frac{X}{2N} \right) \right\} \end{aligned} \quad (62)$$

$$\begin{aligned} \text{Im } A_2 &= - G_F \text{Im} (V_{ts}^* V_{td}) \cdot \\ &\cdot \left\{ \frac{c_1 + c_2}{3} B_{3/2} \left(1 + \frac{1}{N} \right) X + c_7 B_7 \left(\frac{Y}{N} - \frac{X}{2} \right) + \right. \\ &\left. + c_8 B_8 \left(Y - \frac{X}{2N} \right) \right\} \end{aligned} \quad (63)$$

We also write, in preparation for a further brief discussion of $\text{Re} A_2$,

$$\begin{aligned} \text{Re } A_2 &\simeq - G_F \text{Re} (V_{ts}^* V_{td}) \cdot \\ &\cdot \left\{ \frac{a_1 + a_2}{3} B_{3/2} \left(1 + \frac{1}{N} \right) X + a_7 B_7 \left(\frac{Y}{N} - \frac{X}{2} \right) + \right. \\ &\left. + a_8 B_8 \left(Y - \frac{X}{2N} \right) \right\} \end{aligned} \quad (64)$$

In Eq. (64), we have neglected the contribution of the H_c part of the Hamiltonian, since $|V_{ts}^* V_{td}| \approx 10^{-3} |V_{cs}^* V_{cd}|$.

In Eq. (62), we have explicitly written down all the terms. Some of them are, however, much smaller than the others, since c_7 and c_8 are depressed by one power of α (in Figs. 3 and 4 we have plotted $\tilde{c}_{7(8)} = c_{7(8)}/\alpha$) and the matrix element of

Q_6 , Z , is about ten times bigger than X , while their coefficients in $\text{Im}H$ are of similar magnitude. Neglecting the smaller terms in $\text{Im}A_0$ one obtains the final expression for Ω_{em} that has been given in Eq. (9) of Section 1.

Before making a detailed discussion of the electromagnetic correction to ε' and giving numerical results for Ω_{em} , let us first briefly consider the electromagnetic corrections to the real part of the amplitude A_2 . Equation (64) can be put in the following form

$$\text{Re } A_2 = (\text{Re } A_2)_{\alpha=0} (1 + \Xi_{\text{em}}) \quad (65)$$

Defining

$$\delta(a_1 + a_2) = a_1 + a_2 - (a_1 + a_2)_{\alpha=0} \quad (66)$$

one has

$$\begin{aligned} \Xi_{\text{em}} &= \frac{\delta(a_1 + a_2)}{(a_1 + a_2)_{\alpha=0}} + \\ &+ \frac{3N}{(N+1)B_{3/2}(a_1 + a_2)_{\alpha=0}} \left[a_7 B_7 \left(\frac{Y}{NX} - \frac{1}{2} \right) + a_8 B_8 \left(\frac{Y}{X} - \frac{1}{2N} \right) \right] \end{aligned} \quad (67)$$

If one could neglect completely the mixing with electropenguin operators, one would easily obtain the result

$$\frac{\delta(a_1 + a_2)}{(a_1 + a_2)_{\alpha=0}} = \frac{\alpha}{3\pi} \log \frac{m_W^2}{\mu^2} \quad (68)$$

which for $\mu = 1 \text{ GeV}$ would contribute $+7 \times 10^{-3}$ to Ξ_{em} . The electropenguin contribution adds to this small number, but it is also very small [21] since these operators start to be generated at the charm scale, and their coefficients are depressed by the electroweak fine-structure constant α . As a result (in VIA and for $N = 3$),

$$0.007 < \Xi_{\text{em}} \lesssim 0.015 \quad (69)$$

In summary, the electromagnetic corrections to the real part of A_2 are actually negligible.

Let us now come back to Ω_{em} and write Eq. (9) as:

$$\Omega_{em} = \frac{1}{\omega} \frac{\text{Im } A_2}{\text{Im } A_0} = -(a + b + c) \quad (70)$$

with $(\tilde{c}_i = c_i/\alpha)$

$$a = \sqrt{2} \frac{\alpha}{\omega} \frac{Y}{Z} \frac{\tilde{c}_8 B_8 + \frac{1}{N} \tilde{c}_7 B_7}{c_6 B_6 + \frac{1}{N} c_5 B_5 + \dots} \quad (71)$$

$$b = \sqrt{2} \frac{\alpha}{\omega} \left(-\frac{X}{2Z} \right) \frac{\tilde{c}_7 B_7 + \frac{1}{N} \tilde{c}_8 B_8}{c_6 B_6 + \frac{1}{N} c_5 B_5 + \dots} \quad (72)$$

$$c = \sqrt{2} \frac{\alpha}{\omega} \left(\frac{X}{3Z} \right) \frac{\tilde{c}_+ B_{3/2}}{c_6 B_6 + \frac{1}{N} c_5 B_5 + \dots} \left(1 + \frac{1}{N} \right) \quad (73)$$

