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A Useful Angular Variable for the Analysis of Three-Jet Events in $p\bar{p}$ Collisions

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ABSTRACT

We propose the use in analyzing three-jet events in $p\bar{p}$ collisions of an angular variable known from e^+e^- annihilation to be a sensitive probe of the bremsstrahlung nature of QCD multijet final states. The QCD prediction for $p\bar{p}$ collisions is softened, compared with e^+e^- annihilation, by initial state bremsstrahlung, whilst the distributions for different $2 \rightarrow 3$ QCD subprocesses are almost indistinguishable. We find that UA1 data agree with the QCD predictions and disagree with a phase space model. We present predictions of the angular distributions for three-jet events in the CDF detector.

One of the earliest tests of perturbative QCD in e^+e^- annihilation was the investigation of hard gluon bremsstrahlung via the process $e^+e^- \rightarrow q\bar{q}g$. A particularly powerful test was provided by the distribution in an angular variable $\tilde{\theta}$ proposed by two of us [1], which quantified the emission of the gluon in the $q\bar{q}g$ final state [2]. The precise definition of $\tilde{\theta}$ is as follows. Consider a three-jet final state and denote by x_i the scaled jet energies in the e^+e^- centre-of-mass, i.e. $x_i = 2E_i/\sqrt{s}$ and $\sum x_i = 2$. Order the jet energies such that $x_1 > x_2 > x_3$. Boost the final state to the rest-frame of the two least energetic jets: $\tilde{\theta}$ is then the angle between the axis defined by these two jets and the direction of the third jet in this boosted frame. The cosine of the angle, $\xi \equiv \cos \tilde{\theta}$, is related to the scaled jet energies by

$$\xi = \frac{x_2 - x_3}{x_1} \quad (1)$$

Lines of constant ξ in the relevant part of the Dalitz plot are shown in Fig.1. In practice, a cut is imposed on the fastest jet energy, $x_1 < x_1^{max}$, to prevent the slowest jet becoming too soft. This in turn implies a range for ξ of

$$\xi < \xi^{max} = \frac{3x_1^{max} - 2}{x_1^{max}} \quad (2)$$

In QCD, the ξ distribution is obtained by an appropriate (symmetrised) integration over the differential cross-section [3]

$$\frac{1}{\sigma} \frac{d^2\sigma}{dx_q dx_{\bar{q}}} = \frac{2\alpha_s}{3\pi} \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} \quad (3)$$

In the absence of an upper limit on x_1 , the ξ distribution is singular at $\xi = 1$. This is the infra-red singularity from soft gluon emission. Therefore the hardness of the ξ distribution reflects the strength of the gluon emission. In particular, the ξ distribution provides a powerful discrimination between QCD and scalar gluon models [1], and also between QCD and the distribution expected from phase space alone.

The purpose of this Letter is to investigate to what extent the same analysis can be performed for three-jet final states in high-energy $p\bar{p}$ collisions. In particular, is there similar evidence for gluon bremsstrahlung in the final state? The UA1 collaboration have already demonstrated that it is possible to define a clean sample of events with three large p_T jets [4]. For events with $M_{JJJ} > 150 \text{ GeV}$, and with appropriate cuts on the jet angles with respect to the beam direction, the jet energy fractions have been measured and their distribution compared with some of the predictions of perturbative QCD. In this Letter we will use the same sample of events to study the $\tilde{\theta}$ variable in $p\bar{p}$ collisions.*

A fundamental difference between three-jet production in e^+e^- and $p\bar{p}$ collisions is that only for the former does the $q\bar{q}g$ final state ‘decouple’ from the initial state. The matrix element squared (summed and averaged over spins and colours) for the process $e^+e^- \rightarrow q\bar{q}g$ is

$$|M|^2 = 8e^2e_q^2g_s^2 \frac{(q_1 \cdot p_1)^2 + (q_1 \cdot p_2)^2 + (q_2 \cdot p_1)^2 + (q_2 \cdot p_2)^2}{(q_1 \cdot q_2)(p_1 \cdot k)(p_2 \cdot k)} \quad (4)$$

where the incoming lepton momenta are q_1, q_2 , the outgoing quark momenta are p_1, p_2 and the outgoing gluon momentum is k . Only the photon exchange contribution is included. When one integrates over all orientations of the three jets in phase space (the Euler angle integrations) the resulting matrix element depends only on the relative orientation of the $q, \bar{q}$ and g jets i.e. on the scalar products $p_1 \cdot p_2, p_1 \cdot k$ and $p_2 \cdot k$. The result given in eqn.(3) then follows. For $p\bar{p}$ collisions, partons are also radiated from the incoming lines and there is therefore no analogous factorisation.