The numerical values of a , b , c for $B_1 = 1$, $N = 3$ (namely in VIA) and $\mu = 0.8$ GeV are given in the first and fourth columns of the Table, for $(\Lambda = 100$ MeV, $m_t = 83$ GeV) and for $(\Lambda = 300$ MeV, $m_t = 40$ GeV), respectively. As one can see, the new contributions (b and c) are in fact small, and furthermore they cancel each other to a large extent.

In the second and fifth columns of the Table, we report a , b , c evaluated in the large N limit for the matrix elements, but with the same coefficients as before (calculated for $N = 3$). We notice that the new terms are separately more important here, due to the smallness of $\tilde{c}_8$, and again approximately cancel.

This is not a real large N evaluation, since one should also make the small $1/N$ limit (but with $1/N \log M^2/\mu^2$ not small) in the coefficient calculation. If one were to use the values of the coefficients evaluated for $\Lambda = 300$ MeV and $m_t = 40$ GeV in Ref. [6], namely $\tilde{c}_7 = -0.058$, $\tilde{c}_8 = -0.001$ and $c_6 = -0.11$, one would obtain $a = 0.002$ and $b = -0.005$. The b term would now become bigger than a , and the c term should again come into the game to cancel its effect.

We have to note, however, that we do not agree with the results of Ref. [6], even in the framework of the large N approximation. In fact, from their Eq. (37), one finds that they have only kept Γ_{17} and Γ_{27} as non-zero elements in the seventh column of the anomalous dimension matrix. Now, from Eq. (A9) in Appendix A, one can see that also the element Γ_{47} (that in the large N limit should become

approximately opposite to Γ_{17}) is large, and from Ref. [22] we know that in the large N limit the operator Q_4 must be kept. This contribution tends to increase the absolute value of $\tilde{c}_7$, and this in turn (and more importantly) increases also $-\tilde{c}_8$. We obtain, keeping also Γ_{47} (and Γ_{49}), $c_6 = -0.13$, $\tilde{c}_7 = -0.072$, $\tilde{c}_8 = -0.007$ and $\tilde{c}_+ = -0.098$, to be compared to the exact $N = 3$ results $(-0.11, -0.064, -0.011, -0.089)$. The corresponding a, b, c are reported in the sixth column of the Table. We also evaluated the coefficients for several values of μ in the large N approximation, finding that $|\tilde{c}_8|$ is a decreasing function of μ as it also happens for the $N = 3$ calculation, contrary to what is obtained in the other approach (compare with Fig. 2 of Ref. [7]).

We did the same calculation for $(\Lambda = 100 \text{ MeV}, m_t = 83 \text{ GeV})$ and report in the third column the resulting a, b, c , although it is at least dubious to neglect the mixing in the penguin sector ($\Gamma_{44}, \Gamma_{46}, \Gamma_{64} \approx 0$) if the top mass is so high. We note that without neglecting these terms, we would obtain values for the coefficients which are even closer to the values calculated for $N = 3$.

Although larger than previously thought, the large N estimate for Ω_{em} that we obtain is still a small number, as can be seen from the Table.

However, there have been two recent calculations of the matrix elements of the electropenguin operators in lattice QCD, one with staggered fermions [8] and the other with Wilson fermions [9], that do not support the large N limit estimate $\langle Q_7 \rangle \ll \langle Q_8 \rangle$. In fact, the B -parameters at a scale μ of about 2 GeV are consistent with one for both operators.

Using the values of the coefficients at $\mu = 2 \text{ GeV}$ and $B_7 \approx B_8 \approx 1$, one obtains

$$\Omega_{em} = - \frac{0.12 \div 0.29}{B_6} \quad (74)$$

the lower [higher] number corresponding to $(\Lambda, m_t) = (0.3, 40) [0.1, 83] \text{ GeV}$.

The B_6 parameter has not yet been calculated in lattice QCD with Wilson fermions. In the other approach, they estimated [8] $B_6 \approx 0.5$ and therefore a rather large Ω_{em} (but a smaller value for ϵ'/ϵ).

4. - ISOSPIN BREAKING CORRECTIONS

The analysis of these corrections has been done in Refs. [5,6,21] and we do not have much to add on this subject. We repeat it here for completeness, adding our own comments.

The different masses of the quarks provide non-zero off-diagonal matrix elements of the chiral symmetry breaking Hamiltonian

$$\mathcal{H}_{SB} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \quad (75)$$

Defining as usual

$$\begin{aligned} \eta &= \cos \theta \eta_8 - \sin \theta \eta_1 \\ \eta' &= \sin \theta \eta_8 + \cos \theta \eta_1 \end{aligned} \quad (76)$$

and neglecting electromagnetic effects, one has (see Fig. 5)

$$\begin{aligned} \delta A^{\pi^0} &\equiv A(K^0 \rightarrow \pi^0 \pi^0) - [A(K^0 \rightarrow \pi^0 \pi^0)]_{m_u=m_d} = \\ &= 2 \left\{ [\cos \theta A(K^0 \rightarrow \pi^0 \eta_8) - \sin \theta A(K^0 \rightarrow \pi^0 \eta_1)] \cdot \right. \\ &\quad \cdot \frac{1}{M_\pi^2 - M_\eta^2} [\cos \theta \langle \pi^0 | \mathcal{H}_{SB} | \eta_8 \rangle - \sin \theta \langle \pi^0 | \mathcal{H}_{SB} | \eta_1 \rangle] + \\ &\quad \left. + (\cos \theta \rightarrow \sin \theta, \sin \theta \rightarrow -\cos \theta, M_\eta^2 \rightarrow M_{\eta'}^2) \right\} \end{aligned} \quad (77)$$

It is easy to show, using the chiral Lagrangian for $\Delta I = \frac{1}{2}$ weak decays [23], that for an off-shell η_8 , with $p^2 = m_\pi^2$,

$$A(K^0 \rightarrow \pi^0 \eta_8) = -\frac{1}{\sqrt{3}} A(K^0 \rightarrow \pi^0 \pi^0) \quad (78)$$

Moreover, one has

$$\langle \pi^0 | \mathcal{H}_{SB} | \eta_8 \rangle = \sim \frac{m_u - m_d}{\sqrt{3}} \quad (79)$$

The amplitude $A(K^0 \rightarrow \pi^+ \pi^-)$ is plainly unaffected by isospin breaking corrections.

Let us first consider the correction in the chiral limit, in which case π^0 and η become exactly massless Goldstone bosons, while the η' mass is kept non-zero by the anomaly contribution. In this limit, the mixing angle θ goes to zero and the η coincides with the octet member, η_8 . Furthermore, both $\langle \pi^0 | \mathcal{H}_{SB} | \eta_1 \rangle$ and $\langle \pi^0 | \mathcal{H}_{SB} | \eta_8 \rangle$ vanish. The η_8 contribution to the amplitude has, however, a finite limit, resulting in

$$\delta A^{\pi^0} \Big|_{\text{chiral limit}} = \frac{\sqrt{3}}{2} \frac{m_d - m_u}{m_s - (m_u + m_d)/2} A(K^0 \rightarrow \pi^0 \eta_8) \quad (80)$$

Therefore, using Eqs. (4), (48), (49) and (77)

$$\Omega_{IB} \Big|_{\text{chiral limit}} = \frac{1}{3\sqrt{2} \omega} \frac{m_d - m_u}{m_s - (m_u + m_d)/2} \simeq 0.12 \quad (81)$$

A more realistic estimate needs a knowledge of the mixing angle θ , of the matrix element $\langle \pi^0 | \mathcal{H}_{SB} | \eta_1 \rangle$ and of the weak amplitude $A(K^0 \rightarrow \pi^0 \eta_1)$. The analysis by Gasser and Leutwyler [24] of the η - η' mass matrix, made in chiral perturbation theory keeping all effects of $O(p^4)$, suggests a value $\theta \simeq -20^\circ$, in agreement with what is needed to fit the experimental rates for two photon decays of π^0 , η , η' [25].

The relevant matrix elements of the chiral symmetry breaking Hamiltonian can be parametrized as follows:

$$\langle \pi^0 | \mathcal{H}_{SB} | \eta_1 \rangle = v \frac{m_u - m_d}{\sqrt{3}} \left(\sqrt{2} + \frac{a}{N} \right) \quad (82)$$

$$\langle \eta_8 | \mathcal{H}_{SB} | \eta_1 \rangle = -\frac{v}{3} (2m_s - m_u - m_d) \left(\sqrt{2} + \frac{a}{N} \right) \quad (83)$$

and the $O(1/N)$ corrections are the same in the two equations^{*)}. The above-mentioned analysis [24] is compatible with $a/N \simeq 0$ in Eq. (83), namely with the large N limit for $\langle \eta_8 | \mathcal{H}_{SB} | \eta_1 \rangle$. One should not take the large N limit for the η_1 mass term, that would give the disastrous result $m_{\eta'}^2 \simeq m_{\pi}^2$ and a totally wrong mixing angle: it is possible, in fact, to keep different from zero the $O(1/N)$ correction to the η_1 mass, that comes from a totally independent term in the Lagrangian [24].