To quantify this it is convenient to use the following three body phase space variables:

$$d\Phi_3 = (2\pi)^{-5} \frac{\pi s}{16} d\cos\theta_1 d\psi dx_1 dx_2 \quad (5)$$

where θ_1 is the angle between jet 1 (the most energetic jet) and the beam direc-

* This and other three jet variables have also been discussed in reference [5].

tion, and ψ is the angle between the plane defined by jets 2 and 3 and the plane defined by the beam direction and jet 1 – both angles measured in the three-jet centre-of-mass frame. An overall azimuthal angle about the beam direction has been integrated out. To ensure that all three jets are well separated from the beam direction (where the $2 \rightarrow 3$ matrix elements are singular) it is necessary to impose cuts on θ_1 and ψ . For the UA1 detector, for example, the values are:

$$|\cos \theta_1| < 0.6, \quad 30^\circ < \psi < 150^\circ \quad (6)$$

with an additional cut of $x_1 < 0.9$ to ensure that the least energetic jet is not too soft.

The QCD prediction for the distribution in $\tilde{\theta}$ in $p\bar{p}$ collisions is obtained in the usual way by convoluting the $2 \rightarrow 3$ scattering amplitudes with the appropriate combinations of parton distributions, and integrating over the allowed regions of phase space. Fig.2 shows the comparison between theory and the UA1 data. The agreement is very satisfactory. Several points are worth noting. (i) Because of the usual cross-section normalisation uncertainties in both theory and experiment it is more appropriate to compare the *normalised* distributions, as in Fig.2. (ii) Other theoretical ambiguities from the choice of scale in α_s and in the parton distributions, and from the choice of parton distributions themselves, have a minimal effect on the angular distribution. The prediction shown corresponds to $Q = \langle p_T^J \rangle$ and the MRSB parton distributions [6] with $\Lambda_{\overline{MS}} = 200 \text{ MeV}$. (iii) Also shown in Fig.2 is the ‘phase space’ prediction, obtained by replacing the QCD matrix elements by a constant. The data clearly favour the QCD prediction.

Fig.2 raises several interesting questions. How different is the $p\bar{p}$ angular distribution from that in e^+e^- ? How sensitive is the distribution to the cuts on θ_1 and ψ ? Is there any significant difference between the distributions from the different quark and gluon subprocesses? Is there any evidence for the additional initial state radiation expected in $p\bar{p}$ collisions? As has been emphasized in reference [7] the squared matrix elements for all $2 \rightarrow 3$ processes in gauge theories can

be written as that for an underlying $2 \rightarrow 2$ process multiplied by bremsstrahlung factors, and it is desirable for experiment to check this feature.

The remainder of this Letter will attempt to answer the above questions. We begin by comparing, in Fig.3, the QCD prediction of Fig.2 with (a) the e^+e^- distribution obtained from the cross-section of eqn.(3) and (b) the phase space distribution (using the same value of $x_1^{max} = 0.9$). It is immediately apparent that the $p\bar{p}$ distribution lies between the other two, suggesting that the initial state radiation does indeed ‘dilute’ the effect of the final state gluon bremsstrahlung in $e^+e^- \rightarrow q\bar{q}g$. We can illustrate the effect by considering the angular distribution which arises from the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$. This is intermediate between the e^+e^- and $p\bar{p}$ processes, having both initial and final state radiation. The matrix element squared is [7]:

$$|M|^2 = 2e^6((q_1 \cdot p_1)^2 + (q_1 \cdot p_2)^2 + (q_2 \cdot p_1)^2 + (q_2 \cdot p_2)^2) \cdot \left(\frac{1}{(q_1 \cdot q_2)(p_1 \cdot k)(p_2 \cdot k)} + \frac{1}{(p_1 \cdot p_2)(q_1 \cdot k)(q_2 \cdot k)} \right) \quad (7)$$

using a notation similar to that of eqn.(4). Note the additional ‘initial state radiation’ term compared with eqn.(4). The predictions resulting from this matrix element are also shown in Fig.3, imposing exactly the same cuts on the $\mu^+\mu^-\gamma$ final state as on the three-jet final state. Evidently the predictions for $\mu^+\mu^-\gamma$ are very similar to that for the three jets, confirming that the difference between $p\bar{p}$ and e^+e^- is indeed due to the presence of additional initial state bremsstrahlung for the former.

It is straightforward to evaluate the angular distributions from the different $2 \rightarrow 3$ subprocesses. For simplicity we classify these using the initial state only, i.e. $gg \rightarrow 3$, $qg \rightarrow 3$ and $qq \rightarrow 3$, the latter including a sum over all possible quark, antiquark and gluon processes. By explicit calculation we find very little difference between the different subprocesses. Fig.4 shows the normalised ξ distributions for the three classes of subprocesses, for the UA1 cuts described above. The presence of an initial state quark evidently softens the distribution

very slightly. To quantify this further, we show in Table 1 the values of the average ξ and the average ξ^2 for the $2 \rightarrow 3$ processes, including the QED processes discussed above. We conclude that it would be impossible experimentally to observe the small differences between the QCD processes (which arise presumably from the different spin and colour structure for quark and gluon amplitudes). The similarity in the structure of the $2 \rightarrow 3$ matrix elements appears to be a global property of the theory, as discussed recently in reference [8].