If one makes the further assumption that for all the other terms the large N approximation can be made, then one can also estimate the decay amplitude $A(K^0 \rightarrow \pi^0 \eta_1)$. This was done in Ref. [6] with the result

$$\text{Im} A(K^0 \rightarrow \pi^0 \eta_1) \Big|_{N \rightarrow \infty} = -\sqrt{\frac{2}{3}} \text{Im} A(K^0 \rightarrow \pi^0 \pi^0) \quad (84)$$

*) I am indebted to G. Martinelli for this observation.

To obtain this, use has been made of the dominance of the penguin part of the effective Hamiltonian in the imaginary part of the decay amplitude and of the nonet symmetric, large N expression of the operator Q_6 [6,4].

The final outcome of all this is the value [6]

$$\Omega_{IB} \simeq 0.27 \quad (85)$$

to be compared with Eq. (81). This estimate relies heavily on the validity of the large N approximation, and is therefore subject to considerable uncertainties. It shows at any rate that the chiral limit estimate, Eq. (81), can be considered as a lower bound for the true Ω_{IB} .

The isospin breaking corrections also affect the real part of the $\Delta I = 3/2$ amplitude A_2 . In the chiral limit (and for $N = 3$) one obtains, using Eqs. (49) and (80),

$$(\text{Re } A_2)_{IB} = -\frac{\sqrt{2}}{3} \text{Re}(\delta A^{00}) = 0.13 (\text{Re } A_2)_{m_u=m_d} \quad (86)$$

Again, this is a lower bound on the effect.

A more realistic calculation has the same difficulties mentioned above, plus the fact that the real part of the decay amplitude A_0 is not penguin dominated, presumably: this makes more problematic the estimate of $A(K^0 \rightarrow \pi^0 \eta_1)$. In Ref. [21], this amplitude has been evaluated phenomenologically using soft pion reduction and a pole model fit [26] to the decay rate of K_L^0 in two photons^{*)}: the result has been

$$(\text{Re } A_2)_{IB} \simeq 0.20 (\text{Re } A_2)_{m_u=m_d} \quad (87)$$

The calculation could also be done with the method used in Ref. [6] to evaluate Ω_{IB} . One uses in this case the large N representation of the operators Q_i as products of vector and axial currents (or scalar and pseudoscalar densities for Q_6): these currents (and densities) are obtained from the chiral Lagrangian given in Refs. [6] and [4] (that assumes nonet symmetry, except for the η_1 mass term). One obtains in this way at leading order in $1/N$

$$\langle \pi^0 \eta_1 | Q_6 | K^0 \rangle = \sqrt{2} \langle \pi^0 \eta_8 | Q_6 | K^0 \rangle \quad (88)$$

*) A further uncertainty may come here from the possible large SU(3) breaking effects in the $K_L^0 \rightarrow \gamma\gamma$ calculation [27].

$$\langle \pi^0 \eta_1 | Q_1 | K^0 \rangle = -\frac{1}{\sqrt{2}} \langle \pi^0 \eta_8 | Q_1 | K^0 \rangle \quad (89)$$

$$\langle \pi^0 \eta_1 | Q_4 | K^0 \rangle = -2\sqrt{2} \langle \pi^0 \eta_8 | Q_4 | K^0 \rangle \quad (90)$$

and using the values of the coefficients calculated in the large N limit one has

$$(\text{Re } A_2)_{IB} \simeq \left\{ \begin{matrix} 0.15 \\ 0.20 \end{matrix} \right\} (\text{Re } A_2)_{m_u=m_d} \quad (91)$$

The upper (lower) number corresponds to "type II" ("type III") coefficients with $\mu = 0.8$ GeV and $\Lambda_4 = 0.3$ GeV in Ref. [22].

Although both methods of calculation can be criticized on several points, it is reassuring that their results are so close to each other and furthermore not very different from the value obtained in the chiral limit, Eq. (86).

5. - CONCLUSIONS

In this paper, we have analyzed in detail the electromagnetic corrections to the weak $K \rightarrow \pi\pi$ decay amplitudes. We have given the expression for the complete one-loop effective Hamiltonian, including some contributions neglected in the literature up to now. The relevant anomalous dimension matrices have been given in Appendix A.

The electromagnetic corrections may only be of some importance in the calculation of ε' , since the (much larger) QCD penguin corrections do not generate an imaginary part for the $\Delta I = 3/2$ amplitude A_2 . They do in fact increase ε' by an amount which is estimated to be between 8% and 25% in the vacuum insertion approximation. Lattice calculations tend to give an even larger correction, maybe up to 50%. The large N approach of Ref. [6] and [22] would instead predict a smaller increase (less than about 10%), in direct contrast with the lattice estimates. Only in this last approach the newly considered terms would be numerically of some importance.

The isospin breaking corrections give a contribution of opposite sign to $\text{Im}A_2$, and tend to decrease ε' . The precise amount is not easily established: a lower bound to the correction is provided by the chiral limit estimate of 12%; the effect of $\eta-\eta'$ mixing and the very η' contribution tend to increase this number, maybe up

to about 30%. In conclusion, the electromagnetic and isospin breaking corrections tend to cancel each other to a large extent. Given the uncertainty in the evaluation of the (QCD) penguin operator matrix element, it does not seem much worth making an effort to decide whether these corrections do in fact increase or decrease ϵ' by 10 or 20%.

As to the effect of these corrections on the real part of A_2 (they obviously have a negligible effect on A_0), the electromagnetic effect is undoubtedly very small (less than 2%) and essentially negligible. The isospin breaking correction is not negligible (13% to 20%, or may be more) and it actually makes the ratio of the uncorrected amplitudes A_0/A_2 still larger than the experimental number (about 25 instead of 22). This in turn should increase the theoretical estimates given before for Ω_{em} and Ω_{IB} by a factor 1.13, without changing of course the general conclusion.

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	$\Lambda = 100 \text{ MeV}, m_t = 83 \text{ GeV}$			$\Lambda = 300 \text{ MeV}, m_t = 40 \text{ GeV}$		
	$N = 3$	$N \rightarrow \infty$		$N = 3$	$N \rightarrow \infty$	
		(a)	(b)		(a)	(b)
a	0.241	0.087	0.067	0.074	0.025	0.013
b	-0.020	-0.018	-0.017	-0.006	-0.006	-0.005
c	0.023	0.017	0.015	0.007	0.005	0.005
Ω_{em}	0.245	0.086	0.065	0.075	0.024	0.012

TABLE

Contributions to Ω_{em} , see Eqs. (70)-(73), for $\mu = 0.8 \text{ GeV}$ and $B_1 = 1$. For the $N \rightarrow \infty$ calculations: in (a) the coefficients are calculated for $N = 3$, in (b) they have been approximated according to the prescriptions of Ref. [22].

APPENDIX A

We use the operator basis $Q_1, \dots, Q_{10}$. Q_i ($i \leq 6$), Q_9 , Q_{10} are defined in Eqs. (26), (36) and (37). For $f = 3$, Q_7 and Q_8 have been defined in Eqs. (6) and (7). In the general case, one has

$$Q_7 = 6 \bar{s}_L \gamma_\mu d_L \left(\sum_q e_q \bar{q}_R \gamma^\mu q_R \right) \quad (A1)$$

$$Q_8 = -12 \sum_q e_q \bar{s}_L q_R \bar{q}_R d_L \quad (A2)$$

The sum running over all the active quarks (of electric charge e_q). We denote by $f_u(f_d)$ the number of active flavours with charge $+2/3$ ($-1/3$). Plainly

$$f = f_u + f_d \quad (A3)$$

Using the definitions given in Eqs. (18) and (35), the matrices γ_s and γ receive contributions from the graphs exemplified in Fig. 2 and from the penguin diagram, Fig. 1. For $\mu > m_c(m_t)$ the penguin diagram contribution is subleading, and therefore neglected, for the calculation of the coefficients in the $H_c(H_t)$ part of the Hamiltonian, Eq. (13) (this has been criticized in Ref. [22]: it is not evident that the inclusion of a part of subleading terms, done in their calculation, gives a better estimate of the coefficients. At any rate, it makes a very small difference for the calculation of ϵ'). One has, in this case:

$$\begin{aligned} (\gamma_s)_{11} &= (\gamma_s)_{22} = -\frac{3}{N} \\ (\gamma_s)_{12} &= (\gamma_s)_{21} = 3 \\ (\gamma_s)_{1i} &= (\gamma_s)_{2i} = 0 \quad (i \geq 3) \\ \gamma_{1i} &= -\frac{4}{3} \delta_{1i} \\ \gamma_{2i} &= -\frac{4}{3} \delta_{2i} \end{aligned} \quad (A4)$$

and the other elements are unimportant, since the only non-zero bare coefficient is $a_2(b_2)$. For $\mu < m_t(m_c)$, the penguin contribution produces the full mixing in $H_t(H_c)$. For ease of writing, it is convenient to divide into submatrices the full anomalous dimension matrices

$$\gamma_{(s)} = \begin{pmatrix} \gamma_{(s)}^{ss} & \gamma_{(s)}^{se} \\ \gamma_{(s)}^{es} & \gamma_{(s)}^{ee} \end{pmatrix} \quad (A5)$$