We might expect that the effects of the initial state bremsstrahlung would be suppressed somewhat by requiring the three jets to be *transverse* to the beam direction, i.e. maximally far away from the initial state collinear singularities. We therefore define a *transverse* configuration in which the angles θ_1 and ψ are both equal to 90° . Table 1 shows the corresponding values of $\langle \xi \rangle$ and $\langle \xi^2 \rangle$ for the different subprocesses, again with $x_1^{max} = 0.9$. The average values are systematically larger for the transverse configuration, and therefore closer to the $e^+e^- \rightarrow q\bar{q}g$ results. We conclude that the effects of initial state bremsstrahlung are indeed reduced in the transverse configuration. In general the QCD $2 \rightarrow 3$ processes are too complicated to allow a simple analytic treatment of the distributions. However we *can* evaluate the $\mu^+\mu^-\gamma$ matrix element of eqn.(7) in the transverse configuration, in terms of the energy fractions x_i . The result is:

$$|M|_T^2 = \frac{e^6}{(q_1 \cdot q_2)} \left(\frac{x_a^2 + x_b^2}{(1-x_a)(1-x_b)} + \frac{4(x_a^2 + x_b^2)}{x_c^2(1-x_c)} \right) \quad (8)$$

where x_a , x_b and x_c are the energy fractions of the μ^+ , μ^- and γ respectively. Note that the first term on the right-hand-side is the usual $q\bar{q}g$ -type matrix element (eqn.(3)) while the second term arises from initial state bremsstrahlung. It is evidently not singular as either x_a or $x_b \rightarrow 1$, and therefore tends to soften the ξ distribution, as discussed above.

We end this Letter by presenting the QCD predictions for the $\tilde{\theta}$ distribution at the FNAL $p\bar{p}$ collider. Because of the different kinematic range and detector

properties, the three-jet cuts used are slightly different from those used by UA1. We present predictions using the CDF cuts [9]:

$$|\cos \theta_1| < 0.6, \quad 20^\circ < \psi < 160^\circ, \quad x_1 < 0.93, \quad x_2 < 0.84 \quad (9)$$

The fact that the maximum allowed value of x_1 is larger for CDF than for UA1 means that the ξ distribution is harder. This is illustrated in Fig.5 where the $p\bar{p}$ prediction is again compared with the e^+e^- result. It will be interesting to compare these predictions with the data. It should be remembered that any significant deviation from the QCD predictions could signal new physics beyond the Standard Model. For example, one possibility is that with sufficiently high energy a new state X is created which decays into three jets. This might be difficult to detect as an increase in the three-jet rate, but may be more obvious in an angular distribution like the one discussed in this Letter. A more isotropic three-jet decay would soften the ξ distribution for three-jet masses $M_{JJJ} \simeq M_X$.

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subprocess	$\langle \xi \rangle$	$\langle \xi^2 \rangle$	$\langle \xi \rangle_T$	$\langle \xi^2 \rangle_T$
$gg \rightarrow 3$	0.329	0.151	0.336	0.157
$qg \rightarrow 3$	0.328	0.150	0.336	0.156
$qq \rightarrow 3$	0.325	0.148	0.332	0.153
$e^+e^- \rightarrow \mu^+\mu^-\gamma$	0.331	0.152	0.332	0.153
$e^+e^- \rightarrow q\bar{q}g$	0.348	0.165		
phase space	0.278	0.113		

Table 1: Average ξ and ξ^2 values for the various $2 \rightarrow 3$ subprocesses contributing to three jet production in $p\bar{p}$ collisions. The jet cuts are as described in the text. The subscript T refers to the transverse configuration in which the jets emerge at right-angles to the beam direction. Also shown are the values obtained from using the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ matrix element in place of the QCD $2 \rightarrow 3$ matrix elements, and the average values for $e^+e^- \rightarrow q\bar{q}g$ and for phase space alone.

FIGURE CAPTIONS

1. Lines of constant $\xi = \cos \tilde{\theta}$ in the region of the Dalitz plot with $x_1 > x_2 > x_3 = 2 - x_1 - x_2$.
2. The QCD prediction for the normalised ξ distribution compared with data from the UA1 collaboration. The data points have been extracted from the Dalitz plot distribution of the UA1 three-jet sample (Fig.4 of reference [4]). Only the statistical errors are shown [4]. The three jets are required to have an invariant mass greater than 150 *GeV*. Other cuts and theoretical parameters are as described in the text. The solid line is the expectation from a phase space model.
3. The QCD prediction of Fig.2 compared with the e^+e^- distribution (dashed line), the phase space prediction (solid line) and the predictions based on the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ matrix element (dashed histogram), with the same ‘three-jet’ cuts on the $\mu^+\mu^-\gamma$ final state.
4. The normalised ξ distributions from the three classes of $2 \rightarrow 3$ subprocesses, with UA1-type cuts as described in the text.
5. The QCD prediction for the $\xi = \cos \tilde{\theta}$ angular distribution in $p\bar{p}$ collisions at 1.8 *TeV*. The jet cuts and theoretical parameters are as described in the text. The predictions for $e^+e^- \rightarrow q\bar{q}g$ (dashed line) and for phase space (solid line), with the same cuts on the x_i , are also shown.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5