The matrices γ_s^{ss} and γ^{ss} mix among themselves the operator Q_1 , $i < 6$, that form a basis if only gluonic (QCD) corrections are considered. For this reason, the submatrix γ_s^{se} is a null (6x4) matrix

$$(\gamma_s^{se})_{ij} = 0 \quad (A6)$$

The matrix γ_s^{ss} has been known since quite a long time [13,14,22]

$$\gamma_s^{ss} = \begin{pmatrix} -\frac{3}{N} & 3 & 0 & 0 & 0 & 0 \\ 3 & -\frac{3}{N} & -\frac{1}{3N} & \frac{1}{3} & -\frac{1}{3N} & \frac{1}{3} \\ 0 & 0 & -\frac{11}{3N} & \frac{11}{3} & -\frac{2}{3N} & \frac{2}{3} \\ 0 & 0 & 3 - \frac{f}{3N} & \frac{f}{3} - \frac{3}{N} & -\frac{f}{3N} & \frac{f}{3} \\ 0 & 0 & 0 & 0 & \frac{3}{N} & -3 \\ 0 & 0 & -\frac{f}{3N} & \frac{f}{3} & -\frac{f}{3N} & -3N + \frac{3}{N} + \frac{f}{3} \end{pmatrix} \quad (A7)$$

An easy calculation shows that the only non-zero entries in the matrix γ^{ss} are

$$(\gamma^{ss})_{11} = (\gamma^{ss})_{22} = -\frac{4}{3} \quad (A8)$$

The matrix γ^{se} is

$$\gamma^{se} = \begin{pmatrix} \frac{8N}{27} & 0 & \frac{8N}{27} & 0 \\ \frac{8}{27} & 0 & \frac{8}{27} & 0 \\ \frac{8N}{27} \left(f_u - \frac{f_d}{2} - \frac{1}{N} \right) & 0 & \frac{8N}{27} \left(f_u - \frac{f_d}{2} - \frac{1}{N} \right) - \frac{4}{3} & 0 \\ \frac{8N}{27} \left(\frac{f_u}{N} - \frac{f_d}{2N} - 1 \right) & 0 & \frac{8N}{27} \left(\frac{f_u}{N} - \frac{f_d}{2N} - 1 \right) - \frac{4}{3} \\ \frac{8N}{27} \left(f_u - \frac{f_d}{2} \right) + \frac{4}{3} & 0 & \frac{8N}{27} \left(f_u - \frac{f_d}{2} \right) & 0 \\ \frac{8}{27} \left(f_u - \frac{f_d}{2} \right) & \frac{4}{3} & \frac{8}{27} \left(f_u - \frac{f_d}{2} \right) & 0 \end{pmatrix} \quad (A9)$$

With the exception of the contributions of $\pm 4/3$, discussed in the text, the other entries in Eq. (A9) come from the electropenguin diagram.

The remaining submatrices are

$$\gamma_s^{es} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{f_u}{3N} + \frac{f_d}{6N} & \frac{f_u}{3} - \frac{f_d}{6} & -\frac{f_u}{3N} + \frac{f_d}{6N} & \frac{f_u}{3} - \frac{f_d}{6} \\ 0 & 0 & \frac{1}{3N} & -\frac{1}{3} & \frac{1}{3N} & -\frac{1}{3} \\ 0 & 0 & -\frac{f_u}{3N} + \frac{f_d}{6N} & \frac{f_u}{3} - \frac{f_d}{6} & -\frac{f_u}{3N} + \frac{f_d}{6N} & \frac{f_u}{3} - \frac{f_d}{6} \end{pmatrix} \quad (A10)$$

$$\gamma^{es} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{3} & 0 & 0 \end{pmatrix}$$

(A11)

$$\gamma_s^{ee} = \begin{pmatrix} \frac{3}{N} & -3 & 0 & 0 \\ 0 & -3N + \frac{3}{N} & 0 & 0 \\ 0 & 0 & -\frac{3}{N} & 3 \\ 0 & 0 & 3 & -\frac{3}{N} \end{pmatrix} \quad (A12)$$

$$\gamma^{ee} = \begin{pmatrix} \frac{2}{3} + \frac{2N}{27} (4f_u + f_d) & 0 & \frac{2N}{27} (4f_u + f_d) & 0 \\ \frac{2}{27} (4f_u + f_d) & \frac{2}{3} & \frac{2}{27} (4f_u + f_d) & 0 \\ \frac{2N}{27} (4f_u + f_d + \frac{2}{N}) & 0 & -\frac{2}{3} + \frac{2N}{27} (4f_u + f_d + \frac{2}{N}) & 0 \\ \frac{2N}{27} \left(\frac{4f_u + f_d}{N} + 2 \right) & 0 & \frac{2N}{27} \left(\frac{4f_u + f_d}{N} + 2 \right) & -\frac{2}{3} \end{pmatrix} \quad (A13)$$

The submatrices γ^{es} and γ^{ee} have been given only for completeness, since they do not contribute to the $O(\alpha \log M^2)$ correction: their effect would first appear in the $O(\alpha^2 \log^2 M^2)$ corrections that we are neglecting in our calculations.

Using Eqs. (A5), (A6), (A7), (A10) and (A12) with $f_u = 2$ and $f_d = 3$ ($f = 5$), one obtains the matrix γ'_s to be used in Eqs. (42) and (43) of the text. Analogously, the matrix γ' in Eq. (43) is obtained from Eqs. (A5), (A8), (A9), (A11) and (A13).

For $\mu < m_b$ ($f_u = f_d = 2$) the operator basis is restricted to Q_i , $i \leq 9$, because of the relation given in the text, Eq. (38). This relation implies that the 10×9 projection matrix, called A in Eq. (43), has the following non-zero entries:

$$A_{ii} = 1$$

$$- A_{10,3} = A_{10,4} = A_{10,9} = 1 \quad (A14)$$

Furthermore, the anomalous dimension matrices for $f = 4$ are obtained as follows. The tenth row is simply cancelled; the non-zero elements in the tenth column are multiplied by the appropriate coefficient and added to the elements appearing in the other columns; the tenth column is then cancelled. Explicitly, to obtain the

matrix γ'' to be used in Eq. (43), one has to put $f_u = f_d = 2$ in Eqs. (A5)-(A13) and define ($1 \leq i, j \leq 9$)

$$\begin{aligned}\gamma''_{i3} &= \gamma_{i3} - \gamma_{i40} \\ \gamma''_{i4} &= \gamma_{i4} + \gamma_{i40} \\ \gamma''_{i9} &= \gamma_{i9} + \gamma_{i40} \\ \gamma''_{ij} &= \gamma_{ij} \quad (j = 1, 2, 5, 6, 7, 8)\end{aligned}\tag{A15}$$

Analogous relations define the matrix γ'_s .

After decoupling of the charm quark, for $\mu < m_c$, the dimensionality of the matrices is further reduced, as a consequence of Eqs. (27) and (39). The operator basis for $f = 3$ is chosen to be $Q_1, Q_2, Q_3, Q_5, Q_6, Q_7, Q_8$.

The 9×7 projection matrix B in Eq. (43) has the following non-zero entries

$$\begin{aligned}B_{11} &= B_{22} = B_{33} = B_{54} = B_{65} = B_{76} = B_{97} = 1 \\ -B_{41} &= B_{42} = B_{43} = 1 \\ B_{91} &= -3 \quad B_{93} = 3/2\end{aligned}\tag{A16}$$

The anomalous dimension matrices are modified in a way analogous to that previously described for the change of regime from $f = 5$ to $f = 4$. The combined effect of the transition from $f = 5$ to $f = 4$ and then to $f = 3$ is such that, to obtain the matrix γ''' of Eq. (43), one has to put $f_u = 1$ and $f_d = 2$ in Eqs. (A5)-(A13) and define ($1 \leq i, k \leq 7$):

$$\begin{aligned}\gamma'''_{i1} &= \gamma_{j1} - \gamma_{j4} + \frac{3}{2} \gamma_{j9} - \gamma_{j40} \\ \gamma'''_{i2} &= \gamma_{j2} + \gamma_{j4} + \gamma_{j40} \\ \gamma'''_{i3} &= \gamma_{j3} + \gamma_{j4} - \frac{1}{2} \gamma_{j9} + \gamma_{j40} \\ \gamma'''_{ik} &= \gamma_{j, k+1} \quad (k \geq 4)\end{aligned}\tag{A17}$$

with $j = i$ for $i \leq 3$ and $j = i+1$ for $4 \leq i \leq 7$. Similar relations define the matrix γ'''_s .

APPENDIX B

In the basis $Q_1, \dots, Q_{10}$ used in this paper, the structure of the anomalous dimension matrices is [see Eq. (A5)]

$$\gamma_s = \begin{pmatrix} \gamma_s^{ss} & 0 \\ \gamma_s^{es} & \gamma_s^{ee} \end{pmatrix}$$

$$\gamma = \begin{pmatrix} \gamma^{ss} & \gamma^{se} \\ \gamma^{es} & \gamma^{ee} \end{pmatrix} \quad (B1)$$

In the original calculation by Bijnens and Wise [3], a different operator basis was used:

$$Q'_i = Q_i \quad (i \leq 6) \quad (B2)$$

$$Q'_i = \frac{\alpha}{\alpha_s} Q_i \quad (i = 7, 8) \quad (B3)$$

and Q_9, Q_{10} were not included.

We now extend the definition (B3) to Q'_9 and Q'_{10} . In this basis, the anomalous dimension matrices become (at one-loop level)

$$\gamma'_s = \begin{pmatrix} \gamma_s^{ss} & \gamma^{se} \\ 0 & \gamma_s^{ee} - b_s \mathbb{1} \end{pmatrix}$$

$$\gamma' = \begin{pmatrix} \gamma^{ss} & 0 \\ \gamma_s^{es} + \frac{\alpha}{\alpha_s} \gamma^{es} & \gamma^{ee} - b \mathbb{1} \end{pmatrix} \quad (B4)$$

In Eq. (B4), the modifications in the submatrices in the lower right-hand corner are a consequence of the renormalization of the factor α/α_s in the operators Q'_j .

The further approximation evidently made in Ref. [3] and in the other successive calculations [6,7] has been to neglect altogether the effect of γ' , $\gamma' \approx 0$. This simplifies considerably the calculations, see Eq. (43) in the text. It is easy to convince oneself, using Eq. (43), that this procedure reproduces

correctly the coefficients c_7 and c_8 up to corrections of order $\alpha^2 \log^2 M^2$ and it is therefore adequate for their evaluation. At the same time, since one has neglected the electromagnetic corrections to the coefficients that are non-zero for $\alpha = 0$, it is not inconsistent, in the framework of this approximation, to neglect altogether the operators Q'_9 and Q'_{10} , that for $\mu < m_c$, are not independent of the others. In fact, the projection matrices A and B used in Eq. (43) - that in the old basis have been given in Eqs. (A14) and (A16) - become in the new basis:

$$\begin{aligned} A'_{ii} &= A'_{10,9} = 1 \\ -A'_{10,3} &= A'_{10,4} = \frac{\alpha}{\alpha'_s(m_b^2)} \end{aligned} \quad (B5)$$

and

$$\begin{aligned} B'_{9,1} &= -3 B'_{9,3} = \frac{3}{2} \frac{\alpha}{\alpha''_s(m_c^2)} \\ B'_{ij} &= B_{ij} \quad (\text{otherwise}) \end{aligned} \quad (B6)$$

Therefore, the coefficients of the operators Q'_9 and Q'_{10} only induce $O(\alpha \log M^2)$ corrections to the coefficients c_i , $i < 3$.

It was not noticed, however, that these contribute to the imaginary part of the amplitudes, and could therefore be of some relevance, as discussed in this paper.

We may note that the technique used in Ref. [3] was first introduced in Ref. [28] to obtain the effective Hamiltonian for the decay $K \rightarrow \pi e^+ e^-$. There, neglect of $O(\alpha \log M^2)$ corrections to the coefficients of four-quark operators was totally justified, since these operators can only contribute to the process of interest through $O(\alpha)$ corrections of their matrix elements.

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FIGURE CAPTIONS

- Fig. 1 The "penguin" diagram. The wavy line may represent a photon (EMP diagram) or a gluon (strong-interaction penguin).
- Fig. 2 An example of the other diagrams contributing to the anomalous dimensions. The wavy line can be a gluon (QCD corrections) or a photon (e.m. corrections).
- Fig. 3 Results for $c_6(\mu)$ (solid), $\tilde{c}_7(\mu)$ (dashed), $\tilde{c}_8(\mu)$ (dot-dashed) and $\tilde{c}_+(\mu)$ (dotted), calculated with $N = 3$, $\Lambda_4 = 100$ MeV and $m_t = m_w = 83$ GeV.
- Fig. 4 Same as Fig. 3 but for $\Lambda_4 = 300$ MeV and $m_t = 40$ GeV.
- Fig. 5 Diagrams for isospin-breaking contribution. The square (circular) box represents vertices induced by the weak (chiral symmetry breaking) Hamiltonian.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5